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h-Prime and h-Semiprime Ideals in Semirings and Γ-Semirings

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Abstract

The concepts of h-prime ideals and h-semiprime ideals in semirings and Γ-semirings are introduced so that their properties are studied. In particular, the relationships between Γ-semirings and its related operator semirings are described in terms of the h-closure; the h-prime and h-semiprime ideals. These results will be used to obtain some other new results such as the inclusion preserving bijections between the h-prime (or h-semiprime) ideals of a Γ-semiring and its related operator semirings. Moreover, the h-regularity and the H-Noetherian Γ-semirings will be characterized. Some recent results given by T. K. Dutta and S. K. Sardar in semiprime ideals and irreducible ideals of a Γ- Semirings and extended and generlized.

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1 Introduction

It is well known that in the theory of Γ-semirings , the properties of their ideals, prime ideals and semiprime ideals play an important role, however the properties of an ideal in a semiring are sometimes differ from the properties of a ring ideals . In order to amend this gap, the concepts of k-ideals [5] and h-ideals in a semiring were first considered by D. R. LaTorre in 1965 [6]. For the definition of h-ideals in Γ-semirings, the reader is referred to the recent paper of T. K. Dutta and S. K. Sardar in [2]. The notions of operator semirings and Γ-semirings were also introduced by them. Moreover, the properties of prime ideals [1] and semiprime ideals in Γ-semirings were studied and discussed by them in [3].

By using the concept of k-ideal [5], D. M. Olson et al have considered the pre-prime and pre-semiprime ideal of a semiring in [9, 10]. In this aspect, S. K.Sardar and B. C. Saha developed the same notion in Γ-semirings in [11] . In this paper, by using the concept of h-ideals, we also introduce the notions of prime h-ideals and investigate the properties of h-prime ideals and h-semiprime ideals in semirings and Γ-semirings. We therefore derive a new operation by taking the h-closure of a given operation which is commutating with each of the following operations "$*'$, $*$, $+'$, $+$ " given in [2]. Finally, the so called H-Noetherian Γ-semirings will be characterized. Some related results obtained by T. K. Dutta and S. K. Sardar in operator semirings of a Γ-semiring [2,4] will be hence enriched and amplified.

For some basic results and definitions which were not given in this paper, the reader is referred to the monograph of J. S. Golan in semirings, see [5].

2 h-prime and h-semiprime ideals in semirings

Definition 2.1 *[2, 6] An ideal I of a semiring S (respectively Γ-semring) is called an h-ideal of S if $x + y_1 + z = y_2 + z$; $y_1, y_2 \in I$; $x, z \in S$ implies $x \in I$.*

Definition 2.2 *[12] The h-closure $\overline{A}$ of A is defined by $\overline{A} = \{x \in S : x + a_1 + z = a_2 + z \text{ for some } a_1, a_2 \in A, z \in S\}$*

For the sake of convenience, we denote $\overline{A}$ by $H(A)$.

Definition 2.3 *An ideal P of a semiring S is said to be an h-prime ideal of S if $IJ \subseteq H(P)$ implies $I \subseteq H(P)$ or $J \subseteq H(P)$, where I, J are ideals of S.*

Similarly, we have the following definition.

Definition 2.4 *An ideal P of a semiring S is said to be an h-semiprime ideal of S if $I^2 \subseteq H(P)$ implies $I \subseteq H(P)$, where I is an ideal of S.*

The following results follow immediately from the above definitions .

Proposition 2.5 *.The following statements hold in a semiring S.*

- (i) An ideal P of S is h-prime if and only if $H(P)$ is prime in S.
- (ii) An ideal P of S is h-semiprime if and only if H(P) is semiprime in S.

Note. It can be easily seen that an h-prime ideal P of a semiring S is h-semiprime but this statement does not hold conversely.

By Proposition 7.4 in [5] and by Proposition 2.5, we can easily deduce the following characterization theorem for an h-prime ideal in a semiring.

Theorem 2.6 *Let S be a semiring with identity and I an ideal of S. Then the following statements are equivalent:*
i) I is h-prime,
ii) for any $a, b \in S$, $aSb \subseteq H(I)$ implies that $a \in H(I)$ or $b \in H(I)$,
iii) for any $a, b \in S$, $<a>.<b> \subseteq H(I)$ implies that $a \in H(I)$ or $b \in H(I)$.

Analogously, one can prove the following characterization theorem for h-semiprime ideals in semirings.

Theorem 2.7 *Let S be a semiring with unity. Then the following statements are equivalent for any ideal I of S,*
i) I is h-semiprime.
ii) For $a \in S$ such that $aSa \subseteq H(I)$ implies $a \in H(I)$,
iii) $< a >^2 \subseteq H(I)$ implies that $a \in H(I)$.

3 h-prime and h-semiprime ideals in Γ-semirings

Throughout this section, unless otherwise stated, S is a Γ-semiring . We also use $L\, and\, R$ to denote the left and right operator semirings, respectively. Now, we define $H(I) := \{x \in S: x + y_1 + z = y_2 + z$, for some $y_1, y_2 \in I$, $z \in S\}$ and we want to show that $H(I)$ is an h-ideal of S with $I \subseteq H(I)$.

For the sake of brevity, we just denote the h-closure of I in a Γ-semiring S by $H(I)$. Thus if $A\, and\, B$ are ideals of the Γ-semiring S with A⊆B, then

H(A)⊆H(B).

We now give here the definitions of h-prime ideals and h-semiprime ideals in a Γ-semiring s. it is clear that these concepts are more general than the usual concepts of prime and semiprime ideals.

Definition 3.1 *An ideal P of a Γ-semiring S is called an h-prime ideal of S if $I\Gamma J \subseteq H(P)$ implies $I \subseteq H(P)$ or $J \subseteq H(P)$, where I and J are ideals of S.*

Definition 3.2 *An ideal P of a Γ-semiring S is said to be an h-semiprime ideal of S if for any ideal I of S, $I\Gamma I \subseteq H(P)$ implies $I \subseteq H(P)$.*

The concepts of h-prime ideal and h-semiprime ideals of a semiring or a Γ-semiring S are proper generalizations of the concepts of prime ideals and semiprime ideals of S. Thus, an h-prime (h-semiprime) ideal of a semiring or a Γ-semiring$S$ is not necessarily a prime(semiprime) ideal of S and vice-versa. This observation can be easily seen in the following examples:

Example 3.3 *Let $S=Z_0^+$ be the set of all nonnegative integers. Then S is a semiring with respect to the usual addition and multiplication. Let $P=2Z_0^+\setminus \{2, 4, 6\}$. Then P is an ideal of S but not a prime ideal of the semiring S. Now $H(P)=2Z_0^+$ is clearly a prime ideal of S. This example illustrates that there exists an h-prime ideal of S which is not a prime ideal of S.*

Example 3.4 *In the above example, If we let $P=Z_0^+\setminus \{1\}$, then P is a prime ideal of the semiring S. Now it is clear that $H(P)=Z_0^+$ is a not a prime ideal of S because according to the definition a prime ideal of a semiring s, $H(P)$ is a proper ideal [5]) of S, consequently, P is not an h-prime ideal of S.*

Example 3.5 *Let $S=Z_0^-$ be the set of all non-positive integers and $\Gamma = \{a, b\}$. Then with the addition defined as follows:*

$+$	a	b
a	a	b
b	b	a

Now, one can easily see that Γ is a commutative semigroup. If we define the ternary composition
$S \times \Gamma \times S \to S$ as follows:

$$(x, a, y) \mapsto 0 \text{ and}$$
$$(x, b, y) \mapsto -(xy)$$

then S forms a Γ-semiring. Now let $P=2Z_0^- \setminus \{-2\}$. Then P is an ideal of S but not a prime ideal of the Γ-semiring S. But $H(P)=2Z_0^+$ is a prime ideal of S. Hence P is an h-prime ideal of S.

We now state below some known properties of h-prime ideals and h-semiprime ideals of a Γ-semiring.

Proposition 3.6 *An ideal P of a Γ-semiring S is h-prime if and only if $H(P)$ is prime in S.*

Proposition 3.7 *Any ideal P of Γ-semiring S is h-semiprime if and only if $H(P)$ is semiprime in S.*

Note. It is well known that an h-prime ideal I of a Γ-semiring S is h-semiprime but the converse does not hold.

Before we proceed to establish the main results of this paper, we first introduce the concept of the left operator semirings (respectively right operator semiring) of a Γ-semiring which have been described by Dutta and Sardar in [4].

Definition 3.8 *Let S be a Γ-semiring and F a free additive commutative semigroup generated by $S\times\Gamma$. Define the following relation on F :*

$$(\sum_{j=1}^{m}(x_i;\alpha_i),\sum_{j=1}^{n}(y_j;\beta_j))\in\rho$$

if and only if

$$\sum_{j=1}^{m}x_i\alpha_i a=\sum_{j=1}^{n}y_j\beta_j a$$

$\forall a\in S$.

Then the relation ρ can be easily verified to be a congruence on F and so F/ρ forms a semiring under the following multiplication

$$(\sum_{j=1}^{m}[x_i;\alpha_i])\bullet(\sum_{j=1}^{n}([y_j;\beta_j])=\sum_{i,j}[x_i\alpha_i y_j;\beta_j],$$

where $\sum_{j=1}^{m}[x_i;\alpha_i]$, $\sum_{j=1}^{n}[y_j;\beta_j]$ are the classes of ρ which contains $\sum_{j=1}^{m}(x_i;\alpha_i)$ and $\sum_{j=1}^{n}(y_j;\beta_j)$, respectively. We denote this semiring by L which is called the *left operator semiring of the* Γ*-semiring* S.

The right operator semiring R *of the* Γ*-semiring* S *can be dually defined.*

Note. The following Theorems of a Γ-semiring was first stated and proved by Dutta and Sardar in [4].

For the sake of convenience, we cite some of the known THeorems below:

- (A) The lattices of all right (two-sided) ideals of a Γ-semiring S and its left operator semiring L are isomorphic (cf Theorem 6.3 in [4]);

- (B) The lattices of all left (two-sided) ideals of a Γ-semiring S and its right operator semiring R are isomorphic (cf Theorem 6.6 in [4]);

- (C) A Γ-semiring S is Noetherian if and only if its right operator semiring R or left operator semiring L is Noetherian (cf Theorem 7.4 in [4]);

- (D) A Γ-semiring S is Artinian if and only if its right operator semiring R or left operator semiring L is Artinian (cf Theorem 7.5 in [4]).

We now proceed to extend the above results in [4] to a more general situation.

We first prove the following crucial lemmas because it have already been shown in [4] that each of the functions "$*^{'}$, $*$, $+^{'}$,$+$ " [2] is commutative with the operation H. Consequently, we derive the following lemma.

Lemma 3.9 *Let* I *be an ideal of the* Γ*-semiring* S *with unity . Then* $\mathrm{H}(\mathrm{I}^{*^{'}})$ $=\mathrm{H}(\mathrm{I})^{*^{'}}$*, where* $I^{*^{'}}$ *is the corresponding ideal in the right operator semiring* R *(see [2]).*

Proof. Let $\sum_{j=1}^{n}[\gamma_j, f_j]$ be the right unity of the Γ-semiring S. Consider $x \in \mathrm{H}(\mathrm{I}^{*^{'}})$. Then there exist $r_1, r_2 \in \mathrm{I}^{*^{'}}$ and $r \in \mathrm{R}$ such that $x + r_1 + r = r_2 + r$. This implies that $sr_1, sr_2 \in \mathrm{I}$ and $sr \in \mathrm{S}$, for all $s \in \mathrm{S}$.

Now for all $s \in S$, we have $sx + sr_1 + sr = sr_2 + sr$ which gives $sx \in$H(I), for all $s \in$S and whence, $x \in$H(I)$^{*'}$. This shows that H(I$^{*'}$) $\subseteq$ H(I)$^{*'}$. To prove the converse containment, we first let $\sum_k [\alpha_k, y_k] \in$H(I)$^{*'}$. Then, we can see immediately that $\sum_k s\alpha_k y_k \in$H(I), for all s$\in$S , and thereby there exist $t_1, t_2 \in$I and $z \in$S such that $\sum_k s\alpha_k y_k + t_1 + z = t_2 + z$. In particular, we have

$\sum_k f_j \alpha_k y_k + t_1 + z = t_2 + z$, for all $j = 1, 2, ..., n$.

Consequently, we have $[\alpha, \sum_k f_j\alpha_k y_k] + [\alpha, t_1] + [\alpha, z] = [\alpha, t_2] + [\alpha, z]$, for all $\alpha \in \Gamma$ and all j. Hence , we deduce that $[\gamma_j, \sum_k f_j\alpha_k y_k] + [\gamma_j, t_1] + [\gamma_j, z] = [\gamma_j, t_2] + [\gamma_j, z]$, for all j. Now, by summing over j, we conclude that

$\sum_j [\gamma_j, \sum_k f_j\alpha_k y_k] + \sum_j [\gamma_j, t_1] + \sum_j [\gamma_j, z] = \sum_j [\gamma_j, t_2] + \sum_j [\gamma_j, z]$.

In other words, we have $\sum_j [\gamma_j, f_j] \sum_k [\alpha_k, y_k] + \sum_j [\gamma_j, t_1] + \sum_j [\gamma_j, z] = \sum_j [\gamma_j, t_2] + \sum_j [\gamma_j, z]$,

that is, $\sum_k [\alpha_k, y_k] + \sum_j [\gamma_j, t_1] + \sum_j [\gamma_j, z] = \sum_j [\gamma_j, t_2] + \sum_j [\gamma_j, z]$...(A)

Since I is an ideal and $t_1, t_2 \in$I, we have $\sum_j s\gamma_j t_1 and \sum_j s\gamma_j t_2 \in$I, for all $s \in$S.

Hence $\sum_j [\gamma_j, t_1] and \sum_j [\gamma_j, t_2] \in$I$^{*'}$ and thereby, $\sum_j [\gamma_j, z] \in$R. Thus, from (A), we have $\sum_k [\alpha_k, y_k] \in$ H(I$^{*'}$). This shows that H(I)$^{*'}$ $\subseteq$H(I$^{*'}$) and consequently, H(I$^{*'}$)= H(I)$^{*'}$.The proof is completed.

Lemma 3.10 *Let S be a Γ-semiring with unity and Q an ideal of the right operator semiring R of S . Then H(Q^*) =H(Q)*, where Q^* is the corresponding ideal in the Γ-semiring S [2].*

Proof. Let $\sum_{i=1}^{m} [e_i, \delta_i]$ be the left unity of the Γ-semiring S and x$\in$H(Q^*). Then there exist $y_1, y_2 \in Q^*$ and $z \in$S such that $x + y_1 + z = y_2 + z$. Hence $[\alpha, x + y_1 + z] = [\alpha, y_2 + z]$, for all $\alpha \in \Gamma$,

that is, $[\alpha, x] + [\alpha, y_1] + [\alpha, z] = [\alpha, y_2] + [\alpha, z]$, for all $\alpha \in \Gamma$.

Now $[\alpha, y_1], [\alpha, y_2] \in$Q and $[\alpha, x], [\alpha, z] \in$R, for all $\alpha \in \Gamma$. Hence we have $[\alpha, x] \in$H(Q), for all $\alpha \in \Gamma$. This leads to $x \in$H(Q)*. Consequently, H(Q^*) $\subseteq$H(Q)*

Next let $x \in$H(Q)*. Then $[\alpha, x] \in$H(Q), for all $\alpha \in \Gamma$. Now, there exist $q_1, q_2 \in$Q and $r \in$R such that $[\alpha, x]+q_1+r = q_2+r$, for all $\alpha \in \Gamma$. In particular, $[\delta_i, x] + q_1 + r = q_2 + r$, for all $i = 1, 2, ..., m$.

This gives that $s\delta_i x + sq_1 + sr = sq_2 + sr$, for all i and for all $s \in$S.

In particular, we have $e_i\delta_i x + e_i q_1 + e_i r = e_i q_2 + e_i r$, for all i. That is ,

$e_i\delta_i x + e_i q_1 + e_i r = e_i q_2 + e_i r$, for all i.
Summing over i, we have
$\sum\limits_i e_i\delta_i x + \sum\limits_i e_i q_1 + \sum\limits_i e_i r = \sum\limits_i e_i q_2 + \sum\limits_i e_i r$.
that is , $x + \sum\limits_i e_i q_1 + \sum\limits_i e_i r = \sum\limits_i e_i q_2 + \sum\limits_i e_i r$. Now $\sum\limits_i e_i q_1, \sum\limits_i e_i q_2 \in Q^*$ and $\sum\limits_i e_i r \in S$. Hence,$x \in H(Q^*)$. Consequently, $H(Q)^* \subseteq H(Q^*)$ and thus, $H(Q^*) = H(Q)^*$.

