H_2 Filter for Time Delay Systems

Young Soo Suh, Young Shick Ro, and Hee Jun Kang

Abstract: An H_2 filter is derived for time delay systems, where there are time delay terms in the state and in the output. A method to compute the H_2 norm of time delay systems is proposed. Based on the H_2 norm computation method, an H_2 filter design is formulated as a nonlinear optimization problem.

Keywords: H_2 filter, Kalman filter, observer, time delay systems.

1. INTRODUCTION

Many physical systems have time delay elements, which reflect sensor process time, computation time and communication time. For example, in [1], motor speed measurement is time delayed due to characteristics of motor encoders. These systems can be represented by state space equations, where there are time delay terms in the state and in the output. The purpose of this paper is to propose a Kalman-filter-type observer for time delay systems.

In the case of time delay systems, to design an asymptotically stable observer without considering estimation performance is not an easy task. There are several papers on the design of asymptotically stable observers: a modal observer [2], reduced-order observer [3], and output-injection based observer [4]. Recently, an observer [5] is proposed, where the H_∞ norm is used as a performance index. The H_∞ filter using delay independent stability conditions are considered in [6,7], where linear matrix inequalities are used.

However, few observers have been proposed using the H_2 norm despite the utility of the H_2 norm as a performance index for many problems. In [8], an observer for time delay systems has been proposed using delay independent stability conditions. In this paper, an observer whose performance index is an H_2 norm is proposed, where delay dependent stability conditions are used. Note that the optimal H_2 norm observer is the standard Kalman filter when there are no time delay terms. Thus, the proposed filter can be considered as a Kalman filter for time delay systems.

Notation: For a matrix $M \in \mathbb{C}^{n\times n}$ given by

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix}$$

the column string csM is defined by

$$csM \triangleq \left[m_{11}\ m_{12} \cdots m_{1n} \middle| m_{21}\ m_{22} \cdots m_{2n} \middle| \cdots \middle| m_{n1}\ m_{n2} \cdots m_{nn} \right]' \in \mathbb{C}^{n^2\times 1}.$$

2. PROBLEM STATEMENT

Consider linear time-invariant systems described by

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-h) + B_1\omega(t) + B_2 u(t), \\ y(t) &= C_0 x(t) + C_1 x(t-h) + C_2 v(t), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $\omega \in R^p$ is the process noise, $u \in R^q$ is the input, $y \in R^r$ is the measurement, and $v \in R^r$ is the measurement noise. The h is constant known time delay in the states and the outputs.

It is assumed that v and ω are uncorrelated white Gaussian processes, which satisfy

$$\begin{aligned} &E\{\omega(t)\} = 0, E\{\omega(t)\omega(s)'\} = I\delta(t-s), \\ &E\{v(t)\} = 0, E\{v(t)v(s)'\} = I\delta(t-s). \end{aligned} \tag{2}$$

The objective of this paper is to derive an H_2 filter for a time delay system (1), where a filter has the following form:

$$\begin{aligned} \dot{\hat{x}}(t) = {} & A_0\hat{x}(t) + A_1\hat{x}(t-h) \\ & -K\left(C_0\hat{x}(t) + C_1\hat{x}(t-h) - y(t)\right) + B_2 u(t). \end{aligned} \tag{3}$$

Defining the estimation error $e(t)$ as

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$$e(t) \triangleq x(t) - \hat{x}(t),$$

we obtain

$$G_e : \dot{e}(t) = \bar{A}_0 e(t) + \bar{A}_1 e(t-h) + B\xi(t), \tag{4}$$

where

$$\bar{A}_0 \triangleq A - KC_0, \quad \bar{A}_1 \triangleq A - KC_1,$$
$$B \triangleq \left[B_1, -KC_2\right], \quad \xi(t) \triangleq \begin{bmatrix} \omega(t) \\ v(t) \end{bmatrix}.$$

The H_2 norm of the error system is used as the performance index estimate

$$\|G_e\|_2^2 = J(K,h) = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T e'(t)e(t)dt\right\}. \tag{5}$$

If there are no time delay terms (i.e., $A_1 = 0$ and $C_1 = 0$), then (1) becomes

$$\dot{x}(t) = A_0 x(t) + B_1\omega(t) + B_2 u(t),$$
$$y(t) = C_0 x(t) + C_2 v(t),$$

and the filter, minimizing the H_2 norm (5) for this non-delayed system, is the standard Kalman filter. Thus we can call the proposed filter minimizing (5) a Kalman filter for time delay systems.

