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Hadamard-Type Inequalities for s-Convex Functions

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Abstract

In this paper a Hadamard's type inequalities for s–convex functions in both sense and s–convex functions on the co–ordinates are given.

Keywords: Hadamard's inequality, s–convex function, co–ordinated s–convex function

1 Introduction

Let $f : I \subseteq \mathbf{R} \to \mathbf{R}$ be a convex mapping defined on the interval I of real numbers and $a, b \in I$, with $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{2} \tag{1}$$

is known in the literature as Hadamard's inequality for convex mappings.

In [7] Dragomir and Fitzpatrick proved a variant of Hadamard's inequality which holds for s–convex functions in the first sense.

Theorem 1.1 *Suppose that $f : [0,\infty) \to [0,\infty)$ is an s–convex function in the first sense, where $s \in (0,1)$ and let $a, b \in [0,\infty)$, $a < b$. If $f \in L^1[0,1]$, then the following inequalities hold:*

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+sf(b)}{s+1}. \tag{2}$$

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The above inequalities are sharp.

Also, In [7], Dragomir and Fitzpatrick proved a variant of Hadamard's inequality which holds for s–convex functions in the second sense.

Theorem 1.2 *Suppose that $f:[0,\infty)\to[0,\infty)$ is an s–convex function in the second sense, where $s\in(0,1)$ and let $a,b\in[0,\infty)$, $a<b$. If $f\in L^1[0,1]$, then the following inequalities hold:*

$$2^{s-1}f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)\,dx\leq\frac{f(a)+f(b)}{s+1} \tag{3}$$

the constant $k=\frac{1}{s+1}$ is the best possible in the second inequality in (3). The above inequalities are sharp.

After that, in [8], Dragomir established the following similar inequality of Hadamard's type for co–ordinated convex mapping on a rectangle from the plane $\mathbf{R}^2$.

Theorem 1.3 *Suppose that $f:\Delta\to\boldsymbol{R}$ is co-ordinated convex on Δ. Then one has the inequalities*

$$\begin{aligned} f\left(\frac{a+b}{2},\frac{c+d}{2}\right) &\leq \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)\,dydx \\ &\leq \frac{f(a,c)+f(a,d)+f(b,c)+f(b,d)}{4} \end{aligned} \tag{4}$$

The above inequalities are sharp.

In [2], M. Alomari and M. Darus established the following similar inequality of Hadamard–type for co–ordinated s–convex mapping in the second sense on a rectangle from the plane $\mathbf{R}^2$.

Theorem 1.4 *Suppose that $f:\Delta=[a,b]\times[c,d]\subseteq[0,\infty)^2\to[0,\infty)$ is s–convex function on the co–ordinates on Δ. Then one has the inequalities:*

$$\begin{aligned} f\left(\frac{a+b}{2},\frac{c+d}{2}\right) &\leq \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)\,dydx \qquad (5)\\ &\leq \frac{f(a,c)+f(b,c)+f(a,d)+f(b,d)}{(s+1)^2}. \end{aligned}$$

Also, In [5], M. Alomari and M. Darus established the following similar inequality of Hadamard–type for co–ordinated s–convex mapping in the first sense on a rectangle from the plane $\mathbf{R}^2$.

Theorem 1.5 *Suppose that $f : \Delta = [a,b] \times [c,d] \subseteq [0,\infty)^2 \to [0,\infty)$ is s–convex function on the co–ordinates in the first sense on Δ. Then one has the inequalities:*

$$
\begin{aligned}
f\left(\frac{a+b}{2}, \frac{c+d}{2}\right) &\leq \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x,y)\,dydx \qquad (6) \\
&\leq \frac{f(a,c) + sf(b,c) + sf(a,d) + s^2 f(b,d)}{(s+1)^2}.
\end{aligned}
$$

The above inequalities are sharp.

In this paper we will point out a Hadamard–type inequalities of s–convex function on the co–ordinates in the both sense.

For refinements, counterparts, generalizations and new Hadamard's–type inequalities see [1–8].

2 Remarks On A Previous Results

In this section we give some remarks on a previous results for the authors. The following lemma associated with s–convex function (of second sense) was considered by Alomari and Darus in [3].

Lemma 2.1 *Let $f : [a,b] \to \boldsymbol{R}$ be a s–convex function (of second sense). Let $a \leq y_1 \leq x_1 \leq x_2 \leq y_2 \leq b$ with $x_1 + x_2 = y_1 + y_2$. Then*

$$
f(x_1) + f(x_2) \leq f(y_1) + f(y_2). \qquad (7)
$$

Actually, the proof was given in [3] of this property is correct for convex functions but not for s–convex functions. The correction of this proof is given as follows:

Proof. Firstly, we show that $f(x_1) + f(x_2) \leq f(y_1) + f(y_2)$. If $y_1 = y_2$ then we are done. Suppose $y_1 \neq y_2$ and since f is s–convex function of second sense, then for $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$ and for all $0 < s \leq 1$, we have

$$
x_1 = \left(\frac{y_2 - x_1}{y_2 - y_1}\right)^{\frac{1}{s}} y_1 + \left(\frac{x_1 - y_1}{y_2 - y_1}\right)^{\frac{1}{s}} y_2, \ \ x_2 = \left(\frac{y_2 - x_2}{y_2 - y_1}\right)^{\frac{1}{s}} y_1 + \left(\frac{x_2 - y_1}{y_2 - y_1}\right)^{\frac{1}{s}} y_2
$$

without loss of generality, set

$$k_1 = \left(\frac{y_2 - x_1}{y_2 - y_1}\right)^{\frac{1}{s}}, \ k_2 = \left(\frac{x_1 - y_1}{y_2 - y_1}\right)^{\frac{1}{s}}, \ k_3 = \left(\frac{y_2 - x_2}{y_2 - y_1}\right)^{\frac{1}{s}}, \ k_4 = \left(\frac{x_2 - y_1}{y_2 - y_1}\right)^{\frac{1}{s}}$$

such that, $\gamma = k_1^s + k_2^s + k_3^s + k_4^s > 0$; $\alpha = \frac{k_1^s + k_3^s}{\gamma}$ and $\beta = \frac{k_2^s + k_4^s}{\gamma}$. Therefore, $\alpha + \beta = 1$ and by s–convexity, we have

$$\begin{aligned} f(x_1) + f(x_2) &= f\left(\left(\frac{y_2 - x_1}{y_2 - y_1}\right)^{\frac{1}{s}} y_1 + \left(\frac{x_1 - y_1}{y_2 - y_1}\right)^{\frac{1}{s}} y_2\right) \\ &\quad + f\left(\left(\frac{y_2 - x_2}{y_2 - y_1}\right)^{\frac{1}{s}} y_1 + \left(\frac{x_2 - y_1}{y_2 - y_1}\right)^{\frac{1}{s}} y_2\right) \\ &\leq \frac{y_2 - x_1}{y_2 - y_1} f(y_1) + \frac{x_1 - y_1}{y_2 - y_1} f(y_2) \qquad (8) \\ &\quad + \frac{y_2 - x_2}{y_2 - y_1} f(y_1) + \frac{x_2 - y_1}{y_2 - y_1} f(y_2) \\ &= \frac{2y_2 - (x_1 + x_2)}{y_2 - y_1} f(y_1) + \frac{(x_1 + x_2) - 2y_1}{y_2 - y_1} f(y_2) \\ &= f(y_1) + f(y_2). \end{aligned}$$

which completes the proof.

Also, by looking deeply on Theorem 1.4, we find the left side of inequality (5) is incorrect. The correction of Theorem 1.4. pointed out as follows :

Theorem 2.2 *Suppose that $f : \Delta = [a, b] \times [c, d] \subseteq [0, \infty)^2 \to [0, \infty)$ is s–convex function on the co–ordinates on Δ. Then one has the inequalities:*

$$\begin{aligned} 4^{s-1} f\left(\frac{a+b}{2}, \frac{c+d}{2}\right) &\leq \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x, y)\, dy dx \qquad (9) \\ &\leq \frac{f(a, c) + f(b, c) + f(a, d) + f(b, d)}{(s+1)^2}. \end{aligned}$$

Indeed, the difference between (5) and (9) is the left hand side, therefore we will give the proof of the left hand side only, to see the proof of the right hand side see [2].

Proof. Since $f : \Delta \to \mathbf{R}$ is co–ordinated s–convex on Δ it follows that the mapping $g_x : [c, d] \to [0, \infty)$, $g_x(y) = f(x, y)$ is s–convex on $[c, d]$ for all

$x \in [a,b]$. Then, by (3) one has:

$$2^{s-1} g_x\left(\frac{c+d}{2}\right) \leq \frac{1}{d-c}\int_c^d g_x(y)\,dy \leq \frac{g_x(c)+g_x(d)}{s+1}, \quad \forall x \in [a,b].$$

That is,

$$2^{s-1} f\left(x,\frac{c+d}{2}\right) \leq \frac{1}{d-c}\int_c^d f(x,y)\,dy \leq \frac{f(x,c)+f(x,d)}{s+1}, \quad \forall x \in [a,b].$$

