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Hadron-Quark Phase Transition and Statistical Quark Bag Model

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The existence of the intermediate phase between hadron and quark phases, i.e., the multiquark bag phase, is considered. The negative surface energy is introduced phenomenologically on the basis of the statistical quark bag model in order to ensure that the smallest bag is most stable.

However, with the increase of the quark number density in the bag, the surface energy becomes unimportant in comparison with the kinetic energy of the quark, and it is well known that the mass of the bag is proportional to 3/4 power of the quark number in the naive bag model. Thus the multi-quark bag becomes stable at high quark density. The above property suggests a possibility of the hadron-multiquark bag phase transition.

§ 1. Introduction

At high density and high temperature, hadronic matter is predicted to undergo a phase transition to quark matter.[1)~16)] The existence of the intermediate phase, i.e., the multi-quark bag phase, is also predicted in this paper.

In order to know the critical condition the free energies for the both phases of matter must be obtained. The free energy of quark matter is often calculated in perturbative QCD. For nuclear matter the free energy at normal density is extrapolated to the critical density. However hadron-quark phase transition must be a non-perturbative QCD effect and the free energies for the both phases should be obtained from one fundamental theory, QCD. Monte Carlo simulation in lattice gauge theory is a powerful method in the study of such a non-perturbative phenomenon and the existence of the quark deconfining phase transition has been verified by the method numerically.[14)~16)]

On the other hand, the quark bag model[17)~24)] is a very promising approach for treating the hadron-quark phase transition and such a non-perturbative QCD effect from the phenomenological side. In fact the energy density calculated from the bag model describes that from the Monte Carlo simulations very well above the critical temperature.[25]

In the quark bag model the quark field is assumed to be confined in a small domain and the stability of the bag system is ensured by the vacuum pressure (volume energy density) B and the surface tension (surface energy density) σ. The energy of a spherical bag with N quarks is given by

$$E(R)=x\cdot N/R+4\pi R^2\cdot\sigma+4\pi/3\cdot R^3\cdot B\,, \tag{1·1}$$

where R is the bag's radius and x is a dimensionless constant of order 1. Minimizing the bag energy as to the radius R, its mass is obtained for, $\sigma=0$, as

$$M=4/3\cdot(xN)^{3/4}(4\pi B)^{1/4}\,. \tag{1·2}$$

It should be noted that M is proportional to $N^{3/4}$, therefore the most stable bag is the largest one because $(N_1+N_2)^{3/4}<N_1^{3/4}+N_2^{3/4}$. This conclusion is in contradiction with the fact that the nuclei are made of nucleons and do not collapse into one common bag. The positive surface energy was introduced by the Budapest group[20] for the purpose of stabilizing the bag system; however, it does not improve the above contradiction. The negative zero point energy was introduced by the MIT group.[18] This term improves the fitting for hadron mass spectroscopy but is helpless for the improvement of the above difficulty.

Instanton is one of the non-perturbative QCD effects and is expected to cause the bag-type structure of hadrons.[26),27] The contributions of instantons are shown to give not only the vacuum pressure effect but also a negative energy to depend on quark density and the negative energy can improve the above defect of the naive bag model.[28]

The analogy of superconductor of the second kind has been often utilized in the string model.[29] The second kind superconductor forms the multi-vortex structure which is ensured by the negative surface energy of the vortex lines, and the smallest vortex line is realized through quantization. In previous papers[23),24] we proposed the statistical quark bag model, in which a hadron is treated as a quark gas bagged by the pressure of a condensed scalar field. The symmetry of the scalar field is assumed to be recovered inside the bag by means of a high quark density. The negative surface tension σ of the bag may ensure that the nucleus does not collapse into one common bag.

In this paper the negative surface energy is introduced phenomenologically on the basis of the statistical quark bag model. The parameters of the model are estimated by fitting light hadron masses. At high quark density nuclear matter is predicted to undergo a phase transition to the multi (twelve) quark bag matter as well as to quark matter. The both critical densities are estimated by using a solid model for the many body system of the bag (hadron). Then the critical density of the hadron-multi (twelve) quark bag phase transition is shown to be much smaller than that of the multi (twelve)-quark bag-quark phase transition.

In the next section, the Helmholtz free energy of the bagged quark-gluon system is calculated by a statistical approach and the parameters of our model are fixed. In § 3 the possibility of the phase transition is considered and the critical parameters are estimated. In the final section some remarks are given.

§ 2. Statistical quark bag model

As already studied in the previous papers,[23),24)] the thermodynamical potential $\Omega_f(V, T, \mu_\alpha)$ of the free quark gas is given by

$$\Omega_f(V, T, \mu_\alpha) = -TV \sum_\alpha \int \frac{d^3k}{(2\pi)^3} \{\ln[1+\exp(-(E_\alpha-\mu_\alpha)/T)] + \ln[1+\exp(-(E_\alpha+\mu_\alpha)/T)]\} \tag{2·1}$$

with

$$E_\alpha = (\boldsymbol{k}^2 + m_\alpha{}^2)^{1/2}, \tag{2·2}$$

where T is the temperature, V is the volume and μ_α and m_α are the chemical potential and the mass of the quark α, respectively.

In this paper the system of the light quarks only with the flavor quantum number u and d is considered and the masses of the quarks are approximated to be zero. The thermodynamical potential is then calculated at zero temperature as

$$\Omega_f(V, \mu_\alpha) = -\frac{V}{24\pi^2} \sum_\alpha \mu_\alpha{}^4 . \tag{2·3}$$

The Helmholtz free energy is given by the following Legendre transformation:

$$F_f(V, N_\alpha) = \Omega_f(V, \mu_\alpha) + \mu_\alpha N_\alpha = \frac{(6\pi^2)^{4/3}}{8\pi^2} V \sum_\alpha \rho_\alpha^{4/3}, \tag{2·4}$$

where N_α is the number of the quark α in the bag, ρ_α the density, and they are given by the relation:

$$N_\alpha = \rho_\alpha V = -\frac{\partial \Omega_f}{\partial \mu_\alpha} . \tag{2·5}$$

In order to explain the mass splittings of N-Δ and ρ-π, one gluon exchange energy is added. The MIT group have concluded from their analysis[18)] that the dominant effect of the one gluon exchange is the magnetic interaction which contributes to the Helmholtz free energy by the form:

$$F_g=-24\pi\alpha_c V\sum_{\substack{\alpha,\beta\\ \alpha<\beta}}(\boldsymbol{\lambda}_\alpha\cdot\boldsymbol{\lambda}_\beta)(\boldsymbol{s}_\alpha\cdot\boldsymbol{s}_\beta)\frac{\rho_\alpha}{\mu_\alpha}\frac{\rho_\beta}{\mu_\beta} \tag{2·6}$$

with

$$\mu_\alpha=\frac{\partial F_f}{\partial N_\alpha}=(6\pi^2 N_\alpha/V)^{1/3}, \tag{2·7}$$

where $\boldsymbol{\lambda}_\alpha$ and $\boldsymbol{s}_\alpha$ are the color spin and the spin matrices of the quark α, respectively, and α_c is the effective quark-gluon coupling constant.

The total Helmholtz free energy for a spherical bag is given as

$$F(V, N_\alpha)=F_f+F_g+F_s+F_v, \tag{2·8}$$

where F_s and F_v are the surface and the volume energies respectively and they are given by

$$F_s=\sigma\cdot V^{2/3}, \qquad F_v=B\cdot V. \tag{2·9}$$

In our approach the surface energy is assumed to be negative. The origin of the negative surface energy, however, is not considered in this paper.

The mass of a hadron M is defined by the minimum value of the internal energy as

$$M=E(N_\alpha, V)|_{V=\bar{V}} \tag{2·10}$$

with

$$\partial E/\partial V|_{V=\bar{V}}=0. \tag{2·11}$$

At zero temperature, the internal energy E is equal to the Helmholtz free energy F. There are three free parameters in M, i.e., B, σ and α_c, whose values are determined by the masses of Δ, N and ρ meson as follows:

$$B^{1/4}=249\,\text{MeV}, \quad (-\sigma)^{1/3}=311.3\,\text{MeV}, \quad \alpha_c=0.433. \tag{2·12}$$

The value of $B^{1/4}$ is about 2 times larger than the ordinary phenomenological value but is very close to the value estimated from the instanton effect.[26),27)] From the above values of the parameters, the pion mass is calculated as $M_\pi=350$ MeV.

Using the values of Eq. (2·12), the masses of the multi-quark bags including only the light quarks can be calculated in the same way as that of the hadrons and are listed in Table I. Here the multi-quark bags are considered as the color-singlet states. Each predicted value is larger than the threshold value of the corresponding multi-baryon channel. This ensures that the smallest bag is most stable. The di-baryon resonances have been reported and the existence of the ${}^1D_2(2140)$ and ${}^3F_3(2220)$ states in pp seems to be confirmed.[30),31)] The six quark

Table I. Masses of the multi-quark bags quoted in MeV. J is the total spin and N is the total number of quarks and anti-quarks. The values in parentheses in the column of J correspond to the ones of the bags of $N=9$.

J \ N	4	6	9	12
0(1/2)	1292	2069	3148	4125
1(3/2)	1420	2140	3210	4153
2(5/3)	1671	2282	3313	4210
3(7/2)		2491	3456	4296

state of $J=2$ might be a candidate for the ${}^1D_2$ di-baryon resonance.

§ 3. Hadron-quark phase transition

In this section we would like to show the possibility of the hadron-multiquark bag phase transition by using the free energy of Eq. (2·8) and by assuming that the many-body system of the bag forms a solid at zero temperature.

The phase transition point can be calculated from the Gibbs energy. The Gibbs energy $G(N, p)$ is related to the Helmholtz free energy $F(N, V)$ by the following Legendre transformation:

$$G(N, p)=F(N, V)+pV , \tag{3·1}$$

where N is the total number of quarks and p is the pressure of the system. From the free energy of Eq. (2·9) the approximate form of the Gibbs energy for $a_c=0$ is written as

$$G(N, p)=\frac{4}{3}\frac{6\pi^2}{(8\pi^2)^{3/4}}[3(B+p)]^{1/4}(1-\varepsilon_p/N^{1/4})N^{3/4} \tag{3·2}$$

when

$$\varepsilon_p=\frac{3}{4}\frac{(8\pi^2)^{3/4}}{(6\pi^2)^{1/3}}\frac{(-\sigma)}{[3(B+p)]^{3/4}}\ll 1 , \tag{3·3}$$

where $N=\sum_\alpha N_\alpha$ and $N_\alpha=1$. The above Gibbs energy has the limiting form:

$$G(N, p)\xrightarrow[p\to\infty]{}\frac{4}{3}\frac{6\pi^2}{(8\pi^2)^{3/4}}[3(B+p)]^{1/4}N^{3/4} . \tag{3·4}$$

The mass of the multi-quark bag is equal to the Gibbs energy at $p=0$ and it is given in the previous section. Each value of them is larger than the threshold value of the corresponding multi-baryon channel. This means that $G(N, p=0)\sim N^\alpha(\alpha>1)$. The above properties give the possibility of the hadron-multiquark bag phase transition. In order to know the critical point the free energy of the many-body system of the bag must be evaluated. Cold neutron matter is predicted to form a solid at sufficiently high density.[32)~34)] Then we adopt the most simple picture, i.e., the bag is regarded as a classical ball and the many-body system of the bag forms a solid at finite pressure and at zero temperature, in which the kinetic energy of the bag itself and the interaction energy between the bags are ignored and the free energy of the solid is given by the sum of the free energy of the bag system. The mass of the bag becomes larger according to the increase of the pressure. This property gives an effective repulsive force between the bags and ensures a stability of the bag solid. Then the Gibbs energy of the bag solid G_s is related to the Gibbs energy of the bag G_b as

$$G_s(n, N, p)=(N/n)G_b(n, p_b), \tag{3·5}$$

where N is the total number of quarks, n is the number of quarks per one bag, and p and p_b are the pressures to the bag solid and the bag itself respectively. If the highest density packing is assumed for the bag solid,

$$V=1.35(N/n)v, \tag{3·6}$$

$$V=\partial G_s/\partial p, \qquad v=\partial G_b/\partial p_b, \tag{3·7}$$

where $V(v)$ is the volume of the bag solid (the one bag). Thus the following relation between p and p_b is obtained:

$$p_b=1.35p. \tag{3·8}$$

Here the Helmholtz free energy of the bag is given by Eq. (2·9) and the ground state energy is obtained when the number of quarks in the bag is fixed. The bag solid is characterized by the number of quarks per one bag, i.e., the 3, 6, 9, ⋯ quark bags form the baryonic, dibaryonic, tribaryonic, ⋯ solids, respectively and the quark-gluon gas is expressed by the one giant bag including infinite number of quarks.

The Gibbs energy of the compound system of two bag solids is given by

$$G(N, p)=G_s(n, N_n, p)+G_s(l, N_l, p) \tag{3·9}$$

with

$$N=N_n+N_l, \tag{3·10}$$

where $N_n(N_l)$ is the total number of quarks in the $n(l)$ quark bag solid. From the chemical equilibrium condition, the following relation is obtained:

$$\partial G(N, p)/\partial N_n=\partial G_s(n, N_n, p)/\partial N_n-\partial G_s(l, N_l, p)/\partial N_l=0. \tag{3·11}$$

Then Eq. (3·11) gives the Gibbs criteria:

$$\mu_n(p)=\mu_l(p), \tag{3·12}$$

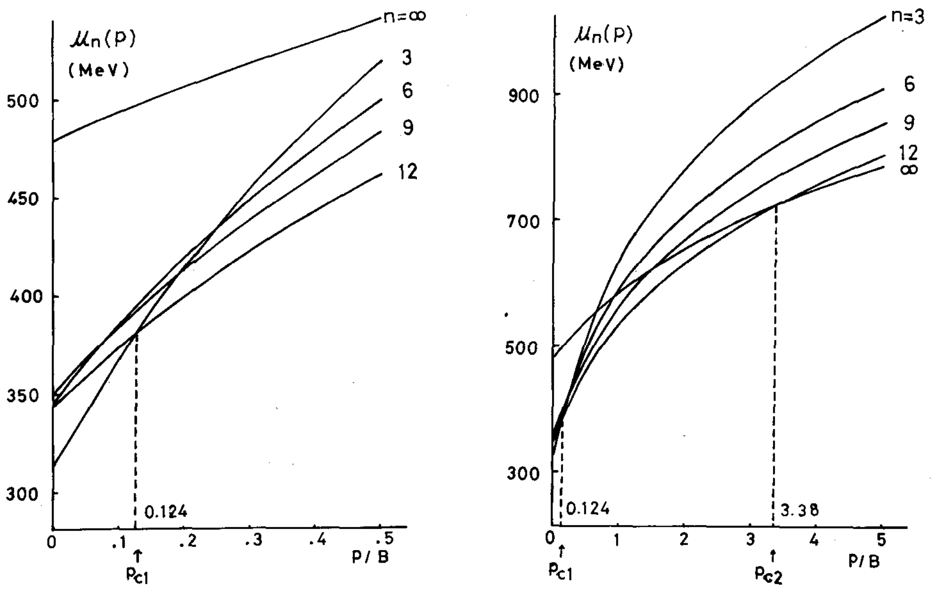

Fig. 1. Gibbs energy per one quark $\mu_n(p)$ as a function of pressure p.

where $\mu_n(\mu_l)$ is the Gibbs energy per one quark for the solid composed of $n(l)$ quark bag and it is given by

$$\mu_n(p) = \partial G_s(n, N, p)/\partial N = G_b(n, 1.35p)/n . \qquad (3\cdot 13)$$

For the quark-gluon gas the Gibbs energy per one quark is easily calculated:

$$\mu_\infty(p) = \lim_{n\to\infty} G_b(n, p)/n . \qquad (3\cdot 14)$$

The Gibbs energy per one quark $\mu_n(p)$ is shown in Fig. 1 as a function of pressure p, where the system is assumed to be composed of only the u and d quarks. In such a system the maximum quantum number of the quark is given by 2·2·3. When the bags including twelve quarks are combined to each other, the energy of the combined system cannot be expected to become lower because of the Pauli principle. Thus it would be enough to consider the solids composed of the 3, 6, 9 and 12 quark bag. The twelve quark bag solid phase can be shown to exist between the two phases of the baryonic solid and the quark-gluon gas.

The critical pressure p_{c1} of the baryonic solid — twelve quark bag solid phase transition is calculated from the Gibbs criteria of Eq. (3·12) as

$$p_{c1}/B = 0.124 , \qquad (3\cdot 15)$$

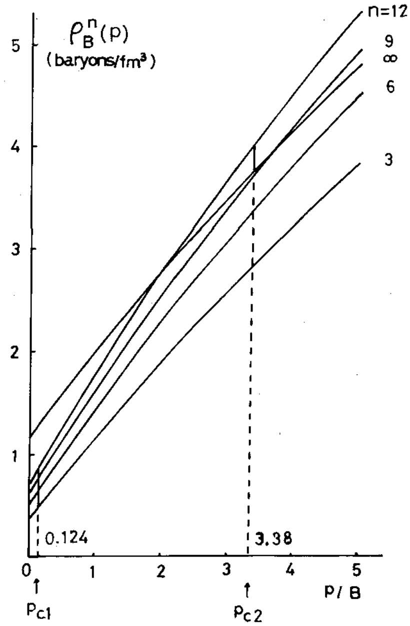

Fig. 2. Average baryon number densities $\rho_B{}^n(p)$ as a function of pressure p.

and the critical pressure p_{c2} of the twelve quark bag solid-quark-gluon gas phase transition is estimated in the same way as

$$p_{c2}/B=3.38\,. \qquad (3\cdot16)$$

The average baryon number densities defined by

$$\rho_B{}^n(p)=N/3V \qquad (3\cdot17)$$

as a function of pressure p are shown in Fig. 2.

§ 4. Remarks

From the results of the previous section the baryon number densities on the both phases of the baryonic and the twelve quark bag solids at p_{c1} are calculated respectively as

$$\rho_B^{(3)}(p_{c1})=0.463 \text{ baryons/ fm}^3\,, \qquad (4\cdot1)$$

$$\rho_B^{(12)}(p_{c1})=0.830 \text{ baryons/ fm}^3\,. \qquad (4\cdot2)$$

The baryon number densities on the both phases of the twelve quark bag and the quark-gluon gas at p_{c2} are given as

$$\rho_B^{(12)}(p_{c2})=4.01 \text{ baryons/ fm}^3\,, \qquad (4\cdot3)$$

$$\rho_B^{(\infty)}(p_{c2})=3.75 \text{ baryons/ fm}^3\,. \qquad (4\cdot4)$$

The baryon number density of the nucleon at $p=0$ is calculated by the inverse value of the nucleon volume as

$$\rho_B^{(3)}(0)=0.360 \text{ baryons/ fm}^3\,. \qquad (4\cdot5)$$

The central baryon number density of the neutron star is estimated as $\rho_c=0.38\sim1.4$ baryons/ fm^3.[34)~36)] Thus it may be unlikely that the quark-gluon gas can be found within the neutron star, but it is much likely that the multi-quark bag to be found.

In our approach the negative surface energy is required for the origin of hadronization, i.e., for the realization of the smallest color-singlet quark-gluon bag at $p=0$. It is also shown that the negative surface energy may play an essential role for the hadron-quark phase transition. The MIT group added the zero point energy:[18)]

$$E_0(R) = -Z_0/R\,, \tag{4·6}$$

to the energy of Eq. (1·1), where Z_0 is a positive constant. Then the Gibbs energy with the negative zero point energy and without the surface energy is calculated:

$$G(N, p) = \frac{4}{3}[4\pi(B+p)]^{1/4}(xN - Z_0)^{3/4}\,. \tag{4·7}$$

The mass of the hadron is given by $G(N, p=0)$. Comparing Eq. (4·7) with Eq. (3·2), the negative zero point energy and the negative surface energy are shown to have similar effects for the mass of the hadron. The Gibbs energy with the zero point energy is, however, factorized into the two functions of p and N so that the phase transition cannot be generated by the negative zero point energy.

References

1) N. Itoh, Prog. Theor. Phys. **44** (1970), 291.
2) J. C. Collins and M. J. Perry, Phys. Rev. Letters **34** (1975), 1353.
3) G. Baym and S. A. Chin, Phys. Letters **62B** (1976), 241.
4) B. D. Keister and L. S. Kisslinger, Phys. Letters **64B** (1976), 117.
5) G. Chapline and M. Nauenberg, Phys. Rev. **D16** (1977), 450.
6) V. Baluni, Phys. Rev. **D17** (1978), 2092.
7) S. A. Chin, Phys. Letters **78B** (1978), 552.
8) J. I. Kapusta, Nucl. Phys. **B148** (1979), 461.
9) O. K. Kalashnikov and V. V. Klimov, Phys. Letters **88B** (1979), 328.
10) J. Kuti, B. Lukács, J. Pdőnyi and K. Szlachanyi, Phys. Letters **95B** (1980), 75.
11) E. Alvarez and R. