Habit Formation and the Equity-Premium Puzzle: a Skeptical View

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Abstract

We argue that ceteris paribus, introducing a habit that resolves the equity premium puzzle is equivalent to increasing the coefficient of relative risk aversion. Thus, if habit is modeled subject to the constraint that the Arrow-Pratt coefficient of relative risk aversion is held at a constant 'acceptable' level, the effect on the equity premium is not quantitatively significant. In a dynamic setting, the fluctuations of the habit increase the equity premium, slightly. However, modest improvement in the model's predictive power comes at a cost of generating unrealistic fluctuations in the risk-free interest rate. Our analysis of these findings yields the following result: a habit is observationally equivalent, up to a first order approximation, to a higher relative risk aversion and to a preference shock. Both these effects are known to be insufficient for resolving the equity-premium puzzle.
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1 Introduction

A 'habit-formation' utility function formalizes an attractive and credible story about human psychology: once an agent gets used to a certain standard of living, his consumption level forms a 'habit', which becomes the benchmark against which he evaluates possible changes in future consumption. Hence, it is the deviations from the habit rather than the absolute level of consumption that matters for the individual decision-maker. Our objective in this paper is neither to examine the nature of the story nor to question its plausibility. Rather, we try to understand to what extent a habit-formation utility function resolves the Mehra-Prescott [1985] equity-premium puzzle, as suggested by several authors; c.f. Constantinides [1990], and Boldrin, Christiano, and Fisher [1997], among many others.

Our skepticism is based on a very simple argument: acquiring a habit is intimately related and hard to distinguish from becoming more risk averse. Thus, as an agent gets used to a certain standard of living, he would be willing to pay a higher premium in order to insure himself against a given uncertainty in the *level* of his consumption. This, under standard theory, amounts to an increase in the Arrow-Pratt measure of relative risk aversion. However, the essence of the equity-premium puzzle is that an artificial economy with a 'plausible' parameter of risk aversion (as estimated from micro data) generates an equity premium that is far below the real-world equity premium. Hence, a formalization can be deemed a resolution of the puzzle only if a realistic equity premium is generated by a model economy with a *low* parameter of the Arrow-Pratt measure of relative risk aversion (AP-RRA). We show that introducing a habit subject to the constraint that the AP-RRA is kept constant at an 'acceptable' level, cannot resolve the puzzle in the sense above.

We start the analysis with a two-period model, where the level of the habit is kept constant over time. In such a model, increasing the level of the habit but holding the AP-RRA constant, does not generate any significant change in the equity premium. Indeed, increasing risk aversion and introducing a habit have a 'very similar' effect on the equity premium. To operationalize the notion of 'very similar', we derive a first-order approximation of the equilibrium equity premium for a model with a general utility function, that nest both higher risk aversion and the habit as special cases. We show that the equity premium equals, approximately, to the AP-RRA times the variance of GNP growth rate. The decomposition of the AP-RRA to a habit parameter and a power parameter is immaterial as far as the equity premium is concerned. Thus, risk aversion and habit formation are *observationally equivalent up to a first order approximation.*

Things differ slightly in a dynamic model, where habit formation has some positive effect on the model's predictive power, although the improvement falls far short of what is required in order to resolve the puzzle. Moreover, it generates problems elsewhere. Using the approximation approach once again, we show that the improvement results entirely from the fluctuations in the level of the habit. (By itself, extending the time horizon from two to infinite number of periods has no effect on the equity premium). Moreover, the fluctuations in the level of the habit are first-order observationally-equivalent to preference shocks. Such fluctuations are known to increase the variability of asset prices and thus the equity premium, but they come at a price of introducing some unrealistic fluctuations in the level of the risk-free rate. We apply this dynamic analysis to both an 'external' and 'internal' habit.

We thus conclude that habit formation is observationally equivalent up to a first-order approximation to other effects that were already shown to be insufficient to resolve the equity-premium puzzle. Obviously, this claim does not imply that habit-formation preferences are unrealistic or 'less interesting' than ordinary power preferences. It is also possible that in some other contexts, where the gap between observation and prediction is not that wide, habit-formation preferences may generate second-order effects that would better explain the data. Evidently, the modeling of the equity-premium has not yet reached the point of second-order refinements.

The structure of the paper is as follows: after a short note on related literature, we analyze the two-period case in Section 2, and the dynamic case, both external and internal habit, in Section 3. We conclude in Section 4.

1.1 Related Literature

A few other authors have recently questioned the extent to which habits may resolve the equity-premium puzzle. Otrok, Ravikumar, and Whiteman [2002] use spectral utility to identify the route through which the habit-formation model resolves the equity-premium puzzle. They find that the habit-formation model is highly sensitive to the specification of the stochastic process according to which output and consumption evolve over time. Once the process is parameterized so as to fit the actual US consumption process, the habit-formation model delivers counterfactual predictions about the time path of the equity premium and risk-free interest rate. Similarly, Chapman [2002] argues that if one uses post war data to estimate the consumption process, the model is unable to predict the desired moments. Addressing a similar question, Lettau and Uhlig [2002] try to mach the Sharpe ratio with a habit-formation model (and some other models). They succeed in doing so with 80% of habitual consumption, however with excessive volatility. A similar point is made in Boldrin, Christiano and Fisher [1997] who match the equity premium with counterfactual volatility, particularly of the risk-free rate.

Hagiwara and Herce [1997] take a different approach. They study whether one can obtain appropriate estimates of the coefficient of relative risk aversion and subjective discount factor using a method-of-moments approach matching the risk-free interest rate and the equity premium in the Euler equations. With consumption data they are unable to do so but with dividend data, they are able to obtain reasonable estimates. Though they show that in the case where they use dividend data rather than consumption data, their estimates of the coefficient of relative risk aversion and subjective discount factor are reasonable, one can see that it is very likely that their second moments are far off. In particular, the volatility of the risk-free interest rate is likely to be too high as their stochastic discount factor is very volatile. We find this to be a general problem with asset pricing models, that once the first moments are matched, it is at the expense of the second moment.

