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Gyroscopic Mode Synthesis in the Dynamic Analysis of a Multi-Shaft Rotor-Bearing System

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ABSTRACT

A mode synthesis method used in the dynamic analysis of multi-shaft rotor-bearing system has been developed in this paper. By introducing the idea of connecting springs and dampers, and using gyroscopic modes instead of complex modes in the mode synthesis , this method differs not only from classical component mode synthesis method, but also from complex mode synthesis. Several numerical examples show the advantages of this method.

NOMENCLATURE

A,B,C	component state vector matrices, modal coordinates
A_s,B_s	system state vector matrices, modal coordinates
C_{BL}	damping matrix of a bearing
D	component damping matrix
f_b	force vector coupled on component boundary coordinates
f_{bn}	nonlinear part of force vector coupled by bearings
f_u	external excitation force vector on component
F_s	external excitation force vector on system, modal coordinates
G	component gyroscopic damping matrix
K	component stiffness matrix
K_{BL}	stiffness matrix of a bearing
M	component mass matrix
N_s	nonlinear force vector coupled of system
q_k	component modal coordinate vector
u	component generalized coordinate vector
v	system generalized coordinate vector
x	component displacement vector
Y,Z	component state vector, physical coordinates
β	transformation matrix of component from physical coordinate to modal coordinate
Λ	component precessional frequency matrix
Φ	constrained mode matrix
Ψ	gyroscopic mode matrix
Ω	spin speed

SUBSCRIPTS

b	boundary
i	interior
s	system

INTRODUCTION

In a series of works by Nelson and others (1,2, 3) dealing with rotor-bearing systems, a mode synthesis method has taken account of the gyroscopic effect of the disks and shafts, and the asymmetry of the stiffness and damping of the bearings. Complex modes consist of left and right vectors with

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biorthogonality which have been adapted in those works. The complex mode synthesis method has been utilized in the work of Hasselman and Kaplan (4). Li and Gunter (5) used real modes in the synthesis of a multiple component system. Childs (6), Lund (7) and others have also suggested the use of modal reduction in the analysis of rotor system.

Considering the gyroscopic effect at a high speed of rotation of the shafts and the factors of viscous and hysteristic damping, as well as the asymmetric stiffness and damping of the bearings, and using the complex mode synthesis method, Nelson and others studied the stability, transient response and nonlinear response of rotor system.The work Of theirs demonstrates several advantages, e.g., it can take into account various complex effects in practice and may reduce the DOF in a large scale and the calculation can still retain the engineering precision.

Since the various complex factors mentioned above have been taken into account in the motion equation of the component, in order to decouple the equations in the state space, it is necessary to seek the approprite left and right complex vectors which are orthonomalized with respect to the generalized matrices. This task is substantial and may take a large amount of computer time. Therefore, it is important and desirable to seek a method which is more efficient when dealing with the modal analysis of the component.

A method of calculated eigenvalues for gyroscopic systems has been presented by Meirovitch (8), transforming the skew symmetric matrix eigenproblem into the generalized symmetric matrix eigenproblem. The real modes obtained provide orthonomality with respect to the generalized matrices, and constitute a set of complete orthogonal vectors which can be used in modal transformation. It is simple and less costly to calculate these gyroscopic modes. Many ready-made programs are available for this computational task.

When applying the mode synthesis method to the complex structures with nonlinear connections (9), we divide the rotor system into a number of components with the fixed boundary points in accordance with its configuration or its natural component constituent, and the bearings are considered to be isolated from the system as separate connection elements. In dealing with the modal analysis of the component, only the most important modes need to be considered, i.e., besides the constrained modes, less gyroscopic modes with fixed boundary coordinates have to be calculated. The matrix composed of the gyroscopic modes and the constrained modes is used as the modal transformation for every component of the system. The bearings are regarded as the coupling forces of the connections, and depend only on the relative displacements and velocities of the boundary points, and may take into account the asymmetry and nonlinearity. The system equation obtained after assembly of the boundary forces will generally be asymmetric and may have nonlinear terms, however, the equations will be purely real.

