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HADRON FINAL STATES IN DEEP INELASTIC PROCESSES*

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I. Introduction

These lectures[1] deal mainly with old physics; in particular the description and discussion of hadron final states in electroproduction, colliding beams, and neutrino reactions from the point of view of the simple parton model, such as one finds described in detail in Feynman's book.[2] We first describe the standard parton model formalism and predictions for distributions of final state hadrons in processes $e^+e^- \rightarrow$ hadrons, $e^-p \rightarrow e^-$ + hadrons, $\mu^-p \rightarrow \mu^-$ + hadrons, $\nu p \rightarrow \mu^-$ + hadrons, $\bar{\nu}p \rightarrow \mu^+$ + hadrons, etc. Once having described in broad terms the predictions and (briefly) the status of the experimental situation we then explore in more detail the various regions of phase space: the so-called fragmentation regions and plateau regions and see how they differ for deep inelastic processes as compared to ordinary processes. Then we consider all this in a more general context which Kogut and I call correspondence.[3] Correspondence is closely related to duality ideas. One argues that for processes in new regions of high Q^2 any hadron distribution is smoothly connected to the regions of low Q^2, and that inclusive processes are always smoothly connected to exclusive processes, as in the Drell-Yan-West connection in electroproduction.[4,5,6] Kogut and I concluded that the smoothest possible behavior consistent with our intuition about exclusive processes, photoproduction processes, or

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other low-Q^2 processes is that at high Q^2 nothing changes drastically as Q^2 is increased as long as the hadron center of mass energy is held fixed, and as long as the distributions one is talking about are normalized to the total cross section. We will see this in more detail later on. Thus far, the correspondence idea seems to be borne out by the data as well.[7] It also meshes nicely with parton model ideas but does not necessarily require them.

As the second major topic of these lectures we will discuss the space-time evolution of final states in the parton model. This is important, in that it bears upon very basic questions of the dynamics of deep inelastic processes. The central question is why, when quarks are struck by leptons or other currents and one would expect to see them in the final states, one does not and only ordinary hadrons come out. Looking in detail at the space-time evolution of the process, say in electroproduction, from the time when the quark is hit to the time when the final hadrons emerge as asymptotic states one should learn something about the dynamics of confinement. In order to do this we first look at the space-time evolution of ordinary collisions in the parton model framework or the framework of theories possessing only short range correlations in rapidity. Then we look, as an aside, at collisions which involve nuclei as targets or as projectiles. These are of interest these days in high energy hadron nucleus collisions. Nuclei may eventually be important in electroproduction at large ω, where the problems of shadowing or antishadowing are significant, fundamental, and at present in a confused state. Finally we look at the processes $e^+e^- \to$ hadrons, and deep inelastic electroproduction or neutrino reactions.

II. The Parton Model Picture

In the parton picture for deep inelastic collisions (or, for that matter for any collision), one views an energetic hadron as equivalent to a beam of partons. That is, for the purpose of calculation the incident hadron beam is replaced by a parton beam. The partons are assumed to be point-like constituents within the hadrons. If, as in electroproduction or neutrino reactions, the other incident projectile is a lepton one assumes that the lepton parton interaction is point-like. In other words the parton has no more structure than a lepton would have. Finally, as an independent hypothesis, it is assumed that the struck parton or final state parton of high transverse momentum (the high transverse momentum essentially defines what one means by deep inelastic processes) evolves into hadrons in a manner which is independent of the rest of the environment, i.e., those partons and/or produced hadrons which are distant from the struck parton in phase space. We now examine this picture in more detail.

A. Colliding Beams: $e^+e^- \to$ hadrons

The easiest example of this is in the colliding beam processes such as $e^+e^- \to$ hadrons [or, equivalently, the decay of a heavy intermediate boson, or the process $e^- + \bar{\nu}_e \to$ hadrons, which of course is a little hard to realize experimentally]. In the colliding beam reaction one first of all presumes that the e^+e^- system annihilates through a virtual photon into a parton-antiparton pair, assumed in fact to be quark-antiquark. Then just after the collision, one has a free quark and antiquark which begin to recede from each other. What happens next is less clear. But at much later times the quark and antiquark have been replaced by a system of hadrons, which are assumed to have the following properties.[2,8,9] First of all, relative to the direction of momentum of the originally produced quarks, there is <u>limited p_T</u> for the produced hadrons.

Secondly, there is assumed to be scaling behavior in the longitudinal momentum variable, as given by the following formula

$$\frac{dN_h}{dz} = \frac{\sum_i e_i^2 D_i^h(z)}{\sum_i e_i^2} \qquad (2.1)$$

where, in the laboratory frame

$$z = p/p_{max} = p/p_{quark} \quad \text{and} \quad zD_i^h(z) \equiv g_i^h(z) \qquad (2.2)$$

Finally, if the energies are high enough (and this means extremely high—at least 30 GeV in the center of mass) there should be a central rapidity plateau. That is, if for a given event one defines the z axis along the direction of the originally produced parton antiparton pair and defines the appropriate rapidity variable, then the rapidity distribution of produced nonleading hadrons should be uniform, just as it is in ordinary reactions. We shall call this flat rapidity distribution of hadrons the "current plateau" to distinguish it from the corresponding plateau region found in ordinary hadron-hadron collisions (which we will call a hadron plateau). The regions near the boundaries of phase space contain the highest momentum hadrons in the event. It is those regions which obey the scaling behavior in Eq. (2.1), and they define the parton fragmentation regions as shown in Fig. 1.

The experimental evidence for the kind of behavior embodied in the above hypotheses is reasonably good. First of all, the longitudinal scaling behavior as in Eq. (2.1) has been checked in the SPEAR experiments.[10] Jet structure has been discovered thanks to the existence of transverse beam polarization. This picture also demands a relatively low multiplicity, logarithmically growing with energy as in ordinary hadron collisions. That is also observed in the colliding beam reactions.

B. Electroproduction and Neutrino Reactions

Next in complexity, with regard to the description of hadron final states, is deep inelastic electroproduction or neutrino reactions. We assume familiarity with the formalism and scaling picture for these processes[11] when hadron final states are summed over. We will first look at the reactions from a relatively naive level, valid for small ω (large x), say $\omega < 10$. We choose to look at the process in the lab frame; in that frame we see first of all that inclusive cross section for deep inelastic electron scattering is given by the formula (for electroproduction)

$$\nu \frac{d\sigma}{dQ^2 d\nu} \cong \frac{4\pi\alpha^2}{Q^4} \left(1 - y + \frac{y^2}{2}\right) \sum_i e_i^2 f_i(x) \tag{2.3}$$

with

$$\begin{aligned} Q^2 &= 4EE' \sin^2 \frac{\theta}{2} = -q^2 \\ \nu &= E - E' \\ x &= Q^2/2m\nu = \omega^{-1} \\ y &= \nu/E \end{aligned} \tag{2.4}$$

In the parton picture, this describes the probability of a fast parton of given type being produced in the collision. Given this ordinary deep inelastic cross section for production of fast partons, we then assume that the produced hadrons evolve from the fast partons in a way that depends only on the nature of the struck parton. Therefore the distribution of hadrons must be very similar to the distribution of the fast hadrons that we just discussed for the colliding beam reaction. Specifically, defining z as equal to the momentum of the produced hadron divided by ν, the virtual photon energy in the laboratory frame, we should have Feynman scaling for the inclusive distribution of fast hadrons measured along the direction of the virtual photon:

$$\frac{dN_h}{dz} = \sum_i \epsilon_i(x)\; D_i^h(z) \tag{2.5}$$

where

$$z = p_{hadron}/\nu = (p/p_{max})_{lab} \tag{2.6}$$

and

$$\epsilon_i = \frac{e_i^2 f_i(x)}{\sum_i e_i^2 f_i(x)} = \text{probability the struck parton is of type i.} \tag{2.7}$$

Secondly, because the u quark has charge 2/3 it should be the quark that is predominantly produced in electroproduction reactions.

$$\epsilon_u + \epsilon_{\bar{u}} \geq 2/3 \tag{2.8}$$

Also for large x (small ω) there should be many more u-quarks than $\bar{u}$-quarks, because we are in the valence region. Likewise for neutrino reactions the u-quarks again dominate, because the predominant subprocess for charged-current reactions is a neutrino striking a d-quark, producing a μ^- and a u-quark. On the other hand, for antineutrino induced reactions the d-quarks should dominate:

$$\bar{\nu}_\mu + u \rightarrow \mu^+ + d \quad . \tag{2.9}$$

The probability of finding a hadron of certain type emerging from a quark of a certain type are given by the D functions of Eq. (2.1). For large z (that is, for leading hadrons) we would expect

$$D_u^{\pi^+}(z) \gg D_u^{\pi^-}(z) \tag{2.10}$$

Furthermore, from isospin conservation there follows

$$D_u^{\pi^+}(z) = D_d^{\pi^-}(z)$$

$$D_i^{\pi^o}(z) = \frac{1}{2}\left(D_i^{\pi^+}(z) + D_i^{\pi^-}(z)\right) \tag{2.11}$$

Therefore, for low ω (large x) we expect that at large z the charge ratio (i.e., the ratio of positively charged produced hadrons to negatively charged hadrons) should strongly favor the positives. In neutrino reactions this is seen very clearly.[12, 13]

For electroproduction at very large ω, the distribution of leading hadrons should be the same function as in e^+e^- annihilation into hadrons, because all partons participate with the same weight as in e^+e^- annihilation. There is one exception: at high energies in the colliding beam reactions about half the total cross section is in new physics. The new physics may not be present in electroproduction to the same degree at presently attainable Q^2 (although for $Q^2 >> 10\ \text{GeV}^2$ it may in fact contribute in the same proportion as for e^+e^- physics). Therefore it may be best to only consider the hadrons produced by old-physics mechanisms in colliding beams, and compare that inclusive distribution with that in electroproduction. Furthermore, to zeroth order of accuracy there should be a plateau structure for small values of z near zero, say $z \ll 0.1$. This however is a more complicated situation than in colliding beams because of the presence of hole fragmentation, a concept we return to later on. However for small ω this is not a complication, and the plateau structure must be the same as the plateau structure which one finds in colliding beams. Finally there should be limited transverse momentum of produced hadrons relative to the virtual photon direction (in the lab frame), just as was in the case for the jets from colliding beams (or for that matter for ordinary processes).

By now the evidence for these properties is in general quite good. In electroproduction there are data[14] from DESY, Cornell, SLAC, and Fermilab which shows approximate scaling of the inclusive distribution of fast hadrons, at least at the factor-of-two level and probably somewhat better. The recent Fermilab data from the 15-foot bubble chamber,[12, 13] with incident neutrinos and antineutrinos show a very large positive charge ratio for leading (large z) hadrons in νp interactions at high Q^2 and ν. In electroproduction, at SLAC energies (10 - 20 GeV) with moderate ω, one sees a positive charge excess for leading hadrons, and in some circumstances even when the target is a neutron. This is in fact in line with parton model expectations. At higher energies ($\sim$ 100 GeV) the FNAL data[15] indicate much less, if any such effect. That data is at considerably larger ω, and a smaller excess is in fact expected, but it is not clear that the situation is good from a quantitative point of view.

Additional evidence that at least semiquantitatively the parton model ideas are working is that the mean multiplicity is roughly independent of Q^2 at fixed W^2, W^2 being the total hadron energy in the final state. This was first seen in a nice experiment from Cornell[16] for $Q^2 \lesssim 8$ GeV^2. The recent data from Fermilab neutrino reactions in the 15-foot bubble chamber[12] have extended this observation to much higher values of ν and also of Q^2. At the highest energies, the mean Q^2 is $\sim$20 or 30 GeV^2, and the multiplicity seems to be typical of that found in ordinary hadron processes. Finally there seems to be some evidence that the mean transverse momentum of produced hadrons is not too different from that in ordinary processes. If the p_T distribution is parametrized as

$$\frac{dN}{dp_\perp^2} \sim e^{-bp_\perp^2} \qquad (2.12)$$

then one finds $b \sim 4$ to 6 GeV^{-2} in experiments from DESY, from Cornell, from SLAC, and from Fermilab.[14, 15] There is only one exception experimentally: Preliminary data[17] from the Santa Barbara group, who measure π^0 electroproduction at SLAC energies (10 - 20 GeV lab energy). They find for Q^2 the order of 5 - 7 GeV^2 a much lower value of b, $\lesssim 1$ GeV^{-2}. This result should soon be checked for the charged particle distributions, which do not yet reach that far in Q^2.

C. Fragmentation Regions

Now we turn in yet more detail to the properties of the plateau and fragmentation regions in the deep inelastic processes.[2, 18, 19]

As we mentioned, for the process $e^+e^- \rightarrow$ hadrons one should be able to define a jet axis along which hadron momenta tend to be large and scale with the incident beam energies and transverse to which the transverse momenta are limited. The rapidity distribution of produced hadrons relative to the jet axis should have the behavior shown in Fig. 1. At the boundaries of the phase space there are the parton fragmentation regions and then in between (if the energy is high enough) the current plateau.

Next, in preparation for examining electroproduction processes, we look at an ordinary hadron-hadron collision,which has a similar kind of structure. Let us take as an example

$$\rho + N \rightarrow \text{hadron} + \text{anything} \quad .$$

The distribution in rapidity of produced hadrons consists again of three regions. The highest rapidity particles are found in the rho fragmentation region. Then there is a hadron plateau of intermediate rapidities. Finally the slow particles in the laboratory frame are fragments of the proton and belong to the proton fragmentation region.

Now for photoproduction of hadrons by real photons, vector dominance would imply a similar distribution as in ρ-nucleon interactions. We may now replace the real photon by a virtual photon, and look at large ω first. Then we find, first of all, that the nucleon fragmentation region and the hadron plateau regions should still be present because all we are changing are the properties of the incident projectile. Therefore by the hypothesis of short range rapidity correlation in ordinary processes, the other regions should not be affected by a change in projectile properties.

However the virtual-photon fragmentation region is different from the real-photon fragmentation region. Even its length in rapidity space changes. Why is this so? It is because at small ω there is no ordinary Pomeranchuk trajectory exchange possible, i.e., no <u>hadron</u> plateau can exist. The Pomeranchuk trajectory has properties similar to a ladder graph in a superrenormalizable theory. We can ask, "Under what circumstances can we exchange ladders in deep inelastic processes?" If we look, for example, at elastic processes, such as virtual $\gamma + N \rightarrow \rho + N$ we can ask for fixed energy at what value of q^2 can we no longer coherently produce the rho. This occurs when the minimum momentum transfer

$$\Delta_{min} = \frac{m_\rho^2 - q^2}{2\nu} = \frac{m_\rho^2 + Q^2}{2\nu} \approx Mx \tag{2.13}$$

exceeds a few hundred MeV. And this occurs when ω becomes smaller than some fixed amount, say 3 or 10 or so. Therefore, when ω is small the hadron plateau must disappear. It follows that its length is of order log ω, as shown in Fig. 2. This leaves for the photon fragmentation region a length in the rapidity space of order log Q^2. This is satisfying inasmuch as the length of the photon fragmentation region should be only a function of Q^2 and therefore we find consistency. These considerations do not require the parton model.

However, assuming the parton model, we find more detailed structure within the virtual photon fragmentation region. In particular, the photon fragmentation region itself divides up into three pieces.[18] For the largest y there is the parton fragmentation region, which is familiar from the colliding beam discussion. Adjacent to it is the current plateau. However, there must be a transition region between the current plateau and the hadron plateau, which we call the hole fragmentation region. The reason for this name is that it is the location in phase space of the original parton before it was struck. This is shown in Fig. 3.

We can see this picture is sensible. In the limit of small ω, the hole fragmentation region merges with the target fragmentation region, leaving the current plateau to separate target and parton fragmentation regions. In the limit of small Q^2, the hole fragmentation region merges into the parton fragmentation region, leaving the hadron plateau to separate the target fragmentation from the photon fragmentation region. An interesting question is whether the rapidity distribution of hadrons in the hadron plateau is equal to the rapidity distribution of hadrons in the current plateau. It is not clear that this should be so. In fact the mean p_T of the produced hadrons may not be the same either. However, from experiment it appears that the height of the hadron plateau and the current plateau are approximately the same, although perhaps the mean p_T may be somewhat larger for the current plateau than it is for the hadron plateau.[13]

III. Vector Dominance

Further insight into these fragmentation regions can be obtained by looking at large-ω electroproduction from the point of view of vector dominance. After all, this concept works well for high energy real-photon physics, and we might expect a generalization to exist for virtual photons as well. This is especially the case in the light of the arguments of Ioffe[20] that large longitudinal distances (proportional to ω) are important in electroproduction at high Q^2 and ω. In considering the vector dominance approach we use the "diagonal approximation" shown in Fig. 4, assuming the intermediate vector states m and n in the forward virtual Compton amplitude to be the same.[21] A simple way of calculating this vector dominance contribution for real photons was given by Gribov[22] several years ago: the probability for the photon to be absorbed by the nucleon is given by the probability that the incident photon fluctuates into a hadron intermediate state, multiplied by the probability that the hadron interacts with the nucleon. These factors are shown below:

$$\sigma_{\gamma N} = (\text{probability } \gamma \text{ is hadron}) \times (\text{probability hadron interacts})$$

$$= \left(\int_0^{\sim s} \frac{dm^2}{m^2} \frac{\alpha}{3\pi} R(m^2) \right) \times \sigma(m^2) \tag{3.1}$$

$$\cong (1 - Z_3)\, \sigma(m^2)$$

where Z_3 is the hadronic charge renormalization constant (the probability a physical γ = bare γ), and where R is again the familiar ratio that appears in the colliding beam cross sections:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{3.2}$$

For virtual photons we go through the same calculation as schematically shown below

$$\sigma_{\gamma N} \sim \sum_n <0|j^\mu \epsilon_\mu|n> \frac{1}{\Delta E_n} \sigma_n \frac{1}{\Delta E_n} <n|j^\mu \epsilon_\mu|0> \qquad (3.3)$$

We see the only change is in the energy denominators. For real photons they are $1/m^2$, while for virtual photons they become $1/(Q^2+m^2)$. Therefore the absorption cross section for a transverse virtual photon is given by the previous formula with this modification made:

$$\sigma^T_{\gamma^* N} = \frac{\alpha}{3\pi} \int_0^{\sim s} \frac{dm^2\, m^2}{(Q^2+m^2)^2} R(m^2)\, \sigma(m^2) \qquad (3.4)$$

However, if we follow our intuition and assume that the absorption cross section for the virtual hadron state n is a constant σ_h independent of n, we reach a disaster:

$$\sigma_{\gamma^* N}(Q^2, \nu) \sim \frac{\alpha}{3\pi} \bar{R} \sigma_h \log \omega \qquad (3.5)$$

Because the absorption cross section σ_h contains an intrinsic dimension (essentially πR^2), there is no scaling of the deep inelastic cross section. Instead of σ_T falling as $1/Q^2$, it behaves roughly like a constant. What went wrong and what must be done to remedy the situation?

There are two main options. The first has been considered in particular by Greco,[23] Schildknecht and Sakurai,[24] and is simply that the absorption cross section for a massive virtual intermediate state is smaller than that for a not-so-massive intermediate state, and falls off like $1/m^2$. Suppose this is the case. Given that the e^+e^- annihilation process yields two jets in the final state, then vector dominance would imply that these jets would also be electroproduced at large ω. Then we should eventually see double jets in electroproduction at large ω. Also, because the inclusive distribution of the hadrons within the jets obeys scaling, the leading hadrons should contain a finite fraction of the $p_\perp$ of the jet. The mean $p_\perp$ of such a jet is of order its mass, which will typically be $\sim\sqrt{Q^2}$. Therefore it is predicted that the mean $p_\perp$ of leading hadrons ($z \sim 0.5$ seems optimum) should be $\sim\sqrt{Q^2}$, if the double-jet option is correct.[25] This is actually the option that is chosen by quantum electrodynamics, as shown by Cheng and Wu.[26] There is a second option. It is that these jets which appear in the virtual intermediate state must be aligned along the virtual photon direction.[27] The argument proceeds as before using vector dominance. But instead of the preceding assumptions, it is assumed that if the transverse momentum of the jets is large (in other words that the partition of longitudinal momentum is balanced between the two jets), then the cross section for absorption of such a system is very small. However, if the transverse momentum of the jets is small relative to the virtual photon direction (with consequent imbalance in the partition of longitudinal momentum), then they are absorbed by the target in the usual way, with a geometrical absorption cross section.

In their own rest frame the virtual-photon jets must have a roughly isotropic angular distribution, because they were produced by a spin 1 photon. Therefore the probability that the transverse momentum of the jets is small is $\Delta\Omega \sim \theta^2_{max} \sim (\langle p_\perp \rangle/\sqrt{Q^2})^2 \sim \text{const}/Q^2$. It follows that in Eq. (3.4) instead of $\sigma(m^2)$ appearing we have to replace it by $\sigma(m^2)$ times the probability that the jet be aligned, and that product is of order $1/m^2$.

What does this option accomplish? First of all, the deep inelastic scaling survives. Secondly, one obtains low-transverse-momentum hadrons in electroproduction at large ω as the parton model would suggest. Finally the hole fragmentation region and the parton fragmentation region can be identified (using vector dominance) from the parton fragmentation regions in collding beams.

I find it very satisfying that the vector-dominance and parton-model ideas can be made to interlock in such a self consistent way. However, one must be on the lookout experimentally for high-p_T leading hadrons in electroproduction, especially at large ω, in the light of these two competing production mechanisms.

IV. Correspondence

The use of the word correspondence I discuss goes back to the idea used by Bohr, in inferring properties of quantum physics from the demand that quantum mechanics must have a smooth limit to classical physics as $h \to 0$. Knowing what the classical physics regime looks like puts constraints on how the quantum mechanical regions behave. In the present case we want to explore unknown regions in phase space—in particular the deep inelastic high Q^2 regions. The physics in these unknown regions most likely has smooth connections with the physics in the known regions. An example of this is duality in strong interactions, where the resonance region is connected smoothly to the continuum region of the Regge trajectories at higher energies. In electroproduction there is the example of the inclusive-exclusive connection of Drell, Yan,[4] West,[5] Bloom, Gilman,[6] and others which connects the shape of the deep inelastic scaling curve for the continuum contribution (extrapolated into the resonance region) to the behavior of the resonance contributions. A simple way of obtaining the above connections is to demand that neither resonance nor continuum should dominate in the resonance region: The ratio of resonance signal to continuum "noise" should be of order one under all circumstances. This is reasonable inasmuch as, if one is given some kind of continuum production mechanism which can be extrapolated into the resonance region, there are a limited number of open channels and partial waves that can comprise it. The amplitude in a resonant channel will be enhanced by some finite factor; hence one must have the ratio of the resonance signal to the total extrapolated inclusive background in the resonance region in general of order 1.

What Kogut and I tried to do for electroproduction and colliding beams processes[3] was to look at all the general connections of this nature that we could think up: inclusive vs. exclusive, a smooth connection between low Q^2 and high Q^2, smoothness in looking at the processes from both the point of view of vector dominance or a parton picture, smoothness in connecting low-s resonances with the high-s Regge behavior. We found that if one demanded all of these connections to be smooth we were almost forced into a unique picture of how the hadron distributions in the deep inelastic processes should look. Although we arrived at this in a rather roundabout way, the answer is in fact direct and very simple. It can be stated as follows:

The smoothest solution for hadron distributions for electroproduction and neutrino reactions at high Q^2 consistent with the known or expected behavior of the processes in the boundary regions (e.g., exclusive processes at high Q^2 and inclusive at low Q^2) is that the normalized distributions of almost anything are independent of Q^2 if W^2 is kept fixed. This assertion is meant to be taken only at the factor of 2 level of accuracy; i.e., there is no systematic behavior with Q^2 (like Q^2 to some power) in any normalized quantity. A second inference Kogut and I found is that the hadron plateau height should be approximately the same as the current plateau height. Finally, at large ω the photon fragmentation region should contain hadron distributions which are the same as those found in e^+e^- annihilation.

There is not the time (and it is probably not so interesting) to go through this logical route that Kogut and I took, but only to point out a couple of surprises that came along the way. First of all, there was a surprise in the behavior of two-body exclusive channels. As an example consider exclusive electroproduction of a π^+ from hydrogen: $\gamma^* p \rightarrow \pi^+ n$. If we go to the real-photon limit at $\overline{Q^2=0}$, we expect a Regge-behaved cross section

$$\sigma(s) \sim s^{2\alpha-2} \tag{4.1}$$

which is (according to ordinary duality) on the average true for all values of $s=W^2$. The prediction of correspondence is that at large Q^2 we should have essentially the same behavior provided we normalize the exclusive cross section to the total cross section with the same value of Q^2 and s. Is this reasonable? For very large ω the Regge behavior should be still good, and only for large ω. This implies that the cross section should go as follows:

$$\frac{\sigma_{tot}(\gamma^* p \to \pi^+ n)}{\sigma_{tot}(\gamma^* p \to \text{all})} \sim s^{2\alpha-2} \equiv (W^2)^{2\alpha-2} \tag{4.2}$$

Hence

$$\sigma_{tot}(\gamma^* p \to \pi^+ n) \sim \frac{1}{Q^2}\left(\frac{W^2}{Q^2}\right)^{2\alpha-2} (Q^2)^{-2+2\alpha} = \frac{1}{Q^2}(\omega')^{2\alpha-2}(Q^2)^{-2+2\alpha} \tag{4.3}$$

The first two factors can be considered a "pointlike" cross section for producing the π by a Reggeon exchange. The γ-π-Reggeon vertex is given a pointlike value and the Regge factor is $\omega' \sim s/Q^2$ to a power rather than s to a power. This leaves the last factor in Eq. (4.3) to be interpreted as the square of the form factor of the Reggeon-pion vertex. This implies that the form factor has the behavior

$$F_\pi(Q^2) \sim (Q^2)^{-1+\alpha} \tag{4.4}$$

Therefore we find a connection between the Regge intercepts that couple to the pion and the transition form factor of the pion to the various mesons on the Regge trajectory. Evidently this result isn't completely trustworthy quantitatively. But at least qualitatively we see that if the Regge intercept were at zero the form factor would have a monopole behavior $\sim 1/Q^2$. On the other hand, if we look at backward photoproduction with the exchange of a baryon, the Regge intercept would be of the order of -1/2 and the form factor would fall more rapidly with Q^2:

$$F \sim (Q^2)^{-3/2} \tag{4.5}$$

If it were possible to be more careful with spin and had better control of the dynamics beyond just this correspondence argument we could expect these to be precise results. But even so the exponents in the above formulae are probably uncertain to no more than 1/2 a unit or so. And just qualitatively, we find the conclusion that baryon form factors should fall off with Q^2 faster than meson form factors because their trajectories lie lower than the meson trajectories. This connection of the behavior of Reggeon intercepts with form factor behavior is found in the massive quark model of Preparata.[28] But beyond that there is no connection with ordinary Reggeon dynamics. We now turn to small ω. For fixed small s we have from correspondence

$$\frac{\sigma_{tot}(\gamma^* p \rightarrow \pi^+ n)}{\sigma_{tot}(\gamma^* p \rightarrow \text{all})} \sim s^{2\alpha-2} \sim \text{const } 0(1) \text{ in the resonance region} \tag{4.6}$$

and this leads back to the Bloom-Gilman type of relationship because at small s the exclusive channels contain a finite fraction of the total cross section. Therefore the situation is consistent and the correspondence arguments work. We can summarize the behavior of this particular exclusive channel at large Q^2 as follows: for small ω (say $\lesssim 3$) we get a total exclusive cross section behavior as

$$\sigma_{tot}(\gamma^* p \rightarrow \pi^+ n) \sim \frac{s^{2\alpha-2}}{Q^2}(\omega'-1)^3 \sim \frac{s^{2\alpha-2}}{Q^2}\left(\frac{s}{Q^2}\right)^3 \sim \frac{s^{2\alpha+1}}{(Q^2)^4} \tag{4.7}$$

For very large ω we find at fixed s a slow falloff with Q^2, $\sim Q^{-2}$. The net behavior is that roughly Q^6 times the exclusive cross section should have scaling behavior, as shown in Fig. 5.

We found another surprise. There is an argument why the hadron plateau should be approximately the same as the current plateau. This uses the assumption of negligible correlation between the hadron secondaries. Then the distribution in multiplicity may be used to connect the exclusive process with the total cross section. The exclusive process provides the low $\bar{n}$ tail of the multiplicity distribution, and is presemed to join smoothly with the bulk of the distribution. We choose large ω (as in Fig. 3a) and ask for the cross section for producing n_h hadrons in the hadron plateau and n_c in the current plateau. This will be given by the usual formulae as follows:

$$\frac{\sigma(n_h, n_c)}{\sigma_{tot}} \sim \left(\frac{\bar{n}_h^{\,n_h} e^{-\bar{n}_h}}{n_h!}\right)\left(\frac{\bar{n}_c^{\,n_c} e^{-\bar{n}_c}}{n_c!}\right) \tag{4.8}$$

The exclusive process can be read off from this formula and is given by

$$\begin{aligned} \frac{\sigma_{excl}}{\sigma_{tot}} \sim \frac{\sigma(0,0)}{\sigma_{tot}} &\sim e^{-(\bar{n}_h+\bar{n}_c)} \\ &\sim e^{-\left(c_h \log \omega + c_c \log Q^2\right)} \\ &\sim s^{-c_h} \left(Q^2\right)^{-(c_c-c_h)} \end{aligned} \tag{4.9}$$

However correspondence demands that the ratio of the exclusive cross section to the total cross section should be independent of Q^2:

$$c_c = c_h \, (\pm 1??) \tag{4.10}$$

This argument clearly isn't very precise but may at least be indicative that if one were to try to make the hadron plateau height and the current plateau height wildly different that there could be trouble in joining the multiplicity distributions smoothly onto the exclusive limits.