2 A two-period model

Let u be a Von-Neuman-Morgenstern habit-formation utility function:

$$u = \frac{1}{1-\alpha}(c-z)^{1-\alpha}, \tag{1}$$

where c is the level of consumption, z the habit and α the power parameter of the utility function. We denote the AP-RRA by θ, and compute:

$$\theta = \alpha \frac{c}{c-z}. \tag{2}$$

Clearly, for any $z > 0$, the power parameter no longer corresponds to the AP-RRA .

The usual methods for estimating the AP-RRA parameter still holds in this case. According to established theory, the premium, p, an agent would be willing to pay in order to fully-insure himself against a certain risk is:

$$p = \frac{1}{2}\theta\sigma^2, \tag{3}$$

where σ^2 is the variance of the (relative) risk to which the agent is exposed. Having observations on p and σ^2, the estimation of θ is immediate.

Now suppose that the habit is modeled as a fraction, λ, of c. Then, the observed premium imposes a certain restriction on the power parameter and the habit, λ and α:

$$\theta = \frac{\alpha}{1-\lambda}, \tag{4}$$

which can no longer be chosen independently. Hence, one cannot argue that a model with a low power, α, and an arbitrarily high habit, λ, resolves the equity-premium puzzle, because such a model might imply that the representative agent has a high level of relative risk aversion and is willing to pay a higher premium, p, than is observed in the micro data. Hence, the constraint (4) implies that whenever the habit parameter λ is increased, the power parameter α should be decreased, so that the AP-RRA remains in harmony with the micro data. There is a good reason to believe that once that is done, habit-formation preferences provide no solution to the equity-premium puzzle.

To see why, consider a two-period representative-agent exchange economy. The price of 'tree' j, in terms of 'apples', is given by the Lucas formula:

$$q^j = E\left[\frac{\beta u'(c_2)}{u'(c_1)} y_2^j\right], \tag{5}$$

where β is the utility discount factor, $t = 1, 2$ is the time index, c_t consumption, and y_2^j the apple-output (a random variable) of tree j. The formula can be applied to the price of riskless debt (typically at zero net supply):

$$\frac{1}{1+r} = E\left[\frac{\beta u'(c_2)}{u'(c_1)}\right], \tag{6}$$

with r being the risk-free rate.

Equation (5) can be applied to derive the price of the whole market, which we denote by q. It follows from the exchange-economy assumption that national output, y_t, is paid as a dividend, so that $c_t = y_t$. Rearranging, and defining $(1 + R) \equiv (y_2 + q_2)/q$ to be the gross rate of return on equity we get:

$$1 = \left[\frac{\beta u'(y_2)}{u'(y_1)} (1 + R) \right]. \tag{7}$$

Note that $q_2 = 0$ here but it is included in the definition for consistency with the dynamic section. Decomposing the expression on the right-hand-side of (7) to the means and covariances of it's two random variables, and rearranging again we get:

$$\frac{1 + E(R)}{1 + r} - 1 = -COV \left\{ \beta \frac{u'(y_2)}{u'(y_1)}, (1 + R) \right\}. \tag{8}$$

We can now linearize the marginal-utility function around $E(y_2)$:

$$u'(c_2) \approx u'(E(y_2)) + u''(E(y_2))(y_2 - E(y_2)), \tag{9}$$

Approximating the left-hand side of (8) and substituting (9) into (8) we obtain:

$$E(R) - r \approx \left\{ -\frac{E(y_2) \cdot u''[E(y_2)]}{u'[E(y_2)]} \right\} \cdot COV \left[\frac{y_2 - E(y_2)}{E(y_2)}, (1 + R) \right], \tag{10}$$

or:

$$E(R) - r \approx \theta \sigma^2, \tag{11}$$

where σ can be reinterpreted as the standard deviation of output growth.

Equation (11) captures the essence of the equity-premium puzzle. Since the standard deviation of consumption growth rate is roughly 3.6% per year, and since most estimates put θ close to one, the equity premium predicted by the model is about 0.1%. Since the actual equity premium is about 6%, we have a puzzle. More so, when we realize the common-sense interpretation of equation (11), a risk-averse agent is indifferent between bearing a risk with variance σ^2 and insuring herself at a premium of σ^2 times her AP-RRA. By its very nature, macro risk cannot be insured, so the representative agent has to bear it. Hence, equilibrium asset prices should be determined so as to bring the agent to the point of indifference between bearing the risk and insuring it. That means that risky assets should yield a premium over safe assets of an order of magnitude of the variance of consumption growth times θ. Crucially, this argument is valid for a very large family of utility functions, including those with a habit. Particularly, equation (11) implies that, at least up to a first-order-approximation, the only parameter relevant to the risk premium is the AP-RRA. The decomposition of θ to a habit and a power parameter is immaterial.

That leaves us with only a second-order effect to hope for. To check this point, we derive a closed-form solution and simulate the model. Suppose that $z = \lambda y_1$; we denote by ξ the random (gross) growth rate (i.e. $y_2 = \xi \cdot y_1$) with mean $(1+g)$ and variance σ^2. Substituting these parameters into equations (5) and (6) we obtain:

$$\frac{q}{E(y_2)} = \beta E\left[\frac{u'\left[y_1\left(\xi-\lambda\right)\right]}{u'\left[y_1\left(1-\lambda\right)\right]} \cdot \frac{\xi y_1}{E(y_2)}\right], \tag{12}$$

and:

$$\frac{1}{1+r} = \beta E\left[\frac{u'\left[y_1\left(\xi-\lambda\right)\right]}{u'\left[y_1\left(1-\lambda\right)\right]}\right]. \tag{13}$$

We parameterize the two period model according to the original Mehra-Prescott [1985] paper. Namely, we assume $g = .018$ and a two-state growth process $\xi = (1+g) \pm 0.036$, each with a probability of 1/2.[1] Table 1 reports the results for the unconditional expected returns and the equity premium against different levels of the habit, when the power, α, is adjusted according to (4) so that the AP-RRA, θ, is kept constant at either one ore three.[2] In order for the model to make sense, we must insure that for any realization of consumption growth, marginal utility is always positive. This implies that $\lambda \in [0, 0.982)$. The results are straight forward: the level of the equity premium is very close to that predicted by the first-order approximation (11), the second order effect insignificantly *decreasing* the explanatory power of the model.

