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H-E-Super magic decomposition of graphs

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Abstract

An H-magic labeling in an H-decomposable graph G is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for every copy H in the decomposition, $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ is constant. The function f is said to be H-E-super magic if $f(E(G)) = \{1, 2, \ldots, q\}$. In this paper, we study some basic properties of m-factor-E-super magic labeling and we provide a necessary and sufficient condition for an even regular graph to be 2-factor-E-super magic decomposable. For this purpose, we use Petersen's theorem and magic squares.

Keywords: H-decomposable graph; H-E-super magic labeling; 2-factor-E-super magic decomposable graph
Mathematics Subject Classification : 05C78

1. Introduction

In this paper, we consider only finite and simple undirected graphs. The vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$ respectively and we let $|V(G)| = p$ and $|E(G)| = q$. For graph theoretic notations, we follow [3, 4]. A *labeling* of a graph G is mapping that carries a set of graph elements, usually vertices and/or edges into a set of numbers, usually integers. Many kinds of labelings have been studied and an excellent survey of graph labeling can be found in [5].

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The notion of an E-super vertex magic labeling was introduced by Swaminathan and Jeyanthi [15] as in the name of super vertex magic labeilng and it was renamed as E-super vertex magic labeling by Marimuthu and Balakrishnan in [10]. A *vertex magic total labeling* is a bijection f from $V(G) \cup E(G)$ to the integers $1, 2, 3, \ldots, p+q$ with the property that for every $u \in V(G), f(u) + \sum_{v \in N(u)} f(uv) = k$ for some constant k. Such a labeling is *E-super* if $f(E(G)) = \{1, 2, 3, \ldots, q\}$. A graph G is called *E-super vertex magic* if it admits an E-super vertex magic labeling. There are many graphs that have been proved to be an E-super vertex magic graph; see for instance [10, 15, 16]. In [10], Marimuthu and Balakrishnan proved that if a graph G of odd order can be decomposed into two Hamilton cycles, then G is an E-super vertex magic graph. The results of the article [10] can be found in [11]. In [17], Tao-Ming Wang and Guang-Hui Zhang gave the generalization of some results stated in [10] using 2-factors.

A *covering* of G is a family of subgraphs $H_1, H_2, \ldots, H_h$ such that each edge of $E(G)$ belongs to at least one of the subgraphs $H_i, 1 \leq i \leq h$. Then, it is said that G admits an $(H_1, H_2, \ldots, H_h)$ *covering*. If every H_i is isomorphic to a given graph H, then G admits an *H-covering*. A family of subgraphs $H_1, H_2, \ldots, H_h$ of G is an *H-decomposition* of G if all the subgraphs are isomorphic to a graph $H, E(H_i) \cap E(H_j) = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{h} E(H_i) = E(G)$. In this case, we write $G = H_1 \oplus H_2 \oplus \cdots \oplus H_h$ and G is said to be *H-decomposable*. Suppose G is H-decomposable. A total labeling $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ is called an *H-magic labeling* of G if there exists a positive integer k (called magic constant) such that for every copy H in the decomposition, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = k$. A graph G that admits such a labeling is called an *H-magic decomposable graph*. An H-magic labeling f is called and an *H-E-super magic labeling* if $f(E(G)) = \{1, 2, \ldots, q\}$. A graph that admits an H-E-super magic labeling is called an *H-E-super magic decomposable graph*. The sum of all vertex and edge labels on H is denoted by $\sum f(H)$.

The notion of H-super magic labeling was first studied by Gutiérrez and Lladó [6] in 2005. They proved that some classes of connected graphs are H-super magic. In 2007, Lladó and Moragas [8] studied the cycle-magic and cyclic-super magic behavior of several classes of connected graphs. They gave several families of C_r-magic graphs for each $r \geq 3$. In 2010, Ngurah, Salman and Susilowati [13] studied the cycle-super magic labeling of chain graphs, fans, triangle ladders, graphs obtained by joining a star $K_{1,n}$ with one isolated vertex, grids and books. Maryati et al. [12] studied the H-super magic labeling of some graphs obtained from k isomorphic copies of a connected graph H. In 2012, Roswitha and Baskoro [9] studied the H-super magic labeling for some classes of trees such as a double star, a caterpillar, a firecracker and a banana tree. In 2013, Kojima [18] studied the C_4-super magic labeling of the Cartesian product of paths and graphs. In 2012, Inayah et al. [7] studied magic and antimagic H-decompositions and Liang [19] studied cycle-super magic decompositions of complete multipartite graphs. In these above results, they call an H-magic labeling as an H-super magic if the smallest labels are assigned to the vertices. Here, we call an H-magic labeling as an H-E-super magic if the smallest labels are assigned to the edges. In many of the results about H-magic graphs, the host graph G is required to be H-decomposable. If $H \cong K_2$, then an H-magic graph is an edge magic graph. The definition of an H-magic decomposition is suggested by this observation. Also it is notable that the notions of super edge magic and E-super edge magic are the same [11].