In any case we can draw the following conclusions from this line of argument. Most importantly, the correspondence idea plus the hypothesis of short range correlation in rapidity for low Q^2 processes, and power-law dependencies on Q^2 (and a little bit more now and then) implies the same qualitative picture of hadron final states as the parton model itself. In other words, it may well be that the predictions may have considerably more generality than that of the parton-model. Under such circumstances, can one therefore claim that the experimental support of the general picture of hadron final states which we have discussed really implies experimental support for the parton picture? Probably the strongest specific support for partons as opposed to the more general ideas is the existence of the jets in colliding beams and, most importantly, that their angular distribution is the same as the angular distribution for the μ pairs, indicating spin 1/2 parton parents. In addition the strongly positive charge ratios in electroproduction (and also in the neutrino reactions) at low ω and high hadron momentum are also strong support. However, whatever the ultimate dynamical picture turns out to be, I believe that the correspondence technique is general. Even if our views of dynamics of hadron states change, there still should be the same correspondence connections between various regions and types of processes as we have discussed here. The specific predictions of course would be different.

V. Space-Time Description of High Energy Processes

The first question to ask about this subject is "Why bother with it?" For someone who was raised in the shadow of the Berkeley bootstrap school, where the S matrix in momentum space contains all of physics, this is not an idle question. However I think there are significant reasons for bothering. First of all the importance of large distances and long time intervals at high energies is well established. This goes back as far as the Landau hydrodynamic model[29] and the work of Landau, Pomeranchuk, and others[30] starting in the 1950's. Therefore one should be able to map the geography of these high energy reactions on a distance scale greater than or the order of a fermi, and this geography should be largely independent of the dynamical details. Indeed models as different as the short-range-correlation models or parton models and the Landau model give very nearly the same space-time evolution.

Secondly because of these large distance and time scales at high energies, nuclear effects become very interesting. By putting additional nuclei downstream of the collision site one probes the structure of fragments of the collision at times short compared to the times required for them to reach their asymptotic configuration of free hadrons.

Third, the problem of quark confinement is relevant. The problem here is not why quarks remain confined in deep inelastic interactions (especially the colliding beam reaction $e^+e^- \to$ hadrons) but how they remain confined. Again it should be possible to trace out the geography of confinement on the large distance and time scales (if indeed such large distance and time scales can be shown to be relevant) for the deep inelastic processes as well as for ordinary collisions. In order to be reasonably specific we shall whenever possible again assume short-range-rapidity correlations only; that is, the parton, multiperipheral, etc. type of picture. However the main results probably have a generality beyond these considerations. We shall first look at ordinary collisions, and then at the effects of the collision occurring in nuclear matter (and therefore the currently interesting problems of multiplicity and inclusive distributions in nucleon-nucleus collisions and nucleus-nucleus collisions). Equipped with this space-time picture of ordinary processes, we will attack colliding beams and then electroproduction, first at low ω and then at high ω. As might be expected, with five different regions in rapidity space for high-ω electroproduction, this turns out to be the most subtle and interesting of the processes with which we deal.

Another way of seeing the need for studying the geography of the final states in space time is by looking at the nonrelativistic prototype of deep inelastic scattering with confinement, that of the scattering of an electron from a single charged particle in a potential well for which $V(x) \to \infty$ as $|x| \to \infty$. The structure function W for this case is a sum of contributions of the square of transition form factors from the ground state to the various discrete excited states. It obeys a scaling law of the following form[31]

$$qW(q^2, \nu) = F\left(\frac{\nu - q^2/2m}{q}\right) \tag{5.1}$$

where

$$W(q^2, \nu) = \sum_n \left| \int d^3x \, \psi_n^*(x) \, e^{i\vec{q}\cdot\vec{x}} \psi_0(x) \right|^2 \delta(E_n - \nu) \tag{5.2}$$

provided we make a coarse-grained average in energy over a group of excited states n. Thus for computing the scaling function W only the region of space where the ground state wave function is nonvanishing is important. The exact levels n can be replaced with free-field levels because the kinetic energy of excitation ν is large compared with the variation in potential energy ΔV over the region of space where the ground state wave packet is nonvanishing. However if we want to know about the final states we must know the behavior of the wave functions ψ_n all the way out to the turning point. Equivalently we must follow the time dependence of the ground state wave function multiplied by $e^{i\vec{q}\cdot\vec{x}}$. It is a wave packet of approximate momentum $\vec{q}$ which is distributed into the final states n. The problem then becomes "What is $\psi(x,t)$?" It will be a packet bouncing back and forth between the classical turning points. Therefore the description of the state becomes semiclassical and also dependent on rather large distances compared to what is required for the structure function W.

A. Structure of a Hadron

First of all, in order to describe ordinary collisions from a short-range-correlation or parton picture, we must have a description in space-time of an individual hadron. For a proton at rest, this is familiar: it is just the nonrelativistic quark model, with 3 quarks confined to a region of the order of a cubic fermi, which have certain levels of excitation with a level spacing typically a few hundred MeV. If one were so naive as to talk about the rapidity distribution of the quarks in the proton when the proton is at rest, it would of course just be concentrated in a low rapidity region, say $|y| \lesssim 1$-2. Naively, we expect that once we know what the wave function of the proton is in the rest frame this would suffice to describe the proton when it is accelerated or when it has high momentum. We just boost the proton; the spherical volume in which the 3 quarks resided would turn into a flattened, pancake shaped region with a thickness varying as $1/p$, where p is the momentum of the proton ($p \gg 1$ GeV). Furthermore the excited levels of this system will all be at large energy but nevertheless for given large fixed p the level density would be higher. This follows simply from kinematics; if $E_n = \sqrt{p^2+m_n^2}$, then the spacing of the levels for a system of given total momentum is

$$\Delta E_n = \frac{\Delta m_n^2}{\sqrt{p^2+m_n^2}} \sim \frac{\Delta m^2}{p} = 0\left(\frac{1}{p}\right) \qquad (5.3)$$

If for example $\gamma \sim 3$ the level spacings instead of being 300 MeV for a proton at rest would be reduced to the order of 100 MeV. This increase of level density with increasing momentum will turn out to be important in following through the dynamics of the high energy collision. In the boosted frame the rapidity distribution of the quarks within the proton is shifted, just displaced to higher values, $\sim \log \gamma$ compared to what they were for a proton at rest.

However, according to the parton picture, this isn't the whole story. In addition to the original three partons we must also consider vacuum fluctuations, let us say quark-antiquark pairs. The important vacuum fluctuations which couple to the boosted proton and which will be relevant for a description of high energy collisions are of relatively low energy, with the excitation energy of the order a few hundred MeV. We will assume that there is a hierarchy of such fluctuations with level spacings of the order of again a few hundred MeV. These low momentum vacuum fluctuations can couple to the boosted proton, provided the proton does not have too much momentum, let us say 3 GeV or less. The moving proton then consists of this conglomerate of an uncontracted vacuum fluctuation coupled to the Lorentz-contracted pancake of original three quarks. Now let us boost to a momentum of the order of 9 GeV. The vacuum fluctuation attached to the proton will now get Lorentz-boosted and Lorentz-contracted as well as the original 3-quark system. Furthermore its level density increases by a factor ~3. However this composite system of vacuum fluctuation and original

3 quarks again can couple to uncontracted vacuum fluctuations which have low momentum in this new laboratory frame, with again the level spacing of a few hundred MeV. In this way we can generalize: for a very energetic proton with momentum p we will have the original 3 quarks plus a number (of order log p) of vacuum fluctuations of geometrically decreasing momentum which are sequentially coupled to the original 3 quark system. The lowest-momentum set of vacuum fluctuations (the wee fluctuations) will be uncontracted. A vacuum fluctuation with momentum $\sim p'$ will have a level density inversely proportional to p' and a thickness in the longitudinal direction inversely proportional to p'. The rapidity distribution of the partons comprising the system of coupled vacuum fluctuations is assumed uniform. Thus the highly energetic proton becomes a quite complex object whose description depends upon its momentum. These extra additional vacuum fluctuations comprise the dx/x spectrum of partons of Feynman. Suppose we were to go back to the laboratory frame. Where do all these extra vacuum fluctuations go? We didn't have them there when we started, but suddenly we have attached them. They will occupy very large longitudinal regions as we undo the Lorentz boost in returning to the lab frame. However, the important feature of the vacuum fluctuations so obtained is that they will have a very low level density. The boost now has the effect of decreasing the level density. The first vacuum fluctuation which attached to the proton may have a first excited level $\sim$1 GeV above its lowest state, whereas other vacuum fluctuations which occupy even larger regions of longitudinal configuration space have even larger excitation energies. Thus these excitation energies are too high to be relevant for ordinary spectroscopic processes involving low excitations and low energies. If there is a high energy projectile available, then they may be relevant and in fact part of the description of the

collision. However as we shall see below it is then more natural to associate these fluctuations with the incoming projectile.

Let us try to summarize all this. First of all we observe that rapidity is a concept which can be used both in momentum space and configuration space. In configuration space it measures the typical longitudinal extent of the system involved; the higher the rapidity the smaller the longitudinal extent as a consequence of Lorentz contraction. For a proton of very high momentum p, we assume the phase space of the partons comprising that proton can be broken up into approximately n cells ($n \sim \log p$). Each cell is labeled by its rapidity y. In the cell with rapidity y we have

Momentum $p \sim e^y$

Energy $E \sim e^y$

Level spacing $\Delta E \sim e^{-y}$

Natural time scale $\tau \sim e^{-y}$ (time dilation)

Thickness (or lag λ) $\sim e^{-y}$ (Lorentz contraction)

There is of course some dynamics involved in this picture. The guiding principle for such a dynamics is the hypothesis of short range correlation in rapidity: (i) couplings should exist only between cells which are neighboring in rapidity, (ii) the couplings between such cells are weak enough so that the concept of levels in the individual cells makes sense, and (iii) in collisions excitation of levels are possible only in those rapidity-cells which are in the same region of phase space (that means both momentum and configuration space). Finally no large amount of momentum or energy is exchanged in any frame at any stage. The subprocesses are as soft as possible. Therefore the flow and exchange of momentum is always $\lesssim 1$ GeV per fermi of elapsed time. [We are as usual letting c=1.]

B. Hadrons in Collision

Consider a high energy hadron-hadron collision in the overall center-of-mass frame. Before the collision there is a right-mover and a left-mover consisting of the previously described conglomeration of partons and their levels. Only the lowest levels in the various cells are occupied. As the hadrons collide with each other we see that only the uncontracted (wee) rapidity cells of the two projectiles will significantly overlap in phase space (momentum space in particular). We may, according to the Feynman picture, expect wee partons to be exchanged between the two hadrons and therefore excitation of the levels existing in those wee rapidity-cells. After the hadrons pass through each other we may expect, say, after a time of the order of 2 or 3 fermis, that these excited levels will have been de-excited, since 1-2 fermis is a natural time scale associated with those excitations. This de-excitation can proceed by the emission of wee hadrons emitted more or less isotropically. However in addition to that mechanism of de-excitation there is also possible de-excitation by excitation of levels in the neighboring cell of higher rapidity. This follows because there is coupling between neighboring rapidity cells and because the level density in the neighboring cell is higher than in the wee cell. Therefore, after this passage of time of 2 or 3 fermis when the hadrons have escaped each other, the event is not over. The next-to-wee rapidity cells of the incoming projectiles are excited and wee hadrons have been emitted. This process can now repeat itself. These not-so-wee cells in rapidity de-excite by emission of not-so-wee hadrons which are emitted primarily along the beam directions because of the large momenta involved and the motion of these cells along the beam directions. Again this is not the only mechanism of de-excitation. There can be de-excitation by excitation of the neighboring cell of still higher rapidity. Notice that this proceeds only in the outward direction in y-space because only in that direction does the level density increases with rapidity. There is negligible re-excitation of the wee cells, because their level density is too coarse. Also, this process of de-excitation and re-excitation takes place on a longer time scale than the original first process of the wee excitation and de-excitation because of time dilātation. The clock is running slower because of the large γ of this subsystem of the proton. The time scale for the de-excitation of a cell is proportional to the momentum of the constituents therein. This process now repeats itself on an exponentially increasing time scale. At a given time t the region of excitation is among those partons of momentum $p \sim (\text{const}) \cdot t$, the constant being $\lesssim 1$ GeV per fermi. Finally after a time proportional to the center-of-mass energy in the collision the leading partons will get excited and then de-excited (on the same time scale as their excitation), after which time the process is over, and the asymptotic hadrons recede toward the detection apparatus.

What are the messages from all of this? The first is that excitation occurs by parton interchange. Secondly, the de-excitation of the excited levels in the various cells occurs by two mechanisms. One is the emission of hadrons and the second the excitation of neighboring cells. This is made possible by a level density which increases with the energy or rapidity of the cells. Third (if the picture makes sense at all) the initial excitation by the collision of the two projectiles is really equivalent to excitation by the "heating" of a neighboring rapidity cell. After all we can look at the process in various frames; some of the cells which belong to one hadron in one frame will belong to the other hadron in another frame. But in any case in any frame the slowest hadrons (that is, the wee hadrons) emerge first from the collision. The fastest, most energetic hadrons emerge last. Relativity is very important. There is no simultaneity in the process and the geography is in fact spread out over a long time scale. Hadrons are emitted and partons are excited at a time t which is proportional to their momentum. Everything happens locally in rapidity; namely if the partons at momentum p at any given time are excited then partons of momenta much different from p are at that time unexcited.

It is important that the description above be a covariant description. We should be able to look at this process in different frames of reference and, even though there will be rather different evolutionary pictures, must still find that the distribution of hadrons which emerge, including the time sequence, be self-consistent. This is an important test of any model which I believe is necessary before it becomes credible. This is in fact one of the weaker points of the Landau hydrodynamic model. It picks out a special frame for the initial conditions, even though the evolution of the dynamics after the initial conditions are set is treated covariantly.

It is in fact instructive to look at what we just went through in another frame which I like to call the Fool's ISR (FISR). The FISR is a double storage ring consisting of very high energy protons going in almost the same direction and colliding with each other with a very small crossing angle 2θ. If we look in at the collision from the upstream direction what is seen is two circular disks of hadronic matter slowly moving through each other with a velocity $\sim\theta$. What happens this time? The rapidity distributions of the partons of the two incoming projectiles look identical; however the transverse momenta in the leading rapidity cells differ. How do we decide which partons are excited as the disks move through each other? First of all, the leading partons will <u>not</u> be immediately excited; because although their distributions overlap in longitudinal phase space they do not overlap in transverse-momentum phase space.