THE MODAL ANALYSIS OF COMPONENT

When analysing the multiple shaft rotor-bearing systems, modal analysis can be executed for every shaft component with the fixed boundary coordinates. The support bearings and intermediary bearings can be regarded as the coupling elements which include elastic and damping forces. Fig.1 (a) illustrates how to divide the system into components I and II , and their connection representing the intermediary bearings and support bearings; (b) is the model for modal analysis of the gyroscopic modes and constrained modes with fixed boundary points; (c) indicates the coupling forces caused by the bearings and how to form the integrate system through the equilibrium requirements.

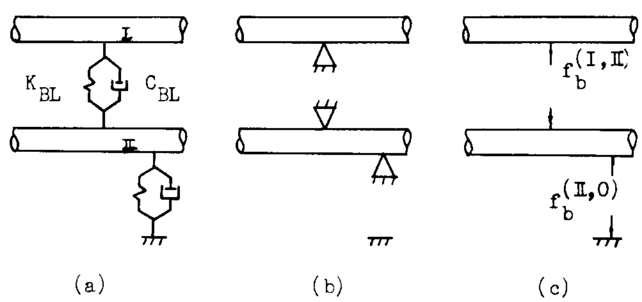

Fig.1 System Divided Into Components

The motion equation of an arbitrary shaft component descreted by FEM can be expressed as follows

$$M\ddot{x} + \Omega G\dot{x} + D\dot{x} + Kx = \bar{f}_b + f_u \qquad (1)$$

According to the interior coordinates x_i and boundary coordinates x_b, Eq.(1) can be arranged in compatible block shape

$$\begin{bmatrix} m_{ii} & m_{ib} \\ m_{bi} & m_{bb} \end{bmatrix}\begin{bmatrix} \ddot{x}_i \\ \ddot{x}_b \end{bmatrix} + \Omega \begin{bmatrix} g_{ii} & g_{ib} \\ g_{bi} & g_{bb} \end{bmatrix}\begin{bmatrix} \dot{x}_i \\ \dot{x}_b \end{bmatrix} + \begin{bmatrix} k_{ii} & k_{ib} \\ k_{bi} & k_{bb} \end{bmatrix}\begin{bmatrix} x_i \\ x_b \end{bmatrix} + \begin{bmatrix} d_{ii} & d_{ib} \\ d_{bi} & d_{bb} \end{bmatrix}\begin{bmatrix} \dot{x}_i \\ \dot{x}_b \end{bmatrix} = \begin{bmatrix} 0 \\ f_b \end{bmatrix} + \begin{bmatrix} f_{ui} \\ f_{ub} \end{bmatrix} \quad (1)'$$

where M, K and D are symmetric matrices, G is a skew symmetric matrix.

The first term on the right side of Eq.(1) represents the boundary force which may be a nonlinear function of spin speed and relative motion of the boundary points between the components. The bearings represent the relative action which will occur between neighbouring components in pair with opposite directions; The second is external excitation, e.g., unbalance force or others.

Introducing the state vector Z and its special arrangement Y, Z and Y have a relation as follow

$$Z = \begin{bmatrix} \dot{x}_i \\ \dot{x}_b \\ x_i \\ x_b \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}\begin{bmatrix} \dot{x}_i \\ x_i \\ \dot{x}_b \\ x_b \end{bmatrix} = \alpha \begin{bmatrix} Y_i \\ Y_b \end{bmatrix} = \alpha Y \quad (2)$$

Eq.(1) can be rewritten as

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}\dot{Z} + \begin{bmatrix} \Omega G & K \\ -K & 0 \end{bmatrix} Z + \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} Z = \begin{bmatrix} \bar{f}_b \\ 0 \end{bmatrix} + \begin{bmatrix} f_u \\ 0 \end{bmatrix} \quad (3)$$

Substituting (2) into (3) and premultiplying by α^T, we obtain

$$\begin{bmatrix} A_{ii} & A_{ib} \\ A_{bi} & A_{bb} \end{bmatrix}\begin{bmatrix} \dot{Y}_i \\ \dot{Y}_b \end{bmatrix} + \begin{bmatrix} B_{ii} & B_{ib} \\ B_{bi} & B_{bb} \end{bmatrix}\begin{bmatrix} Y_i \\ Y_b \end{bmatrix} + \begin{bmatrix} D_{ii} & D_{ib} \\ D_{bi} & D_{bb} \end{bmatrix}\begin{bmatrix} Y_i \\ Y_b \end{bmatrix} = F_u + \bar{F}_b \quad (4)$$