3. H_2 NORM COMPUTATION

The H_2 norm of G_e is expressed in terms of the matrix function $P(s)$ in the next theorem.

Theorem 1: If G_e is stable, then

$$\|G_e\|_2^2 = Tr\left(B'P(0)B\right), \tag{6}$$

where $P(s),\ 0 \le s \le h$ is continuously differentiable and satisfies

$$P(0) = P'(0),$$
$$\dot{P}(s) = \bar{A}_0' P(s) + \bar{A}_1' P'(h-s), \quad 0 \le s \le h, \tag{7}$$
$$\dot{P}(0) + \dot{P}'(0) + I = 0.$$

Remark 1: $P(s)$ is related to the Lyapunov functional of state delay system (4). Let $V(\phi),\ \phi \in C\left[-h, 0\right]$ be defined by

$$V(\phi) \triangleq \phi'(0)P(0)\phi(0) + 2\phi'(0)\int_0^h P(r)\bar{A}_1\phi(-h+r)dr + \int_0^h \phi'(-h+r)\int_0^h \bar{A}_1' P(r-s)\bar{A}_1\phi(-h+r)dsdr, \tag{8}$$

where $P(s) \triangleq P'(-s)$ if s < 0. Equation (7) is derived from

$$\frac{d}{dt}V(x_t) = -x'(t)x(t), \tag{9}$$

where $x_t(r) \triangleq x(t+r), \quad r \in \left[-h, 0\right]$.

Remark 2: If there are no time delay terms, the result in Theorem 1 becomes a standard H_2 norm computation. See, for example, Theorem 3.3.1 in [9]: the H_2 norm of a stable non-delay system is given by

$$\|G_e\|_2^2 = Tr\left(B'PB\right), \tag{10}$$

where

$$\bar{A}_0' P + P\bar{A}_0 + I = 0.$$

Note that conditions (7) are equivalent to those in (10) if $h = 0$, $A_1 = 0$ and $C_1 = 0$.

The proof of Theorem 1 will be given using Lemma 1 and 2.

Lemma 1: If system G_e is stable, then

$$\|G_e\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Tr\left(G_e(j\omega)G_e'(-j\omega)\right)d\omega. \tag{11}$$

Proof: The result is standard (see Chap 3.3 in [9]).

Lemma 2: If G_e is stable and $P(s),\ 0 \le s \le h$ satisfies (7), then

$$P(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Delta^{-1}(j\omega)'\Delta^{-1}(j\omega)d\omega, \tag{12}$$

where

$$\Delta(j\omega) \triangleq j\omega I - \bar{A}_0 - \bar{A}_1 e^{-j\omega h}. \tag{13}$$

Proof: See [10].

(Proof of Theorem 1) From Lemma 1,

$$Tr\left(B'P(0)B\right) = Tr\left\{\frac{1}{2\pi}\int_{-\infty}^{+\infty} B'\Delta^{-1}(j\omega)'\Delta^{-1}(-j\omega)Bd\omega\right\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Tr\left\{B'\Delta^{-1}(j\omega)'\Delta^{-1}(-j\omega)B\right\}d\omega.$$

Since $\int_{-\infty}^{+\infty} f(j\omega)d\omega = \int_{-\infty}^{+\infty} f(-j\omega)d\omega$, we have

$$Tr\left(B'P(0)B\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Tr\left\{B'\Delta^{-1}(-j\omega)'\Delta^{-1}(j\omega)B\right\}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Tr\left\{G_e'(-j\omega)G_e(j\omega)\right\}d\omega.$$

Since $Tr(AB) = Tr(BA)$ whenever AB and BA are square matrices, we have

$$Tr\left(B'P(0)B\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Tr\left\{G_e(j\omega)G_e'(-j\omega)\right\}d\omega = \|G_e\|_2^2.$$

The last equality is from (11).

