MARCH 1979

PPPL-1527
UC-20g

GUIDING CENTER DRIFT EQUATIONS

BY

A. H. BOOZER

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by the U. S. Department of Energy Contract No. EY-76-C-02-3073.

Guiding Center Drift Equations

Allen H. Boozer

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

ABSTRACT

The equations for particle guiding center drift orbits are given in a new magnetic coordinate system. This form of the equations not only separates the fast motion along the lines from the slow motion across, but also requires less information about the magnetic field than many other formulations of the problem.

I. Introduction

The close relation between particle drift orbits and transport in high temperature plasmas is well known. However, the evaluation of the drift orbits is not easily accomplished, even computationally, if there are no symmetry directions. Even in ideally symmetric systems, like the tokamak, small symmetry breaking terms which occur in real devices have significant transport effects. These effects are larger and often more subtle in non-symmetric systems like the stellarator.

In this paper a simple form for the guiding center velocity in steady state fields is given. A new magnetic coordinate system is developed and the drift orbit equations are given in this system. These drift orbit equations not only separate the slow and fast particle motion, but also require minimal information about the magnetic field.

II. Drift Velocity

The expression for the guiding center drift velocity across a steady state magnetic field is well-known

$$\bar{v}_{\perp} = \frac{c\bar{B}}{eB^2} \times [\mu\bar{\nabla}B + e\bar{\nabla}\Phi + m v_{\parallel}^{2}\hat{b}\cdot\bar{\nabla}\hat{b}] \tag{1}$$

with $\hat{b}=\bar{B}/B$ a unit vector along the magnetic field $\bar{B}$. The electric potential is Φ, μ is the magnetic moment, and $v_{\parallel}$ is the particle's velocity along the magnetic field. In a steady-state field, energy conservation

$$E = \tfrac{1}{2}mv_{\parallel}^{2} + \mu B + e\Phi \qquad (2)$$

permits the guiding center velocity to be written in a simple and often more useful form.

To derive the desired expression, we note that

$$\bar{\nabla}(\mu B+e\Phi) = -\bar{\nabla}\tfrac{1}{2}mv_{\parallel}^{2} \qquad (3)$$

with the particle's energy E taken to be a constant. The expression $\hat{b} \times (\hat{b}\cdot\bar{\nabla}\hat{b})$ can be rewritten using a vector identity for unit vectors

$$\hat{b}\cdot\bar{\nabla}\hat{b} = -\hat{b} \times (\bar{\nabla}\times\hat{b}) \qquad (4)$$

and

$$\bar{\nabla}\times\bar{B} = B\bar{\nabla}\times\hat{b} - \hat{b}\times\bar{\nabla}B \qquad (5)$$

to yield

$$\hat{b} \times (\hat{b}\cdot\bar{\nabla}\hat{b}) = \frac{1}{B}\hat{b} \times \bar{\nabla}B + \frac{1}{B}(\bar{\nabla}\times\bar{B})_{\perp} \qquad (6)$$

It is then simple algebra to show

$$\bar{v}_{\perp} = -\frac{c\bar{B}}{eB^{2}} \times \left[mv_{\parallel}\bar{\nabla}v_{\parallel} - mv_{\parallel}^{2}\frac{1}{B}\bar{\nabla}B\right] + mv_{\parallel}^{2}\frac{c}{eB^{2}}(\bar{\nabla}\times B)_{\perp}$$

$$= \frac{v_{\parallel}}{B}\left[\bar{\nabla}\times(\rho_{\parallel}\bar{B})\right]_{\perp} \qquad (7)$$

with

$$\rho_{\parallel} \equiv mcv_{\parallel}/eB. \tag{8}$$

The total guiding center drift velocity is, of course, the sum of the parallel and perpendicular terms. It is tempting to write the parallel component as just $v_{\parallel}\,\bar{B}/B$, but this turns out not to be quite consistent. Consider the drift kinetic equation