Although this second-order effect is not quantitatively significant, we seek to explain it. Consider again the two-state case:

$$q = \beta\left[\pi_1 \frac{u'\left(y_2^1\right)}{u'\left(y_1\right)} y_2^1 + \pi_2 \frac{u'\left(y_2^2\right)}{u'\left(y_1\right)} y_2^2\right], \tag{14}$$

and:

$$\frac{1}{1+r} = \beta\left[\pi_1 \frac{u'\left(y_2^1\right)}{u'\left(y_1\right)} + \pi_2 \frac{u'\left(y_2^2\right)}{u'\left(y_1\right)}\right], \tag{15}$$

where the superscript in y_t^i is an index for the state with $i = 1, 2$ and π_i is its probability. We can now compute the equity factor (one plus the equity premium):

$$\frac{E(y_2)/q}{1+r} = \frac{\pi_1 u'\left(y_2^1\right) E(y_2) + \pi_2 u'\left(y_2^2\right) E(y_2)}{\pi_1 u'\left(y_2^1\right) y_2^1 + \pi_2 u'\left(y_2^2\right) y_2^2}, \tag{16}$$

where:

$$E(y_2) = \pi_1 y_2^1 + \pi_2 y_2^2. \tag{17}$$

Rewriting the premium in terms of the (absolute value) marginal (subjective) rate of substitution (MRS) we obtain:

[1] Mehra and Prescott [1985] account for an autocorrelation in the growth process of -0.14. Following Abel [1990], and for the sake of simplicity, we ignore this effect.

[2] It is common to set α equal to one; see Constantinides [1990] and Boldrin, Christiano and Fisher [1997]. Campbell and Cochrane [1999] set it equal to two.

$$\frac{(E(y_2))/q}{1+r} = \frac{MRS\,(y_2^1, y_2^2)\cdot E(y_2) + E(y_2)}{MRS\,(y_2^1, y_2^2)\cdot y_2^1 + y_2^2} \tag{18}$$

where:

$$MRS\left(y_2^1, y_2^2\right) = \frac{\pi_1 u'\left(y_2^1\right)}{\pi_2 u'\left(y_2^2\right)}. \tag{19}$$

Equation (18) has a simple diagrammatical exposition: it is expected consumption, $E(y_2)$, evaluated at the MRS at the state-contingent consumption point (y_2^1, y_2^2), divided by state-contingent consumption evaluated at the same MRS; namely, the ratio between segment (0,A) and segment (0,B) in Figure 1. The steeper is the relevant MRS, the lower is the premium (as the vertical distance, a, in Figure 1 falls and OB increases.)

We have:

$$MRS\left(y_2^1, y_2^2\right) = \left(\frac{\pi_1 y_2^2}{\pi_2 y_2^1}\right)^{\theta}, \tag{20}$$

while with a habit:

$$MRSH\left(y_2^1, y_2^2\right) = \left[\frac{\pi_1\left(y_2^2 - \lambda y_1\right)}{\pi_2\left(y_2^1 - \lambda y_1\right)}\right]^{\theta}. \tag{21}$$

It follows that:

$$\frac{MRSH\,(y_2^1, y_2^2)}{MRS\,(y_2^1, y_2^2)} = \left\{\frac{\left[\left(y_2^2 - \lambda y_1\right)/\left(y_2^1 - \lambda y_1\right)\right]^{(1-\lambda)}}{y_2^2/y_2^1}\right\}^{\theta}. \tag{22}$$

The magnitude of the ratio relative to one is ambiguous: while (for the case of Figure 1):

$$\frac{\left(y_2^2 - \lambda y_1\right)}{\left(y_2^1 - \lambda y_1\right)} < \frac{y_2^2}{y_2^1}, \tag{23}$$

raising the left-hand side of (23) to the power of $(1-\lambda) < 1$ may reverse its direction.

Hence, the main moral of this section is that the habit and the power parameters (λ and α, respectively) are two alternative formalizations of risk aversion, which are observationally equivalent up to a first-order approximation. Any two time-separable utility functions, with or without a habit, *but with the same* AP-RRA, span a 'similar' map of indifference curves, and generate approximately the same equity premium. However, the curve that goes through the critical (y_2^1, y_2^2) point may be slightly steeper or slightly flatter (at that point) relative to the no-habit indifference curve. That may generate a certain effect on the equity-premium, which may be either negative or positive. This second-order effect is of minor quantitative significance, but for the canonical Mehra-Prescott set of parameters, it happens to be negative and thus exacerbates the equity-premium puzzle.

3 Infinite-horizon Model

It is well known that in a model without a habit, extending the time-horizon of the analysis (i.e. adding more periods) affects the price-earnings ratio but not the equity premium. As the analysis of the previous section concludes that a habit is just an alternative formulation of risk aversion, one is tempted to stipulate that extending the time horizon of a habit-formation model would have no effect on the equity premium. It turns out that this is not quite the case. As we shall see, the results in this section differ somewhat from those of the previous section, the main novelty being the dynamics of the habit itself. Although the effect on the equity premium is far from sufficient to resolve the puzzle, the improvement could be considered a modest progress towards a resolution; if only it did not create problems on other dimensions. As we shall see, the fluctuations in the habit are first-order observationally equivalent to a preference shock, which does raise the volatility of the risk-free rate in a manner that is inconsistent with the data.