Any spanning subgraph of a graph G is referred to as a *factor* of G. An m-regular factor is called an *m-factor*. A graph G is said to be *factorable* into the factors $G_1, G_2, \dots, G_h$ if these factors are pairwise edge-disjoint and $\bigcup_{i=1}^{h} E(G_i) = E(G)$. If G is factored into $G_1, G_2, \dots, G_h$, then we represent this by $G = G_1 \oplus G_2 \oplus \cdots \oplus G_h$, which is called a *factorization* of G. It is nothing but the factor-decomposition. If there exists a factor-decomposition of a graph G such that each factor is a m-factor, then G is *m-factor-decomposable*. If G is a m-factor-decomposable graph, then necessarily G is r-regular for some integer r that is a multiple of m. Of course, for a graph to be 2-factor-decomposable, it is necessary that it be $2r$-regular for some integer $r \geq 1$. Petersen [14] showed that this obvious necessary condition is sufficient as well.

Theorem 1.1. [14] *Every $2r$-regular graph has a $2k$-factor for every integer $k, 0 < k < r$.*

Magic squares are among the more popular mathematical recreations. A classical reference on magic squares is [1], while one of the better recent book is [2]. A *magic square* of side n is an $n \times n$ array whose entries are an arrangement of integers $\{1, 2, \dots, n^2\}$ in which all elements in any row, any column or either main diagonal or back-diagonal, add to the same sum. Furthermore, we denote this sum as magic number (MN) and also we observe that the value of the magic number is $MN = \frac{1}{2}n(n^2 + 1)$.

In this paper, first we study the elementary properties of m-factor-E-super magic graphs and then we present a necessary and sufficient condition for an even regular graph to be 2-factor-E-super magic decomposable. To prove these results, we use Petersen's theorem and magic squares.

2. m-factor-E-Super magic graphs

This section will explore the basic properties of m-factor-E-super magic graphs.

Lemma 2.1. *If a non-trivial m-factor-decomposable graph G is m-factor-E-super magic decomposable, then the magic constant k is $\frac{q(q+1)}{2h} + pq + \frac{p(p+1)}{2}$, where h is the number of m-factors of G.*

Proof. Let f be an m-factor-E-super magic labeling of a graph G with the magic constant k. Then $f(E(G)) = \{1, 2, \dots, q\}$, $f(V(G)) = \{q+1, q+2, \dots, q+p\}$, and $k = \sum_{v \in V(G')} f(v) + \sum_{e \in E(G')} f(e)$ for every factor G' in the decomposition of G. Then,

$$\begin{aligned} hk &= \sum_{e \in E(G)} f(e) + h \sum_{v \in V(G)} f(v) \\ &= [1 + 2 + \cdots + q] + h[q + 1 + q + 2 + \cdots + q + p] \\ &= \frac{q(q+1)}{2} + h\left[pq + \frac{p(p+1)}{2}\right] \end{aligned}$$

Thus, $k = \frac{q(q+1)}{2h} + pq + \frac{p(p+1)}{2}$. □

If G is an m-factor-decomposable graph and G possesses an m-factor-E-super magic labeling, then we can easily find the sum of the vertex labels (denoted by k_v) in each factor and are the same. This gives the following result.