$$\bar{v}\cdot\bar{\nabla}f = 0 \tag{9}$$

with the guiding center velocity $\bar{v}$ and the distribution function considered to be functions of E, μ, and position $\bar{x}$. For $\bar{v}$ to be physical, the drift kinetic equation must imply particle conservation or

$$\bar{\nabla}\cdot\left(\int \bar{v}fd^3v\right) = 0 \tag{10}$$

Expressing d^3v in terms of E and μ gives

$$d^3v = \sum_{\sigma} \frac{2\pi B}{m^2|v_{\parallel}|}\, dEd\mu \tag{11}$$

with $\sigma = v_{\parallel}/|v_{\parallel}|$. That is a sum is taken of $v_{\parallel}>0$ and $v_{\parallel}<0$ terms. To obtain particle conservation from the drift kinetic equation for arbitrary distribution functions f, it is clear that

$$\bar{\nabla}\cdot\left(\frac{B}{v_{\parallel}}\bar{v}\right) = 0 \qquad (12)$$

The appropriate expression for $\bar{v}$ is then

$$\bar{v} = \frac{v_{\parallel}}{B}\left[\bar{B}+\bar{\nabla}\times(\rho_{\parallel}\bar{B})\right] \qquad (13)$$

Related expressions have been given by a number of authors[1,2,3] for the $\bar{\nabla}\times\bar{B}=0$ case. However, the divergence condition for a physically reasonable $\bar{v}$ and its implications for the velocity appear to be new.

The expression for the guiding center velocity can be written as $\bar{v} = (v_{\parallel}/B)\bar{H}$ with

$$\bar{H} = \bar{B} + \bar{\nabla}\times(\rho_{\parallel}\bar{B}) \qquad (14)$$

in some sense a "real" magnetic field. That is $\bar{\nabla}\cdot\bar{H}=0$. The field $\bar{H}$ does have the unfortunate feature of being singular at turning points ($v_{\parallel}=0$). At these points, the $\bar{H}$ field with $v_{\parallel}>0$ is to be joined to the $\bar{H}$ field with $v_{\parallel}<0$ to obtain the drift orbit. The $\bar{H}$ field does allow a simple evaluation of constants of the drift motion in symmetric fields. Let $\bar{\zeta}$ be a vector pointing in the symmetry direction chosen so $\bar{\nabla}\cdot\bar{\zeta}=0$. Then if the curl of $\bar{\zeta}$ is of the form $\bar{\nabla}\times\bar{\zeta}=\gamma\bar{\zeta}$ the magnetic field and its vector potential can be written in the form

$$\begin{aligned}\bar{B} &= g\bar{\zeta} + \bar{\zeta}\times\bar{\nabla}\psi\\ \bar{A} &= -\psi\,\bar{\zeta} + \bar{a}\end{aligned} \qquad (15)$$

The vector $\bar{H}$ can, of course, be written in a similar form with its vector potential $\bar{A}_* = -\psi_* \bar{\zeta} + \bar{a}_*$. Using the definition of $\bar{H}$

$$\psi_* = \psi - g\rho_{||} \tag{16}$$

Since $\bar{H}\cdot\bar{\nabla}\psi_* = 0$, ψ_* is a constant of the motion. In toroidal symmetry, $\bar{\zeta} = \bar{\nabla}\phi = \hat{\phi}/R$ and ψ_* conservation is essentially p_ϕ conservation. In helical symmetry,

$$\bar{\zeta} = \frac{hr\hat{\theta} + \ell\hat{z}}{\ell^2 + h^2 r^2} \tag{17}$$

III. Magnetic Geometry

When solving the drift orbit equations it is clearly advantageous to go to a magnetic coordinate system. By a magnetic coordinate system we mean one in which magnetic field lines serve as coordinate lines. In this coordinate system the rapid particle motion along the lines is separated from the slow motion across the lines. The magnetic coordinates used in this paper are α, ψ, and χ, which are three functions of position chosen so the magnetic field can be written in a contravariant and a covariant form

$$\bar{B} = \bar{\nabla}\alpha \times \bar{\nabla}\psi \tag{18}$$

$$= \bar{\nabla}\chi + \beta\bar{\nabla}\psi + \gamma\bar{\nabla}\alpha \tag{19}$$

The first or contravariant form for $\bar{B}$ is well-known as the Clebsch representation and can be used to describe any divergent-free field. The second or covariant form is not so familiar but is completely general so long as $\bar{\nabla}\chi\cdot(\bar{\nabla}\alpha x\bar{\nabla}\psi)$, the inverse of the Jacobian, is not zero. The inverse of the Jacobian of the α, ψ, χ coordinates is especially simple

$$\bar{\nabla}\chi \cdot (\bar{\nabla}\alpha x\bar{\nabla}\psi) = B^2 \tag{20}$$

When there is a scaler pressure, the physical interpretation of ψ, χ, and α is simple. The coordinate ψ labels the constant pressure surfaces being essentially the magnetic flux within a surface. The coordinate α is an angle within a pressure surface labeling the various field lines and χ is in some sense the distance along a line. Actually, the differential distance along a field line is $d\chi/B$.

The equations for the guiding center drift orbits can be simply expressed for the general magnetic field. However, the equations are simpler and the covariant form of the magnetic field has an interesting structure in the scaler pressure case

$$\bar{\nabla}P = \frac{1}{c}\,\bar{j} \times \bar{B} \tag{21}$$

Since $\bar{B}\cdot\bar{\nabla}P=0$, one can always choose ψ so the pressure is a function of ψ alone. This choice of ψ will be assumed. Using $\bar{j}\cdot\bar{\nabla}P=0$, it is easy to show that one can redefine β and γ so $\gamma=0$. The current density in a plasma with a scaler pressure is then

$$\bar{j} = \frac{c}{4\pi}\bar{\nabla}xB = \frac{c}{4\pi}\left[\bar{B}\frac{\partial\beta}{\partial\alpha} - (\bar{\nabla}\psi \times \bar{\nabla}\chi)\frac{\partial\beta}{\partial\chi}\right] \tag{22}$$

which implies

$$\frac{\partial\beta}{\partial\alpha} = \frac{4\pi}{c}\frac{j_{\parallel}}{B} \tag{23}$$

with $j_{\parallel}$ the parallel current density. Evaluating $\bar{j}x\bar{B}$ using contravariant form of $\bar{B}$, Eq. (18), one finds

$$\frac{\partial\beta}{\partial\chi} = \frac{4\pi}{B^2}\frac{dP}{d\psi} \tag{24}$$

There is some arbitrariness in the definition of β and χ which can be eliminated by the boundary condition

$$\beta(\alpha=0,\ \psi,\ \chi=0) = 0. \tag{25}$$

With this boundary condition, $\bar{B} = \bar{\nabla}\chi$ when the magnetic field is curl free on a constant ψ surface.

IV. Drift Orbit Equations

The expressions derived in the last two sections for the magnetic field and the guiding center drift velocity permit a very simple derivation of the guiding center orbit equations. These equations are expressions for $d\alpha/dt$, $d\psi/dt$, and $d\chi/dt$ along the drift trajectory of a particle and are derived using $d\alpha/dt = \bar{v}\cdot\bar{\nabla}\alpha$, $d\psi/dt=\bar{v}\cdot\bar{\nabla}\psi$, and $d\chi/dt=\bar{v}\cdot\bar{\nabla}\chi$.