Remark 3.11 *The results for the left operator which are analogous to the above two lemmas also hold.*

Proposition 3.12 *Let S be a Γ-semiring with unity and Q an h-prime(h-semiprime) ideal of the right operator semiring R of the Γ-semiring S. Then Q^* is an h-prime (respectively h-semiprime) ideal of S.*

Proof. Let Q be an h-prime ideal of R. Then by Proposition 2.5, H(Q) is a prime ideal of R .Now by Theorem 3.18 in [1] , one can easily show that $H(Q)^*$ is a prime ideal of S. By applying Lemma 3.10, $H(Q^*)$ is a prime ideal of S. Now, it is obvious that Q^* is an h-prime ideal of S by Proposition 3.6. By applying Remark 3.34 in [3] and Lemma 3.10, we can also similarly prove that Q^* is an h-semiprime ideal of S when Q is an h-semiprime ideal.

Proposition 3.13 *Let S be a Γ-semiring with unity and I an h-prime (h-semiprime) ideal of the Γ-semiring S. Then $I^{*'}$ is an h-prime (respectively h-semiprime) ideal of the right operator semiring R of the Γ-semiring S.*

Proof Let I be an h-prime ideal of S. Then , by Proposition 3.6,H(I) is a prime ideal of S. By applying Theorem 3.18 of [1], we can prove that $H(I)^{*'}$ is a prime ideal of R, that is , $H(I^{*'})$ is a prime ideal of R (see Lemma 3.9). Hence, by Proposition 2.5, $I^{*'}$ is an h-prime ideal of R. By Remark 3.34 in [3] and Lemma 3.9, we can similarly prove that $I^{*'}$ is h-semiprime ideal when I is an h-semiprime ideals.

Remark 3.14 *Similar to Propositions in 3.12 3.13, we can establish similar results forthe left operator semiring L.*

By Theorem 6.6 in [2], Proposition 3.12, 3.13 and Remark 3.14, we can deduce the following theorem.

Theorem 3.15 *Let S be a Γ-semiring with unity and R(respectively L) the right (respectively left) operator semiring of S. Then there exists an inclusion preserving bijection $Q \longrightarrow Q^{*'}$ $(Q \longrightarrow Q^{+'})$ between the h-prime(h-semiprime) ideals of the Γ-semiring S and that of R (respectively L).*

Since for an h-prime ideal I, H(I) is prime and so by Theorem 3.6 in [1], we can obtain the following theorems of h-prime ideals in a Γ-semiring.

Theorem 3.16 *Let S be a Γ-semiring with unity and I an ideal of S. Then the following statements are equivalent:*
i) I is h-prime.
ii) For a, b $\in S$ such that $a\Gamma S\Gamma b \subseteq$H(I)implies either a$\in$H(I) or b$\in$H(I).
iii) If <a>, <b> are principle ideals of S such that <a>Γ<b>$\subseteq$H(I) then either a$\in$H(I) or b$\in$H(I).
iv) If U and V are two right(left) ideals of S such that $U\Gamma V\subseteq$H(I), then either U$\subseteq$H(I) or V$\subseteq$H(I).

Theorem 3.17 *Let S be a Γ-semiring with unities and I an ideal of S. Then the following statements are equivalent*
i) I is h-semiprime.
ii) For a$\in$S, $a\Gamma S\Gamma a\subseteq$H(I) implies a$\in$H(I).
iii) If <a> is a principle ideal of S such that <a>Γ<a>$\subseteq$H(I) then a$\in$H(I).
iv) If U is a right(left) ideal of S such that $U\Gamma U\subseteq$H(I), then U$\subseteq$H(I).

Proof. Since I is h-semiprime, $H($ I $)$ is semiprime. Hence, the Theorem follows immediately from Theorem 3.6 in [3].