Lemma 2.2. *If a non-trivial m-factor-decomposable graph G is m-factor-E-super magic decomposable, then the sum of the edge labels, denoted by k_e, is a constant and it is given by $k_e = \frac{q(q+1)}{2h}$, where h is the number of m-factors of G.*

Proof. Suppose that G is m-factor-decomposable and G has an m-factor-E-super magic labeling f. Then, by Lemma 2.1, the magic constant k is given by $k = \frac{q(q+1)}{2h} + pq + \frac{p(p+1)}{2}$ for every m-factor G' in the decomposition of G. Since G is m-factor-decomposable, every m-factor G' in the decomposition of G is a spanning subgraph of G. It follows that k_v is constant for every m-factor G' of G. Since $k = k_e + k_v$, then k_e must be a constant. Also, $hk_e = \sum_{e \in E(G)} f(e) = 1 + 2 + \cdots + q = \frac{q(q+1)}{2}$ and hence $k_e = \frac{q(q+1)}{2h}$. □

In addition, the following lemma gives a necessary and sufficient condition for an m-factor-decomposable graph to be m-factor-E-super magic decomposable. This lemma is helpful in deciding whether a particular graph has an m-factor-E-super magic labeling.

Lemma 2.3. *Let G be a m-factor-decomposable graph and let g be a bijection from $E(G)$ onto $\{1, 2, \ldots, q\}$. Then g can be extended to an m-factor-E-super magic labeling of G if and only if $k_e = \sum_{e \in E(G)} g(e)$ is constant for every m-factor G' in the decomposition of G.*

Proof. Suppose that G can be decomposed into some m-factors. Assume that $k_e = \sum_{e \in E(G)} f(e)$ is constant for every m-factor G' in the decomposition of G. Define $f : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ as $f(uv) = g(uv)$ for $uv \in E(G)$ and $f(v_i) = q + i$ for all $i = 1, 2, \ldots, p$. Then $f(E(G)) = \{1, 2, \ldots, q\}$ and $f(V(G)) = \{q+1, q+2, \ldots, q+p\}$. Since every m-factor G' of G is a spanning subgraph of G, $k_v = \sum_{v \in V(G')} f(v)$ is constant for every m-factor G' in the decomposition of G. Therefore $k_v + k_e = \sum_{v \in V(G')} f(v) + \sum_{e \in E(G')} f(e)$ is a constant for every m-factor G' in the decomposition of G. Thus, we have that f is an m-factor-E-super magic labeling of G. Suppose g can be extended to a m-factor-E-super magic labeling f of G with a magic constant k. Then, $k = \sum_{v \in V(G')} f(v) + \sum_{e \in E(G')} f(e)$ for every m-factor G' in the decomposition of G. Since G is m-factor-decomposable, $k_v = \sum_{v \in V(G')} f(v)$ is constant and it follows that $k_e = \sum_{e \in E(G')} f(e)$ is also a constant for every m-factor G' in the decomposition of G. □

3. Necessary and sufficient condition

Based on the lemmas stated in the previous section, the problem of finding an m-factor-E-super magic labeling of m-factor-decomposable graphs is difficult. So, we restrict our attention to 2-factor-decomposable graphs. In this section, we discuss the 2-factor-E-super magic labeling of

2-factor-decomposable graphs. The following theorem is useful in finding classes of graphs that are not 2-factor-E-super magic.

Theorem 3.1. *An even regular graph G of odd order is not 2-factor-E-super magic decomposable, when the number of factors h is even.*

Proof. Let G be an even regular graph of odd order. Then by Petersen's theorem, G is 2-factor-decomposable. Suppose G is a 2-factor-E-super magic decomposable graph. Then G has a 2-factor-E-super magic labeling. By Lemma 2.2, we have $k_e = \frac{q(q+1)}{2h}$.

Since G is 2-factor-decomposable with h 2-factors, $q = ph$. Therefore, $k_e = \frac{ph(ph+1)}{2h} = \frac{p(ph+1)}{2}$. It is given that G is of odd order. We take $p = 2t+1$. Therefore,

$$\begin{aligned} k_e &= \frac{(2t+1)[(2t+1)h+1]}{2} \\ &= 2t^2h + 2th + t + \frac{h+1}{2}, \end{aligned}$$

which is an integer only if h is odd and hence G is not a 2-factor-E-super magic decomposable if h is even. □

The following theorem provides a necessary and sufficient condition for an even regular graph G of odd order to be 2-factor-E-super magic decomposable.

