h-gamma: An RC Delay Metric Based on a Gamma Distribution Approximation of the Homogeneous Response

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Abstract

Recently a probability interpretation of moments was proposed as a compromise between the Elmore delay and higher order moment matching for RC timing estimation[5]. By modeling RC impulses as time-shifted incomplete gamma distribution functions, the delays could be obtained via table lookup using a gamma integral table and the first three moments of the impulse response. However, while this approximation works well for many examples, it struggles with responses when the metal resistance becomes dominant, and produces results with impractical time shift values.

In this paper the probability interpretation is extended to the circuit homogeneous response, without requiring the time shift parameter. The gamma distribution is used to characterize the normalized homogeneous portion of the step response. For a generalized RC interconnect model (RC tree or mesh), the stability of the homogeneous-gamma distribution model is guaranteed. It is demonstrated that when a table model is carefully constructed, the h-gamma approximation provides for excellent improvement over the Elmore delay in terms of accuracy, with very little additional cost in terms of CPU time.

1: Introduction

With the advent of deep submicron technologies, the delay due to the RC interconnect is becoming a more dominant portion of the overall path delay for digital integrated circuits. Model order reduction methods [2][7][8][11][15] have been widely used to control the overwhelming complexity of the interconnect circuit models by representing the responses in terms of their dominant poles. However, while model order reduction methods are extremely efficient for verification purposes, the computational complexity required to solve the transcendental equations to obtain the delay points is unacceptable for use as a delay metric for the early phases of design. Therefore, due to the explicit nature of the Elmore delay approximation, it remains a popular inner-loop metric for performance driven design optimization. Unfortunately, this metric is of limited efficacy for the DSM technologies for which interconnect resistance and delays are dominant[3][10].

While the Elmore delay is provably an upperbound for the 50% delay of a large class of RC tree responses[3], the tightness of the bound varies significantly from one node to the next. For this reason, attempts have been made to create higher order (2-pole) moment matching models from which the delays can be approximated explicitly[4][16]. But these models are lacking in terms of generality and accuracy[5].

As a compromise between model order reduction via moment matching and Elmore's distribution interpretation of RC impulse responses to approximate the median (50% delay) by the mean (first moment of the impulse response), a probability interpretation of moments was used in [5] which combines the benefit of both. By modeling the RC impulse responses in terms of time-shifted incomplete gamma functions, the distribution parameters are fitted in a provably stable way (for RC trees) by matching the first three moments of the impulse response[5]. The accuracy of this approach was generally superior to an Elmore approximation, particularly for nodes at the far ends of RC trees. However, large time shifts can occur when the impulse is not accurately captured in terms of a gamma function, and delay approximation errors comparable to those for the Elmore delay can result[5].

This paper extends the probability interpretation to fit the homogeneous portion of the step response. The gamma distribution is selected to model the normalized homogeneous portion of the step response. The first two (central) moments of the homogeneous response are matched to determine the parameter values for this distribution model. The moment matching is shown to be provably stable and realizable for a general class of RC interconnect circuits; namely, RC meshes. Once the gamma distribution parameters are obtained via moment matching, the step and ramp response delays are estimated using table lookup techniques. By selecting appropriate table entries, the lookup tables can be limited to moderate size, which makes this approach efficient in both terms of runtime and memory. The result is a delay metric in terms of the first three moments of the impulse response which provides accuracy similar to two-pole models, but with computational complexity comparable to the Elmore delay approximation.

This paper is organized as follows: Section 2 presents the necessary background and review of the probability interpretation of moments and gamma distributions. Section 3 proposes the application of the gamma distribution for modeling the normalized

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homogeneous portion of the step response and the delay calculation of the step and ramp responses. Properties of the moment fitting and the lookup tables are presented in Section 4. The experimental results are shown in Section 5, followed by the conclusions in Section 6.