Therefore we must find longitudinal momenta sufficiently small so that the transverse momenta are of the typical several-hundred-MeV and that there is overlap in transverse-momentum space as well as in longitudinal momentum space. It is easily checked that this occurs for those partons which, in the center-of-mass description, we found were first excited by the collision. But now we may ask about the partons which have less rapidity than those which were excited: will they be excited as well? The answer is no, because they are moving through each other very slowly and, although the transverse momentum distributions overlap, the level densities of these very-low-rapidity cells are so small that there can be no excitation. There simply isn't enough kinetic energy imparted by the partons as they go through each other to significantly excite those levels. The situation in fact has a very nice analogue in atom-atom collisions. Collisions between leading partons correspond to very energetic electrons in two atoms colliding with each other. Such collisions are rare because the interaction is weak ($0(\alpha)$) when the relative velocity is very large. The strongest interaction in atom-atom collisions occurs at velocities for which the two atoms can be considered as two Thomas-Fermi gases interpenetrating and for which their momentum-space distributions overlap significantly and for which there can be electron exchange and transfer of momentum as well (and thereby excitation). This case is analogous to the Feynman wee parton exchange. The collisions of the very wee partons with each other are analogous to atom-atom collisions at very low velocities (much less than the velocities of the electrons within the atom). Under such circumstances the adiabatic approximation for the collision is applicable. As the atoms go through each other the position of levels change. But they change so slowly and so slightly that if the systems were in their ground state before the collision they would remain in the ground state after the collision, provided there are no level crossings.

The space time aspects of this picture can be summarized in terms of space-time diagrams where the world lines of initial projectiles and all final produced hadrons lie approximately along the light cone. That is, they will be rays emerging in various directions along the light cone since most produced hadrons are travelling at the speed of light, and there are no special features associated with the finite velocities of the hadrons which we have used. Therefore if we look from the top of the light cone (this is shown in Fig. 6) we see all the rays emanating in the transverse directions. The real dynamical activity occurs near the t-z plane (defined as x=y=0). Define the region $|x_\perp| = \sqrt{x^2+y^2} \leq 1f$ as the interaction cylinder. Rays outside the interaction cylinder describe essentially asymptotic hadrons, unless two rays are so close to each other that one must wait a little longer for them to separate by a fermi from each other. Projecting the rays of emitted hadrons onto the zt plane, we see that the time at which the hadrons emerge from the interaction cylinder as asymptotic particles is when their proper time τ is ~1 fermi:

$$\tau^2 \sim (t-z)(t+z) - x_\perp^2 \sim \frac{1}{p} \cdot p - x_\perp^2 \sim 0(1) \tag{5.4}$$

Thus the boundary surface that distinguishes the asymptotic hadrons from the still-interacting hadrons is a hyperbola given by the approximate equation

$$t^2 - z^2 \sim 1 \text{ fermi}^2 \quad . \tag{5.5}$$

Thus at some intermediate time, we see the picture as in Fig. 6a. The hadrons going along the beam direction within 1 fermi of the beam direction or so will be still excited. The thickness of the excited region will be inversely proportional to the time at which we observe this intermediate state; this time is assumed large compared to 1 fermi. Momenta in the excited region will be proportional to the time t. The highese momentum of the emitted hadrons will also be proportional to t. The rapidity distribution of the produced hadrons will be a plateau starting from the inside out (wees first) and growing at the ends to larger and larger rapidities as time goes on, as shown in Fig. 6c. The rapidity distribution of the partons in the original projectiles is shown in Fig. 6b, and the excited parton will have momentum proportional to the time. This picture of the collision is geometrical and what one would expect given only classical considerations. But it also seems to be consistent with anything quantum mechanical that one imposes upon it.

We now turn to nuclear collisions and see what the implications of this picture are in that case.

C. Nucleon-Nucleus Collisions

The nucleus at rest is generally considered a collection of nucleons inside of which reside a collection of quarks of low rapidity. When this nucleus is boosted to sufficiently high rapidities, naively one obtains again a pancake-shaped object containing all the quarks. If the boost is sufficiently high, the thickness of the pancake will be $\lesssim 10^{-13}$ cm. Under such circumstances we must again consider the effect of vacuum fluctuations on the structure of the moving nucleus; in such a case we must attach one layer of wee vacuum fluctuations over the entire surface of the moving disk. As the nucleus is boosted still further, the same process as for a single nucleon repeats itself. The original vacuum fluctuations attach to vacuum fluctuations of lesser rapidity, until the last layer of vacuum fluctuations are again the wees. An important consequence of this is that the thickness of a moving nucleus (in the longitudinal direction) never is less than a fermi, just as the case for a single nucleon. Thus the number of wee and not-so-energetic partons will be proportional to the surface area of the nucleus $\sim A^{2/3}$ rather than strictly proportional to A. This occurs for boosts greater than a critical velocity or γ, with the critical gamma $\gamma_c \sim 2.4\ A^{1/3}$, as follows just from our knowledge of the nuclear size. Partons of rapidity $y \gtrsim y_{max} - \log \gamma_{crit}$ belong to the "naive" nucleus ($n \sim A$), while the other partons belong to the vacuum polarization "fur" ($n \sim A^{2/3}$) which coats the surface of the Lorentz-contracted nucleus.

Now let us look at a very high energy nucleon-nucleus collision in some center-of-mass frame for which the γ of the nucleus exceeds the critical value. Then the nucleon projectile is excited by the layer of wee partons around the "naive" nucleus in the same way as it would be excited were there only a single nucleon as target instead of the entire nucleus. Wee partons in the projectile are excited and wee hadrons in the lab frame are emitted. As far as the fragments of the nucleon projectile are concerned, the same sequence of events then occurs as for an ordinary nucleon-nucleon collision. Therefore the inclusive spectrum of leading hadrons from the nucleon projectile is at sufficiently high energies the same as for an ordinary hadron-hadron collision. This is shown in Fig. 7. This situation persists into the central-plateau region, until a critical rapidity is reached, corresponding to the critical gamma which we described before. Then for rapidities beyond this critical value we must change our considerations and study afresh the distribution of these particles, which comprise the nucleus-fragmentation region. This is easiest to do in the rest frame of the nucleus and is most interesting in the somewhat artificial case of dilute nuclear matter (interaction length >> 1 f) where we can watch in some detail the time evolution of the process. What happens? In the rest frame of the nucleus the nucleon projectile enters and interacts with stationary nucleons within the nucleus. Wee partons in the projectile are excited, wee hadrons are emitted into the nuclear matter at large angles, and the excited projectile continues on its way. After a few fermis the wee partons are de-excited, re-exciting not-so-wee partons in the projectile, which in turn emit not-so-wee hadrons, which excite even more energetic partons in the projectile and so on. This continues on until a subsequent collision occurs downstream with another nucleon in the nuclear matter. At that time there will be re-excitation of the wee partons in the projectile (recall that those wee partons which were excited in the first encounter have already cooled off). Wee hadrons will be emitted at large angles from this second nuclear site, and the process will repeat itself: the wee partons in the projectile excite the not-so-wee partons, etc., until the parton excitation from the second collision overtakes (in rapidity-space) the parton excitation still remaining from the first collision. The two excitations merge into one and proceed onward toward higher rapidity until the next nuclear collision occurs and the process is repeated. This is all shown in Fig. 8, which illustrates the configuration as the excited nucleon leaves the nucleus. There will be various stars, separated on the average by an interaction length and containing the large-angle low-rapidity hadrons which were previously emitted. In the forward cone, as roughly defined by an angle $\lesssim$ 1 fermi/(nuclear diameter), there will exist the excited projectile which will have not yet reached its asymptotic cooled-off condition. It will contain excited partons which have momenta or rapidity of the order of the critical rapidity ($\sim \log \theta^{-1} \sim \log A^{1/3}$) which we discussed previously. Therefore by looking in these two reference frames, we have accounted for the entire hadron distribution. We see that the multiplicity of slow nucleons emitted in the nucleus will be proportional to $A^{1/3}$ (the number of stars) times an additional factor coming from the secondary cascade of the slow hadrons as they proceed in a transverse direction through the nuclear matter. This must be calculated by a straightforward but complex cascade model. But in terms of the multiplicity of particles which within the nuclear matter leave the interaction cylinder (radius ~1 fermi and axis along the trajectory of the incident projectile), we have a multiplicity, relative to an ordinary nucleon collision, of the order $A^{1/3}$.

D. Nucleus-Nucleus Collisions at Fantastic Energies

Before leaving this subject it is fun to consider the collision of two nuclei at energies sufficiently high so that in addition to the fragmentation regions, a central plateau region can develop. Let us consider a central collision of a relatively small nucleus, say carbon, with a big one, say lead. Let us look at this collision in a center-of-mass frame for which the rapidities of both of the nucleus projectiles exceeds the critical rapidity. In such a frame they both possess the fur coat of wee-parton vacuum fluctuations. In such a central collision we see that the collision initially occurs between the fur of wee partons in each of the projectiles. Therefore the number of independent collisions will be of order of the area of overlap of the two projectiles; namely the cross-sectional area of the smaller nucleus.

It follows that the number dN/dy of emitted pions will also be of the order of the area of the smaller nucleus; namely $A_<^{2/3}$, where $A_<$ is the atomic number of the smaller nucleus. This determines the height of the hadron plateau in the central region, relative to what it is in an ordinary nucleon-nucleon collision. There are in addition the two target fragmentation regions. The easiest way to study their properties is to go into the appropriate rest frames. For the fragmentation region of the smaller nucleus we go to its rest frame. In this frame we see the big pancake of the larger nucleus (with its fur coat of wee partons) sweeping through the entire volume of the smaller nucleus. Therefore all nucleons in the smaller nucleus can be excited and the distribution of produced hadrons will therefore be proportional to $A_<^{2/3}$, multiplied by the distribution of produced hadrons for a nucleon-nucleus collision. For the fragmentation region of the larger nucleus, we go into its rest frame. What we see there is a smaller pancake (with wee partons attached) sweeping through the central volume of the large nucleus. The volume swept will be proportional to the cross-sectional area of the smaller nucleus times the diameter of the larger nucleus. Therefore the rapidity distribution of slow hadrons in the fragmentation region of the big nucleus will again be given by $\sim A_<^{2/3}$ times the distribution of slow hadrons for a nucleon incident on the larger nucleus. This can be made a little more quantitative by just estimating the number of independent vacuum fluctuations that can be attached to the incoming nuclei so that there is no overlap in the transverse directions. Doing that I found the semiquantitative guess for the rapidity distribution which is shown in Fig. 9. Much more professional studies along the same line of initial assumptions can be found in the work of O. Kancheli,[32] E. Lehman and G. Winbow,[33] J. Koplik and A. Mueller,[34] and A. Goldhaber.[35]

E. Colliding Beams

We now turn to the real subject at hand, which is the space-time evolution of produced hadrons in the deep inelastic processes, first for the colliding beams. We assume the existence of the jet-like structures discussed earlier and suggested by correspondence or the parton model. Specifically the hadron final states are assumed to be similar to, say $\pi\pi \rightarrow$ hadrons, with the axis of the $\pi\pi$ collision distributed almost isotropically. This is supported experimentally both by the jet observations and by the scaling behavior of the inclusive cross sections. [In making this comparison we must, however, leave out $\pi\pi$ final states involving diffractive excitation.] If the final state in colliding beams really is so similar to that in a $\pi\pi$ collision, it is very natural to suppose that the time evolution of the final state is similar in detail as well, at least at the geographical level that we have discussed. However, we run into a problem. In the colliding beam process, just after the e^+e^- collision, there exist only two produced partons, and they have widely different rapidities. There is no time before they escape from each other for vacuum fluctuations to attach to them or for anything else to happen. The natural time scale for these energetic partons to do something is, as we discussed, proportional to their momentum which in turn is proportional to the center-of-mass energy and can be made in principle as large as we like by going to higher and higher energy. Furthermore we cannot let the intrinsic time scale for these partons to do something be arbitrarily short because we want to protect the free field behavior of the parton propagation at short distances in order that the total cross section behavior comes out right; namely R (the ratio of hadron production to μ-pair production) to be a constant with energy. The simplest picture that comes to mind is that the emission of the mesons be sequential. First an energetic meson is emitted and then a not so energetic one, etc., i.e., the cascade starts with the leading parton dividing, then redividing and so on. I advocated this for a while,[25] but now I think it is wrong. It was also advocated by Drell and Yan,[8] who found it to be a consequence of their cutoff field-theory parton model. However, the trouble with this is that first of all there is no confinement. If there is just division of the original quark and the original antiquark without any communication between them some part of the final state jets will contain fractional charge. Furthermore a finite hadron multiplicity even at infinite energies is predicted. This is unacceptable because if the multiplicity were really finite at infinite energies and the total cross section scaled, then some partial cross section would have to comprise a finite fraction of the total cross section. Therefore some exclusive channel, composed of a finite number of hadrons would have to exhibit scaling behavior with respect to its total cross section. Then the problem is which hadron final state is it? If it did exist we would essentially find the partons directly.

What about adding a ladder between the two groups of partons associated with the quark and the antiquark? There is still trouble. This was discovered by Kogut, Sinclair, and Susskind,[36] and by Craigie, Kraemmer, and Rothe.[37] Again the trouble is the large time scale for the original parent partons to do something: they are too far apart by the time they start emitting their ladders. When one studies the diagrams in momentum space, what happens is that in ordinary collisions the ladder graph with n rungs corresponds to an amplitude behaving as $(\log s)^n$. Upon summation one generates the Regge trajectory. However in the case of colliding beams the $i\epsilon$'s in the propagators are in the wrong places to generate these logarithms, which come from pinches in certain integration contours. There are no $(\log s)^n$ factors generated in nth order, and in fact the ladders are rather inconsequential corrections to the lowest order diagram. The fact that the $i\epsilon$'s are in the wrong place is related to causality considerations, which again have to do with the space-time development. In the case of hadron-hadron collisions, there is a long time before the collision for virtual processes to prepare the long chain of virtual partons (including the wees) which initiate the kind of final state generation which we already discussed. However when everything starts at $t=0$ instead of $t=-\infty$, there is no time for all of that preparation.