4 H-Noetherian semirings and Γ-semirings

Definition 4.1 *A Γ-semiring (respectively semiring) S is said to be an H-Noetherian Γ-semiring (respectively semiring) if for any ascending chain $I_1 \subseteq I_2 \subseteq ...$ of ideals of S the corresponding ascending chain $H(I_1) \subseteq H(I_2) \subseteq ...$ of ideals terminates.*

Note 4.2 *If a Γ-semiring (respectively semiring) S is Noetherian* [4]*,then S is called H-Noetherian.*

Theorem 4.3 *Let S be a Γ-semiring with an unity. Then S is H-Noetherian if and only if its corresponding left operator semiring L (respectively R) of S is H-Noetherian.*

Proof. Let S be H-Noetherian and $P_1 \subseteq P_2 \subseteq ...$ an ascending chain of ideals of L. Then, by Theorem 6.3 in [2], $P_1^+ \subseteq P_2^+ \subseteq ...$ is an ascending chain of ideals of S, that is, $H(P_1^+) \subseteq H(P_2^+) \subseteq ...$ is an ascending chain of ideals of S. Since S is H-Noetherian, there exists a positive integer m such that $H(P_n^+) = H(P_m^+)$=..., for all $n \geq m$, that is , $\left(H(P_n^+)\right)^{+'} = \left(H(P_m^+)\right)^{+'}$=... for

all $n \geq m$. In other words, we have $H(\left(P_n^+\right)^{+'}) = H(\left(P_m^+\right)^{+'})$, for all $n \geq m$(cf. Remark 3.11). Now, by Theorem 6.3 in [2], we have $H(P_n) = H(P_m)$=... in L, for all $n \geq m$. This shows that L is H-Noetherian. Similarly, by Theorem 6.6 in [2], we can also show that R is H-Noetherian.
By reversing the above argument and using Remark 3.11 , the converse of the Theorem can be easily proved. We omit the details.

Corollary 4.4 *Let S be a Γ-semiring with unity. Then the left operator semiring L of S is H-Noetherian if and only if the right operator semiring R of S is H-Noetherian .*

Regular Γ-semirings were first studied by S. Kyuno, N. Nobusawa and Misoo B. Smith in 1987 (see [8]). We now characterize the h-regular Γ-semirings (respectively semirings) by using its semiprime h-ideals.

Definition 4.5 *A Γ-semiring (respectively semiring) S is said to be h-regular if every h-ideal of S is semiprime.*

Theorem 4.6 *A Γ-semiring (respectively semiring) S is h-regular if and only if every ideal of S is h-semiprime.*

Proof. Let S be an h-regular Γ-semiring(respectively semiring) and I an ideal of S. Then H(I) is an h-ideal of S. Now, by the definition of h-regularity, H(I) must be semiprime. Consequently, I is h-semiprime.

Conversely, if every ideal of S is h-semiprime and I is an h-ideal of S, then by hypothesis, I is h-semiprime and thereby, $H(I)$ is semiprime. This shows that I is semiprime because H(I) =I. Hence, the Γ-semiring (respectively semiring) S must be h-regular.

In closing this paper, we characterize the h-regular Γ-semi rings.

Theorem 4.7 *Let S be a Γ-semiring with unity. Then S is h-regular if and only if the left operator semiring L of S (respectively R) is h-regular.*

Proof. Let S be a h-regular Γ-semiring with unity and L be the left operator semiring of S. If I is an ideal of L, then by Proposition 6.1 in [2], we can easily prove that I^+ is an ideal of S. By Theorem 4.6, we can easily deduce that I^+ is an h-semiprime ideal of S. Now, by using the Remark 3.14, $(I^+)^{+'}$ is h-semiprime, hence by Theorem 6.3 in [2], I is an h-semiprime ideal in L. Thus, by Theorem 4.6, L itself is indeed an h-regular Γ-semiring . The converse part of this theorem can be proved by reversing the above arguments.

Note 4.8 *The above Theorem can also be proved by using Theorem 3.15.*

Conclusion 4.9 . *It is noteworthy that the relationships between a* Γ-*semiring and its operator semiring in terms of h-prime ideals and h-semiprime ideals may be useful in the study of the structure of matrix semiring* $\begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$ *over a Nobusawa* Γ-*semiring [7]. Also, J.Zhan et al have recently considered the fuzzy h-ideals of fuzzy hemirings and* BCK*algebas (see* [12]-[14]*). Some of the results in fuzzy algebras on fuzzy h-ideals related to soft computing have been obtained.*

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