# $\gamma Z$ Box Corrections to Weak Charges of Heavy Nuclei in Atomic Parity Violation

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We present a new dispersive formulation of the $\gamma Z$ box radiative corrections to weak charges of bound protons and neutrons in atomic parity violation measurements on heavy nuclei such as $^{133}$Cs and $^{213}$Ra. We evaluate for the first time a small but important additional correction arising from Pauli blocking of nucleons in a heavy nucleus. Overall, we find a significant shift in the $\gamma Z$ correction to the weak charge of $^{133}$Cs, approximately 4 times larger than the current uncertainty on the value of $\sin^2\hat{\theta}_W$, but with a reduced error compared to earlier estimates.

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In the search for physics beyond the Standard Model one of the most important indirect methods involves a high precision test of the evolution of the Weinberg angle with scale. In particular, the comparison between the value of $\sin^2\hat{\theta}_W$ measured at the $Z$ pole at LEP and the value extracted from parity violation in atomic systems has provided a very strong confirmation of the radiative corrections calculated within the Standard Model. The atomic system with the most accurate current measurement is the parity-violating $S$-$S$ transition in neutral Cs [1,2]. The predicted enhancement factor of the atomic parity violation (APV) effect in the $S$-$D$ transition in $\mathrm{Ra}^+$ is about 50 times larger [3]. It is critical that the calculations of the radiative corrections for these systems incorporate the latest theoretical developments and match the precision of the experimental data.

Driven by the demand for accurate radiative corrections for high energy parity-violating electron scattering, notably the $Q_{\rm weak}$ experiment at Jefferson Lab [4], there have recently been new evaluations of the $\gamma Z$ box diagram [5–9]. In particular, Blunden *et al.* [9] developed a formulation of this correction in terms of dispersion relations and moments of the inclusive $\gamma Z$ interference structure functions. This approach provides a systematic method for improving the accuracy of the calculation.

In this Letter we use the dispersive relations methods to compute a new value for the radiative correction associated with the $\gamma Z$ box diagram for both protons and neutrons in $^{133}$Cs and $^{213}$Ra. Compared with previous estimates that were computed some three decades ago [10], the new corrections give $\gamma Z$ contributions that are 15% smaller for free nucleons, and 20% smaller for nucleons bound in a heavy nucleus (due to Pauli blocking). This shifts the theoretical value of $Q_W(\mathrm{Cs})$ from $-73.14(6)$ to $-73.26(4)$. This agrees with the experimental value of $-73.16(29)_{\rm exp}(20)_{\rm th}$ given in Ref. [2], and is within $1.5\sigma$ of the value $-72.58(29)_{\rm exp}(32)_{\rm th}$ given in Ref. [11].

Including radiative corrections, the weak charges of the proton and neutron can be written as [10,12]

$$Q_W^p = (\rho + \Delta_e)(1 - 4\kappa(0)\hat{s}^2 + \Delta_e') + \Box_{WW}^p + \Box_{ZZ}^p + \Box_{\gamma Z}^p, \quad (1)$$

$$Q_W^n = -(\rho + \Delta_e) + \Box_{WW}^n \Box +_{ZZ}^n + \Box_{\gamma Z}^n, \quad (2)$$

with $\hat{s}^2 \equiv \sin^2\hat{\theta}_W(M_Z^2) = 0.23116(13)$ in the $\overline{\rm MS}$ scheme. $\Delta_e$ and $\Delta_e'$ are $Zee$ and $\gamma ee$ vertex corrections, given in Refs. [10,12], and expressions for the universal parameters $\rho$ and $\kappa(Q^2)$ are given in Ref. [13].

The proton weak charge $Q_W^p$ is sensitive to the running of the Weinberg angle, as embodied in $\kappa(Q^2)$, whereas the neutron weak charge $Q_W^n$ is primarily sensitive to $\rho$. At the one-loop level, $\rho$ has a quadratic dependence on the top quark mass. This dependence is modified by significant higher-order QCD and electroweak corrections [14], some of which have been evaluated to four loops.