What about other very exotic alternatives? One can imagine elastic rubber bands connecting the partons, which first separate a long way, emit some hadrons, come to a stop, then get pulled together by the rubber band, then oscillate back and forth with some damping, eventually annihilating. In terms of the space-time development, that looks bizarre relative to what we have considered. One might think about violent discharge in the vacuum, something analogous to lightning bolts. That again looks a little ridiculous. The best bet seems just to imitate as best as possible the ordinary processes we discussed, where the wee hadrons emerge first, the not-so-wee later on, etc., with all the activity concentrated on a proper time surface as shown in Fig. 6. But we must look at this option in more detail to see what the implications are. At a time, after the birth of the parton anti-parton pair, small compared to a fermi there is nothing but original quark-antiquark: no produced hadrons and no produced partons. After a time of the order of a fermi we somehow have wee hadrons produced, if we are to imitate what happens in ordinary collisions. If wee hadrons are produced, there can also be produced an extra wee quark-antiquark pair which follow respectively the originally produced antiquark and quark directions and begin to screen the fractional charge of the antiquark-quark. As time goes on, by hypothesis the time evolution is the same as discussed for ordinary collisions. Therefore later on, not-so-wee hadrons are produced and the original polarization clouds of produced parton and antiparton which were initially following the parent antiquark-quark get accelerated; as time goes on they gain momentum in such a way as to be in the same region of phase space as excited partons in an ordinary collision. This is all illustrated in Fig. 10. The time scale for this kind of evolution is assumed to be the same as for the ordinary collision. Therefore the polarization cloud of partons which follow the original parents attains a rapidity (or momentum) comparable to their parents at a time proportional to the original momentum of the partons; namely proportional to the center-of-mass energy. At that time they can annihilate into a final leading hadron system.

What are the messages this time? First of all, we see that in this process there must necessarily be long range correlations in rapidity in order to screen the fractional charge. This is in fact true even were we to invoke rubber bands or lightning bolts. There seems to be no way of avoiding it. Secondly, confinement of a quark occurs at a time proportional to the momentum of the quark, when the polarization cloud overtakes (in momentum space) its parent sufficiently to annihilate it. In the intermediate stage there is always a polarization cloud accompanying the quark. It will have a lag or a typical longitudinal thickness inversely proportional to its momentum. This cloud is gently accelerated by the leading quark as time goes on. In the c.m.s. frame of the quark and the polarization cloud, fractional charge is never separated by more than a fermi. As time goes on the leading quark gets slightly decelerated in order to feed energy into the produced hadrons as well as to feed energy into the accelerating polarization cloud so that at any given time energy can be approximately conserved. Since hadrons (i.e., quark pairs) are being emitted from the polarization cloud, it need not be a unique quark in the cloud which is accelerated. The current in the cloud is polarization current.

The rate of change of the momentum of the leading quark with time dp/dt is a gentle constant, say ~1 GeV per fermi of travel, involving no large mass scale. Furthermore in order of magnitude this is also the time rate of change of the momentum of the polarization cloud as well as the time rate of change of the total energy of the produced hadrons. The free-field behavior at short distances is clearly protected because the number of soft interactions per unit of time is of the order of a constant. We need at most a few soft interactions per fermi in order to change the momentum of the quark or create the produced hadrons, and this number is independent of the quark momentum. As the total energy W increases one only needs the free field behavior to be true at distances of the order of W^{-1} in order to protect the total cross section behavior. Finally, if the only effect on the original parent quark is to be gently decelerated by the polarization cloud which it is dragging along and by the hadrons which are being emitted, then the wave function of the parent quark should just be a free field wave function times an eikonal phase:

$$\psi_{quark}(x, t) \sim \psi_{free}(x, t)\, e^{iS(x, t)} \tag{5.6}$$

This is suggestive of an underlying gauge theory (however, in real life we may well want that gauge theory to be non-abelian; the presence of simple eikonal phases is a property more of Abelian rather than non-Abelian theories). But in any case a gauge theory for the confinement mechanism, either Abelian or non-Abelian, certainly is strongly suggested by this line of argument, both by the above feature of the gentle deceleration being associated with eikonal phases and also by the existence in the dynamics of long range correlations in rapidity, again suggesting the existence of spin 1 quanta in the field theory.

There is some formal support for this handwaving from the behavior of two-dimensional quantum electrodynamics, as found by Casher, Kogut and Susskind.[38] There is some question as to whether that theory is too simple to really be a good prototype, but in any case it does give some encouragement. Furthermore, one-dimensional concepts may be relevant even in the three-dimensional case, because we saw that for the system of the quark and polarization cloud the transverse extent is of the order of 1 fermi whereas the longitudinal extent of the important components of the system is much smaller than 1 fermi. The pancake-shaped polarization cloud may in fact produce something like a uniform electric field which then acts upon the leading quark. The relevance of such one-dimensional motions and approximations has already been exhibited in the Landau hydrodynamic model, which has in fact a realistic space-time development similar (but not identical) to what we have described.

Before leaving the subject of colliding beams, it will be useful to check the consistency of this picture by looking at the same process in the Fool's ISR. In this case electron and positron have comparable longitudinal momentum and collide at a small crossing angle θ at fantastically high energies. To describe this situation we simply have to change the old solution by boosting it in the transverse direction. Before we boost it, the characteristic time at which the first hadrons emerged from the collision was of the order of 1 fermi. Now in the boosted frame this time will be increased by the γ of the boost; $\gamma \sim \theta^{-1}$. Then the characteristic time will be 1 fermi/θ; much longer. No appreciable number of hadrons are emitted which are wee. The wee hadrons in the original frame now have momentum which are ~(hundreds of MeV)/θ. An analog in very high energy electrodynamics is the Chudakov effect,[39] which inhibits the ionization produced in electromagnetic pair production at sufficiently high energy because the electron and positron do not have sufficient transverse separation to create a dipole moment strong enough to ionize the atoms until considerably downstream from the production point. For our case the quark and the antiquark must have a transverse separation (or an invariant separation in space-time) of the order of a fermi before there is any appreciable hadron production. In the FISR configuration that leads to a delay in the appearance of the hadrons until a time considerably longer than 1 fermi.

F. Electroproduction (or Neutrino Reactions)

The simplest case to study first is for small ω, where there is a reasonably straightforward generalization of what we have just discussed. We look at the process in the laboratory frame. The incident virtual photon strikes a wee quark and imparts to it all of its momentum ν. Just after this happens, the parton rapidity distribution will consist of wee partons and the struck parton of high energy, with a large rapidity gap ($\sim \log \nu$) between the two systems. Therefore the parton leaves the proton by a distance of ~1 fermi, wee hadrons and the wee polarization cloud accompanying the struck parton will again be produced. At some later time (~10 f. ?) we will have what is shown in Fig. 11, with the rapidity distribution of produced hadrons moving from the wee region outward. The rapidity distribution of partons in the polarization cloud terminates at a momentum comparable to the momentum of the most energetic produced hadron which in turn is proportional to the elapsed time. Finally the original parent parton (the one that was struck by the virtual photon) is still isolated in phase space (i.e., momentum space) and has been essentially unaffected except to be gently decelerated by the polarization cloud.

What are the messages? First of all, the time evolution resembles the colliding beam evolution. The polarization cloud exists and therefore for the rapidity distribution of produced hadrons there should be the same current plateau as there was in e^+e^- annihilation. Secondly the reaction is complete only when the elapsed time in the laboratory frame is of order ν, the energy of the virtual photon. Therefore if we were dealing with electron-nucleus scattering at large ν, the reaction terminates only after the quark has left the nucleus. I don't have a complete picture of what will happen under these circumstances. But at least it is clear that an experimental study of the

A-dependence of the spectrum of energetic hadrons produced in electroproduction could turn out to be quite interesting.

Finally, let us again look at electroproduction (in the laboratory frame) but when ω is very large (>> 100?). This is a difficult case; the problem is how the hole fragmentation phenomenon appears and whether we can see how vector dominance is involved. For large ω, the general considerations of Ioffe[20] imply that the process of virtual-photon dissociation into quark-antiquark will start before the virtual photon arrives at the target proton. There is some time for the quark-antiquark system to at least become partially dressed before arrival at the target. Let us look at this in detail. The kinematics is shown in Fig. 12a. At the naive classical level (which should not be too bad for very light quarks) we might expect the following picture: First of all let us suppose that the virtual photon dissociates into a quark-antiquark pair with comparable longitudinal momenta. If the virtual photon has virtual (spacelike) mass ~Q, the typical mass (now timelike) of the quark-antiquark system will be again ~Q. Therefore, for a symmetrical dissociation the transverse momenta of the dissociated quark-antiquark will also be ~Q/2. The time Δt this fluctuation lives cannot be indefinitely long because we have not conserved energy. In the usual way we can estimate using the uncertainty relations

$$z \sim \Delta t \sim \frac{1}{\Delta E} \sim \frac{2\nu}{Q^2 + m_n^2} \sim \frac{\omega}{M} \tag{5.7}$$

The distance z (or the time) the fluctuation survives is proportional to ω: when ω is large we have this kind of upstream phenomenon, when ω is small we do not.

We may also estimate the separation of the quark-antiquark in impact parameter when it arrives at the nucleon. The calculation gives

$$\Delta x_\perp \sim z\theta \sim z\left(\frac{p_\perp}{\nu}\right) \sim \left(\frac{\nu}{Q^2}\right)\left(\frac{\sqrt{Q^2}}{\nu}\right) \sim \frac{1}{\sqrt{Q^2}} \tag{5.8}$$

This is the "shrinking photon". When Q^2 is large the transverse separation is much less than a fermi and decreases with increasing $Q^2 \sim Q^{-1}$. This is too small to produce any wee partons because of the "Chudakov effect" which we discussed in the previous section. With no wee partons in the virtual state there is no way that this system can be absorbed by the target nucleon and nothing happens, in line with the discussion in Section I. However, if the fluctuation is asymmetric in the parton momenta then we can obtain a transverse separation ~1 fermi within the duration of the fluctuation. The kinematics is shown in Fig. 12b, and we see that the pair mass can be found by computing $E-p_\parallel$ for the pair and its components

$$E - p_\parallel \cong \frac{m^2}{2\nu} \cong \frac{p_\perp^2}{2p} + \frac{p_\perp^2}{2(\nu - p)} \tag{5.9}$$

With a pair mass $m \sim Q$ and $p \ll \nu$ we can calculate the transverse momentum $p_\perp$ of the partons; it follows from Eq. (5.9)

$$p_\perp^2 \sim \frac{p}{\nu}\, Q^2 = \frac{2Mp}{\omega} \tag{5.10}$$

Now we repeat the calculation leading to Eq. (5.8). The important longitudinal distance depends only on the virtual mass of the quark-antiquark system and on Q^2 and therefore is again proportional to ω. The transverse separation $\Delta x_\perp$ is given by

$$\Delta x_\perp \sim z\theta = z\left(\frac{p_\perp}{p}\right) = \frac{\nu}{pQ^2}\sqrt{\frac{2Mp}{\omega}} \sim \sqrt{\frac{\omega}{2Mp}} \tag{5.11}$$

and the condition $\Delta x_\perp \gtrsim 1f$ leads to

$$p \lesssim \omega \qquad (5.12)$$

From Eq. (5.10) we see, not unexpectedly, that this is equivalent to the condition $p_\perp \lesssim$ const. Probably (5.10) is the most reliable numerical estimate of p, using $\langle p_\perp^2 \rangle \sim 0.1 - 0.2 \text{ GeV}^2$. Therefore the typical rapidity for the slow quark is going to be of order log $\omega/10$. If the rapidity is larger than this, the transverse separation of the quark-antiquark at arrival is too small and we expect very little interaction. If, on the other hand, the rapidity of the slow quark is less than the characteristic value ~log $\omega/10$ there is no phase space. The slow quark has not a dx/x spectrum, but a dx spectrum corresponding to an isotropic decay of a parent (namely the virtual photon) into two secondaries. This situation leads to an exponentially decreasing contribution for smaller rapidities. We conclude that it is necessary for the slow parton to have a characteristic momentum $\sim(\omega/10)$ GeV in order to induce some partons out of the vacuum by arrival time. If parton pairs are induced they will also have momentum $\lesssim(\omega/10)$ GeV. And there is enough time for all of these virtual quarks to have coupled to a sequence of ordinary vacuum fluctuations just like an ordinary hadron would. Therefore when this whole system arrives at the nucleon the virtual photon has the structure shown in Figs. 12c and 12d. We see that the ordinary vacuum fluctuations will interact just as in an ordinary hadron collision, provided the momentum scale is $\lesssim(\omega/10)$ GeV and the distance scale downstream is $\lesssim(\omega/10)$ fermi. At collision the cloud of vacuum fluctuations excites the nucleon. Then wee hadrons are emitted and again excitations in the virtual-photon cloud move upward in momentum (or rapidity) space in the way appropriate to a normal hadron-hadron collision. This continues until a characteristic time $t \sim \omega/10$ f.

Remember that (at sufficiently large ω) this is a long time and therefore in an electron-nucleus collision this mechanism should govern what is going on in the nuclear matter.

Finally for $t > \omega/10$ f , the leading parton in the polarization cloud (of momentum $\sim\omega/10$ GeV) begins to get accelerated by the leading quark up to momentum ν. In this period of time the hadron emission occurs in the same way as for small ω, because the polarization cloud of fractional charge is being accelerated by the energetic quark. This process generates a "current plateau" and completes the picture of hadron production in a way which is totally consistent with what we discussed in more general terms without use of the space-time picture. In particular the jet picture again emerges. There is limited transverse momentum of the produced hadrons, there is a current plateau for produced-hadron rapidities large compared with log $(\omega/10)$, and there is a hadron plateau for rapidities $\ll$ log $(\omega/10)$. The hole fragmentation region divides the two plateaux, and we have even identified some of the dynamics associated with the hole fragmentation region itself.

G. Conclusions

In summary, we have found that the picture of space-time evolution of hadron final states in deep inelastic processes isn't totally trivial, and also have found that it can be made consistent with the hypotheses of the parton model (and therefore also with the trends of the data at present). This is only the case provided that there exist long-range rapidity correlations in the deep inelastic dynamics. This is in fact what is indicated in two dimensional quantum electrodynamics, as discovered by Casher, Kogut and Susskind. It strongly suggests the relevance of some kind of gauge theory as a necessary element in the interpretation of the confinement problem. Finally it is clear that the discussion above is so qualitative that while hopefully providing some insight into the space-time geography of what is going on, it is a far cry from what we should like to have in order to make quantitative comparisons with experiment. In fact, I see the above description as a solution to a problem which hasn't been well defined. The problem remaining is to try to sharpen the above considerations by specifying precisely what the problem is that is to be solved. Perhaps it is a variant of the Landau hydrodynamic model, but one which incorporates the concepts of short range correlation in rapidity instead of Landau's concept of total arrest of hadronic matter in hadron-hadron collisions. If such a hydrodynamic picture could be worked out it might lead to some further insights into the dynamics of confinement and of deep inelastic processes.