3.1 A Model with an External Habit

Consider a representative agent with an 'external' habit:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t - z_t\right), \tag{24}$$

$$u\left(c_t - z_t\right) = \frac{1}{1-\alpha}\left(c_t - z_t\right)^{1-\alpha}, \quad z_t = \lambda c_{t-1}. \tag{25}$$

By external we mean that the agent does not internalize the effect of her selected level of consumption on the habit formed next period. A possible interpretation of this formalization is that the agent represents a wide (homogeneous) population where habits are determined by the population's mean-consumption level on which an individual decision-maker has no effect. Note that in such an economy, the Lucas formula still holds:

$$q_t = \beta E_t\left[\frac{u'\left(c_{t+1} - z_{t+1}\right)}{u'\left(c_t - z_t\right)}\left(q_{t+1} + y_{t+1}\right)\right]. \tag{26}$$

Substituting the price recursively and applying the law of iterative expectation, we can derive the Lucas formula in its usual form:

$$q_t = E_t \sum_{j=1}^{\infty}\left[\beta^j \frac{u'\left(c_{t+j} - z_{t+j}\right)}{u'\left(c_t - z_t\right)} y_{t+j}\right]. \tag{27}$$

The following no bubble condition must also hold:

$$\lim_{j\to\infty} E_t\left[\beta^j \frac{u'\left(c_{t+j} - z_{t+j}\right)}{u'\left(c_t - z_t\right)} y_{t+j}\right] = 0. \tag{28}$$

Equation (26), can be readily applied to price riskless debt (in zero net supply):

$$\frac{1}{1+r_t} = E_t\left[\beta \frac{u'(c_{t+1}-h_{t+1})}{u'(c_t-h_t)}\right]. \tag{29}$$

We derive a closed-form solution using a two-state growth process, $y_{t+1} = y_t\xi_{t+1}$, and denote the states as 'low' and 'high' (namely $\xi_{t+1} \in \{l, h\}$ respectively), with a probability of 1/2 for each state. Substituting the exchange-economy condition $c_t = y_t$ into equation (26) and using the homotheticity of preferences, we obtain:

$$q_t = y_t\beta E_t\left[\frac{u'(\xi_{t+1}-\lambda)}{u'\left(1-\frac{\lambda}{\xi_t}\right)}\left(\frac{q_{t+1}}{y_{t+1}}+1\right)\xi_{t+1}\right], \tag{30}$$

which can be expressed as a multiplicative function of the two state variables y_t and ξ_t:

$$q_t = y_t K(\xi_t). \tag{31}$$

Substituting (31) into (30) we obtain:

$$K(h) = \frac{\beta}{u'\left(1-\frac{\lambda}{h}\right)}(A+B), \tag{32}$$

$$K(l) = \frac{\beta}{u'\left(1-\frac{\lambda}{l}\right)}(A+B), \tag{33}$$

$$A = \frac{u'(h-\lambda)\,h + u'(l-\lambda)\,l}{2}, \tag{34}$$

$$B = \frac{u'(h-\lambda)\,h \cdot K(h) + u'(l-\lambda)\,l \cdot K(l)}{2}. \tag{35}$$

Substituting (32) and (33) into (35) we obtain:

$$B = (A+B)\,C, \quad C \equiv \frac{\beta}{2}\left[\frac{u'(h-\lambda)\,h}{u'\left(h-\frac{\lambda}{h}\right)} + \frac{u'(l-\lambda)\,l}{u'\left(l-\frac{\lambda}{l}\right)}\right], \tag{36}$$

which allows us to solve for B and thus for:

$$K(h) = \frac{\beta}{u'\left(1-\frac{\lambda}{h}\right)}\frac{A}{1-C}, \tag{37}$$

$$K(l) = \frac{\beta}{u'\left(1-\frac{\lambda}{l}\right)}\frac{A}{1-C}. \tag{38}$$

More specifically:

$$A \equiv \frac{1}{2}\left[\frac{h}{(h-\lambda)^{\alpha}} + \frac{l}{(l-\lambda)^{\alpha}}\right], \tag{39}$$

$$C \equiv \frac{\beta}{2}\left[h^{1-\alpha} + l^{1-\alpha}\right]. \tag{40}$$

The expected rate of return on equity is thus:

$$E(1+R_t) = E\left(\frac{q_t + y_t}{q_{t-1}}\right) = E\left[\frac{\xi_t\left[\beta\left(1-\frac{\lambda}{\xi_t}\right)^{\alpha}\frac{A}{1-C}+1\right]}{\beta\left(1-\frac{\lambda}{\xi_{t-1}}\right)^{\alpha}\frac{A}{1-C}}\right], \tag{41}$$

where R_t is the rate of return on equity. Taking unconditional expectations over the four possible realizations of ξ_t and ξ_{t-1}, we compute the unconditional expected rate of return on equity in an artificial economy:

$$\begin{aligned} E(1+R_t) &= \frac{1-C}{4A}\left\{\left[h\left(\frac{h-\lambda}{h}\right)^{\alpha} + l\left(\frac{l-\lambda}{l}\right)^{\alpha}\right]\frac{A}{1-C} + 2(1+g)\right\} \times \\ &\quad \left[\left(\frac{h}{h-\lambda}\right)^{\alpha} + \left(\frac{l}{l-\lambda}\right)^{\alpha}\right]. \end{aligned} \tag{42}$$

We can also solve for the risk-free rate as:

$$\frac{1}{1+r_t} = \left(1-\frac{\lambda}{\xi_t}\right)^{\alpha}\frac{\beta}{2}\left[\frac{1}{(h-\lambda)^{\alpha}} + \frac{1}{(l-\lambda)^{\alpha}}\right]. \tag{43}$$

Taking unconditional expectations over the two possible realizations of ξ_t we get the expected risk-free rate in the model economy:

$$E\left(1+r_t\right) = \frac{1}{\beta}\left[\left(\frac{h}{h-\lambda}\right)^{\alpha} + \left(\frac{l}{l-\lambda}\right)^{\alpha}\right]\left[\frac{1}{(h-\lambda)^{\alpha}} + \frac{1}{(l-\lambda)^{\alpha}}\right]^{-1}. \tag{44}$$