The correction $\kappa(Q^2)$ includes boson self-energy contributions from $\gamma Z$ mixing. In particular, it has a hadronic uncertainty from the quark contributions to fermion loops, denoted by $\Delta\kappa_{\rm had}^{(5)}$ for five quark flavors, which is strongly correlated with the analogous contribution $\Delta\alpha_{\rm had}^{(5)}$ to the running of $\alpha(Q^2)$. A reduction of $\Delta\alpha_{\rm had}^{(5)}$ to 0.02772 in the most recent analysis implies a corresponding reduction in $\Delta\kappa_{\rm had}^{(5)}$ compared with Ref. [12], leading to $\Delta\kappa_{\rm had}^{(5)}\hat{s}^2 = 7.87(8)\times 10^{-3}$. Using the most recent Standard Model parameters [15], we obtain $\rho = 1.0006(2)$ and $\kappa(0)\hat{s}^2 = 0.23807(15)$, in essential agreement with the values quoted in Ref. [15].

The remaining terms in Eqs. (1) and (2) arise from the (numerically dominant) $WW$, $ZZ$, and $\gamma Z$ boxes. We take the expressions for $WW$ and $ZZ$ boxes from Erler *et al.* [12], including the leading order QCD correction to the original expressions of Ref. [10]. The box diagrams with two heavy bosons are dominated by high momentum scales

 

[16]. By contrast, the $\gamma Z$ box diagram contains both high and low momentum scales, and is therefore sensitive to hadronic corrections. Following convention we write the $\gamma Z$ contribution for protons and neutrons in terms of a parameter $B^N$,

$$\square^N_{\gamma Z} = \frac{3\alpha}{2\pi}\hat{v}_e \mathcal{Q}^N B^N, \tag{3}$$

where $\hat{v}_e \equiv 1 - 4\hat{s}^2$ and $\mathcal{Q}^N = 5/3(4/3)$ for the proton (neutron). A free-quark model gives $B^p = \ln(M_Z^2/m^2) + 3/2$, where $m$ is an undetermined hadronic mass scale (such as a constituent quark mass); however, this does not adequately describe either the long-range or short-range behavior of the $\gamma Z$ box. Marciano and Sirlin (MS) [10] give a more refined estimate by separately modeling the low and high energy contributions, but an equivalent logarithmic dependence on the hadronic mass parameter $m$ remains.

Using Eqs. (1) and (2), we find numerically

$$Q_W^p = 0.0664(6) + 0.00044B^p, \tag{4}$$

$$Q_W^n = -0.9922(2) + 0.00035B^n. \tag{5}$$

The error in $Q_W^p$ (excluding the $\gamma Z$ boxes) arises from $\kappa(0)\hat{s}^2$ ($\pm$ 0.0006) and the $WW$ boxes ($\pm$ 0.0001), while the error in $Q_W^n$ arises from $\rho$ ($\pm$ 0.0002). The results in the column labeled MS in Table I use the most recent estimates of $B^p$ and $B^n$ from Refs. [12,16]. For $^{133}$Cs, we have $Q_W(\mathrm{Cs}) = 55Q_W^p + 78Q_W^n$. Adding the independent errors in quadrature (the errors in $B^p$ and $B^n$ are not independent), we find $-73.14(6)$. The $\gamma Z$ boxes contribute $\pm0.052$ to this total, while all other errors combined contribute $\pm0.037$. (See also the recent review of APV, including $^{133}$Cs, in Ref. [17].)

To proceed beyond the work of MS [10] we use the dispersion methods developed in Refs. [5–9], writing the imaginary part of $\square_{\gamma Z}(E)$ in terms of structure functions $F^N_{i,\gamma Z}$ ($i = 1, 2, 3$) that can be obtained from inclusive lepton-nucleon scattering. A dispersion integral over energy then gives the real part of $\square_{\gamma Z}(E)$, which contributes to the weak charge. The energy dependence of $\square_{\gamma Z}(E)$ was evaluated in Refs. [6,9]. At $E = 0$, relevant for APV, only the axial-vector $ZN$ coupling involving the $F^N_{3,\gamma Z}$ structure function gives a nonzero contribution. Performing the dispersion energy integral analytically, the real part of $\square_{\gamma Z}$ can be written

$$\square^N_{\gamma Z} = \frac{2}{\pi}\int_0^\infty dQ^2 \frac{\alpha(Q^2)v_e(Q^2)}{Q^2(1+Q^2/M_Z^2)} \times \int_0^1 dx F^N_{3,\gamma Z}(x,Q^2)\frac{1+2\gamma}{(1+\gamma)^2}, \tag{6}$$

where $\gamma = \sqrt{1+4M^2x^2/Q^2}$ and $x = Q^2/(W^2 - M^2 + Q^2)$, with $W$ the invariant mass of the intermediate hadronic state. Following Ref. [9], we include the running with $Q^2$ of $\alpha(Q^2)$ and $v_e(Q^2) \equiv 1 - 4\kappa(Q^2)\hat{s}^2$ in Eq. (6) due to boson self-energy contributions. Both quantities vary significantly over the relevant $Q^2$ range.

The contributions to $\square_{\gamma Z}$ can be split into three kinematic regions: (i) elastic, with $W^2 = M^2$; (ii) resonances, with $(M + m_\pi)^2 \le W^2 \lesssim 4$ GeV$^2$; and (iii) deep-inelastic scattering (DIS), with $W^2 > 4$ GeV$^2$. Contributions from region (i) depend on the nucleon's elastic magnetic $G_M^N$ and axial-vector $G_A^{Z,N}$ form factors,

$$F^{N(\mathrm{el})}_{3,\gamma Z}(x,Q^2) = -G_M^N(Q^2)G_A^{Z,N}(Q^2)x\delta(1-x). \tag{7}$$

We set $G_M^N(Q^2) = \mu^N F_V(Q^2)$, and $G_A^{Z,N}(Q^2) = -g_A^N F_A(Q^2)$, with $\mu^N$ the nucleon magnetic moment, and $g_A^p = -g_A^n = 1.267$. A dipole $Q^2$ dependence, $F_{V,A}(Q^2) = 1/(1 + Q^2/\Lambda^2_{V,A})^2$, suffices for both $V$ and $A$ form factors, with $\Lambda_V = 0.84$ GeV, and $\Lambda_A = 1.0$ GeV. More sophisticated form factors give essentially identical numerical results.

For the resonance contributions we use the parametrizations of the transition form factors from Lalakulich *et al.* [18], but with modified isospin factors appropriate to $\gamma Z$. These form factors have been fit to pion production data in $\nu$ and $\bar{\nu}$ scattering, and include the lowest four spin 1/2 and 3/2 states.

For the DIS region, we divide the $Q^2$ integral of Eq. (6) into a low-$Q^2$ part $Q^2 < Q_0^2$, where the structure function $F^N_{3,\gamma Z}$ is relatively unknown, and a high-$Q^2$ part ($Q^2 > Q_0^2$), where at leading order the structure function can be expressed in terms of valence quark distributions [15]. At high $Q^2$ the $\gamma Z$ contribution can be expanded in powers of $x^2/Q^2$, yielding a series whose coefficients are structure function moments of increasing rank,

$$\square^{(\mathrm{DIS})}_{\gamma Z} = \frac{3}{2\pi}\int_{Q_0^2}^\infty dQ^2 \frac{\alpha(Q^2)v_e(Q^2)}{Q^2(1+Q^2/M_Z^2)} \times \left[M_3^{(1)}(Q^2) - \frac{2M^2}{3Q^2}M_3^{(3)}(Q^2) + \cdots\right], \tag{8}$$

where the $n$th moment of the $F_3^{\gamma Z}$ structure function is $M_3^{(n)}(Q^2) = \int_0^1 dx x^{n-1}F_3^{\gamma Z}(x,Q^2)$. Numerically, the $n = 1$ moment dominates, with the $n \ge 3$ contributions to the integral of Eq. (8) less than 0.1%.