2: Background

2.1: Probability Interpretation of Moments

Let $h(t)$ be the time-domain impulse response for an RC circuit. The corresponding transfer function $H(s)$, can be expanded in a Taylor series about s=0:

$$H(s) = \int_0^{\infty} h(t)e^{-st}dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} s^k \int_0^{\infty} t^k h(t)dt \quad (1)$$

We define the k-th coefficient, $\tilde{m}_k$, and refer to it as the response waveform moment following the terminology in [11]:

$$\tilde{m}_k = \frac{(-1)^k}{k!}\int_0^{\infty} t^k h(t)dt \quad (2)$$

It follows that $H(s)$ can be written as

$$H(s) = \tilde{m}_0 + \tilde{m}_1 s + \tilde{m}_2 s^2 + \tilde{m}_3 s^3 + \tilde{m}_4 s^4 + \ldots \quad (3)$$

If $h(t)$ satisfies the following conditions,

$$\begin{cases} h(t) \geq 0 & \forall t \\ \int_0^{\infty} h(t)dt = 1 \end{cases} \quad (4)$$

then we can also express the transfer function in terms of moments of a probability function. For example, if the conditions in (4) are true, $h(t)$ can be treated as a p.d.f. (probability density function)[6], where the *k-th* moment m_k exists for any $k > 0$,

$$m_k = \int_0^{\infty} t^k h(t)dt \quad (5)$$

Elmore used this distribution interpretation of moments to propose the approximation of the median by the first moment, or mean[1]. Penfield and Rubenstein demonstrated that RC tree impulse responses satisfied the conditions in (4)[9], and the Elmore delay has been used for delay estimation ever since.

Higher order moments of the distribution are often translated into shape characteristics as described by *central moments*[6]. Noting the relationship between the moments of a p.d.f. and the circuit-response moments using equations (2) and (5),

$$m_k = (-1)^k k! \tilde{m}_k \quad (6)$$

we can express the *central moments* of $h(t)$ in terms of the moments of the circuit response as follows[3]:

$$\begin{aligned} \mu &= m_1 = -\tilde{m}_1 \\ \mu_2 &= m_2 - m_1^2 = 2\tilde{m}_2 - \tilde{m}_1^2 \\ \mu_3 &= m_3 - 3m_1m_2 + 2m_1^3 = -6\tilde{m}_3 + 6\tilde{m}_1\tilde{m}_2 - 2\tilde{m}_1^3 \end{aligned} \quad (7)$$

Following typical central moment definitions, μ is the mean of the distribution, μ_2 represents the variance, or spread of the distribution, and μ_3 is a measure of the skew, or asymmetry of the distribution.

In [3], the central moment properties where used to prove that the Elmore delay is an absolute upper bound on the 50% delay of RC trees. But if one can select a representative distribution family for RC tree impulse responses, the next step is to use central moments to characterize a higher order estimation of the median point of the function.

2.2: Gamma Distribution Model

One family of the distributions suggested in [5] is the gamma distribution. The probability density function $g_{\lambda,n}(t)$ has the form:

$$g_{\lambda,n}(t) = \frac{\lambda^n t^{n-1} e^{-\lambda t}}{\Gamma(n)} \quad (t{>}0),\ \Gamma(x) = \int_0^{\infty} y^{x-1}e^{-y}dy \quad (8)$$

The variable *n* is the shape parameter, while λ is the scale factor. The central moments of the gamma function can be expressed as[5]:

$$m_1 = \frac{n}{\lambda},\quad m_2 = \frac{n(n+1)}{\lambda^2},\quad m_3 = \frac{n(n+1)(n+2)}{\lambda^3},\qquad \mu_2 = \frac{n}{\lambda^2},\quad \mu_3 = \frac{2n}{\lambda^3} \quad (9)$$