We simulate the model with the same Mehra-Prescott parameters used in the previous section, namely $g = 0.018$, $h = (1+g) + 0.036$, and $l = (1+g) - 0.036$. The no bubble condition (28) can be written as $\frac{1}{2}\beta(h^{1-\alpha} + l^{1-\alpha}) < 1$, which implies that no equilibrium exists for some parameter values. As before, when we increase the habit parameter, we adjust the power so as to preserve a constant AP-RRA. However, in the dynamic setting, the AP-RRA varies over time according to the realization of the growth rate:

$$\theta_t = \alpha\frac{\xi_t}{\xi_t - \lambda}. \tag{45}$$

We thus impose on the $\alpha - \lambda$ parameters the constraint that the *mean* AP-RRA:

$$E(\theta) = \frac{\alpha}{2}\left(\frac{h}{h-\lambda} + \frac{l}{l-\lambda}\right), \tag{46}$$

remains constant over different levels of the habit. We set the mean AP- RRA to either one or three. We use βs of either 0.99 or 0.95. The results are presented in Tables 2 and 3, and seem to indicate some improvement in the model's predictive power, although too small to resolve the equity-premium puzzle.

In the previous section we have argued that the power and the habit are two alternative formulations of risk aversion, which are observationally equivalent up to a first-order approximation.

Thus, the modest improvement in the model's predictive power deserves an explanation. We derive a first-order approximation of the premium, similar to the one we derived in the previous section. By moving q_t to the other side of equation (26):

$$1 = \beta E_t \left[\frac{u'(y_{t+1} - \lambda y_t)}{u'(y_t - \lambda y_{t-1})} (1 + R_{t+1}) \right]. \tag{47}$$

Taking similar steps to the previous section we obtain:

$$E(R_{t+1} - r_t) \approx \theta \cdot COV(\xi_{t+1}, R_{t+1}). \tag{48}$$

or:

$$E(R_{t+1} - r_t) \approx \theta \cdot COV \left\{ \xi_{t+1}, \xi_{t+1} \left[\frac{1 + K(\xi_{t+1})}{K(\xi_t)} \right] \right\}. \tag{49}$$

Evidently, (and given that, typically, the equilibrium $K(\xi_t)$, the price-dividend ratio, is much larger than 1), it is not the habit *per se* that generates the modest improvement in the model's performance but its fluctuations over time. Without these fluctuations, the price-dividend ratio would remain constant over time (see equations (31), (32), and (33)), and the model's predicted equity premium would remain very close to that which is obtained in a two-period model, namely the AP-RRA times the variance of the growth rate.

To better appreciate this point, we express equation (49) as follows:

$$E(R_{t+1} - r_t) \approx \theta \cdot COV \left\{ \xi_{t+1}, \left[\frac{\xi_{t+1}}{K(\xi_t)} \right] \right\} + \theta \cdot COV \left\{ \xi_{t+1}, \xi_{t+1} \left[\frac{K(\xi_{t+1})}{K(\xi_t)} \right] \right\}, \tag{50}$$

or, using equations (30), and (31) as:

$$E(R_{t+1} - r_t) \approx \theta \cdot COV \left\{ \xi_{t+1}, \left[\frac{\xi_{t+1}}{K(\xi_t)} \right] \right\} + \theta \cdot COV \left\{ \xi_{t+1}, \xi_{t+1} \left[\frac{\frac{1}{u'(1-\frac{1}{\xi_{t+1}})}}{\frac{1}{u'(1-\frac{\lambda}{\xi_t})}} \right] \right\}. \tag{51}$$

By further approximations, one may derive:

$$E(R_{t+1} - r_t) \approx \frac{\theta}{K(\xi_t)} \sigma^2 + \theta \left(1 + \theta \frac{\lambda}{1 - \lambda} \right) \sigma^2. \tag{52}$$

The first term on the right-hand side is the dividend uncertainty priced according to the AP-RRA. As we have already shown in the previous section, the small magnitude of this term is the very source of the equity premium puzzle. The problem is exacerbated once that magnitude is divided by the price-dividend ratio — typically a number much bigger than one. The second term is the capital-gains uncertainty resulting from the fluctuations in the price-dividend ratio, which is also priced according to the AP-RRA. As noted above, when the habit is zero, that expression collapses again to $\theta \cdot \sigma^2$, but non-zero habits would have a first-order effect on the equity premium.

Similar to the argument made in the previous section, we argue that the fluctuations in the price-dividend ratio are observationally equivalent to some other, well-known, factor. Indeed, the similarity between habit fluctuations and a (transitory) preference shock is illuminating. This similarity can be identified first on a formal level. Consider a general homothetic Von-Neuman-Morgenstern utility function $v(c_t, \epsilon_t)$ where ϵ_t is a *transitory* preference shock (with a positive partial derivative). Then, following the same steps as above, one may verify that the equity premium can be approximated by:

$$E(R_{t+1} - r_t) \approx \theta \cdot COV \left\{ \xi_{t+1}, \left[\frac{\xi_{t+1}}{K(\xi_t)} \right] \right\} + \theta \cdot COV \left\{ \xi_{t+1}, \xi_{t+1} \left[\frac{\frac{1}{v'(1,\epsilon_{t+1})}}{\frac{1}{v'(1,\epsilon_t)}} \right] \right\}. \tag{53}$$

Equation (53) may be thought of as a special case of equation (51), with ξ_t playing the same technical role as ϵ_t. Moreover, it is possible to calibrate the preference shock such that it would be observationally equivalent to the habit, up to a first-order approximation. The only characteristic that may distinguish the habit from a general preference shock is its 'built in' perfect correlation with the one-period lagged output shock. However, the habit — like many other preference shocks that have been tried before — cannot resolve the equity premium puzzle. The reason is that in order to achieve even the modest improvement shown in Tables 2 and 3, the generated fluctuations in the risk-free rate are already of an implausible magnitude.[3]