Integrating this inequality on $[a,b]$, we have

$$\begin{aligned} \frac{2^{s-1}}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx &\leq \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)\,dydx \\ &\leq \frac{1}{s+1}\left[\frac{1}{b-a}\int_a^b f(x,c)\,dx + \frac{1}{b-a}\int_a^b f(x,d)\,dx\right]. \end{aligned} \tag{10}$$

A similar arguments applied for the mapping $g_y : [a,b] \to [0,\infty)$, $g_y(x) = f(x,y)$, we get

$$\begin{aligned} \frac{2^{s-1}}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy &\leq \frac{1}{(d-c)(b-a)}\int_c^d\int_a^b f(x,y)\,dxdy \\ &\leq \frac{1}{s+1}\left[\frac{1}{d-c}\int_c^d f(a,y)\,dy + \frac{1}{d-c}\int_c^d f(b,y)\,dy\right]. \end{aligned} \tag{11}$$

Summing the inequalities (10) and (11), we get the second and the third inequalities in (9).

Therefore, by (3), we have

$$4^{s-1} f\left(\frac{a+b}{2},\frac{c+d}{2}\right) \leq \frac{2^{s-1}}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy \tag{12}$$

and

$$4^{s-1} f\left(\frac{a+b}{2},\frac{c+d}{2}\right) \leq \frac{2^{s-1}}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx \tag{13}$$

which give, by addition the first inequality in (9).

The definition of s–convex function (in both sense) in a rectangle from the plane, was defined by Alomari and Darus in [4]. In the next section some Hadamard–type inequalities are considered.

3 Some Hadamard–Type Inequalities

Consider the bidimensional interval $\Delta := [a,b] \times [c,d]$ in $[0,\infty)^2$ with $a < b$ and $c < d$. A mapping $f : \Delta \to \mathbf{R}$ is called *s–convex of first sense* on Δ if there exists $s_1, s_2 \in (0,1]$ with $s = \frac{s_1+s_2}{2}$, such that

$$f(\alpha x + \beta z, \alpha y + \beta w) \leq \alpha^{s_1} f(x,y) + \beta^{s_2} f(z,w) \tag{14}$$

holds for all $(x,y),(z,w) \in \Delta$, $\alpha, \beta \geq 0$ with $\alpha^{s_1} + \beta^{s_2} = 1$ and for all fixed $s_1, s_2 \in (0,1]$. We denote this class of functions by $MWO^1_{s_1,s_2}$.

Let $f : \Delta \to \mathbf{R}$ be s–convex on Δ, then f is called co–ordinated s–convex of first sense on Δ if the partial mappings $f_y : [a,b] \to \mathbf{R}$, $f_y(u) = f(u,y)$ and $f_x : [c,d] \to \mathbf{R}$, $f_x(v) = f(x,v)$, are s_1–, s_2–convex functions in the first sense for all $s_1, s_2 \in (0,1]$, $y \in [c,d]$ and $x \in [a,b]$; respectively, with $s = \frac{s_1+s_2}{2} \in (0,1]$.

Also, a mapping $f : \Delta \to \mathbf{R}$ is called *s–convex of second sense* on Δ if there exists $s_1, s_2 \in (0,1]$ with $s = \frac{s_1+s_2}{2}$, such that (14) holds for all $(x,y),(z,w) \in \Delta$, $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$ and for all fixed $s_1, s_2 \in (0,1]$. We denote this class of functions by $MWO^2_{s_1,s_2}$.

Similarly, we define the s–convex function of second sense on the co–ordinates, *i.e.,* a function f is called co–ordinated s–convex of second sense on Δ if the partial mappings $f_y : [a,b] \to \mathbf{R}$, $f_y(u) = f(u,y)$ and $f_x : [c,d] \to \mathbf{R}$, $f_x(v) = f(x,v)$, are s_1–, s_2–convex functions in the second sense for all $s_1, s_2 \in (0,1]$, $y \in [c,d]$ and $x \in [a,b]$; respectively, with $s = \frac{s_1+s_2}{2} \in (0,1]$.

The following inequalities is considered as a Hadamard–type inequalities connected with inequality (14) for s–convex function in the second sense on the co–ordinates.