$$F = (f_{ui}^T , 0 , f_{ub}^T , 0)^T, \quad \bar{F} = (0 , 0 , f_b^T , 0)^T$$

From Eq.(4) the undamped free vibration equation with fixed boundary coordinates can be obtain as

$$A_{ii}\dot{Y}_i + B_{ii} Y_i = 0 \quad (5)$$

$$A_{ii} = \begin{bmatrix} m_{ii} & 0 \\ 0 & k_{ii} \end{bmatrix}, \qquad B_{ii} = \begin{bmatrix} \Omega g_{ii} & k_{ii} \\ -k_{ii} & 0 \end{bmatrix}$$

where A_{ii} is symmetric and positive definite. Eq.(5) represents a gyroscopic eigenproblem. According to the method of solving a gyroscopic eigenproblem (8), every precessional frequency ω_l and its related gyroscopic modes Φ_l and Ψ_l (l = 1,2, ··· , k) can be obtained and arranged to form a tri-diagonal skew symmetric frequency matrix Λ_k and a gyroscopic modal matrix Ψ_k as follow

$$\Lambda_k = \text{block-diag}\begin{pmatrix} 0 & \omega_l \\ -\omega_l & 0 \end{pmatrix} \quad (6)$$

$$\Psi_k = \left[\Phi_1, \Psi_1, \cdots, \Phi_k, \Psi_k\right] = \begin{bmatrix} \Psi_{kv} \\ \Psi_{kx} \end{bmatrix} \quad (7)$$

where Ψ_k has been normalized, satisfying the following generalized orthogonal conditions

$$\Psi_k^T A_{ii} \Psi_k = I \quad , \quad \Psi_k^T B_{ii} \Psi_k = \Lambda_k \quad (8)$$

As for the constrained modes, they can be obtained as follow

$$\Phi = -k_{ii}^{-1} k_{ib} \quad , \quad \bar{\Phi} = \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} \quad (9)$$

The modal transformation relation may then be written as

$$\begin{bmatrix} Y_i \\ Y_b \end{bmatrix} = \begin{bmatrix} \Psi_k & \bar{\Phi} \\ 0 & I \end{bmatrix}\begin{bmatrix} q_k \\ Y_b \end{bmatrix} = \begin{bmatrix} \Psi_{kv} & \Phi & 0 \\ \Psi_{kx} & 0 & \Phi \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} q_k \\ \dot{x}_b \\ x_b \end{bmatrix} = \beta u \quad (10)$$

In order to reduce the DOF, matrix Ψ_k includes only the first k order (pairs) modes of the component.

Executing the β matrix transformation for Eq.(4), gives

$$A\dot{u} + (B + C)u = F + F_b \quad (11)$$

$$A = \begin{bmatrix} I & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}, \qquad B = \begin{bmatrix} \Lambda_k & B_{12} & 0 \\ B_{21} & B_{22} & B_{23} \\ 0 & B_{32} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad F = \begin{bmatrix} \Psi_{kv}^T f_{ui} \\ \Phi^T f_{ui} + f_{ub} \\ 0 \end{bmatrix}$$

$$F_b = \begin{bmatrix} 0 \\ f_b \\ 0 \end{bmatrix}, \qquad u = \begin{bmatrix} q_k \\ \dot{x}_b \\ x_b \end{bmatrix}$$

where A is symmetric, and B is skew symmetric. The sub-matrices are expressed from the subelements of M, D, G and K as follow

$$A_{12} = \Psi_{kv}^T m \Phi + \Psi_{kv}^T m_{ib}$$

$$A_{22} = \Phi^T m_{ii} \Phi + \Phi^T m_{ib} + m_{bb} + m_{bi} \Phi$$

$$A_{33} = B_{23} = \Phi^T k_{ib} + K_{bb}$$

$$C_{11} = \Psi_{kv}^T d_{ii} \Psi_{kv}$$

B_{12} , C_{12} and B_{22} , C_{22} have the same form as A_{12} and A_{22} but using m instead of g or d.