It might help to provide a more intuitive explanation of the similarity between a preference-shock and the habit. A representative individual who faces a temporary increase in her marginal utility of consumption, would try to move consumption from the future to the present. In an exchange economy like ours, no such substitution is possible, and the shock must affect asset prices alone. Thus, a simultaneous attempt by all agents to borrow against future income would increase the interest rate up to the point that agents are no longer interested in substituting present for future consumption. Equivalently, a negative realization of the growth rate would take the agent closer to her habit and increase the marginal utility of consumption. Crucially, the agent knows that this *increase in the marginal utility of consumption is temporary,* because soon enough she would get used to the lower level of consumption. Hence, it is only due to the temporary nature of the effect that she tries to borrow against future income.[4]

To summarize, the modest improvement in the model's predictive power is due to the fluctuations in the habit rather than the habit itself. The fluctuations in the habit operate in a manner that is very similar to a preference shock. Like a preference shock, the improvement is due to the greater risk in the capital-gains component of asset returns, and like a preference shock the improvement is obtained at a cost of violating the observed variability of the risk-free rate.

[3]This is a point that goes back to Hansen and Jaganathan [1991] who measures the variability in the IMRS necessary to resolve the equity-premium puzzle. This amounts to increasing the variability in the risk-free rate.

[4]Note that a negative autocorrelation in the output process, as is actually observed in the data, tends to alter the effects somewhat since the shocks have some persistence. It is important to realize that in the end, one will still have the same problem with matching the equity premium, see for example Boldrin, Christiano and Fisher [1997], and Chapman [2002].

3.2 A Model with Internal Habit Formation

For the sake of completeness, we also include the case of an 'internal habit', which is very similar to the external case discussed above. By 'internal habit' we mean that the agent internalizes the effect of her selected level of consumption on the habit formed next period; note however that the agent is still a price taker with respect to the risk-free rate and equity prices. Consider a representative agent with an 'internal' habit:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t - \lambda c_{t-1}\right), \tag{54}$$

$$u\left(c_t - \lambda c_{t-1}\right) = \frac{1}{1-\alpha}\left(c_t - \lambda c_{t-1}\right)^{1-\alpha}. \tag{55}$$

The Lucas formula still holds as follows:

$$q_t = \beta E_t \left[\frac{u'\left(c_{t+1} - \lambda c_t\right) + \beta E_{t+1} u'\left(c_{t+2} - \lambda c_{t+1}\right)}{u'\left(c_t - \lambda c_{t-1}\right) + \beta E_t u'\left(c_{t+1} - \lambda c_t\right)} (q_{t+1} + y_{t+1}) \right]. \tag{56}$$

Substituting the price recursively and applying the law of iterative expectations, we can derive the Lucas formula in its usual form:

$$q_t = E_t \sum_{j=1}^{\infty} \left[\beta^j \frac{u'\left(c_{t+j} - \lambda c_{t+j-1}\right) + \beta E_{t+1} u'\left(c_{t+j+1} - \lambda c_{t+j}\right)}{u'\left(c_t - \lambda c_{t-1}\right) + \beta E_t u'\left(c_{t+1} - \lambda c_t\right)} y_{t+j} \right]. \tag{57}$$

The following no bubble condition must also hold:

$$\lim_{j\to\infty} E_t \left[\beta^j \frac{u'\left(c_{t+j} - \lambda c_{t+j-1}\right) + \beta E_{t+1} u'\left(c_{t+j+1} - \lambda c_{t+j}\right)}{u'\left(c_t - \lambda c_{t-1}\right) + \beta E_t u'\left(c_{t+1} - \lambda c_t\right)} y_{t+j} \right] = 0. \tag{58}$$

This no bubble condition can be rewritten as $\frac{1}{2}\beta(u^{1-\alpha} + d^{1-\alpha}) < 1$. The formula, equation (56), can be applied to price riskless debt (in zero net supply):

$$\frac{1}{1+r_t} = \beta E_t \left[\frac{u'\left(c_{t+1} - \lambda c_t\right) + \beta E_{t+1} u'\left(c_{t+2} - \lambda c_{t+1}\right)}{u'\left(c_t - \lambda c_{t-1}\right) + \beta E_t u'\left(c_{t+1} - \lambda c_t\right)} \right]. \tag{59}$$

Substituting $c_t = y_t$ and $y_{t+1} = y_t \xi_{t+1}$, into (56) and using the homotheticity of preferences, we obtain:

$$q_t = y_t \beta E_t \left[\xi_t^{-\alpha} \frac{u'\left(\xi_{t+1} - \lambda\right) - \beta\lambda\xi_{t+1}^{-\alpha} E_{t+1} u'\left(\xi_{t+2} - \lambda\right)}{u'\left(\xi_t - \lambda\right) - \beta\lambda\xi_t^{-\alpha} E_t u'\left(\xi_{t+1} - \lambda\right)} \left(\frac{q_{t+1}}{y_{t+1}} + 1\right)\xi_{t+1} \right]. \tag{60}$$

Like before:

$$q_t = y_t K\left(\xi_t\right), \tag{61}$$

Substituting (61) into (60) we obtain:

$$K\left(h\right) = \frac{\beta h^{-\alpha}}{(h-\lambda)^{-\alpha} - \beta\lambda h^{-\alpha} D}\left(A + B\right), \tag{62}$$

$$K(l) = \frac{\beta l^{-\alpha}}{(l-\lambda)^{-\alpha} - \beta\lambda l^{-\alpha} D} (A+B), \tag{63}$$

$$A \equiv \frac{1}{2}\left[(h-\lambda)^{-\alpha} h - \beta\lambda h^{1-\alpha} D + (l-\lambda)^{-\alpha} l - \beta\lambda l^{1-\alpha} D\right], \tag{64}$$

$$B \equiv \frac{1}{2}\left[(h-\lambda)^{-\alpha} h K(h) - \beta\lambda h^{1-\alpha} D K(h) + (l-\lambda)^{-\alpha} l K(l) - \beta\lambda l^{1-\alpha} D K(l)\right], \tag{65}$$

and:

$$D \equiv \frac{1}{2}\left[(h-\lambda)^{-\alpha} + (l-\lambda)^{-\alpha}\right]. \tag{66}$$