Theorem 3.2. *An even regular graph G of odd order is 2-factor-E-super magic decomposable if and only if h is odd, where h is the number of 2-factors of G.*

Proof. Let G be an even regular graph of odd order p. If h is even, by Theorem 3.1, G is not 2-factor-E-super magic. Suppose that h is odd. Then, by Petersen's theorem, G can be decomposed into 2-factors which is the sum say $G = F_1 \oplus F_2 \oplus \cdots \oplus F_h$ where F_i is a 2-factor for each $i, 1 \leq i \leq h$. Now, the edges of G can be labeled as shown in Table 1.

F_1	F_2	F_3	$\dots$	F_{h-1}	F_h
			$h \times h$ magic square		
h^2+1	h^2+2	h^2+3	$\dots$	h^2+h-1	h^2+h
h^2+2h	h^2+2h-1	h^2+2h-2	$\dots$	h^2+h+2	h^2+h+1
h^2+2h+1	h^2+2h+2	h^2+2h+3	$\dots$	h^2+3h-1	h^2+3h
h^2+4h	h^2+4h-1	h^2+4h-2	$\dots$	h^2+3h+2	h^2+3h+1
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$(p-2)h+1$	$(p-2)h+2$	$(p-2)h+3$	$\dots$	$(p-1)h-1$	$(p-1)h$
ph	$ph-1$	$ph-2$	$\dots$	$(p-1)h+2$	$(p-1)h+1$

Table 1. The edge label of an odd order 2-factor-decomposable graph G if h is odd.

From Table 1, the sum of the edge labels at the 2-factor F_1 in the decomposition is calculated as follows:

$$\begin{aligned}\sum_{e\in E(F_1)} f(e) &= MN + h^2 + 1 + h^2 + 2h + h^2 + 2h + 1 + h^2 + 4h + h^2 + 4h + 1\\ &\quad + \cdots + h^2 + (p-h-2)h + 1 + h^2 + (p-h)h,\\ &\quad \text{where } MN = \text{ magic number of } h\times h \text{ magic square}\\ &= MN + (p-h)h^2 + 2(2h + 4h + \cdots + (p-h-2)h)\\ &\quad + \underbrace{[1+1+\cdots+1]}_{\frac{p-h}{2}} + (p-h)h\\ &= MN + (p-h)h^2 + 4h\left[\frac{\left(\frac{p-h-2}{2}\right)\left(\frac{p-h-2}{2}+1\right)}{2}\right] + \frac{p-h}{2} + (p-h)h\\ &= MN + (p-h)h^2 + (ph - h^2 - 2h)\left(\frac{p-h}{2}\right) + \frac{p-h}{2} + (p-h)h\\ &= \frac{h^3}{2} + \frac{h}{2} + ph^2 - h^3 + \frac{p^2h}{2} - \frac{ph^2}{2} - \frac{ph^2}{2} + \frac{h^3}{2} - \frac{2ph}{2}\\ &\quad + \frac{2h^2}{2} + \frac{p}{2} - \frac{h}{2} + ph - h^2\\ &= \frac{p^2h + p}{2}.\end{aligned}$$

In a similar way, we can calculate that the sum of the edge labels at each 2-factor in the decomposition is the constant $k_e = \frac{p^2h+p}{2}$. Then, by Lemma 2.3, this labeling can be extended to a 2-factor-E-super magic labeling. □

Examples 1 and 2 illustrate Theorem 3.2.

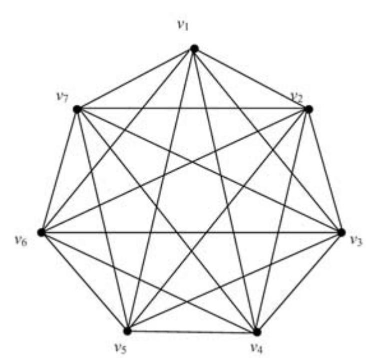

Figure 1. The complete graph K_7 is 2-factor-E-super magic.