If we choose $g_{\lambda,n}(t)$ to approximate the RC impulse response, then the corresponding *s* domain transfer function, $G_{\lambda,n}(s)$, would follow via the Laplace transformation of $g_{\lambda,n}(t)$:

$$\begin{aligned} G_{\lambda,n}(s) &= \int_0^{\infty} g_{\lambda,n}(t)e^{-st}dt = \int_0^{\infty} \frac{\lambda^n t^{n-1} e^{-(\lambda+s)t}}{\Gamma(n)}dt \\ &= \left(\frac{\lambda}{\lambda+s}\right)^n \end{aligned} \quad (10)$$

The frequency domain model denoted by (10) may be interpreted as a unique pole with a real number order. Notice that when $n = 1$, the gamma distribution model can naturally degrade to the dominant pole model in both frequency domain and time domain. However, the existence of the parameter *n* increases the degree of freedom of the model.

2.3: PRIMO using the Gamma Distribution Model

Since the gamma function has only two variables, it can be uniquely characterized by fitting it with two moments. However, three moments are generally required to capture essential waveform response characteristics[3]. Therefore, in [5] a third parameter, namely a time-shift value Δ was added to the gamma function so that three moments could be matched. The first three moments of the impulse response, $(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3)$, were used to uniquely specify the three time-shifted gamma parameters.

One can think of this fitting as matching the central moments (μ_2, μ_3) of the gamma distribution with the corresponding central moments of the circuit impulse response,

$$\lambda = \frac{2\mu_2}{\mu_3}, n = \frac{4(\mu_2)^3}{(\mu_3)^2} \tag{11}$$

then shifting the resulting p.d.f in time domain so that it has the same mean value μ as the impulse response

$$\Delta = m_1 - \frac{n}{\lambda} \tag{12}$$

It follows that the step response is approximated by:

$$y(t) = \int_0^t g_{\lambda, n}(\tau - \Delta) d\tau = \int_0^{\lambda(t-\Delta)} g_{1, n}(\tau) d\tau \tag{13}$$

Denoting the cumulative distribution function (c.d.f) of the unscaled gamma distribution by *P(n, t):*

$$P(n, t) = \int_0^t g_{1, n}(\tau) d\tau \tag{14}$$

Then we can also express the step response as

$$y(t) = P(n, \lambda(t - \Delta)). \tag{15}$$

where $P(n, t)$ is also an incomplete gamma function.

To calculate the delay at a particular percentage point α, one needs only to use a one dimensional lookup table for the unscaled gamma distribution (with p.d.f $g_{1, n}(t)$) percentiles to get the value $\lambda(t - \Delta)$. Then with simple shifting and scaling the delay approximation, *t* is obtained.

For saturated ramp responses similar table lookup techniques can also be adopted. Assuming the input ramp has a maximum voltage of 1, and rise time of t_r, the ramp response expressed in terms of integration of the step response is

$$r(t) = \begin{cases} \frac{1}{t_r}\int_0^t y(\tau) d\tau & t \le t_r \\ \frac{1}{t_r}\left(\int_0^t y(\tau) d\tau - \int_0^{t - t_r} y(\tau) d\tau\right) & t > t_r \end{cases} \tag{16}$$

Substituting $y(\tau)$ by (15), and letting $x = \lambda(t - \Delta)$

$$r(t) = \begin{cases} \frac{1}{\lambda t_r}\int_0^x P(n, \tau) d\tau & x \le \lambda t_r \\ \frac{1}{\lambda t_r}\int_{x - \lambda t_r}^x P(n, \tau) d\tau & x > \lambda t_r \end{cases} \tag{17}$$

Notice that the right hand side of (17) is a function of variable *x* with two parameters *n,* and λt_r. Thus, for any percentage point α, we only need a precompiled two dimensional table with *n* and λt_r as the entries, for the *x* values satisfying

$$\alpha = \begin{cases} \frac{1}{\lambda t_r}\int_0^x P(n, \tau) d\tau & x \le \lambda t_r \\ \frac{1}{\lambda t_r}\int_{x - \lambda t_r}^x P(n, \tau) d\tau & x > \lambda t_r \end{cases} \tag{18}$$

After looking up the 2D table for $x = \lambda(t - \Delta)$, the delay value $t - \alpha t_r$ is computed by simple transformations.