Substituting (62) and (63) into (65) we obtain:

$$B \equiv (A+B)\,C, \quad C \equiv \frac{\beta}{2}\left[h^{1-\alpha} + l^{1-\alpha}\right], \tag{67}$$

which allows us to solve for B and thus for:

$$K(h) = \frac{\beta h^{-\alpha}}{(h-\lambda)^{-\alpha} - \beta\lambda h^{-\alpha} D}\frac{A}{1-C}, \tag{68}$$

$$K(l) = \frac{\beta l^{-\alpha}}{(l-\lambda)^{-\alpha} - \beta\lambda l^{-\alpha} D}\frac{A}{1-C}. \tag{69}$$

The expected rate of return on equity is thus:

$$E(1+R_t) = E\left[\frac{q_t + y_t}{q_{t-1}}\right] = E\left[\xi_t \frac{K(\xi_t)+1}{K(\xi_{t-1})}\right], \tag{70}$$

where R_t is the rate of return on equity. Taking unconditional expectations over the four possible realizations of ξ_t and ξ_{t-1} we obtain the unconditional expected return to the stock. We can also solve for the risk-free rate:

$$\frac{1}{1+r_t} = \frac{\beta\xi_t^{-\alpha}}{(\xi_t-\lambda)^{-\alpha} - \beta\lambda D\xi_t^{-\alpha}}\left(D - \beta\lambda D E_t(\xi_{t+1}^{-\alpha})\right). \tag{71}$$

Taking unconditional expectations over the two possible realizations of ξ_t we obtain the unconditional expected risk-free rate in the model economy:

$$E(1+r_t) = \frac{1}{2\beta}\left[\left(\frac{h}{h-\lambda}\right)^{\alpha} + \left(\frac{l}{l-\lambda}\right)^{\alpha} - 2\lambda D\right]\left[\frac{1}{D - \beta\lambda D E_t(\xi_{t+1}^{-\alpha})}\right]. \tag{72}$$

We simulate the model for the same parameter values as above. Since the AP-RRA varies according to the realization of GNP growth rate, the constraint which is imposed on the power and the habits is:

$$\theta_t = \xi_t \frac{\alpha(\xi_t-\lambda)^{-\alpha-1} + \beta\alpha\lambda^2 G\xi_t^{-\alpha-1}}{(\xi_t-\lambda)^{-\alpha} - \beta\lambda D\xi_t^{-\alpha}}, \tag{73}$$

where $G = E_{t+2}((\xi_{t+1} - \lambda)^{-\alpha-1})$. We set the mean AP-RRA to either one or three. We use βs of either 0.99 or 0.95. The results are presented in Tables 4 and 5. Evidently, the improvement due to the habit is even more modest than before.

4 Conclusion

In this paper we articulate a skeptical argument regarding the effectiveness of the habit-formation hypothesis in resolving the equity-premium puzzle. Essentially, we argue that habit formation is observationally equivalent, up to a first-order approximation, to some other effects that were already shown to be insufficient in resolving the equity-premium puzzle.

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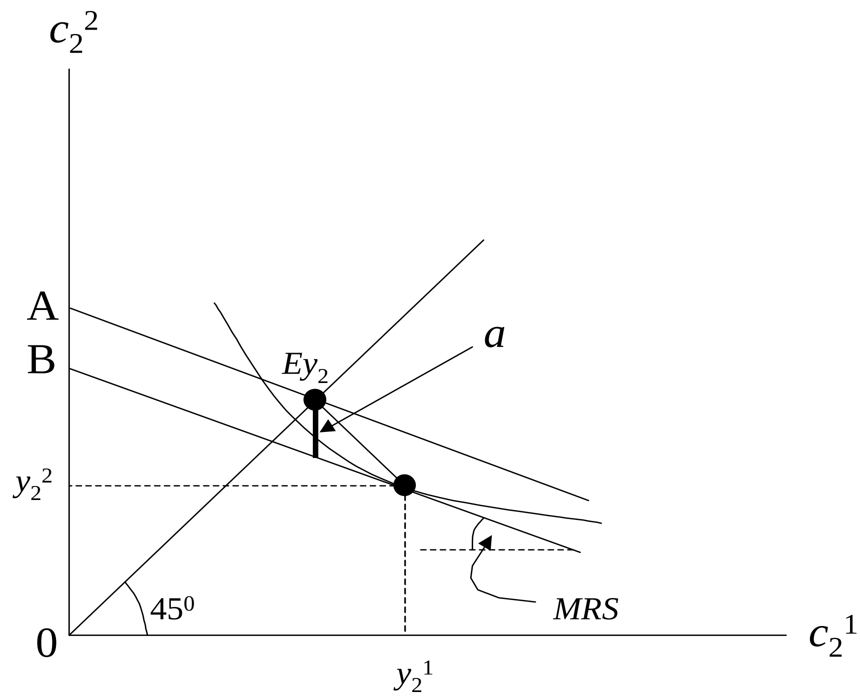
Figure 1



Tables

Table 1: Expected Returns and Equity Premium

	$\theta = 1$			$\theta = 3$		
λ	$E(R-r)$	$E(R)$	E(r)	$E(R-r)$	$E(R)$	$E(r)$
0	0.125%	2.83%	2.70%	0.375%	6.16%	5.77%
0.5	0.123%	2.75%	2.63%	0.369%	5.94%	5.55%
0.8	0.118%	2.55%	2.43%	0.354%	5.34%	4.97%
0.9	0.112%	2.26%	2.15%	0.334%	4.49%	4.14%
0.95	0.104%	1.80%	1.69%	0.313%	3.12%	2.80%
0.965	0.103%	1.45%	1.34%	0.308%	2.06%	1.75%

Notes : In this table we calculate the unconditional expected returns and equity premium for consumption growth with $\xi = 1.018 \pm 0.036$, each with a probability of 50%. We set $\beta = 0.99$ and $\theta = 1$ and 3. We adjust α so that as we change λ, θ remains constant.