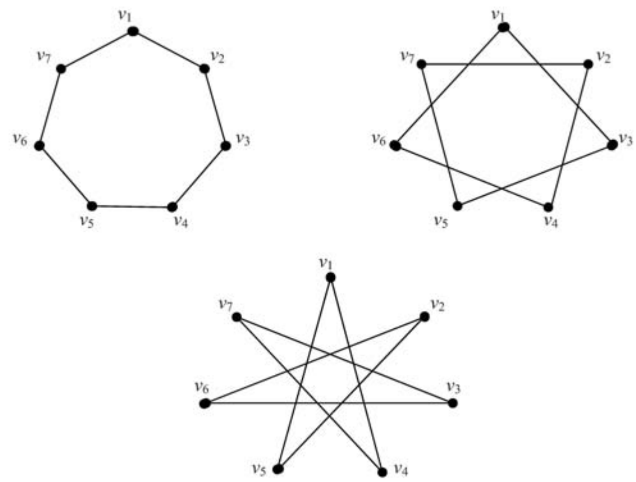

Figure 2. The 2-factor-decomposition of K_7.

F_1	F_2	F_3
4	3	8
9	5	1
2	7	6
10	11	12
15	14	13
16	17	18
21	20	19

Table 2. A 2-factor-E-super magic labeling of K_7.

Example 1. *The complete graph K_7 is decomposed into three 2-factors, namely $K_7 = F_1 \oplus F_2 \oplus F_3$. The edges of each factor-decomposition of Figure 2 are labeled as shown in the Table 2.*

In Table 2, the sum of the edge labels at each factor is $k_e = 77$. Then, by using Lemma 2.3, we extend this edge labeling to a 2-factor-E-super magic labeling.

Example 2. *The complete graph K_{11} can be decomposed into five 2-factors say $K_{11} = F_1 \oplus F_2 \oplus F_3 \oplus F_4 \oplus F_5$. The edge labels of each factor of K_{11} are shown above in Table 3. In Table 3, the sum of the edge labels at each factor is $k_e = 308$. Then, by using Lemma 2.3, we extend this edge labeling to a 2-factor-E-super magic labeling.*

Theorem 3.3. *An even regular graph G of even order is 2-factor-E-super magic decomposable.*

Proof. Let G be an even regular graph of even order p. By Petersen's theorem G can be decom-

F_1	F_2	F_3	F_4	F_5
1	7	13	19	25
18	24	5	6	12
10	11	17	23	4
22	3	9	15	16
14	20	21	2	8
26	27	28	29	30
35	34	33	32	31
36	37	38	39	40
45	44	43	42	41
46	47	48	49	50
55	54	53	52	51

Table 3. A 2-factor-E-super magic labeling of K_{11}.

F_1	F_2	F_3	$\dots$	F_{h-1}	F_h
1	2	3	$\dots$	$h-1$	h
$2h$	$2h-1$	$2h-2$	$\dots$	$h+2$	$h+1$
$2h+1$	$2h+2$	$2h+3$	$\dots$	$3h-1$	$3h$
$4h$	$4h-1$	$4h-2$	$\dots$	$3h+2$	$3h+1$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$(p-2)h+1$	$(p-2)h+2$	$(p-2)h+3$	$\dots$	$(p-1)h-1$	$(p-1)h$
ph	$ph-1$	$ph-2$	$\dots$	$(p-1)h+2$	$(p-1)h+1$

Table 4. The edge label of an even order 2-factor-decomposable graph G.

posed into 2-factors which is the sum, say $G = F_1 \oplus F_2 \oplus F_3 \oplus \cdots \oplus F_h$, where F_i is a 2-factor for each $i, 1 \leq i \leq h$. Now, the edges of G can be labeled as shown in the Table 4.

From Table 4, the sum of the edge labels at the 2-factor F_1 in the decomposition is calculated as follows:

$$\begin{aligned}
\sum_{e \in E(F_1)} f(e) &= 1 + 2h + 2h + 1 + 4h + 4h + 1 + \cdots + ph \\
&= 1 + 2[2h + 4h + \cdots + (p-2)h] + \underbrace{[1 + 1 + \cdots + 1]}_{\frac{p}{2}-1} + ph \\
&= 4h\left[1 + 2 + \cdots + \frac{p-2}{2}\right] + \frac{p}{2} + ph \\
&= \frac{p^2h}{2} - ph + \frac{p}{2} + ph = \frac{p^2h + p}{2}.
\end{aligned}$$

In similar way, we can calculate that the sum of the edge lables at each factor-decomposition is the constant $k_e = \frac{p^2h+p}{2}$. Then, by Lemma 2.3, this labeling can be extended to a 2-factor-E-

super magic labeling and thus every even regular graph of even order is 2-factor-E-super magic decomposable. □

Examples 3 and 4 illustrate Theorem 3.3.