If G_e is stable, then $\|G_e\|_2^2$ can be computed from $P(0)$ in Theorem 1. How to check the stability of G_e will be considered later in Theorem 2; first we will consider how to compute $P(0)$ in the next lemma.

Lemma 3: If G_e is stable, then $P(0)$ and $P(h)$ satisfying (7) are given by

$$\begin{bmatrix} (I\otimes \bar{A}_0') + (\bar{A}_0'\otimes I) & (I\otimes \bar{A}_1')E + (\bar{A}_1'\otimes I) \\ R_1 & R_2 \end{bmatrix} \cdot \begin{bmatrix} csP(0) \\ csP(h) \end{bmatrix} = \begin{bmatrix} -csI \\ 0 \end{bmatrix}, \tag{14}$$

where

$$[R_1 R_2] \triangleq [\Sigma_1 \ \ 0]V^*.$$

Matrices Σ_1 and V^* are from the singular value decomposition of the following

$$(I - J\exp(Hh)) = U\begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} V^*, \tag{15}$$

where U and V are unitary matrices, and $\Sigma_1 \in R^{n^2\times n^2}$ is a diagonal matrix whose diagonal elements are nonzero singular values of $(I - J\exp(Hh))$. Let E_{ij} denote an $n\times n$ matrix with (i, j)-entry equal to 1 and all other entries equal to zero, and let $E\in R^{n^2\times n^2}$ be the block matrix E, $[E_{ji}]$ (i.e., the (i, j)-block of E is E_{ji}). Matrices H and J are defined by

$$H \triangleq \begin{bmatrix} (I\otimes \bar{A}_0') & (I\otimes \bar{A}_1')E \\ -(I\otimes \bar{A}_0')E & -(I\otimes \bar{A}_1') \end{bmatrix}, \quad J \triangleq \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}.$$

Proof: See [11].

Note that $P(0)$ can be computed from the matrix exponential (15) and a simple linear equation (14). Thus if G_e is stable, then we can easily compute H_2 norm: see (6).

Now the stability of G_e is considered in Theorem 2, where a stability condition for interval delay $h\in[0,\bar{h})$ is provided.

Theorem 2: Suppose G_e is stable for $h = 0$. If H has imaginary eigenvalues $\{j\omega_1, \cdots, j\omega_k\}$ and their corresponding eigenvectors are given by

$$v_1 = \begin{bmatrix} v_{1,1} \\ v_{1,2} \\ \vdots \\ v_{1,2n^2} \end{bmatrix}, \cdots, v_k = \begin{bmatrix} v_{k,1} \\ v_{k,2} \\ \vdots \\ v_{k,2n^2} \end{bmatrix},$$

then G_e is stable for $h\in[0,\bar{h})$ where $\bar{h}$ is defined by

$$\bar{h} = \min_{1\le i\le k} \left| \frac{1}{j\omega} \ln\left(\frac{v_{i,l}}{v_{i,l+n^2}}\right) \right|, \tag{16}$$

where $v_{i,l}, 0\le l\le n^2$ is any nonzero element of v_l.

Theorem 2 is proved using Lemma 4 and 5. Lemma 4 is based on the fact that if G_e is stable for $h = 0$ and G_e does not have any imaginary poles for $h\in[0,\bar{h})$, then G_e is stable for $h\in[0,\bar{h})$.

Lemma 4: G_e is stable for $h\in[0,\bar{h})$ if

- G_e is stable for $h = 0$.
- The following equation does not have any roots for $h\in[0,\bar{h})$:

$$\det(j\omega I - \bar{A}_0 - \bar{A}_1 e^{-j\omega h}) = 0 \tag{17}$$

Proof: See [12].