3: Gamma Fitting of Homogeneous Response

3.1: Homogeneous Response for RC Meshes

Since the Laplace transform of the circuit impulse response can be expressed in the form shown in (3), the Laplace transform of the unit step response *Y*(s) can be expressed as

$$Y(s) = \frac{1}{s}H(s) = \frac{\tilde{m}_0}{s} + \tilde{m}_1 + \tilde{m}_2 s^1 + \tilde{m}_3 s^2 + \tilde{m}_4 s^3 + \dots \tag{19}$$

Referring to (19), we can decompose the step response into the forced response, $\frac{\tilde{m}_0}{s}$, and the homogeneous response $Y_h(s) = \tilde{m}_1 + \tilde{m}_2 s^1 + \tilde{m}_3 s^2 + \tilde{m}_4 s^3 + \dots$. Therefore, we can write the time-domain unit step response as:

$$y(t) = \tilde{m}_0 + \tilde{m}_1 y_h(t) \tag{20}$$

The components of *y(t)* are displayed in Fig.1.

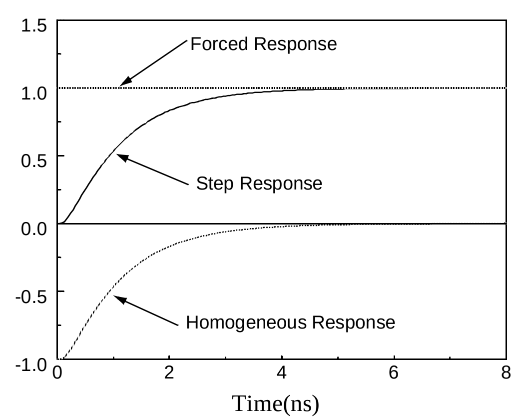

FIGURE 1. Decomposition of a unit step response.

Since the unit step response, *y(t)*, of an RC mesh [14] monotonically increases from 0 to 1, and $\tilde{m}_0 = 1, \tilde{m}_1 < 0$, the normalized homogeneous response $y_h(t)$ satisfies the following:

$$\begin{cases} y_h(t) = \dfrac{1 - y(t)}{-\tilde{m}_1} \geq 0 & \forall t \\ \int_0^{\infty} y_h(t)dt = \dfrac{1}{-\tilde{m}_1}\int_0^{\infty} 1 - y(t)dt = 1 \end{cases} \tag{21}$$

Therefore, in a manner similar to what was done for the impulse response, we can treat $y_h(t)$ as a probability density function and use a gamma distribution to fit the homogeneous response.

3.2: h-gamma Approximations

Unlike PRIMO in [5] which fits the impulse response with the shifted gamma distribution, we propose here to fit the gamma distribution to the normalized homogeneous response without time-shifting. Without the extra parameter, Δ, can completely characterize the gamma distribution representation of the homogeneous response by matching two central moments.

We begin by writing the Laplace transform of the normalized homogeneous function as:

$$Y_h(s) = 1 + \frac{\tilde{m}_2}{\tilde{m}_1}s + \frac{\tilde{m}_3}{\tilde{m}_1}s^2 + \frac{\tilde{m}_4}{\tilde{m}_1}s^3 + \ldots \tag{22}$$

We can express the mean μ and the variance μ_2 in terms of the circuit response moments using (22) and the expressions in [3]:

$$\begin{aligned} \mu &= -\frac{\tilde{m}_2}{\tilde{m}_1} \\ \mu_2 &= 2\left(\frac{\tilde{m}_3}{\tilde{m}_1}\right) - \left(\frac{\tilde{m}_2}{\tilde{m}_1}\right)^2 \end{aligned} \tag{23}$$