For an arbitrary bearing between components I and II , the linear characteristic of stiffness and damping can be expressed by the following matrices

$$K_{BL} = \begin{bmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{bmatrix} , \qquad C_{BL} = \begin{bmatrix} C_{yy} & C_{yz} \\ C_{zy} & C_{zz} \end{bmatrix}$$

For a system with several bearings, the matrices can be augmented to form the following matrices

$$K_B = \text{block-diag}(\ K_{BL}\ ) \ , \ C_B = \text{block-diag}(\ C_{BL}\ )$$

Thus, the boundary forces between two components may be expressed by

$$\begin{Bmatrix} f_b^{(I,II)} \\ f_b^{(II,I)} \end{Bmatrix} = - \begin{bmatrix} C_B & K_B & -C_B & -K_B \\ -C_B & -K_B & C_B & K_B \end{bmatrix} \begin{Bmatrix} \dot{x}_b^{(I)} \\ x_b^{(I)} \\ \dot{x}_b^{(II)} \\ x_b^{(II)} \end{Bmatrix} + \begin{Bmatrix} f_{bn}^{(I,II)} \\ f_{bn}^{(II,I)} \end{Bmatrix}$$

where the second term on the right side of the equation represents nonlinear term.

ESTABLISHMENT OF THE SYNTHESIS EQUATION ITS SOLUTION

Collection of all the modal coordinates of each component and the boundary coordinates, and arranged them in sequence, yield the overall set of independent coordinates of the system

$$v = (\ q_k^{(I)T}, q_k^{(II)T}, \cdots, q_k^{(s)T}, \ Y_b^{(I,II)T}, Y_b^{(II,I)T}, \cdots, Y_b^{(s-1,s)T}, Y_b^{(s,s-1)T})^T$$

Thus, the elements of each component motion equation can be arranged according to their relative position to form the overall equation of the system, i.e.

$$A_s \dot{v} + B_s v = F_s + N_s \tag{12}$$

where A_s is the assemblage of all component matrices A , B_s is the collection of all component matrices B and C plusing contribution from the linear part of the right side boundary force. The matrices of linear stiffness and damping of the bearings are arranged in the lower right area corresponding to the boundary coordinates. F_s is the collection of the external force vectors with modal coordinates such as unbalance force and inertia force caused by the base motion. Whereas N_s is the nonlinear part of bearing coupling force. When the nonlinearities are presented in the component, those can also be included into N_s. The details can be consulted in author's works (10)(11). Because the linear term has been moved to the left side of the equation, the nonlinear term which is the function of the system state vector appears in a relatively small number of locations. This nonlinear equation can be solved by using an iterative method.

We have obtained the overall system equation in which various complex factors have been considered. Owing to the seperation of the system into subsystems , every component can be analized in detail by FEM and the asymmetricity and nonlinearity of the bearings can be fully taken account. Using the gyroscopic modal synthesis method, the DOF of the system can be reduced sufficiently and calculation can be simpfied.

From Eq.(12), neglecting the nonlinear and forcing terms, we can obtain the free vibration equation as follows

$$A_s \dot{v} + B_s v = 0 \tag{13}$$

Let $v = V e^{\lambda t}$, to obtain

$$(\ B_s + \lambda A_s\ )\ V = 0 \tag{14}$$

Using a complex eigenvalue program of real coefficient matrices, the complex eigenvalues λ_r and associate complex modes V_r can also be obtained. The eigenvalue λ_r can be expressed as

$$\lambda_r = n_r + j\ p_r$$

where n_r is the exponent damping coefficient, and p_r is the damped precessional frequency. If p_r is a forward precessional frequency, frequency curve $p_r - \Omega$ intersect with the 45° line at the point $p_r = \Omega$, and spin speed Ω is some critical speed of the system.

Because the external force vector F_s is known, it is easy to obtain the linear response. For example, in the case of unbalance response, let

$$F_s = F_y \cos\Omega t + F_z \sin\Omega t = F_1 e^{j\Omega t} + F_2 e^{j\Omega t} \quad (15)$$

and substitute (15) into Eq.(12), to obtain

$$(j\Omega A_s + B_s) x_1 = F_1$$

$$(-j\Omega A_s + B_s) x_2 = F_2$$

Then, x_1 and x_2 solved from above equation provide conjugate vector pair.

As for the nonlinear response , it can be obtained by means of numerical integration.