Theorem 3.1 *Suppose that $f : \Delta = [a,b] \times [c,d] \subseteq [0,\infty)^2 \to [0,\infty)$ is s–convex function of second sense on the co–ordinates on Δ. Then one has the inequalities:*

$$\begin{aligned}
&\frac{(4^{s_1-1} + 4^{s_2-1})}{2} f\left(\frac{a+b}{2}, \frac{c+d}{2}\right) \\
\leq\ & \frac{2^{s_1-2}}{b-a} \int_a^b f\left(x, \frac{c+d}{2}\right) dx + \frac{2^{s_2-2}}{d-c} \int_c^d f\left(\frac{a+b}{2}, y\right) dy \\
\leq\ & \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x,y)\, dy dx
\end{aligned} \tag{15}$$

$$
\begin{aligned}
&\leq \frac{1}{2(s_1+1)(b-a)}\int_a^b [f(x,c)+f(x,d)]\,dx \\
&\quad + \frac{1}{2(s_2+1)(d-c)}\int_c^d [f(a,y)+f(b,y)]\,dy \\
&\leq \frac{1}{2}\left(\frac{1}{(s_1+1)^2}+\frac{1}{(s_2+1)^2}\right)[f(a,c)+f(a,d)+f(b,c)+f(b,d)]
\end{aligned}
$$

The above inequalities are sharp.

Proof. Since $f:\Delta\to\mathbf{R}$ is co–ordinated s–convex on Δ it follows that the mapping $g_x:[c,d]\to[0,\infty)$, $g_x(y)=f(x,y)$ is s_1–convex on $[c,d]$ for all $x\in[a,b]$ with $s_1\in(0,1]$. Then by s–Hadamard's inequality (3) one has:

$$2^{s_1-1}g_x\left(\frac{c+d}{2}\right)\leq\frac{1}{d-c}\int_c^d g_x(y)\,dy\leq\frac{g_x(c)+g_x(d)}{s_1+1},\quad\forall x\in[a,b].$$

That is,

$$2^{s_1-1}f\left(x,\frac{c+d}{2}\right)\leq\frac{1}{d-c}\int_c^d f(x,y)\,dy\leq\frac{f(x,c)+f(x,d)}{s_1+1},\quad\forall x\in[a,b].$$

Integrating this inequality on $[a,b]$, we have

$$
\begin{aligned}
\frac{2^{s_1-1}}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx &\leq \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)\,dydx \qquad (16)\\
&\leq \frac{1}{s_1+1}\left[\frac{1}{b-a}\int_a^b f(x,c)\,dx+\frac{1}{b-a}\int_a^b f(x,d)\,dx\right].
\end{aligned}
$$

A similar arguments applied for the mapping $g_y:[a,b]\to[0,\infty)$, $g_y(x)=f(x,y)$, where, g_y is s_2–convex on $[a,b]$ for all $y\in[c,d]$ with $s_2\in(0,1]$

$$
\begin{aligned}
\frac{2^{s_2-1}}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy &\leq \frac{1}{(d-c)(b-a)}\int_c^d\int_a^b f(x,y)\,dxdy \qquad (17)\\
&\leq \frac{1}{s_2+1}\left[\frac{1}{d-c}\int_c^d f(a,y)\,dy+\frac{1}{d-c}\int_c^d f(b,y)\,dy\right].
\end{aligned}
$$

Summing the inequalities (16) and (17), we get the second and the third inequalities in (15).

Therefore, by (3), we have

$$\frac{4^{s_2-1}}{2} f\left(\frac{a+b}{2}, \frac{c+d}{2}\right) \leq \frac{2^{s_2-2}}{d-c} \int_c^d f\left(\frac{a+b}{2}, y\right) dy \tag{18}$$

and

$$\frac{4^{s_1-1}}{2} f\left(\frac{a+b}{2}, \frac{c+d}{2}\right) \leq \frac{2^{s_1-2}}{b-a} \int_a^b f\left(x, \frac{c+d}{2}\right) dx \tag{19}$$

which give, by addition the first inequality in (15).

Finally, by the same inequality we can also state:

$$\frac{1}{b-a} \int_a^b f(x,c)\, dx \leq \frac{f(a,c)+f(b,c)}{s_1+1}$$
$$\frac{1}{b-a} \int_a^b f(x,d)\, dx \leq \frac{f(a,d)+f(b,d)}{s_1+1}$$
$$\frac{1}{d-c} \int_c^d f(a,y)\, dy \leq \frac{f(a,c)+f(a,d)}{s_2+1}$$

and

$$\frac{1}{d-c} \int_c^d f(b,y)\, dy \leq \frac{f(b,c)+f(b,d)}{s_2+1}$$

which give, by addition the last inequality in (15).