To fit the gamma distribution parameters n and λ, we force the model to match the μ and μ_2 in (23) via moment matching:

$$\begin{aligned} \mu &= \frac{n}{\lambda} \\ \mu_2 &= \frac{n}{\lambda^2} \end{aligned} \tag{24}$$

Rearranging (24),

$$n = \frac{\mu^2}{\mu_2}; \ \lambda = \frac{\mu}{\mu_2} \tag{25}$$

Once we solve for the *n* and λ, the approximate step response is

$$y(t) = 1 + \tilde{m}_1 g_{\lambda, n}(t) \tag{26}$$

It is important to note that while it appears that we are matching only the first two central moments, it is evident from (23) that we are using the first three moments of the impulse response. Moreover, by fitting the complete response with the summation of the forced response and the gamma distribution approximation, it is apparent from (3) and (19) that the first three moments of the impulse response are implicitly matched.

3.3: Delay Calculation

To find the step delay for any threshold percentage point α using (26) we must evaluate

$$\alpha = 1 - \frac{(-\tilde{m}_1\lambda)(\lambda t)^{n-1}e^{-\lambda t}}{\Gamma(n)} \tag{27}$$

By defining the following two parameters

$$k = -\tilde{m}_1\lambda, \ x = \lambda t \tag{28}$$

and using the expression,

$$y_{k,n}(x) = 1 - \frac{kx^{n-1}e^{-x}}{\Gamma(n)} \tag{29}$$

we can rewrite (27) as

$$\alpha = y_{k,n}(x) \tag{30}$$

Instead of solving for *t* using the nonlinear expression in (27), we follow the p.d.f. approach and construct a table lookup model to evaluate (30). For each predetermined percentage point α, we precompile a 2 dimensional table with *k* and *n* as its entries, and *x* as its outcome. Thus when *n* and λ are obtained by matching the first three moments, we compute *k* by (28), and use the *n* and *k* to get *x* via the table lookup. The delay value, *t* is obtained by scaling *x* back by λ, i.e., $t = \frac{x}{\lambda}$.

3.4: Ramp Response Delay

The response of a saturated ramp with maximum voltage *1*, and rise time t_r can be expressed in terms of integration of the unit step response model:

$$r(t) = \begin{cases} \dfrac{1}{t_r}\int_0^t y(\tau)d\tau & t \leq t_r \\ \dfrac{1}{t_r}\left(\int_0^t y(\tau)d\tau - \int_0^{t-t_r} y(\tau)d\tau\right) & t > t_r \end{cases} \tag{31}$$

Defining

$$l = \frac{t_r}{-\tilde{m}_1} \tag{32}$$

we can express the ramp response as:

$$r(t) = \begin{cases} \dfrac{1}{kl}\int_0^x y_{k,n}(\tau)d\tau & x \leq kl \\ \dfrac{1}{kl}\left(\int_0^x y_{k,n}(\tau)d\tau - \int_0^{x-kl} y_{k,n}(\tau)d\tau\right) & x > kl \end{cases} \tag{33}$$

Following the approach proposed for the step delay evaluation, we can write $r(t)$ in (33) as $r_{k,l,n}(x)$. Then, for a given threshold percentage point α, the nonlinear equation of the form $a = r(t)$ can be transformed to

$$a = r_{k,l,n}(x) \tag{34}$$

A 3-dimensional table with entries *k*, *l*, and *n*, and outcome *x*, can be pre-compiled for the solution of (34). Then, for delay computation purposes we need only to compute *k*, *l*, and *n*, use the 3D table to get *x*, then scale it by λ to get *t*. The delay relative to the input ramp is then calculated as $t - \alpha T_r$.

4: Properties and Implementation Issues

4.1: Stability

The stability of the gamma distribution model for the homogeneous response refers to the realizability of the model; namely, that the parameters *n* and λ should satisfy $n > 0, \lambda > 0$.