Example 3. *The following graph G can be decomposed into three 2-factors say $G = F_1 \oplus F_2 \oplus F_3$. Note that one of the factors is disconnected.*

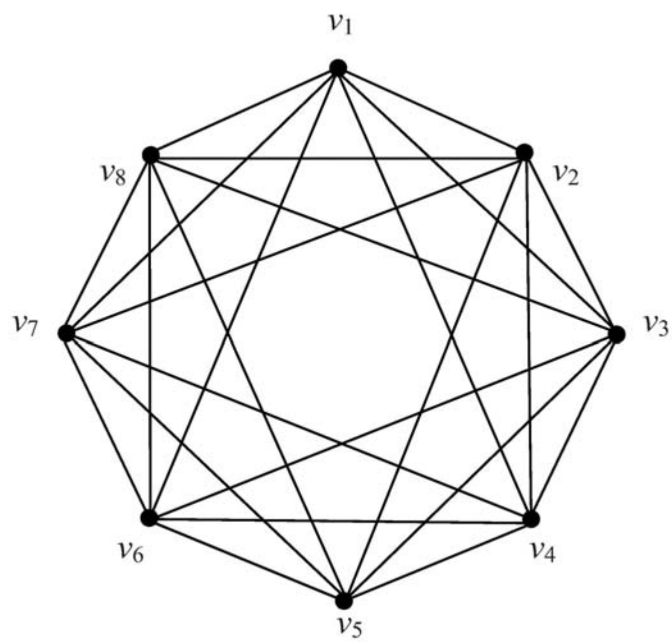

Figure 3. The graph G is 2-factor-E-super magic.

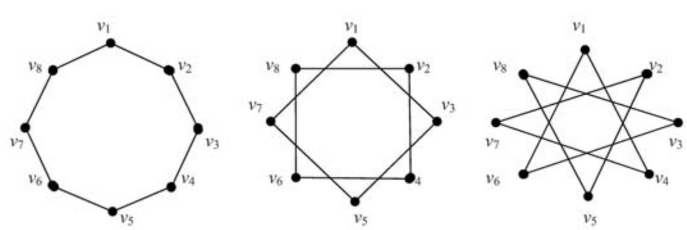

Figure 4. The 2-factor-decomposition of the graph G.

The edges of each factor-decomposition of Figure 4 are labeled as shown in Table 5.

In Table 5, the sum of the edge labels at each factor-decomposition is $k_e = 100$. Then, by Lemma 2.3, we extend this edge labeling to a 2-factor-E-super magic labeling.

F_1	F_2	F_3
1	2	3
6	5	4
7	8	9
12	11	10
13	14	15
18	17	16
19	20	21
24	23	22

Table 5. 2-factor-E-super magic labeling of G.

F_1	F_2	F_3	F_4
1	2	3	4
8	7	6	5
9	10	11	12
16	15	14	13
17	18	19	20
24	23	22	21
25	26	27	28
32	31	30	29
33	34	35	36
40	39	38	37

Table 6. 2-factor-E-super magic labeling of G.

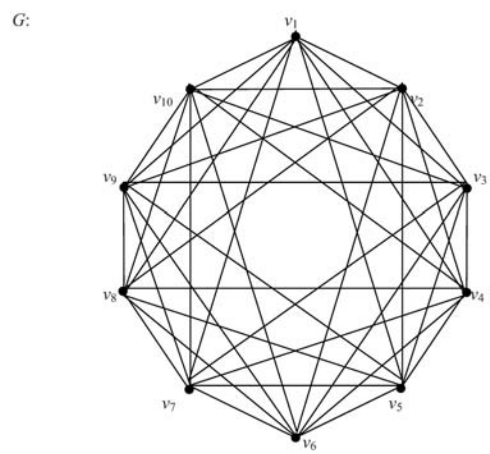

Figure 5. The graph G is 2-factor-E-super magic decomposable.