Table 2: Expected Returns and Equity Premium
External Habit Persistence, $E(\theta) = 3$

$\beta = 0.99$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.397%	6.16%	5.77%	3.75%	0%	3.00
0.2	0.618%	5.34%	4.72%	6.25%	2.19%	2.41
0.4	.840%	4.57%	3.73%	9.07%	4.33%	1.82
0.6	1.056%	3.85%	2.79%	11.97%	6.42%	1.23
0.7	1.166%	3.51%	2.34%	13.41%	7.43%	0.929
0.8	1.261%	3.16%	1.90%	14.80%	8.39%	0.629
0.9	non-existence					

$\beta = 0.95$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.414%	10.63%	10.22%	3.91%	0%	3.00
0.2	0.635%	9.77%	9.13%	6.43%	2.28%	2.41
0.4	0.856%	8.95%	8.10%	9.29%	4.51%	1.82
0.6	1.076%	8.19%	7.12%	12.25%	6.69%	1.23
0.7	1.18%	7.83%	6.65%	13.72%	7.74%	0.929
0.8	1.277%	7.46%	6.19%	15.13%	8.74%	0.629
0.9	1.29%	7.02%	5.73%	16.07%	9.40%	0.319
0.95	1.044%	6.53%	5.48%	14.89%	8.57%	0.147

Notes : In this table we calculate the unconditional expected returns and equity premium. We have consumption growth, $\xi = 1.018 \pm 0.036$, each with a probability of 50%. We adjust α so that as we change λ, $E(\theta)$ remains constant. Non-existence means the no bubble condition is violated for that parameterization.

Table 3: Expected Returns and Equity Premium
External Habit Persistence, $E(\theta) = 1$

$\beta = 0.99$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.129%	2.828%	2.700%	3.63%	0%	1.000
0.2	0.153%	2.508%	2.36%	4.39%	0.71%	0.803
0.4	0.178%	2.194%	2.016%	5.23%	1.42%	0.606
0.6	non-existence					
0.7	non-existence					
0.8	non-existence					
0.9	non-existence					
$\beta = 0.95$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.134%	7.158%	7.024%	3.79%	0%	1.000
0.2	0.159%	6.823%	6.665%	4.55%	0.74%	0.803
0.4	0.183%	6.494%	6.311%	5.39%	1.48%	0.606
0.6	0.2072%	6.171%	5.964%	6.28%	2.21%	0.409
0.7	0.219%	6.01%	5.79%	6.73%	2.56%	0.310
0.8	0.2283%	5.847%	5.619%	7.16%	2.90%	0.210
0.9	0.2263%	5.67%	5.44%	7.46%	3.13%	0.106

Notes : In this table we calculate the unconditional expected returns and equity premium. We have consumption growth, $\xi = 1.018 \pm 0.036$, each with a probability of 50%. We adjust α so that as we change λ, $E(\theta)$ remains constant. Non-existence means the no bubble condition is violated for that parameterization.

Table 4: Expected Returns and Equity Premium
Internal Habit Persistence, $E(\theta) = 3$

$\beta = 0.99$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.397%	6.164%	5.767%	3.75%	0%	3.00
0.2	0.493%	4.42%	3.95%	6.1%	2.1%	1.87
0.4	0.512%	3.02%	2.51%	8.21%	3.70%	0.96
0.6		non-existence				
0.7		non-existence				
0.8		non-existence				
0.9		non-existence				

$\beta = 0.95$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.414%	10.634%	10.221%	3.9%	0%	3.00
0.2	0.515%	8.89%	8.37%	6.3%	2.2%	1.90
0.4	0.533%	7.40%	6.87%	8.4%	5.6%	0.99
0.6	0.523%	6.43%	5.91%	9.9%	5.0%	0.40
0.7	0.527%	6.14%	5.62%	10.48%	5.39%	0.22
0.8	0.539%	5.96%	5.42%	11.0%	5.7%	0.10
0.9	0.566%	5.88%	5.31%	11.4%	6.1%	0.03

Notes : In this table we calculate the unconditional expected returns and equity premium. We have consumption growth, $\xi = 1.018 \pm 0.036$, each with a probability of 50%. We adjust α so that as we change λ, $E(\theta)$ remains constant. Non-existence means the no bubble condition is violated for that parameterization.

Table 5: Expected Returns and Equity Premium
Internal Habit Persistence, $E(\theta) = 1$

$\beta = 0.99$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.129%	2.828%	2.700%	3.6%	0%	1.00
0.2	0.112%	2.181%	2.058%	4.4%	0.7%	0.62
0.4	non-existence					
0.6	non-existence					
0.7	non-existence					
0.8	non-existence					
0.9	non-existence					

$\beta = 0.95$						
λ	$E(R-r)$	$E(R)$	$E(r)$	σ_R	σ_r	α
0	0.134%	7.158%	7.024%	3.79%	0%	1.00
0.2	0.129%	6.498%	6.369%	4.5%	0.8%	0.63
0.4	0.1179%	5.96%	5.84%	5.1%	1.3%	0.33
0.6	0.108%	5.60%	5.49%	5.6%	1.7%	0.13
0.7	0.103%	5.99%	5.39%	5.7%	1.8%	0.07
0.8	0.098%	5.42%	5.32%	5.7%	1.8%	0.03
0.9	0.097%	5.38%	5.28%	5.8%	1.8%	0.01

Notes : In this table we calculate the unconditional expected returns and equity premium. We have consumption growth, $\xi = 1.018 \pm 0.036$, each with a probability of 50%. We adjust α so that as we change λ, $E(\theta)$ remains constant. Non-existence means the no bubble condition is violated for that parameterization.