Theorem: *The gamma distribution model for the homogeneous portion of an RC mesh step response is stable for any RC mesh.*

Proof:
The circuit response starts at a zero state and ends with the dc value of the step input for all nodes of the RC mesh; namely, $y(0) = 0, y(\infty) = 1$ for a unit step response. It was stated in [14] that the step response of an RC mesh is monotonically increasing as an obvious extension of the Rubenstein, Penfield and Horowitz proof in [13]. It follows that the step response for any node in an RC mesh is bounded between 0 and 1.

Therefore, given that

$$0 \le y(t) \le 1 \qquad \forall t \ge 0 \tag{35}$$

The normalized homogeneous response is

$$y_h(t) = \frac{1 - y(t)}{-\tilde{m}_1} \ge 0 \qquad \forall t \ge 0 \tag{36}$$

And since μ and μ_2 are defined as

$$\begin{aligned} \mu &= \int_0^\infty t y_h(t) dt \\ \mu_2 &= \int_0^\infty (t-\mu)^2 y_h(t) dt \end{aligned} \tag{37}$$

Obviously μ and μ_2 are always positive for any nonnegative normalized homogeneous response $y_h(t)$. From the equations for the gamma parameters in (25), it follows that the model must always satisfy $n > 0$, $\lambda > 0$. Q.E.D

4.2: Properties of the Tables

One important issue with regard to the utility of this method is the runtime and storage efficiency of the table model. Note that among the 3 parameters, *k* and *n* are not associated with the input ramp rise time, and can be expressed with the circuit-response moments exclusively. From equations (23), (25) and (28), we have

$$\begin{aligned} k &= \frac{\tilde{m}_1^2 \tilde{m}_2}{2\tilde{m}_3\tilde{m}_1 - \tilde{m}_2^2} \\ n &= \frac{\tilde{m}_2^2}{2\tilde{m}_3\tilde{m}_1 - \tilde{m}_2^2} \end{aligned} \tag{38}$$

To determine the expected range for these values, first consider the case when the response is dominated by a single pole *p*, such that $\tilde{m}_1 \approx \frac{\tilde{m}_2}{\tilde{m}_1} \approx \frac{\tilde{m}_3}{\tilde{m}_2} \approx \frac{1}{p}$. Using this relation it is apparent from (38) that *k* and *n* should be close to 1. Our empirical results validate this, where we find that *k* is between 0.3~1.8, and *n* is between 0.5~1.5.

To determine the range of *l* it should be noted that when $l > 10$, i.e, $t_r > 10(-\tilde{m}_1)$, the response will be ramp follower and the 50% percentage delay point will approach the Elmore delay upperbound[3]. When $l < 0.1$, i.e., $t_r < 0.1(-\tilde{m}_1)$, the ramp is steep enough to be considered as a step input, which means the delay can be approximated by the outcome of the lookup table for step response delay with entries *k* and *n*. Thus we only need to make the table with the value of *l* in the range of 0.1~10.

5: Experimental Results

5.1: signal bus example

The first example is a 0.25 micron example from a commercial microprocessor. The RC tree has 50 fanouts, including nodes close to the driver as well as nodes far from the driving point. The h-gamma model is applied to estimate the 50% delay of the ramp response. We compared the results with the extension of PRIMO[5] described in Section 2.3, and the Elmore approximation when used as a dominant time constant. The exact response was measured in terms of a 5th order RICE approximation which showed no difference from SPICE when a sufficiently small time step was chosen in SPICE.

To provide for a fair comparison of all three methods over representative best and worst case conditions for all 50 responses, we plotted histograms of the errors when the rise times were 0.2, 1.0 and 5 times that of the Elmore delay, *for each node*. That is, each histogram in Figures 2, 3 and 4 represents the delays for each of the 50 nodes using 50 different input signal rise times. From the histograms we observe that large errors sometimes result for both the Elmore approximation and the time-shifted gamma, while the h-gamma results are consistently more accurate, with smaller average and maximum errors.