REFERENCES

1. Much of these lectures are also presented in the 1973 Proceedings of the Summer Institute on Particle Physics, edited by M. Zipf, SLAC-167 (1973).
2. R. P. Feynman, Photon-Hadron Interactions (W. A. Benjamin, New York, 1972).
3. J. Bjorken and J. Kogut, Phys. Rev. D 8, 1341 (1973).
4. S. Drell and T. Yan, Phys. Rev. Letters 24, 181 (1970).
5. G. West, Phys. Rev. Letters 24, 1206 (1970).
6. E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970).
7. In addition to the contributions to this volume, which we do not explicitly cite, a valuable source of recent data is found in the Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, ed. W. T. Kirk, Stanford Linear Accelerator Center (1975).
8. S. Drell, D. Levy and T. Yan, Phys. Rev. D 1, 1617 (1970).
9. S. Berman, J. Bjorken and J. Kogut, Phys. Rev. D 4, 3388 (1971).
10. R. Schwitters, Ref. 2, p. 5.
11. A review of the kinematics for these processes can be found in F. Gilman, Phys. Reports C 4, 98 (1972).
12. B. Roe, Ref. 2, p. 551.
13. A recent and informative discussion is found in J. Van der Velde; talk presented at IVth International Winter Meeting on Fundamental Physics, Salardu, Spain (1976); University of Michigan preprint.
14. G. Wolf, Ref. 2, p. 795. Also R. Mozley, Ref. 2, p. 783.
15. L. Mo, Ref. 2, p. 651.
16. P. Garbinicus et al., Phys. Rev. Letters 32, 328 (1974); A. J. Sadoff et al., Phys. Rev. Letters 32, 955 (1974).

17. See the report of G. Wolf, Ref. 2, especially p. 846.

18. J. Bjorken, Phys. Rev. D 7, 2747 (1973).

19. R. Cahn and E. Colglazier, Phys. Rev. D 9, 2658 (1974).

20. B. L. Ioffe, Phys. Letters B 30, 123 (1969).

21. The relaxation of this assumption is studied by H. Fraas, B. Read and D. Schildknecht, Nucl. Phys. B 86, 346 (1975).

22. V. Gribov, Fourth Winter Seminar on the Theory of the Nucleus and the Physics of High Energies, 1969, Ioffe Institute of Engineering Physics, Acad. Sci., USSR.

23. M. Greco, Nucl. Phys. B 63, 398 (1973).

24. J. J. Sakurai and D. Schildknecht, Phys. Letters B 40, 121 (1972); 41, 489 (1972); 42, 216 (1972).

25. J. Bjorken, Proceedings of the International Conference on Duality and Symmetry in Hadron Physics, ed. E. Gotsman (Weizmann Science Press, Jerusalem, 1971).

26. H. Cheng and T. T. Wu, Phys. Rev. 183, 1324 (1969).

27. J. Bjorken, Particle Physics (Irvine Conference, 1971), AIP Conference Proceedings No. 6, Particles and Fields Subseries, No. 2, ed. M. Bander, G. Shaw and D. Wong (American Institute of Physics, New York, 1972).

28. G. Preparata, Phys. Rev. D 7, 2973 (1973) and these proceedings.

29. L. D. Landau, Izv. Acad. Nauk. SSSR 17, 31 (1953).

30. E. Feinberg and I. Ya. Pomeranchuk, Nuovo Cimento Suppl. 3, Series 10, 652 (1956).

31. A recent discussion of the relationship of parton concepts to nonrelativistic quantum physics is given by G. West, Physics Reports 18, 263 (1975).

32. O. Kancheli, JETP Letters 18, 274 (1973).

33. E. Lehman and G. Winbow, Phys. Rev. D 10, 2962 (1974).

34. J. Koplik and A. Mueller, Phys. Rev., Preprint CO-2271-59 (to be published).

35. A. Goldhaber, Phys. Rev. Letters 33, 47 (1974).

36. J. Kogut, D. Sinclair, and L. Susskind, Phys. Rev. D 7, 3637 (1973).

37. N. Craigie, A. Kraemmer, and K. Rothe, Z. Phys. 259, 1 (1973).

38. A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Letters 31, 792 (1973).

39. A. E. Chudahov, Isvestia Akad. Nauk. SSSR, Ser. Fiz. 19, 650 (1955).

FIGURE CAPTIONS

1. Expected rapidity distribution of hadrons produced in very high energy e^+e^- annihilation or ω-decay, as measured along the jet axis.
2. Expected rapidity distribution of electroproduced or neutrino-produced hadrons at very high ν: (a) Q^2 small or zero; (b) ω, Q^2 large; (c) ω small, Q^2 large.
3. Rapidity distribution of electroproduced or neutrino-produced hadrons according to the parton model: (a) ω large; (b) ω small.
4. Virtual forward Compton amplitude according to the vector dominance model.
5. Estimated scaling behavior of single-pion electroproduction according to correspondence arguments.
6. Space-time diagram of a hadron-hadron collision: (a) view down the light cone from time t; (b) view in the z-t plane ($x_\perp$ small); (c) parton rapidity distribution at time t; (d) rapidity distribution of produced hadrons at time t.
7. Rapidity distribution of produced hadrons in a nucleon-nucleus collision at very high energy.
8. Intermediate state of a nucleon projectile interacting in nuclear matter: (a) picture of the collision; (b) rapidity distribution of projectile partons; (c) rapidity distribution of produced hadrons.
9. Estimated rapidity distribution of produced hadrons for a central collision of a light nucleus of atomic weight $A_<$ and a heavy nucleus of atomic weight $A_>$.
10. Space-time picture of hadron production in e^+e^- annihilation: (a) view down the light cone at time t; (b) view in the z-t plane; (c) parton rapidity distribution at time t; (d) rapidity distribution of emitted hadrons at time t.

11. Electroproduction at small ω (laboratory frame): (a) configuration of produced hadrons at time t; (b) rapidity distributions of partons at time t; (c) rapidity distribution of produced hadrons.

12. Electroproduction at large ω: (a) dissociation of virtual γ into $q\bar{q}$ pair of comparable longitudinal momenta; (b) dissociation of virtual γ into $q\bar{q}$ pair with asymmetric partition of longitudinal momenta; (c) structure of virtual photon in rapidity space upon arrival at target (for asymmetric momentum partition); (d) structure of virtual photon in impact parameter (x-y) space upon arrival at target; (e) rapidity distribution of partons and produced hadrons at a time t after collision, $t \ll \omega$; (f) rapidity distribution of partons and produced hadrons at a time t after collision with $t \gg \omega$.

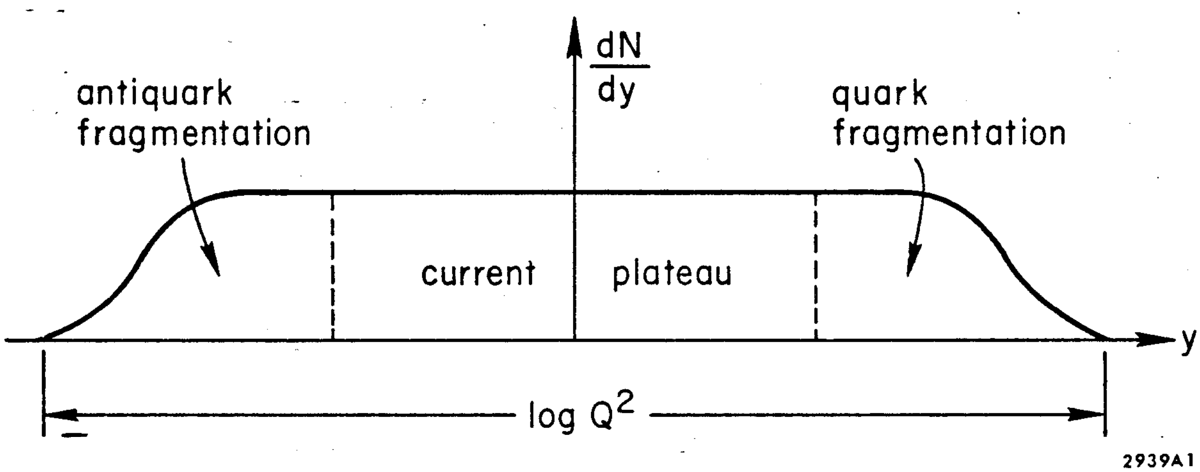
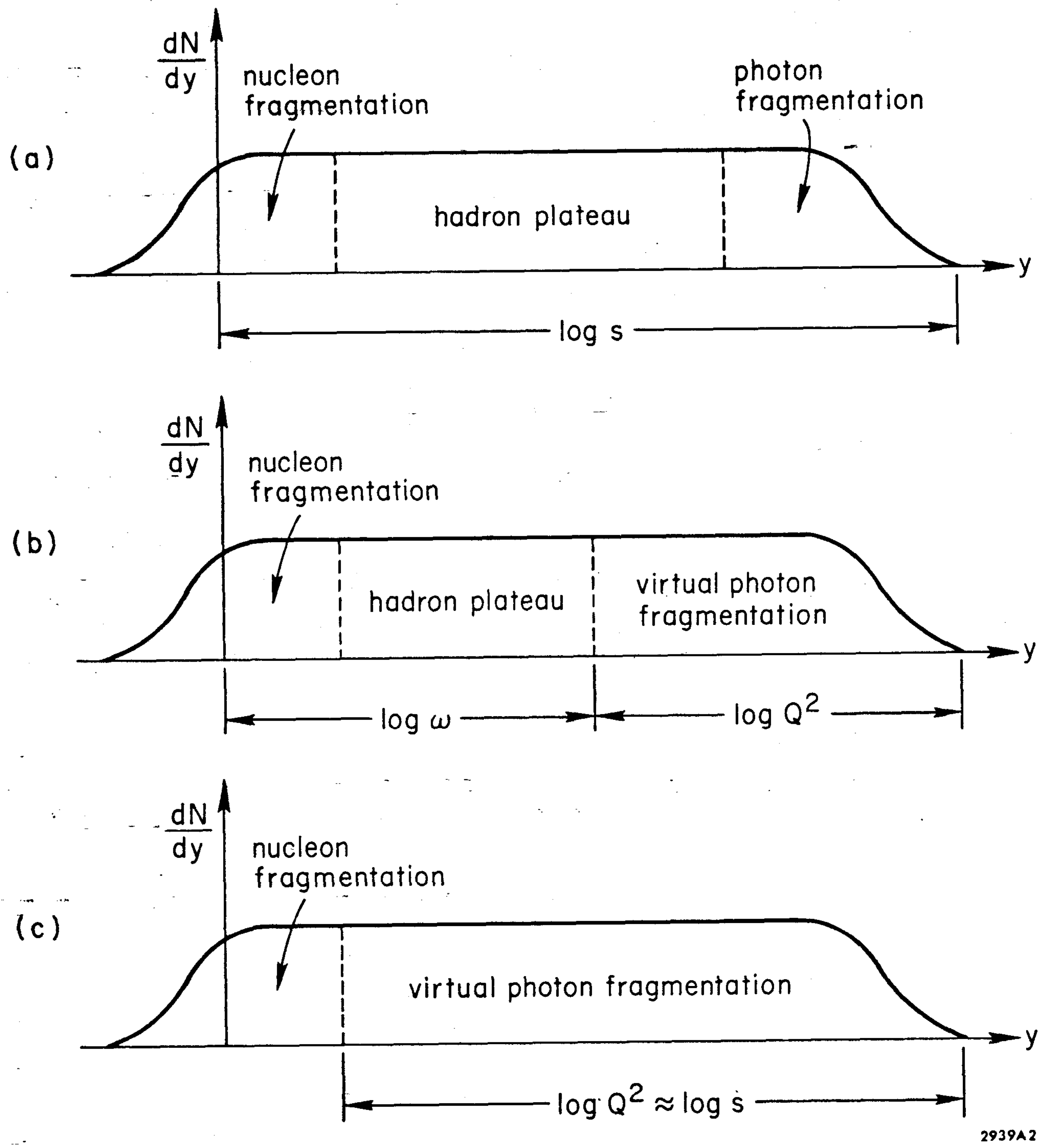
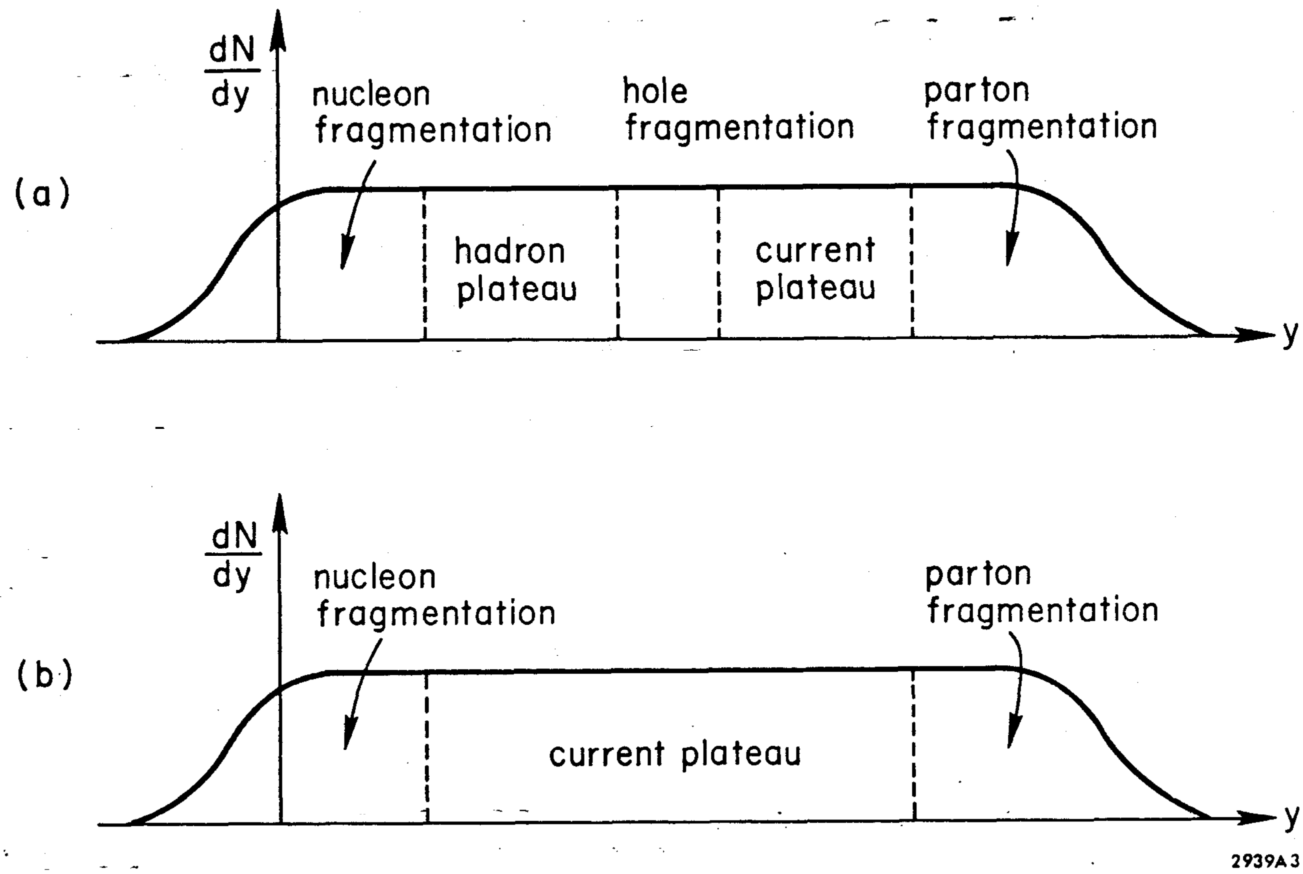


Fig. 1



dN/dy
nucleon fragmentation
photon fragmentation
(a)
hadron plateau
y
log s
dN/dy
nucleon fragmentation
(b)
hadron plateau
virtual photon fragmentation
y
log ω
log Q2
dN/dy
nucleon fragmentation
(c)
virtual photon fragmentation
y
log Q2 ≈ log s
2939A2

Fig. 2



dN/dy
nucleon fragmentation
hole fragmentation
parton fragmentation
(a)
hadron plateau
current plateau
y
dN/dy
nucleon fragmentation
parton fragmentation
(b)
current plateau
y
2939A3

Fig. 3

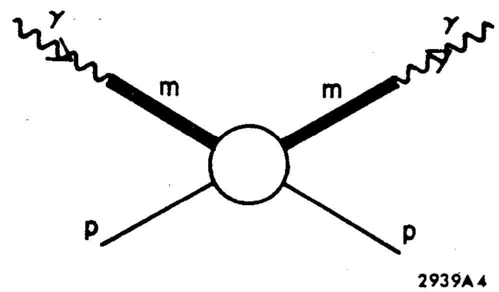

γ
γ
m
m
p
p
2939A4

Fig. 4

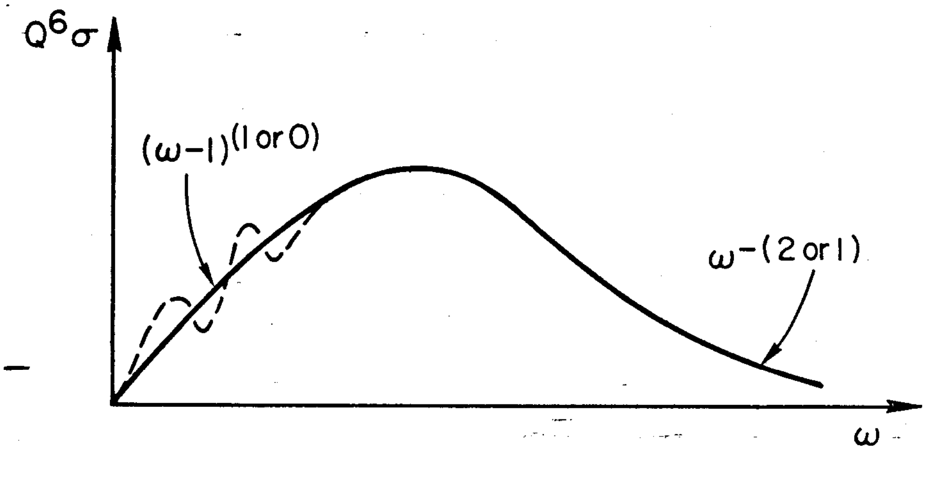

$Q^6\sigma$
$(\omega-1)^{(1 \text{ or } 0)}$
$\omega^{-(2 \text{ or } 1)}$
ω
2239A7

Fig. 5

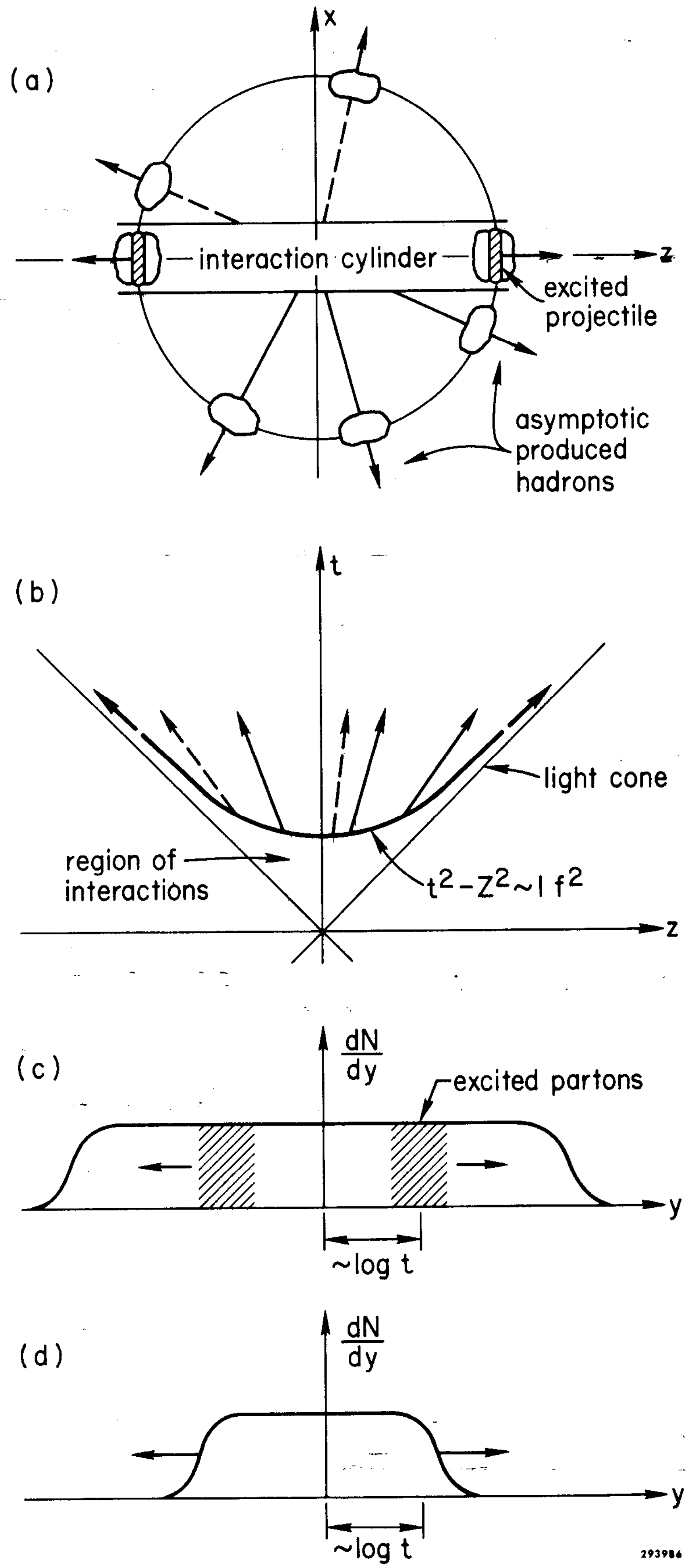

(a)
x
interaction cylinder
z
excited projectile
asymptotic produced hadrons
(b)
t
light cone
region of interactions
$t^2 - Z^2 \sim 1\ f^2$
z
(c)
$\frac{dN}{dy}$
excited partons
y
~log t
(d)
$\frac{dN}{dy}$
y
~log t
293986

Fig. 6

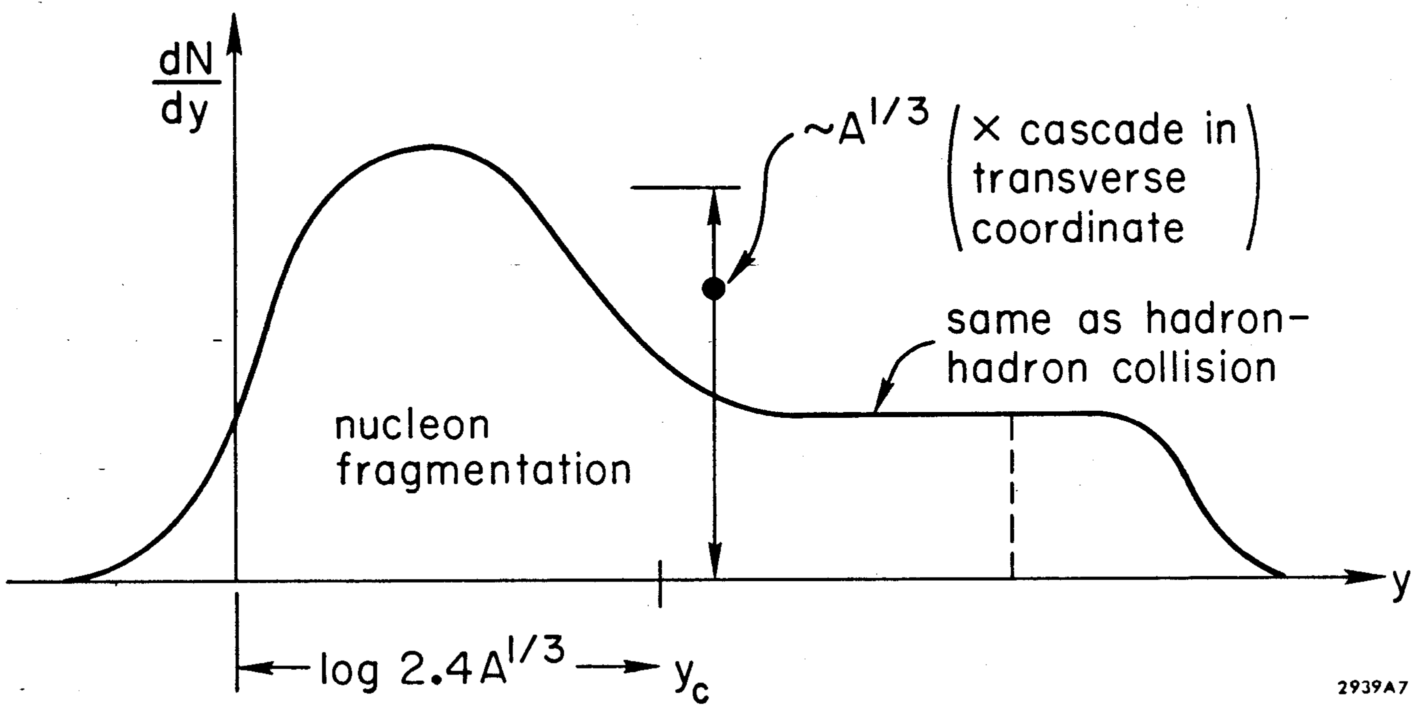

dN/dy
~$A^{1/3}$
(× cascade in transverse coordinate)
same as hadron-hadron collision
nucleon fragmentation
log $2.4A^{1/3}$
y_c
y
2939A7

Fig. 7

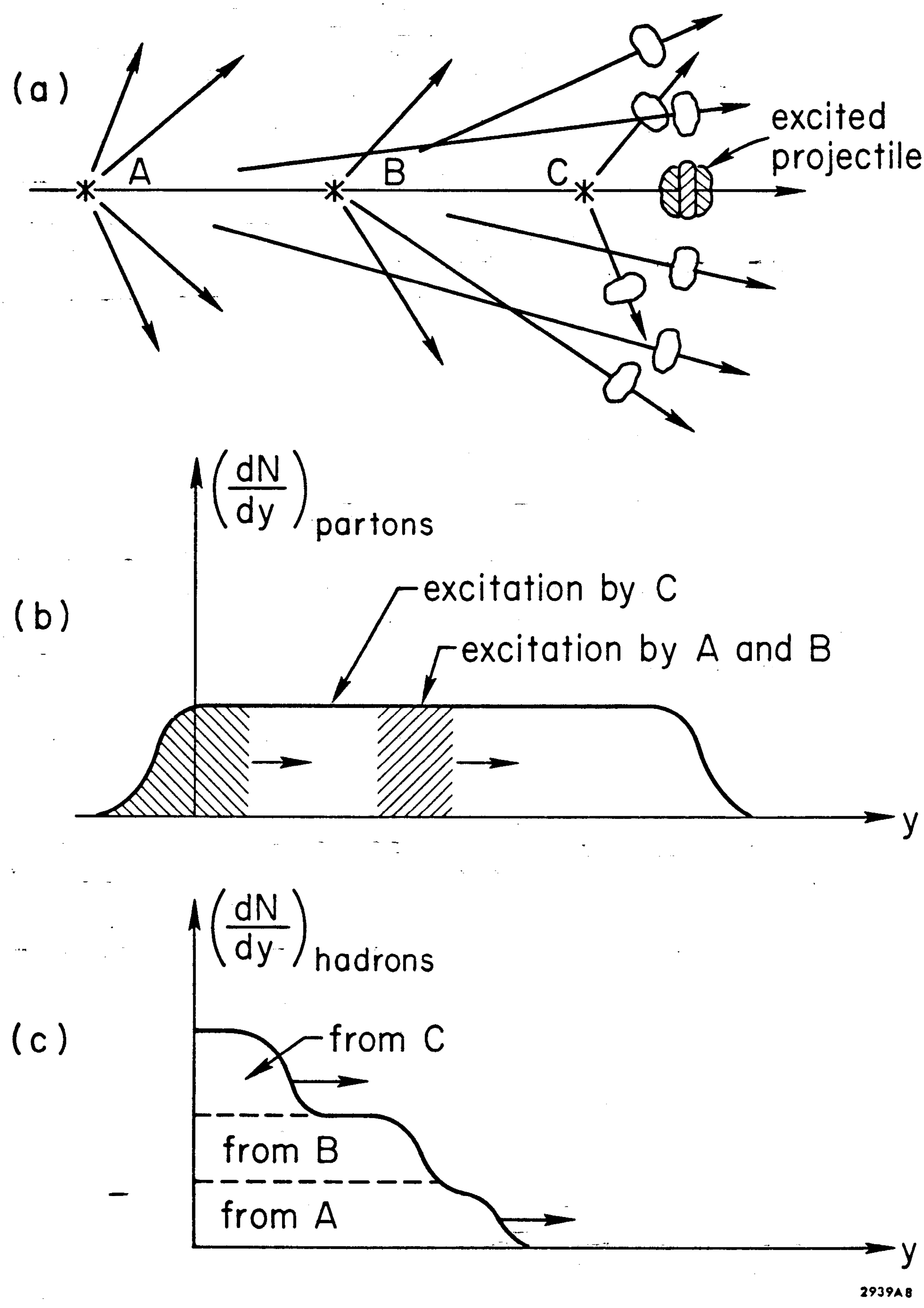

(a)
A
B
C
excited projectile
(b)
dN/dy partons
excitation by C
excitation by A and B
y
(c)
dN/dy hadrons
from C
from B
from A
y
2939A8