To derive the drift orbit equations, we note that the drift velocity, Eq. 13, can be expressed using Eqs. 18 and 19 for the magnetic field as

$$\bar{v} = \frac{v_{||}}{B}[\bar{\nabla}\alpha x\bar{\nabla}\psi+\bar{\nabla}x(\rho_{||}\ \bar{\nabla}\chi+\beta\rho_{||}\ \bar{\nabla}\psi+\gamma\rho_{||}\ \bar{\nabla}\alpha)] \quad (26)$$

Expressions like $\bar{\nabla}x(\rho_{||}\ \bar{\nabla}\chi)$ are rewritten as

$$\bar{\nabla}x(\rho_{||}\ \bar{\nabla}\chi) = -(\bar{\nabla}\chi x\bar{\nabla}\alpha)\frac{\partial\rho_{||}}{\partial\alpha} -(\bar{\nabla}\chi x\bar{\nabla}\psi)\ \frac{\partial\rho_{||}}{\partial\psi} \quad (27)$$

then with repeated use of $\bar{\nabla}\chi\cdot(\bar{\nabla}\alpha x\bar{\nabla}\psi) = B^2$, one finds

$$\frac{d\alpha}{dt} = v_{||}\ B\left(\frac{\partial\rho_{||}}{\partial\psi} - \frac{\partial\beta\rho_{||}}{\partial\chi}\right) \quad (28)$$

$$\frac{d\psi}{dt} =-v_{||}\ B\left(\frac{\partial\rho_{||}}{\partial\alpha} - \frac{\partial\gamma\rho_{||}}{\partial\chi}\right) \quad (29)$$

$$\frac{d\chi}{dt} = v_{||}\ B\left(1-\frac{\partial\gamma\rho_{||}}{\partial\psi} + \frac{\partial\beta\rho_{||}}{\partial\alpha}\right) \quad (30)$$

These equations determine the drift orbit of a particle of given energy E and magnetic moment μ once four functions of α, ψ, χ are specified. These functions are B, β, γ, and Φ.

There at first appear to be a number of difficulties with integrating Eqs. 28-30 to obtain drift orbits due to the question of the sign of $v_{||}$ and the singular nature of its derivatives at the turning points. Actually these problems can be easily dealt with. The problem of the sign of $v_{||}$ is of importance only in the equation for $d\chi/dt$ and then only near turning points.

This problem comes from evaluating $v_{\parallel}$ using energy conservation (Eq. 2) and is avoided by evaluating $v_{\parallel}$, at least near turning points, using a differential equation for $dv_{\parallel}/dt$. It is simpler to use the differential equation for $d\rho_{\parallel}/dt$ which clearly serves the same purpose ($\rho_{\parallel} = mcv_{\parallel}/eB$).

$$\frac{d\rho_{\parallel}}{dt} = \bar{v}\cdot\bar{\nabla}\rho_{\parallel} = v_{\parallel} B\left[\frac{\partial\rho_{\parallel}}{\partial\chi} - \rho_{\parallel}\left(\frac{\partial\rho_{\parallel}}{\partial\alpha}\frac{\partial\beta}{\partial\chi} - \frac{\partial\rho_{\parallel}}{\partial\chi}\frac{\partial\beta}{\partial\alpha}\right) + \rho_{\parallel}\left(\frac{\partial\rho_{\parallel}}{\partial\psi}\frac{\partial\gamma}{\partial\chi} - \frac{\partial\rho_{\parallel}}{\partial\chi}\frac{\partial\gamma}{\partial\psi}\right)\right] \quad (31)$$

This expression is especially simple in the curl-free field case with $d\rho_{\parallel}/dt=v_{\parallel}B(\partial\rho_{\parallel}/\partial\chi)$ of a similar form as the other orbit equations. The problem of singular derivatives of $v_{\parallel}$ at turning points is of no fundamental importance to the drift orbit equations for derivatives of $v_{\parallel}$ are always multiplied by a $v_{\parallel}$ factor which gives a finite product. The following easily derived but useful expression illustrates this with ξ equal to α, ψ, or χ.

$$v_{\parallel} B\frac{\partial\rho_{\parallel}}{\partial\xi} = -c\frac{\partial\Phi}{\partial\xi} - \left[\frac{c}{e}\mu + \frac{eB}{mc}\rho_{\parallel}^2\right]\frac{\partial B}{\partial\xi} \quad (32)$$