Remark 3.2 *In (15), if $s_1 = s_2 = 1$, then (15) reduced to inequality (4). Also, in (15), if $s_1 = s_2$, then (15) reduced to inequality (9).*

The following inequalities is considered as a Hadamard–type inequalities connected with inequality (14) for s–convex function in the first sense on the co–ordinates.

Theorem 3.3 *Suppose that $f : \Delta = [a,b] \times [c,d] \subseteq [0,\infty)^2 \to [0,\infty)$ is s–convex function on the co–ordinates in the first sense on Δ. Then one has the inequalities:*

$$f\left(\frac{a+b}{2}, \frac{c+d}{2}\right)$$

$$
\begin{aligned}
&\leq \frac{1}{2}\left[\frac{1}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx+\frac{1}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy\right] \\
&\leq \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)\,dydx \qquad (20)\\
&\leq \frac{1}{2(s_1+1)(b-a)}\int_a^b [f(x,c)+s_1f(x,d)]\,dx \\
&\quad +\frac{1}{2(s_2+1)(d-c)}\int_c^d [f(a,y)+s_2f(b,y)]\,dy \\
&\leq \frac{f(a,c)+s_1f(a,d)+s_1f(b,c)+s_1^2f(b,d)}{2(s_1+1)^2} \\
&\quad +\frac{f(a,c)+s_2f(a,d)+s_2f(b,c)+s_2^2f(b,d)}{2(s_2+1)^2}
\end{aligned}
$$

The above inequalities are sharp.

Proof. Since $f:\Delta\to\mathbf{R}$ is co–ordinated s–convex in first sense on Δ it follows that the mapping $g_x:[c,d]\to[0,\infty)$, $g_x(y)=f(x,y)$ is s_1–convex on $[c,d]$ for all $x\in[a,b]$. Then by s–Hadamard's inequality (2) one has:

$$g_x\left(\frac{c+d}{2}\right)\leq\frac{1}{d-c}\int_c^d g_x(y)\,dy\leq\frac{g_x(c)+s_1g_x(d)}{s_1+1},\quad \forall x\in[a,b].$$

That is,

$$f\left(x,\frac{c+d}{2}\right)\leq\frac{1}{d-c}\int_c^d f(x,y)\,dy\leq\frac{f(x,c)+s_1f(x,d)}{s_1+1},\quad \forall x\in[a,b].$$

Integrating this inequality on $[a,b]$, we have

$$
\begin{aligned}
\frac{1}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx &\leq \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)\,dydx \qquad (21)\\
&\leq \frac{1}{s_1+1}\left[\frac{1}{b-a}\int_a^b f(x,c)\,dx+\frac{s_1}{b-a}\int_a^b f(x,d)\,dx\right].
\end{aligned}
$$

A similar arguments applied for the mapping $g_y:[a,b]\to[0,\infty)$, $g_y(x)=f(x,y)$, we get

$$\frac{1}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy \leq \frac{1}{(d-c)(b-a)}\int_c^d\int_a^b f(x,y)\,dxdy \qquad (22)$$

$$\leq \frac{1}{s_2+1}\left[\frac{1}{d-c}\int_c^d f(a,y)\,dy+\frac{s_2}{d-c}\int_c^d f(b,y)\,dy\right].$$

Summing the inequalities (21) and (22), we get the second and the third inequalities in (20).

Therefore, by Hadamard's inequality (2), we also have:

$$f\left(\frac{a+b}{2},\frac{c+d}{2}\right)\leq\frac{1}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy \tag{23}$$

and

$$f\left(\frac{a+b}{2},\frac{c+d}{2}\right)\leq\frac{1}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx \tag{24}$$

which give, by addition the first inequality in (20).

Finally, by the same inequality we can also state:

$$\frac{1}{b-a}\int_a^b f(x,c)\,dx\leq\frac{f(a,c)+s_1 f(b,c)}{s_1+1}$$

$$\frac{1}{b-a}\int_a^b f(x,d)\,dx\leq\frac{f(a,d)+s_1 f(b,d)}{s_1+1}$$

$$\frac{1}{d-c}\int_c^d f(a,y)\,dy\leq\frac{f(a,c)+s_2 f(a,d)}{s_2+1}$$

and

$$\frac{1}{d-c}\int_c^d f(b,y)\,dy\leq\frac{f(b,c)+s_2 f(b,d)}{s_2+1}$$

which give, by addition the last inequality in (20).

Remark 3.4 *In (20), if $s_1=s_2=1$, then (20) reduced to inequality (4). Also, in (20), if $s_1=s_2$, then (20) reduced to inequality (6).*

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