Stability of G_e for $h = 0$ can be easily checked from eigenvalues of $\bar{A}_0 + \bar{A}_1$. On the other hand, checking whether (17) has any roots for $h\in[0,\bar{h})$ is not easy: (17) should be checked for all $0\le\omega<\infty$ and $0\le h<\bar{h}$. In the next lemma, it is shown that a root $j\omega$ of (17) (if any) is an eigenvalue of H.

Lemma 5: If (17) has a root ω, then it is an eigenvalue of H.

Proof: Suppose (17) has a root $j\omega$ for h; then there exists $x(\in C^n)\ne 0$ such that

$$x'(j\omega I - \bar{A}_0 - \bar{A}_1 e^{-j\omega h}) = 0.$$

Taking the transpose (not complex conjugate), we obtain

$$(j\omega I - \bar{A}_0 - \bar{A}_1 e^{-j\omega h})x = 0. \tag{18}$$

Let $\alpha\in C^n$ be defined by

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \triangleq x e^{-\frac{j\omega h}{2}}, \tag{19}$$

where $\alpha_i, 1\le i\le n$ is a complex number. Let v be defined by ($\bar{u}$ is the complex conjugate of u)

$$v \triangleq \begin{bmatrix} u \\ \bar{u} \end{bmatrix}, \tag{20}$$

where

$$u \triangleq \begin{bmatrix} \bar{\alpha}_1 x \\ \bar{\alpha}_2 x \\ \vdots \\ \bar{\alpha}_n x \end{bmatrix} \in C^{n^2}. \quad (21)$$

The theorem is proved if we show that this v ($v \neq 0$ from the construction) satisfies $(j\omega I - H)v = 0$: that is, $j\omega$ is an eigenvalue of H. From the definition of H, we obtain

$$(j\omega I - H)v = \begin{bmatrix} j\omega I - (I \otimes \bar{A}_0') & -(I \otimes \bar{A}_1')E \\ (I \otimes \bar{A}_1')E & j\omega I + (I \otimes \bar{A}_0') \end{bmatrix} v$$
$$= \begin{bmatrix} \left(j\omega I - (I \otimes \bar{A}_0')\right)u - (I \otimes \bar{A}_1')E\bar{u} \\ \left(j\omega I + (I \otimes \bar{A}_0')\right)\bar{u} + (I \otimes \bar{A}_1')Eu \end{bmatrix}. \quad (22)$$

Partition $(j\omega I - H)v$ into $2n$ complex vectors and let the i-th block of $(j\omega I - H)v$ be denoted by $r_i \in C^n$. Then $r_i, 1 \le i \le n$ is given by

$$r_i = (j\omega I - \bar{A}_0')\bar{\alpha}_i x - \bar{A}_1'(E_{1i}\bar{\alpha}_1 + E_{2i}\bar{\alpha}_2 + \cdots + E_{ni}\bar{\alpha}_n)\bar{x}.$$

Noting the following relation

$$(E_{1i}\bar{\alpha}_1 + E_{2i}\bar{\alpha}_2 + \cdots + E_{ni}\bar{\alpha}_n)\bar{x}$$
$$= (E_{1i}\bar{\alpha}_1 + E_{2i}\bar{\alpha}_2 + \cdots + E_{ni}\bar{\alpha}_n)\bar{\alpha} e^{-\frac{j\omega h}{2}}$$
$$= e^{-\frac{j\omega h}{2}} \bar{\alpha}_i \alpha.$$

We obtain

$$r_i = (j\omega I - \bar{A}_0')\bar{\alpha}_i \alpha e^{\frac{j\omega h}{2}} - \bar{\alpha}_i \bar{A}_1' \alpha e^{-\frac{j\omega h}{2}}$$
$$= \bar{\alpha}_i e^{\frac{j\omega h}{2}} (j\omega I - \bar{A}_0' - \bar{A}_1' e^{-j\omega h})\alpha$$
$$= \bar{\alpha}_i (j\omega I - \bar{A}_0' - \bar{A}_1' e^{-j\omega h})x = 0, \quad 1 \le i \le n.$$

The last equality is from (18). Since $r_{i+n} = -\bar{r}_i, 1 \le i \le n$ (see (22)), we have $r_i = 0$, $n+1 \le i \le 2n$. Hence, $(j\omega I - H)v = 0$, where $v \neq 0$ (since $x \neq 0$).

Proof of Theorem 2: From the proof of Lemma 5, if (17) has a root ω_i for h_i $(1 \le i \le k)$, then ω_i is an eigenvalue of H. Furthermore, the corresponding eigenvector of H is of the form:

$$v_i = \begin{bmatrix} \bar{x}_1 x e^{\frac{j\omega_i h_i}{2}} & \bar{x}_2 x e^{\frac{j\omega_i h_i}{2}} & \ldots & \bar{x}_n x e^{\frac{j\omega_i h_i}{2}} & x_1 \bar{x} e^{-\frac{j\omega_i h_i}{2}} & x_2 \bar{x} e^{-\frac{j\omega_i h_i}{2}} & \ldots & x_n \bar{x} e^{-\frac{j\omega_i h_i}{2}} \end{bmatrix}^T.$$

Thus h_i can be computed as follows:

$$h_i = \left| \frac{1}{j\omega} \ln\left(\frac{v_{i,l}}{v_{i,l+n^2}}\right) \right|,$$

where $v_{i,l}, 1 \le l \le n^2$ is any nonzero element of v_i. If the minimum value of h_i $(1 \le i \le k)$ is $\bar{h}$, then (17) does not have a root for $h \in \left[0, \bar{h}\right)$. From Lemma 4, this proves the theorem.

Remark 3: Once a filter gain K is determined, we can check the stability of the error system (4) (Theorem 2) and compute its H_2 norm (Theorem 1).

4. FILTER DESIGN

In this section, the synthesis algorithm of an H_2 filter (3) is proposed, where the algorithm is formulated as a constrained nonlinear optimization problem. When minimizing H_2 norm of G_e over K using Theorem 1, it should be guaranteed that G_e is stable. If the filter gain K is given, the stability of G_e can be checked using Theorem 2, which provides a upper stability bound $\bar{h}(K)$ (i.e., $G_e(K, h)$ is stable as long as $h < \bar{h}$). Thus finding an optimal K, which stabilizes G_e and minimizes $\|G_e(K,h)\|_2$, can be formulated as follows:

$$\min_K J(K,h) \triangleq \|G_e(K,h)\|_2^2 \quad (23)$$
$$\textit{subject to } h < \bar{h}(K).$$

(23) is a constrained nonlinear optimization problem whose global solution is difficult to find. A suboptimal approach is proposed to compute K using penalty methods [13]. A penalty function is defined by

$$p(K,h) \triangleq \begin{cases} 0 & \textit{if } \ h < \bar{h}(K) \\ \alpha(h - \bar{h})^2 & \textit{if } \ h \ge \bar{h}(K), \end{cases}$$

where α is a constant and is chosen so that $p(K, h) \gg J(K,h)$ when $h \gg \bar{h}(K)$. With this penalty function, a constrained optimization problem (23) can be replaced by the following unconstrained optimization problem:

$$\min_K J_p(K,h) \triangleq \|G_e(K,h)\|_2^2 + p(K,h). \quad (24)$$

Note that if $h < \bar{h}(K)$ (i.e., G_e is stable), then $J_p(K,h) = J(K,h)$. Also note that if $h \ge \bar{h}(K)$, then $J_p(K,h)$ is dominated by the penalty function $p(K,h)$. Thus the penalty function $p(K,h)$ prevents unstable region searching when the H_2 norm is being minimized.

An initial value of K can be chosen by minimizing $J(K, 0)$: the initial value corresponds to the Kalman filter gain for a non-delayed system. Minimization problem (24) can be solved, for example, using an unconstrained nonlinear optimization function fminunc in MATLAB optimization toolbox.

5. NUMERICAL EXAMPLE

Consider the following system

$$\dot{x}(t)=\begin{bmatrix}-2 & 1\\ 0 & -1\end{bmatrix}x(t)+\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}x(t-h)+\begin{bmatrix}0.2\\ 0.2\end{bmatrix}\omega(t)+\begin{bmatrix}1\\ 1\end{bmatrix}u(t),$$
$$y(t)=\begin{bmatrix}0 & 1\end{bmatrix}x(t)+\begin{bmatrix}1 & 1\end{bmatrix}x(t-h)+0.5v(t), \qquad (25)$$

where $\omega(t)$ and $v(t)$ are zero-mean, uncorrelated white Gaussian processes satisfying (2). The time delay is set to be $h = 0.3$.

Optimization problem (24) was solved using Matlab optimization toolbox. The initial value of the filter gain K is computed using $h = 0$, and α in the penalty function is set to 100. The computed values are as follows:

$$K=\begin{bmatrix}0.0208\\ 0.0072\end{bmatrix},\ \bar{h}=1.6309,\ \|G_e(K,h)\|_2^2=0.0243.$$

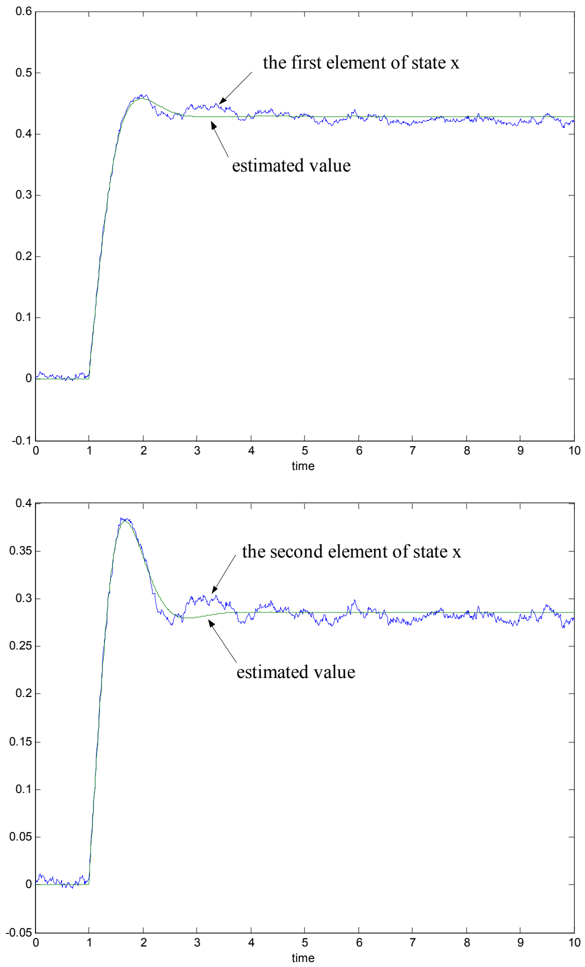

Fig. 1. Simulation results: true state and estimated value.

Table 1. Time delay effects on estimation performance.

	h=0.1	h=0.3	h=0.5	h=0.7
$\|G_e(K,h)\|_2^2$	0.0180	0.0243	0.0321	0.0424
Variance of actual estimation error	0.000088	0.00011	0.00013	0.00015

Using the computed filter gain, state estimation simulation was done, where a unit step signal was applied to the control input $u(t)$ at time 1s. The simulation result is given in Fig. 1: it can be seen that the proposed H_2 filter estimates system states well.

To see how the time delay affects estimation performance, H_2 filters were designed for different h values.

As seen in Table 1, computed H_2 norm increases as time delay h increases. Variance of actual estimation error, which was computed from a simulation, also increases as time delay h increases. This verifies a common belief that the time delay adversely affects on estimation performance.

6. CONCLUSION

In this paper, an H_2 observer design method for time delay systems has been proposed. The proposed filter coincides with the standard Kalman filter when there are no time delay terms. As the popularity of a Kalman filter proves, in many practical situations an H_2 norm observer provides most satisfactory results.

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