In symmetric systems, the drift equations conserve ψ_* $= \psi - g\rho_{\parallel}$ as demonstrated in Sec. II. The relation between this conservation law and Eqs. 28 to 30 is quite fascinating in the scaler pressure case. Remembering $\bar{\zeta}$ is a vector in the symmetry direction and $\bar{\zeta}\cdot\bar{\nabla}\psi=0$, one has $\bar{\zeta}\cdot\bar{B}= \bar{\zeta}\cdot\bar{\nabla}\chi$ using Eq. 19 for $\bar{B}$. Using Eq. 18 for $\bar{B}$, one obtains $\bar{\zeta}x\bar{B}= -(\bar{\zeta}\cdot\bar{\nabla}\alpha)\bar{\nabla}\psi$. Finally using Eq. 15 for $\bar{B}$, one finds $\bar{\zeta}\cdot\bar{B}=g\zeta^2$ and $\bar{\zeta}x\bar{B} = -\zeta^2\bar{\nabla}\psi$. These results imply $\bar{\zeta}\cdot\bar{\nabla}\chi = g\bar{\zeta}\cdot\bar{\nabla}\alpha$ or if f is any function such that $\bar{\zeta}\cdot\bar{\nabla}f=0$

then

$$\frac{\partial f}{\partial \alpha} = -g \frac{\partial f}{\partial \chi} \tag{33}$$

The conservation of ψ_* means $d\psi_*/dt=\bar{v}\cdot\bar{\nabla}\psi_*=0$, but evaluating $d\psi_*/dt$ one finds

$$\frac{d\psi_*}{dt} = v_{\parallel} B\rho_{\parallel}\left[-g\frac{\partial\rho_{\parallel}}{\partial\chi}\left(\frac{\partial g}{\partial\psi}+g\frac{\partial\beta}{\partial\chi}+\frac{\partial\beta}{\partial\alpha}\right)+g\frac{\partial\rho_{\parallel}}{\partial\psi}\frac{\partial g}{\partial\chi}-\rho_{\parallel}\frac{\partial g}{\partial\chi}\left(g\frac{\partial\beta}{\partial\chi}+\frac{\partial\beta}{\partial\alpha}\right)-\frac{\partial g}{\partial\chi}\right] \tag{34}$$

This expression must be zero for any $\rho_{\parallel}$ which obeys the symmetry ($\bar{\zeta}\cdot\bar{\nabla}\rho_{\parallel}=0$) which implies g is only a function of ψ alone and

$$\frac{dg}{d\psi} + g\frac{\partial\beta}{\partial\chi} + \frac{\partial\beta}{\partial\alpha} = 0 \tag{35}$$

This is equivalent to

$$j_{\parallel} = -\frac{c}{4\pi} B\frac{dg}{d\psi} - c\,\frac{g}{B}\,\frac{dP}{d\psi} \tag{36}$$

These results appear quite remarkable. The symmetry conditions applied to the drift orbit equations give us information about the magnetic field rather than the other way around. Actually the results are not so amazing if one looks at the formal operations involved. The result comes from being able to write any divergence free field as $\bar{B}=g\bar{\zeta}+\bar{\zeta}x\bar{\nabla}\psi$. Consequently $\bar{H} = \bar{B}+\bar{\nabla}x(\rho_{\parallel}\bar{B})$ can be written in this form with the only condition on $\rho_{\parallel}$ being $\bar{\zeta}\cdot\bar{\nabla}\rho_{\parallel}=0$.

The longitudinal adiabatic invariant J is of course conserved by Eqs 28 to 30 provided the parallel motion is fast enough compared to the cross field drifts. To prove this, let us construct a function f such that $\bar{v}\cdot\bar{\nabla}f=0$. We let $f=f_o+f_1,+\ldots$ with the subscripts representing orders in the small parameter $\rho_{\parallel}$. The zeroth order is

$$\bar{v}\cdot\bar{\nabla}f_o = v_{\parallel} B \frac{\partial f_o}{\partial \chi} = 0 \tag{37}$$

and $f_o=f_o(\alpha,\psi)$. In first order

$$\frac{\partial f_1}{\partial \chi} + \left(\frac{\partial \rho_{\parallel}}{\partial \psi} - \frac{\partial \beta \rho_{\parallel}}{\partial \chi}\right)\frac{\partial f_o}{\partial \alpha} - \left(\frac{\partial \rho_{\parallel}}{\partial \alpha} - \frac{\partial \gamma \rho_{\parallel}}{\partial \chi}\right)\frac{\partial f_o}{\partial \psi} = 0 \tag{38}$$

The consistency condition on this equation for trapped particles is

$$\left(\oint \frac{\partial \rho_{\parallel}}{\partial \psi} d\chi\right) \frac{\partial f_o}{\partial \alpha} = \left(\oint \frac{\partial \rho_{\parallel}}{\partial \alpha} d\chi\right) \frac{\partial f_o}{\partial \psi} \tag{39}$$

with the loop integral implying an integral at constant α and ψ from a point where $\rho_{\parallel}=0$ to another where $\rho_{\parallel}=0$ and back, but

$$\oint \frac{\partial \rho_{\parallel}}{\partial \psi} d\chi = \frac{\partial}{\partial \psi} \oint \rho_{\parallel} d\chi = \frac{\partial}{\partial \psi} \frac{c}{e} m \oint v_{\parallel} d\ell = \frac{c}{e} \frac{\partial J}{\partial \psi} \tag{40}$$

with J the longitudinal invariant (we used $d\ell=d\chi/B$). It is then obvious the $f_o=f_o(J)$ so J is conserved at least to lowest order.

V. Summary

In a reactor grade plasma even thermal particles can travel 10 km. between collisions, the ratio of the cross field to the parallel velocity can be 10^{-3}, and a particle can have 10^7 cyclotron orbits per collision.These parameters imply a brute force technique may not be sufficiently accurate to calculate particle orbits. In this paper a method of finding particle orbits has been developed which is based on the drift kinetic equation and magnetic field line coordinates. This system has a number of advantages. First, the fast particle motion along the field lines is separated from the slow drift across the lines. Second, far less information is required about the magnetic field than in other formulations. Third, if there is a scaler pressure, the constant pressure surfaces can be used as a coordinate. Since the transport is determined by the distance particles stray from the constant pressure surfaces, the use of these surfaces as a coordinate greatly simplifies the interpretation of the results.

Although the drift equations are given in the paper for an arbitrary magnetic field, the most important case is the locally curl-free field due to its simplicity. For this case the magnetic field can be written

$$\bar{B} = \bar{\nabla}\alpha \times \bar{\nabla}\psi = \bar{\nabla}\chi$$

Using α, ψ, and χ as coordinates, the particle equations of motion are

$$\frac{d\alpha}{dt} = -c\frac{\partial\Phi}{\partial\psi} - \left(\frac{c}{e}\mu + \frac{eB}{mc}\rho_{\parallel}^{2}\right)\frac{\partial B}{\partial\psi}$$

$$\frac{d\psi}{dt} = c\frac{\partial\Phi}{\partial\alpha} + \left(\frac{c}{e}\mu + \frac{eB}{mc}\rho_{\parallel}^{2}\right)\frac{\partial B}{\partial\alpha}$$

$$\frac{d\chi}{dt} = \frac{e}{mc}\rho_{\parallel}B^{2}$$

$$\frac{d\rho_{\parallel}}{dt} = -c\frac{\partial\Phi}{\partial\chi} - \left(\frac{c}{e}\mu + \frac{eB}{mc}\rho_{\parallel}^{2}\right)\frac{\partial B}{\partial\chi}$$

In these equations $\Phi(\alpha,\psi,\chi)$ is the electric potential, $B(\alpha,\psi,\chi)$ the magnetic field strength, and e, m, and μ the particle's charge, mass, and magnetic moment. The quantity $\rho_{\parallel} = mcv_{\parallel}/eB$. Actually, the equations for a curl-free field can be stated more elegantly by defining a Hamiltonian

$$H(\rho_{\parallel},\alpha,\psi,\chi) = \frac{1}{2}\rho_{\parallel}^{2}\frac{eB^{2}}{mc} + \frac{\mu c}{e}B + c\Phi$$

then

$$\frac{d\chi}{dt} = \frac{\partial H}{\partial\rho_{\parallel}}, \quad \frac{d\rho_{\parallel}}{dt} = -\frac{\partial H}{\partial\chi}$$

$$\frac{d\psi}{dt} = \frac{\partial H}{\partial\alpha}, \quad \frac{d\alpha}{dt} = -\frac{\partial H}{\partial\psi}$$

The adiabatic invariance of J

$$J = \frac{e}{c}\oint\rho_{\parallel}\,d\chi = \oint mv_{\parallel}\,d\ell$$

then follows from the standard classical mechanics[4] treatment. Since $dH/dt=0$, the Hamiltonian is just the energy E times c/e and it is conserved.

ACKNOWLEDGMENTS

This work was supported by United States Department of Energy, Contract No. EY-76-C-02-3073.

REFERENCES

[1]A. I. Morozov and L. S. Solov'ev, Reviews of Plasma Physics Vol. 2 (M. A. Leontovich editor, Consultants Bureau, N.Y. 1966) see page 228.

[2]P. H. Rutherford, Phys. Fluids 13, 482 (1970).

[3]D. Dobrott and E. A. Frieman, Phys. Fluids 14, 349 (1971).

[4]L. D. Landau and E. M. Lifshitz, Mechanics (Pergamon Press, N.Y., 3rd ed., 1976) see p. 154-162.

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

ALL CATEGORIES

R. Askew, Auburn University, Alabama
S. T. Wu, Univ. of Alabama
Geophysical Institute, Univ. of Alaska
G.L. Johnston, Sonoma State Univ, California
H. H. Kuehl, Univ. of S. California
Institute for Energy Studies, Stanford University
H. D. Campbell, University of Florida
N. L. Oleson, University of South Florida
W. M. Stacey, Georgia Institute of Technology
Benjamin Ma, Iowa State University
Magne Kristiansen, Texas Tech. University
W. L. Wiese, Nat'l Bureau of Standards, Wash., D.C.
Australian National University, Canberra
C.N. Watson-Munro, Univ. of Sydney, Australia
F. Cap, Inst. for Theo. Physics, Austria
Ecole Royale Militaire, Bruxelles, Belgium
D. Palumbo, C. European Comm. B-1049-Brussels
P.H. Sakanaka, Instituto de Fisica, Campinas, Brazil
C. R. James, University of Alberta, Canada
T.W. Johnston, INRS-Energie, Vareenes, Quebec
H. M. Skarsgard, Univ. of Saskatchewan, Canada
Librarian, Culham Laboratory, Abingdon, England (2)
A.M. Dupas Library, C.E.N.-G, Grenoble, France
Central Res. Inst. for Physics, Hungary
R. Shingal, Meerut College, India
A.K. Sundaram, Phys. Res. Lab., India
M. Naraghi, Atomic Energy Org. of Iran
Biblioteca, Frascati, Italy
Biblioteca, Milano, Italy
G. Rostagni, Univ. Di Padova, Padova, Italy
Preprint Library, Inst. de Fisica, Pisa, Italy
Library, Plasma Physics Lab., Gokasho, Uji, Japan
S. Mori, Japan Atomic Energy Res. Inst., Tokai-Mura
Research Information Center, Nagoya Univ., Japan
S. Shioda, Tokyo Inst. of Tech., Japan
Inst. of Space & Aero. Sci., Univ. of Tokyo
T. Uchida, Univ. of Tokyo, Japan
H. Yamato, Toshiba R. & D. Center, Japan
M. Yoshikawa, JAERI, Tokai Res. Est., Japan
N. Yajima, Kyushu Univ., Japan
R. England, Univ. Nacional Auto-noma de Mexico
B. S. Liley, Univ. of Waikato, New Zealand
S. A. Moss, Saab Univas Norge, Norway
J.A.C. Cabral, Univ. de Lisboa, Portugal
O. Petrus, AL.I. CUZA Univ., Romania
J. de Villiers, Atomic Energy Bd., South Africa
A. Maurech, Comisaria De La Energy y Recoursos Minerales, Spain
Library, Royal Institute of Technology, Sweden
Cen. de Res. En Phys.Des Plasmas, Switzerland
Librarian, Fom-Instituut Voor Plasma-Fysica, The Netherlands
V. E. Golant, A.F. Ioffe Physical-Tech. Inst., USSR
B.B. Kadomtsev, Kurchatov Inst. of Atomic Energy, USSR
The Kharkov Physical-Tech. Inst., USSR
M. S. Rabinovich, Academy of Sci, USSR
Bibliothek, Stuttgart, West Germany
R.D. Buhler, Univ. of Stuttgart, West Germany
Max-Planck-Inst. fur Plasmaphysik, W. Germany
Nucl. Res. Estab., Julich, West Germany
K. Schindler, Inst. Fur Theo. Physik, W. Germany