Example 4. *The graph G shown in Figure 5 can be decomposed into four 2-factors say $G = F_1 \oplus F_2 \oplus F_3 \oplus F_4$.*

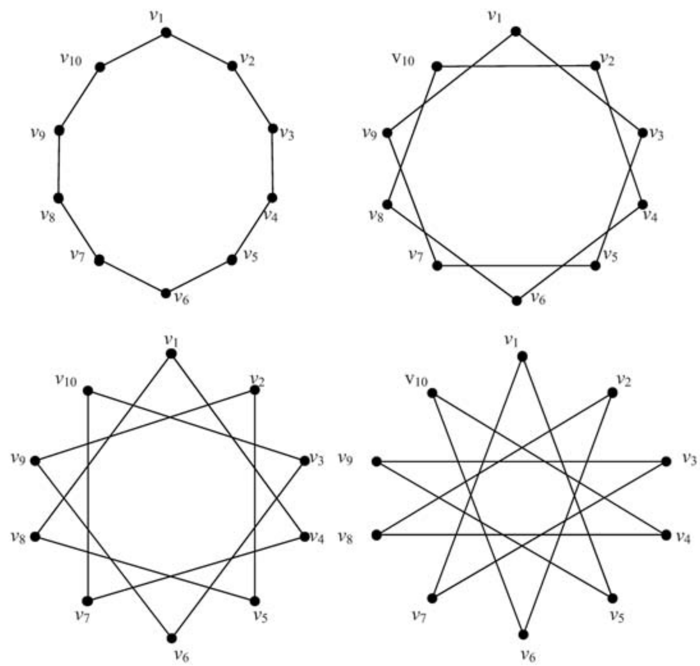

Figure 6. The 2-factor-decomposition of the graph G.

The edges of each factors of Figure 6 are labeled as shown in Table 6. In Table 6, the sum of the edge labels at each factor-decomposition is $k_e = 205$. Then, by using Lemma 2.3, we extend this edge labeling to a 2-factor-E-super magic labeling.

4. Conclusion

In this paper, we have given a complete characterization of 2-factor-E-super magic decomposable graphs. Furthermore, we can find some examples of 1-factor-E-super magic decomposable graphs (see Figures 7 and 8). The complete graph K_6 can be decomposed into five 1-factors say $K_6 = F_1 \oplus F_2 \oplus F_3 \oplus F_4 \oplus F_5$.

In Figure 8, the sum of the edge labels at each factor-decomposition is $k_e = 24$. Since, every 1-factor-decomposition is a spanning subgraph of K_6, then sum of the labels on edges and vertices of each factor is $k_v + k_e$ is constant and hence K_6 is 1-factor-E-super magic decomposable.

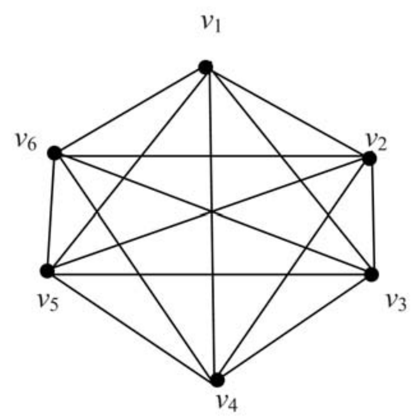

Figure 7. The graph K_6 is 1-factor-E-super magic.

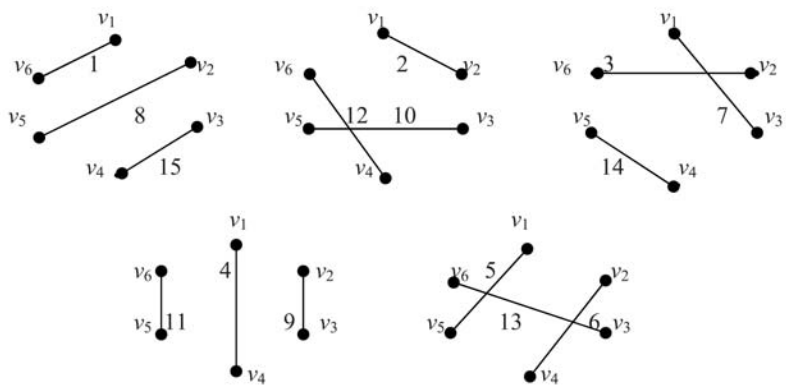

Figure 8. A 1-factor-decomposition of K_6.

Thus, we conclude this paper with the following open problem.

Open Problem 1. Characterize all m-factor-E-super magic decomposable graphs, $m \neq 2$.

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