5.2: Clock Tree Example

The second example is a balanced clock tree with 188 leaf nodes. The relative errors from the three approximation methods are summarized in Table 1, once again measured relative to a 5-th order RICE approximation. Since all the output nodes are far-end leaf nodes, all of the metrics provided reasonable accuracy, however, the h-gamma results are clearly superior.

TABLE 1. Comparison of the 50% step delay errors for a clock tree example.

Statistics	Dominant Pole	PRIMO	h-gamma
Mean error	2.06%	0.22%	0.17%
Standard deviation	1.04%	0.25%	0.11%
Maximal error	4.78%	0.73%	0.37%

5.3: Statistics on Large Industry RC Mesh

As a final example we analyzed a large industry example containing 753 RC nets (actually RC meshes) with a combined total of 1221 fanout nodes. It is demonstrated in Table 2 that the h-gamma results are clearly superior in all cases once again in terms of the mean, maximum and standard deviation of the error distribution.

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TABLE 2. Comparison of the 50% delay errors for step and ramp responses. To consider ramp inputs that were comparable to delays, all of the ramps were set to be equal to the Elmore delay of the node under test.

Step Input	Dominant Pole	PRIMO	h-gamma
Mean error	4.87%	1.06%	0.62%
Standard deviation	10.55%	3.99%	1.96%
Maximal error	117.24%	49.14%	24.02%
Tr = Elmore delay*	Dominant Pole	PRIMO	h-gamma
Mean error	4.15%	0.59%	0.36%
Standard deviation	6.03%	1.17%	0.41%
Maximal error	54.4%	10.24%	3.07%

Of further interest, from an implementation perspective, are the statistics associated with the parameter values k and n. For this example, the results are summarized in Table 3. As expected, the values do not change significantly in a range that is very close to 1.0. Because of the limited range of these values, we were able to construct a sufficiently accurate table with moderate storage space.

6: Conclusion

By modeling the normalized homogeneous portion of the step response as a probability density function in terms of the gamma distribution, a new delay metric using the first three moments of the impulse response is proposed. This model is shown to be provably stable for any RC mesh response, and the accuracy of this metric was demonstrated on a large number of industrial examples. Importantly, the table model which is required for the distribution function is shown to span a very small range of element values such that high accuracy can be achieved with moderate-sized tables. This results in a metric that provides accuracy close to that from model order reduction, but with runtime complexity comparable to the Elmore delay approximation.

TABLE 3. The distribution of k and n.

Statistics	k	n
Maximum	1.3592	1.1046
Minimum	0.5841	0.8537
Mean	1.0375	0.9913
Median	1.0321	1.0015
Standard Deviation	0.1046	0.0708

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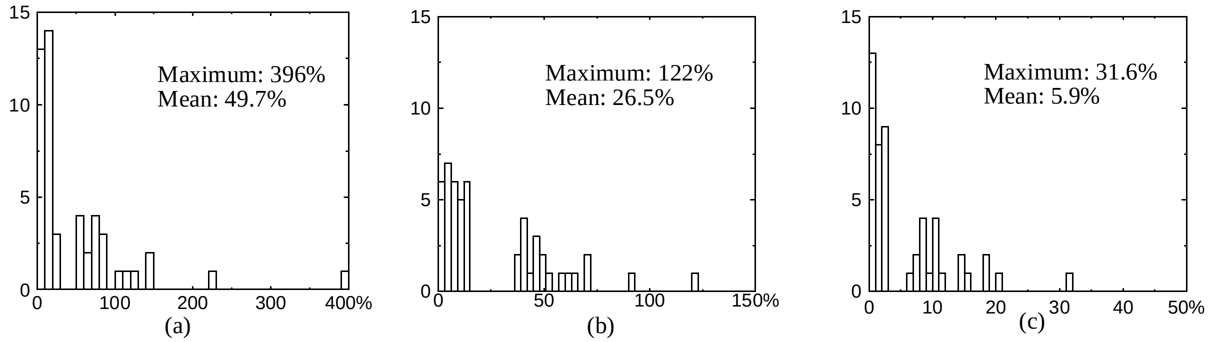

Figure 2: The relative errors of the 50% delay via a single dominant time constant approximation for an (a) input rise time of 0.2 times the Elmore delay for each node; (b) An input rise time of 1.0 times the Elmore delay for each node; (c) And an input rise time of 5 times the Elmore delay for each node.