Fig. 8

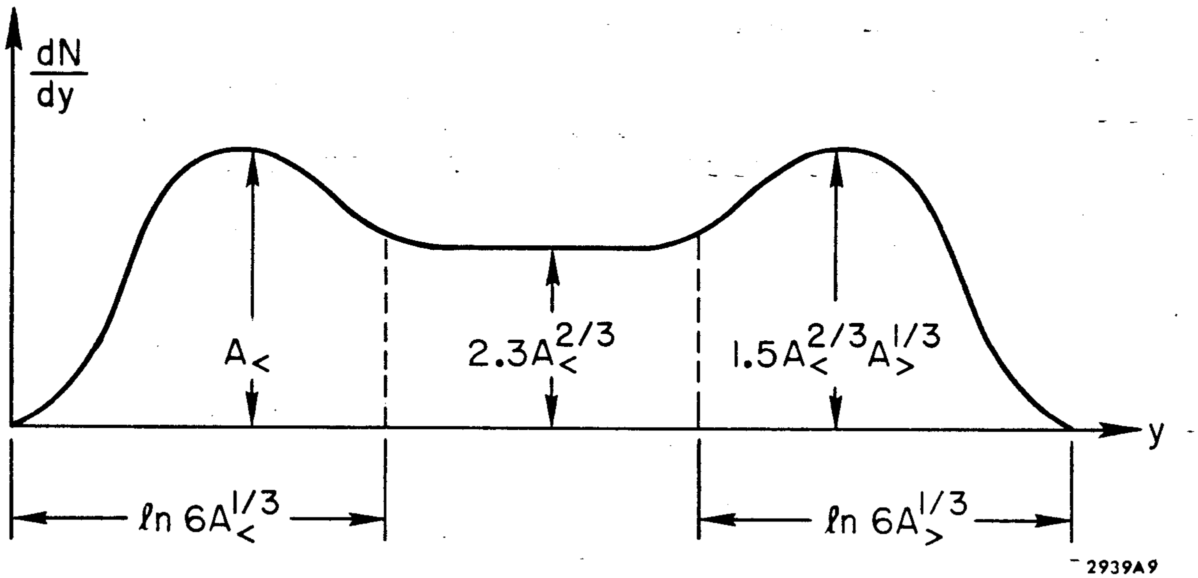


Fig. 9

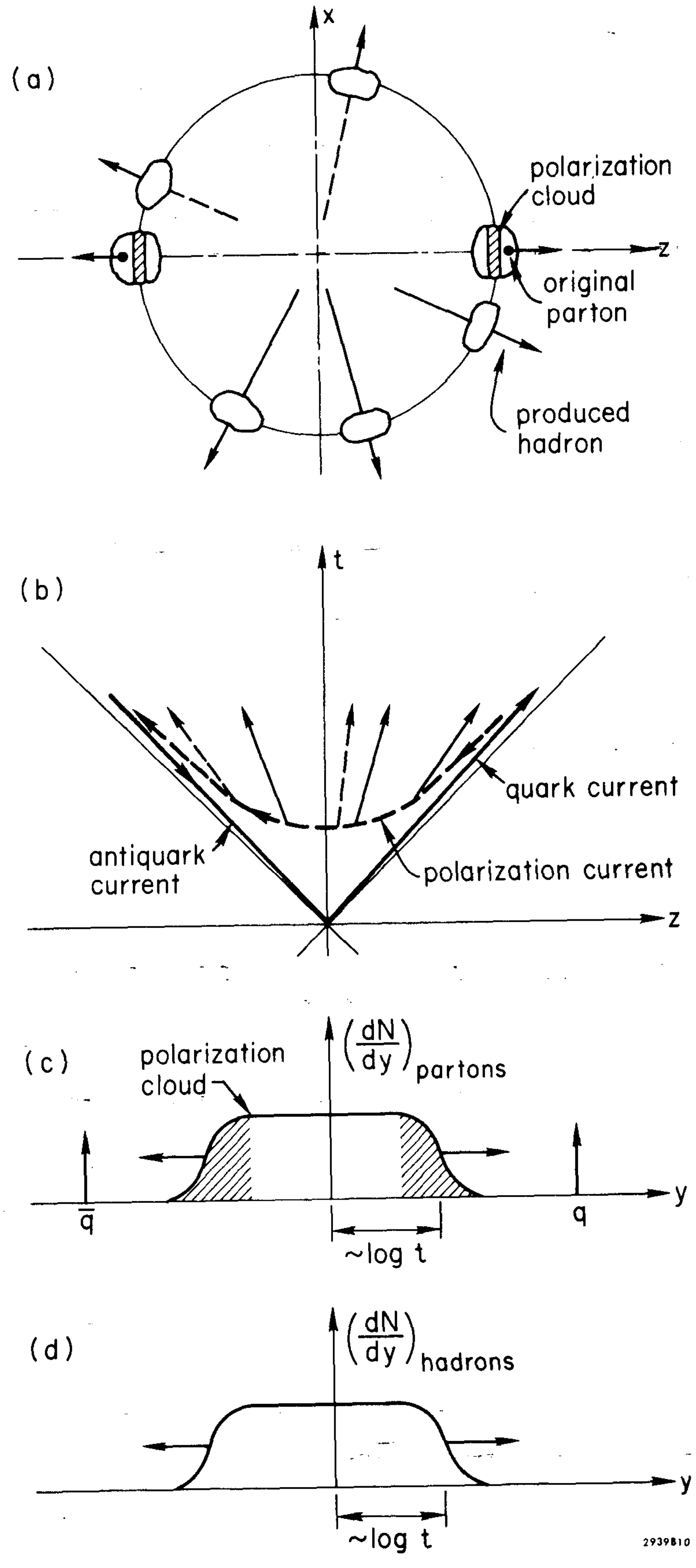

(a)
x
polarization cloud
z
original parton
produced hadron
(b)
t
quark current
antiquark current
polarization current
z
(c)
polarization cloud
$\left(\frac{dN}{dy}\right)_{partons}$
y
$\bar{q}$
q
~log t
(d)
$\left(\frac{dN}{dy}\right)_{hadrons}$
y
~log t
2939B10

Fig. 10

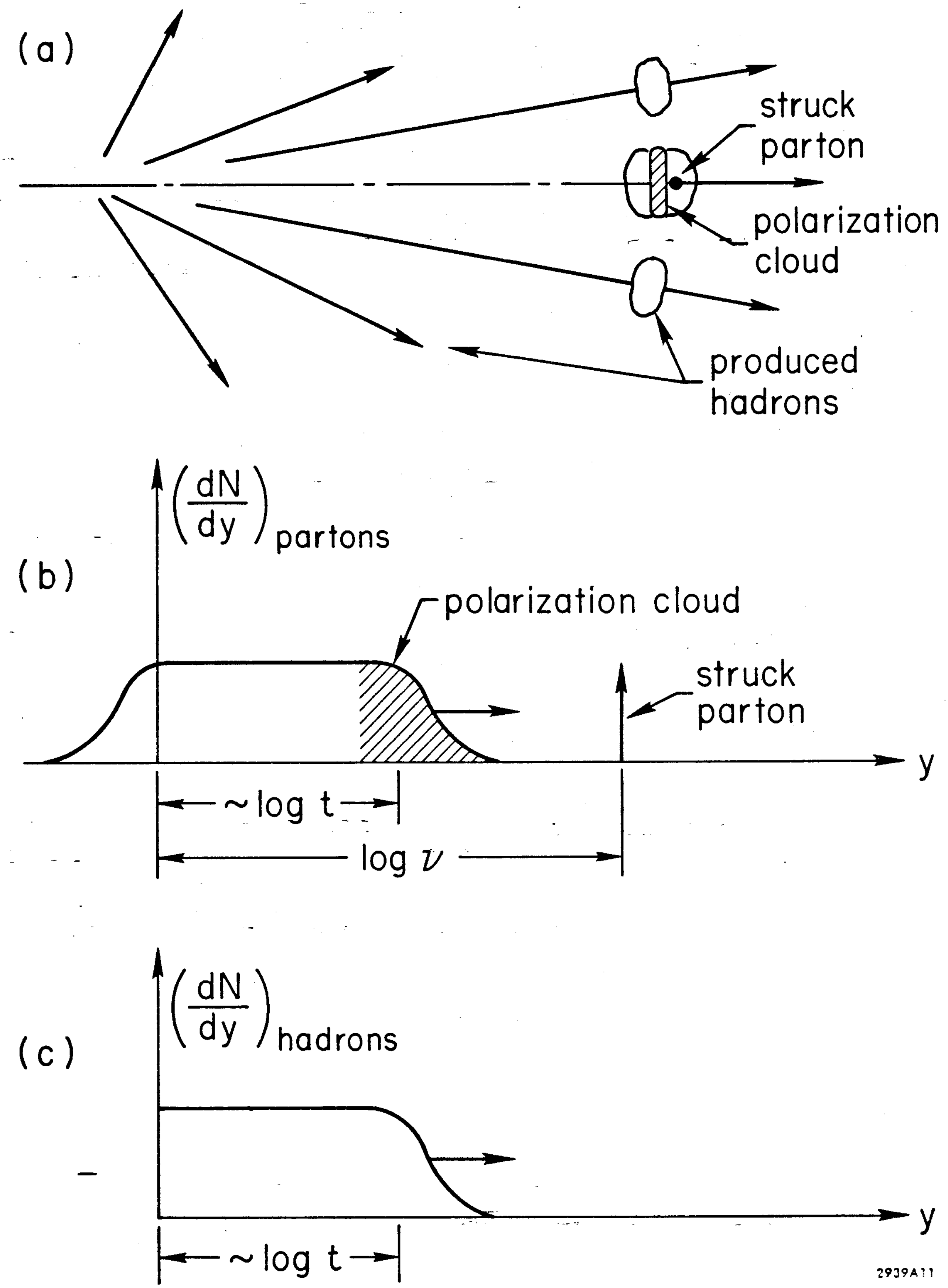

(a)
struck
parton
polarization
cloud
produced
hadrons
(b)
$\left(\frac{dN}{dy}\right)_{\text{partons}}$
polarization cloud
struck
parton
y
~log t
log ν
(c)
$\left(\frac{dN}{dy}\right)_{\text{hadrons}}$
y
~log t
2939A11

Fig. 11

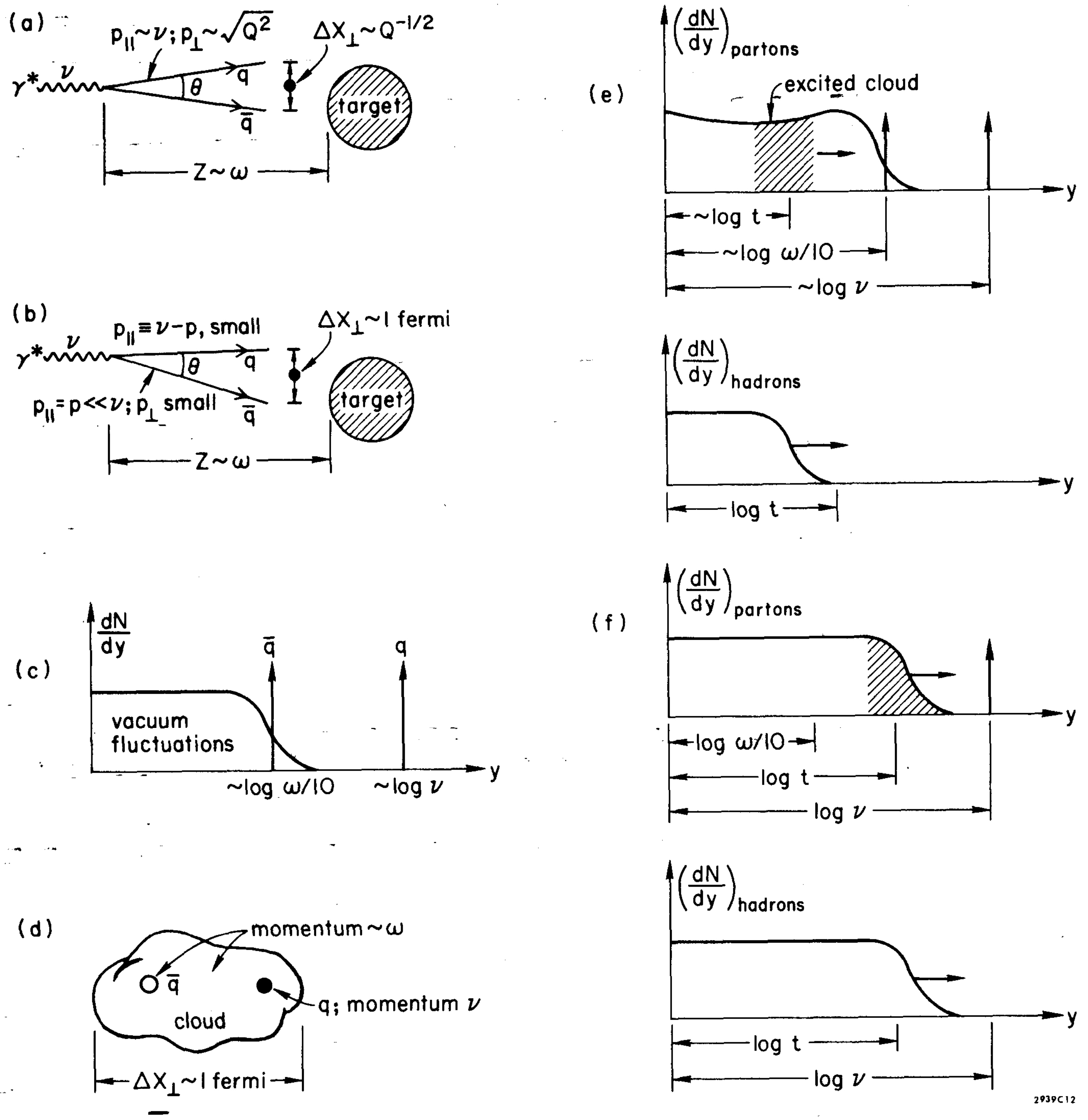

(a)
$p_{\parallel} \sim \nu ; p_{\perp} \sim \sqrt{Q^2}$
$\Delta X_{\perp} \sim Q^{-1/2}$
γ^* ν θ q $\bar{q}$
target
$Z \sim \omega$
(b)
$p_{\parallel} \equiv \nu - p$, small
$\Delta X_{\perp} \sim 1$ fermi
γ^* ν θ q $\bar{q}$
$p_{\parallel} = p \ll \nu ; p_{\perp}$ small
target
$Z \sim \omega$
(c)
$\frac{dN}{dy}$
$\bar{q}$ q
vacuum fluctuations
$\sim \log \omega/10$ $\sim \log \nu$ y
(d)
momentum $\sim \omega$
$\bar{q}$
q; momentum ν
cloud
$\Delta X_{\perp} \sim 1$ fermi
(e)
$\left(\frac{dN}{dy}\right)_{partons}$
excited cloud
y
$\sim \log t$
$\sim \log \omega/10$
$\sim \log \nu$
$\left(\frac{dN}{dy}\right)_{hadrons}$
y
$\log t$
(f)
$\left(\frac{dN}{dy}\right)_{partons}$
y
$\log \omega/10$
$\log t$
$\log \nu$
$\left(\frac{dN}{dy}\right)_{hadrons}$
y
$\log t$
$\log \nu$
2939C12

Fig. 12