NUMERICAL EXAMPLES

Example 1. A Uniform Shaft Supported on Springs

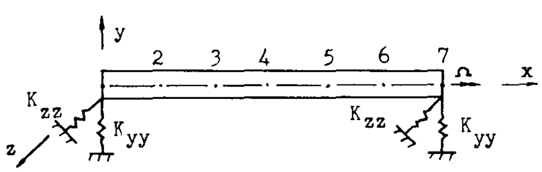

Fig.2 A Uniform Shaft

diameter	1	cm	elastic modulus	
length	30	cm	E	= 2000000.0 kg/cm^2
mass density	0.0078	kg/cm^3	$K_{yy} = K_{zz}$	= 10^4 kg/cm
Poisson ratio	0.3		Ω	= 10^4 RPM

A uniform shaft which is divided by FEM into 6 elements and has 28 DOF is shown in Fig.2 . There is a unbalance amount me = 0.01 kg-cm at node 4. The structure is symmetric. The component with fixed boundary coordinates has 24 DOF.

The precessional frequencies of the system with different component modes retained is given in Table 1. Among other things, the results with all component modes (24 pairs) are agreed with the results calculated by FEM (12). It is interesting that , the first k pairs of the component modes being retained, the errors of the first k/2 pairs of the precessional frequencies of the system are less than 1% in comparison with that by FEM. The node orbit radii excited by the unbalance amount with the different component modes retained is given in Table 2. It is seen that effect of the high order modes of the component for unbalance response is very small. The results with only 4 pairs of the modes retained are less than 1% in comparison with the results with all component modes retained.

Table 1. Precessional Frequencies with Different Component Modes Unit: rad/sec

r \ k, p_r		24	16	8	4
1	B	216.8958	216.8958	216.8958	216.8958
	F	217.1168	217.1168	217.1168	217.1168
2	B	846.6977	846.6981	846.6995	846.7191
	F	847.4984	847.4987	847.5002	847.5201
3	B	1829.799	1829.817	1830.464	3799.219
	F	1831.322	1831.340	1831.990	3800.194
4	B	3067.097	3067.329	3072.816	4663.915
	F	3069.285	3069.519	3075.031	4666.095
5	B	4445.738	4450.488	5574.485	
	F	4448.822	4453.596	5578.530	
6	B	5959.879	5971.478	6680.882	
	F	5964.950	5976.601	6687.524	
7	B	7485.165	7538.220		
	F	7492.756	7546.011		
8	B	10652.76	10719.59		
	F	10664.57	10731.70		
9	B	14034.54	14666.10		
	F	14050.38	14685.44		
10	B	18537.92	19229.15		
	F	18558.21	19253.60		

Table 2. Orbit Radii of Unbalance Response with Different Component Modes Unit: cm

node \ k, R		24	16	8	4
1	10^{-3}x	1.578244	1.578240	1.578125	1.579892
2	10^{-2}x	7.766927	7.767475	7.760528	7.841186
3	10^{-1}x	1.345331	1.345320	1.346523	1.346568
4	10^{-1}x	1.561873	1.561809	1.560422	1.552438

Example 2. Lund' Rotor

lund and Orcutt (13) performed an unbalance response experiment of a rotor system, and gave a theoretical calculation by transfer matrix method. A group of complete parameters of the structure was given in their paper. In reference (12), Lund' Rotor was divided by FEM into 14 elements. The first 8 pairs of the component modes have been used in synthesis. The results calculated were marked " + " in the figues of the paper (13) as a comparison. Only four figues from the paper (12) are cited, from Fig. 3 to Fig. 6 seen. The results of this paper well agree with the experimental results (13).

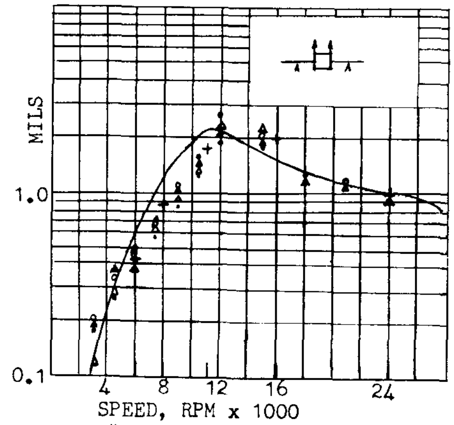

Fig.3* Fig.3 in Paper (13)

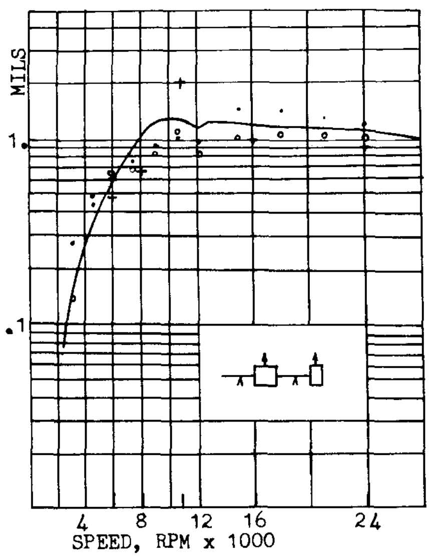

Fig.4* Fig.6 in Paper (13)

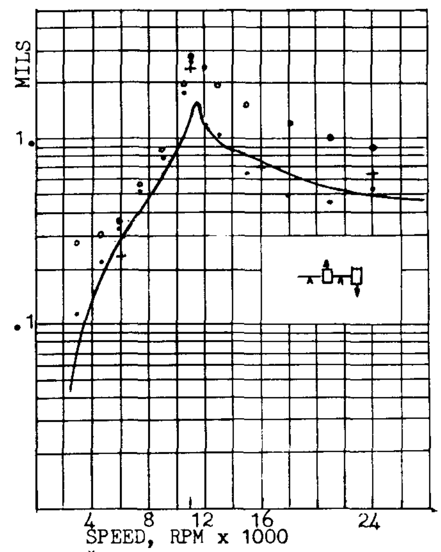

Fig.5* Fig.11 in Paper (13)

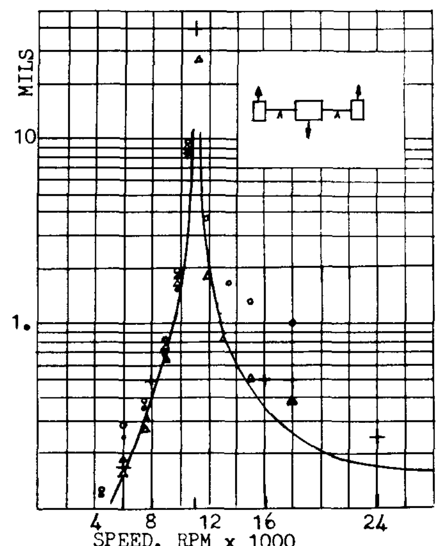

Fig.6* Fig.16 in Paper (13)

Example 3. Dual Rotor System

In a dual rotor system shown in fig.7 , inner shaft is treated as component I, and outer shaft the component II . Both of them are divided into 10 elements. There is a unbalance amount at node 5 on the outer shaft. In working process, spin speed of the inner shaft is a constant, i.e., $\Omega^{(\mathrm{I})}$ = 10100 RPM. The first 6 pairs of modes for every component are used in synthesis. The first 3 forward frequencies of the system are given in Table 3. The critical speeds determined by 45° line method are

n_{cr1} = 10104 RPM n_{cr2} = 15230 RPM

n_{cr3} = 20206 RPM

The approximate modes for 1st and 2nd critical speeds are shown in Fig.8 and Fig.9 respectively. Fig.10 and Fig.11 show the orbit radius curves of unbalance response of the 6th node on the inner shaft and the 5th node on the outer shaft. It can be seen that response crests well correspond with critical speeds.

* —— THEORETICAL RESULTS

▲▲ •• EXPERIMENTAL DATA

\+ PRESENT PAPER RESULTS

Table 3. The First 3 Forward Precessional Frequencies of Dual Rotor System   Unit: RPM

r \ p_r \ Ω	10100	10500	15000	16000	18000	19000	20000
1	10104	10118	10237	10260	10303	10323	10341
2	13128	13334	15169	15434	15850	15977	16099
3	16080	16208	17778	18206	19146	19632	20108

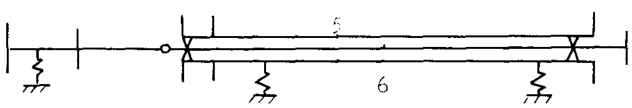

Fig.7 Dual Rotor System Configuration

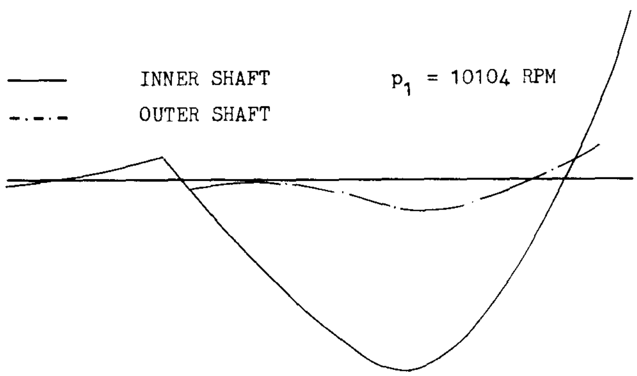

Fig.8 Mode Corresponding to 1st Critical Speed

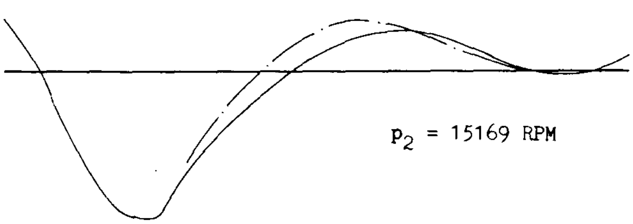

Fig.9 Mode Corresponding to 2nd Critical Speed

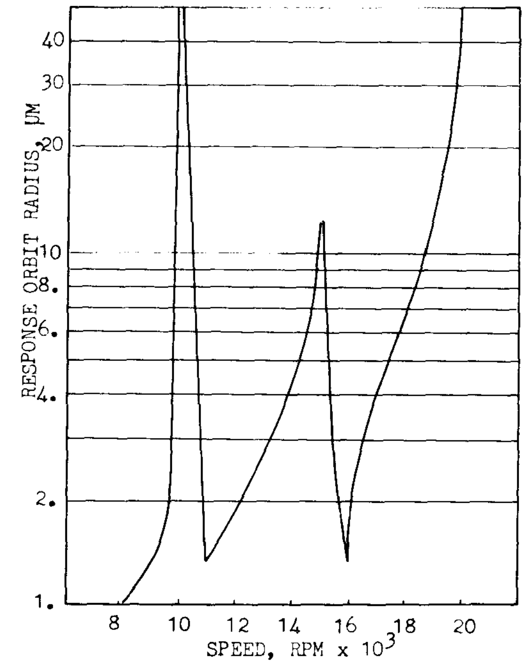

Fig.10 Unbalance Response of Dual Rotor, 6th Node on Inner Shaft

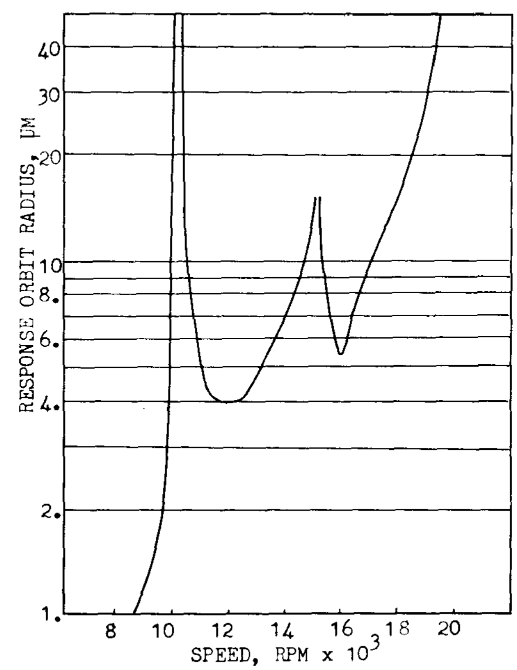

Fig.11 Unbalance Response of Dual Rotor, 5th Node on Outer Shaft

CONCLUSION

In this paper, the use of gyroscopic modes in the modal synthesis of multiple shaft rotor-bearing systems allows for the including of various factors such as the nonlinearity and asymmetricity of bearings , gyroscopic moments of shafts and disks, damping and others. Through modal reduction the size of system equation can be very small before solving the precessional frequencies, complex modes, and linear and nonlinear response for the system.

There are the following advantages in the use of gyroscopic mode synthesis: (i) It can use real mode programs for calculation of the gyroscopic modes in component modal analysis; (ii) The synthesis equation of the system is an asymmetric matrix equation with real coefficients. Therefore, the important advantage is saving computer time and memory in comparison with the complex mode synthesis method.

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