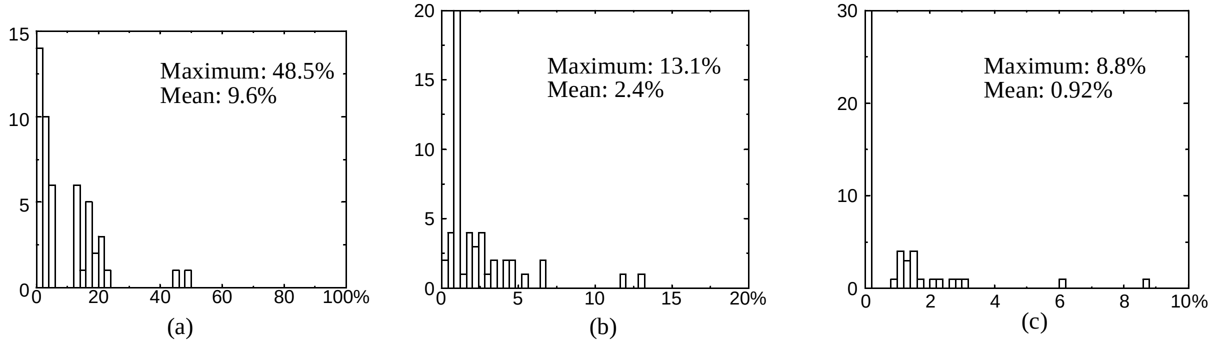

Figure 3: The relative errors of the 50% delay via a time-shifted gamma (PRIMO) approximation for an (a) input rise time of 0.2 times the Elmore delay for each node; (b) An input rise time of 1.0 times the Elmore delay for each node; (c) And an input rise time of 5 times the Elmore delay for each node.

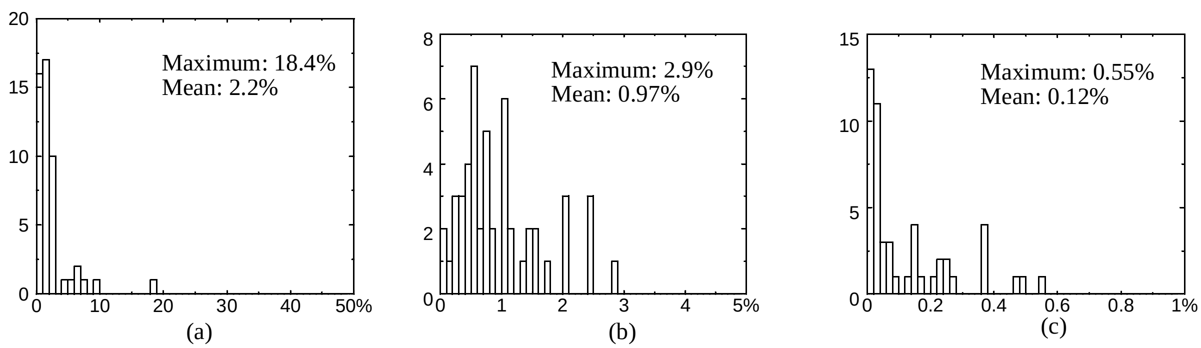

Figure 4: The relative errors of the 50% delay via an h-gamma approximation for an (a) input rise time of 0.2 times the Elmore delay for each node; (b) An input rise time of 1.0 times the Elmore delay for each node; (c) And an input rise time of 5 times the Elmore delay for each node.