EXPERIMENTAL THEORETICAL

M. H. Brennan, Flinders Univ. Australia
H. Barnard, Univ. of British Columbia, Canada
S. Screenivasan, Univ. of Calgary, Canada
J. Radet, C.E.N.-B.P., Fontenay-aux-Roses, France
Prof. Schatzman, Observatoire de Nice, France
S. C. Sharma, Univ. of Cape Coast, Ghana
R. N. Aiyer, Laser Section, India
B. Buti, Physical Res. Lab., India
L. K. Chavda, S. Gujarat Univ., India
I.M. Las Das, Banaras Hindu Univ., India
S. Cuperman, Tel Aviv Univ., Israel
E. Greenspan, Nuc. Res. Center, Israel
P. Rosenau, Israel Inst. of Tech., Israel
Int'l. Center for Theo. Physics, Trieste, Italy
I. Kawakami, Nihon University, Japan
T. Nakayama, Ritsumeikan Univ., Japan
S. Nagao, Tohoku Univ., Japan
J.I. Sakai, Toyama Univ., Japan
S. Tjotta, Univ. I Bergen, Norway
M.A. Hellberg, Univ. of Natal, South Africa
H. Wilhelmson, Chalmers Univ. of Tech., Sweden
Astro. Inst., Sonnenborgh Obs., The Netherlands
N.G. Tsintsadze, Academy of Sci GSSR, USSR
T. J. Boyd, Univ. College of North Wales
K. Hubner, Univ. Heidelberg, W.Germany
H. J. Kaeppeler, Univ. of Stuttgart, West Germany
K. H. Spatschek, Univ. Essen, West Germany

EXPERIMENTAL ENGINEERING

B. Grek, Univ. du Quebec, Canada
P. Lukac, Komenskeho Univ., Czechoslovakia
G. Horikoshi, Nat'l Lab for High Energy Physics, *Tsukuba-Gun, Japan*
V. A. Glukhikh, D.V. Efremov Sci. Res. Instit.of Elect. App., USSR

EXPERIMENTAL

F. J. Paoloni, Univ. of Wollongong, Australia
J. Kistemaker, Fom Inst. for Atomic & Molec. Physics, The Netherlands

THEORETICAL

F. Verheest, Inst. Vor Theo. Mech., Belgium
J. Teichmann, Univ. of Montreal, Canada
T. Kahan, Univ. Paris VII, France
R. K. Chhajlani, India
S. K. Trehan, Panjab Univ., India
T. Namikawa, Osaka City Univ., Japan
H. Narumi, Univ. of Hiroshima, Japan
Korea Atomic Energy Res. Inst., Korea
E. T. Karlson, Uppsala Univ., Sweden
L. Stenflo, Univ. of UMEA, Sweden
J. R. Saraf, New Univ., United Kingdom