TABLE I. Weak charges of the proton, neutron, $^{133}$Cs and $^{213}$Ra, comparing the previous MS estimates [10,16] with our new results for free and bound nucleons.

| | MS | Free nucleons | Bound nucleons |
|---|---|---|---|
| $B^p$ | 11.8(1.0) | 9.95(40) | 9.36(40) |
| $B^n$ | 11.5(1.0) | 9.82(40) | 9.32(40) |
| $Q_W^p$ | 0.0716(8) | 0.0708(6) | 0.0705(6) |
| $Q_W^n$ | −0.9882(4) | −0.9888(2) | −0.9890(2) |
| $Q_W$(Cs) | −73.14(6) | −73.23(4) | −73.26(4) |
| $Q_W$(Ra) | −117.22(10) | −117.37(7) | −117.42(7) |

The lowest moment is in fact the $\gamma Z$ analog of the GLS sum rule [19] for $\nu N$ DIS, which at leading order counts the number of valence quarks in the nucleon. The corresponding quantity for $\gamma Z$ is $\mathcal{Q}^p = \sum_q 2e_q g_A^q = 5/3$ for the proton and $\mathcal{Q}^n = 4/3$ for the neutron. Including the next-to-leading order strong interaction correction in the $\overline{\rm MS}$ scheme, the $n = 1$ contribution is

$$M_3^{N(1)}(Q^2) = \mathcal{Q}^N\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right). \tag{9}$$

Combined with Eq. (8), this is identical to the high energy result of MS [10], but with $Q_0$ replacing the arbitrary hadronic mass parameter $m$. In our case the scale $Q_0$ corresponds to the momentum above which a partonic representation of the nonresonant structure functions is valid, and above which the $Q^2$ evolution of parton distribution functions (PDFs) via the $Q^2$ evolution equations is applicable. We vary $Q_0^2$ between 1 and 2 $\text{GeV}^2$, which coincides with the typical lower limit of recent sets of PDFs.

The contribution for $Q^2 < Q_0^2$ can in principle be obtained from data. There is limited information on $F_{3,W}$ from neutrino scattering, but little or no existing data on $F_{3,\gamma Z}$ at low $Q^2$. As in Ref. [9], we use two different models to smoothly interpolate in $Q^2$ between $Q_0^2$ and 0: one vanishes in the $Q^2 \to 0$ limit, and the other approaches a constant. The differences between the models are of the order of 5% to 15%.

The relation between proton and neutron contributions from the different kinematic regions is

$$\Box_{\gamma Z}^{n(\text{el})} = -\frac{\mu^n}{\mu^p}\Box_{\gamma Z}^{p(\text{el})}, \qquad \Box_{\gamma Z}^{n(\text{res})} \approx \Box_{\gamma Z}^{p(\text{res})},$$
$$\Box_{\gamma Z}^{n(\text{DIS})} = \frac{4}{5}\Box_{\gamma Z}^{p(\text{DIS})}. \tag{10}$$

The near equality of the resonance contributions (within 3%) is due to the dominance of isovector resonances, which contribute equally for protons and neutrons.

The full results are summarized in Table II. The largest contributions come from the DIS region. Fortunately, the results show only a mild sensitivity to the parameter $Q_0$

TABLE II. Contributions to $B^p$ from different kinematic regions for two different values of the matching scale $Q_0^2$. The range of values for the DIS ($Q^2 < Q_0^2$) contribution is for the two models described in Ref. [9].

| $Q_0^2$ | 1 $\text{GeV}^2$ | 2 $\text{GeV}^2$ |
|---|---|---|
| Elastic | 1.47 | 1.47 |
| Resonance | 0.59 | 0.59 |
| DIS | | |
| $Q^2 > Q_0^2$ | 7.50 | 7.05 |
| $Q^2 < Q_0^2$ | 0.42–0.48 | 0.78–0.82 |
| Total | 9.98–10.04 | 9.89–9.93 |

that separates the model dependent low-$Q^2$ extrapolation from the high-$Q^2$ partonic region. The main part of the uncertainty arises from the model dependence of this extrapolation to low $Q^2$. We therefore assign $B^p = 9.95(40)$, equal to the average of the four values in Table II, with a very conservative error given by the DIS contribution from the region $Q^2 < 1$ $\text{GeV}^2$. This is the value appearing in the column labeled 'Free nucleons' of Table I, together with the neutron contribution using Eq. (10).

Because the nucleons in a heavy nucleus are bound, properties such as their weak charge can differ from those of free nucleons. In particular, for the elastic $\gamma Z$ box contribution, transitions to occupied states are forbidden by the Pauli exclusion principle. To estimate this effect of the nuclear medium on the weak charges, we consider the expression

$$\Box_{\gamma Z}^{N(\text{el})} = \frac{2\alpha}{\pi} v_e \mu^N g_A^N \int_0^\infty dQ^2 F_V(Q^2) F_A(Q^2) f(Q^2), \tag{11}$$

where

$$f(Q^2) = \frac{1 + 2\gamma_1}{Q^2(1+\gamma_1)^2}, \qquad \gamma_1 \equiv \gamma(x = 1). \tag{12}$$

Here $v_e$ is taken at an appropriate low-momentum scale, and the $Q^2$ dependence of the $Z$ propagator has been dropped.

Pauli blocking is important because the integrand in Eq. (11) is heavily weighted towards low $Q^2$,

$$f(Q^2) \overset{Q^2\to 0}{\longrightarrow} \frac{1}{MQ} - \frac{3}{4M^2} + \cdots. \tag{13}$$

Since the form factors introduce corrections of order $Q^2/\Lambda^2$, the dominant low-$Q^2$ contribution is largely independent of nucleon structure.

To allow for Pauli blocking of the intermediate nucleon in a heavy nucleus we use the Fermi gas model, where nucleon states are occupied below the Fermi momentum $k_F$ (typically $\sim$0.25 GeV). This estimate should suffice in a heavy nucleus like $^{133}$Cs with a low surface to volume ratio. For momentum transfer $\boldsymbol{q}$ to a bound nucleon of momentum $\boldsymbol{p}$, we must exclude from the integral of Eq. (11) all intermediate nucleon states of momentum $|\boldsymbol{p} + \boldsymbol{q}| < k_F$. Introducing the occupation number $n_p = \Theta(k_F - p)$, we therefore have a factor

$$C(q) = \frac{\int d^3p\, n_p n_{|\boldsymbol{p}+\boldsymbol{q}|}}{\int d^3p\, n_p} \tag{14}$$

to be folded into the integrand of Eq. (11), with $\int d^3p\, n_p = (4\pi/3)k_F^3$. This represents the fractional volume of occupied states that cannot reach the Fermi surface for a given value of $\boldsymbol{q}$. Since $k_F^2 \ll M^2$, the affected states have nonrelativistic energies, and so $Q^2 \approx \boldsymbol{q}^2$. From simple geometry, we find

TABLE III. The parameters of the proton and neutron Woods-Saxon densities $\rho(r)$ for $^{133}$Cs, together with the mean square radii. The Fermi momenta $k_F(0)$ are determined from the central nucleon densities, with $\langle k_F \rangle$ the corresponding mean.

| | $c$ (fm) | $a$ (fm) | $\langle r^2 \rangle$ (fm$^2$) | $k_F(0)$ (GeV) | $\langle k_F \rangle$ (GeV) |
|---|---|---|---|---|---|
| $p$ | 5.8895 | 0.4010 | 23.03 | 0.241 | 0.218 |
| $n$ | 5.9482 | 0.4946 | 24.61 | 0.266 | 0.236 |

$$C(q) = \begin{cases} 1 - \frac{1}{2}\left(3\frac{q}{2k_F} - \left(\frac{q}{2k_F}\right)^3\right), & 0 < q < 2k_F, \\ 0, & q \geq 2k_F \end{cases}. \quad (15)$$

The expression $1 - C(q)$ is also the Coulomb sum rule for longitudinal quasielastic scattering in a nonrelativistic Fermi gas.

The Pauli blocking effect can be introduced as a correction $1 - \Delta(k_F)$ to the value of $\Box_{\gamma Z}^{N(\mathrm{el})}$ in Eq. (11), with

$$\Delta(k_F) = \frac{\int_0^{2k_F} dQ^2 F_V(Q^2) F_A(Q^2) f(Q^2) C(Q)}{\int_0^{\infty} dQ^2 F_V(Q^2) F_A(Q^2) f(Q^2)} \quad (16)$$

representing the fractional contribution from the excluded states. This correction depends on $k_F$, and has only a weak dependence on nucleon structure through the form factor parameters $\Lambda$, with other parameters dropping out in the ratio. In the numerator, the leading terms are of order $\mathcal{O}(k_F)$ and $\mathcal{O}(k_F^2)$, with the form factor corrections only appearing at order $\mathcal{O}(k_F^3)$ and higher.

The values of $k_F$ for protons and neutrons are taken from Hartree-Fock calculations for $^{133}$Cs, which reproduce the experimental charge density [20]. Fitting the Hartree-Fock proton and neutron distributions to a standard Woods-Saxon form $\rho(r) = \rho_0/(1 + \exp[(r - c)/a])$, normalized to $Z$ and $N$, respectively, leads to the parameters given in Table III. From these one can compute the Fermi momentum in the local density approximation, $\rho = k_F^3/(3\pi^2)$. The central and average values of $k_F$ are given in Table III. The latter, which are used in our calculations, are consistent with those obtained in fits to experimental quasielastic electron scattering data using a simple Fermi gas model [21].

We find the Pauli blocking correction factor $1 - \Delta(k_F)$ is approximately linear in $k_F$ over the range 0.2–0.3 GeV, and well approximated by the expression

$$1 - \Delta(k_F) \approx 0.83 - 1.04 k_F, \quad (17)$$

with $k_F$ in GeV. Specifically, using the volume-averaged values of $\langle k_F \rangle$ in Table III, we find a correction factor of 0.61 to $\Box_{\gamma Z}^{p(\mathrm{el})}$ and 0.59 to $\Box_{\gamma Z}^{n(\mathrm{el})}$. For reasonable values of $k_F$ up to 0.27 GeV in a very heavy nucleus, the Pauli blocking factors will therefore fall into the narrow range 0.55–0.60, suggesting a relatively insignificant variation in the total value of $\Box_{\gamma Z}^{N}$ for nuclei beyond $^{133}$Cs.

The effect on $Q_W$(Cs) is shown in final column of Table I. There is a small shift of $-0.03$ compared to the free nucleon values, giving $-73.26(4)$. The individual terms contributing to the uncertainty are $\kappa(0)\hat{s}^2$ ($\pm$ 0.033), $WW$ boxes ($\pm$ 0.006), $\rho$ ($\pm$ 0.016), and $\gamma Z$ boxes ($\pm$ 0.021). Also included in Table I is our theoretical value for the weak charge of $^{213}$Ra.

In summary, we have computed the effect of $\gamma Z$ exchange corrections on the weak charge of heavy nuclei such as $^{133}$Cs and $^{213}$Ra, using a recently developed formalism based on dispersion relations and an expansion of the $\gamma Z$ interference structure function moments. The results improve earlier estimates based on a quark model description of the $\gamma Z$ box contributions, allowing a significant reduction in the theoretical uncertainty. Compared with the pioneering early estimates of Marciano and Sirlin [10], the new corrections enhance the magnitude of the weak charge by $\approx 0.16\%$, which is approximately 4 times larger than the current uncertainty on $\sin^2\hat{\theta}_W$, and will therefore affect future high-precision determinations of the weak angle.

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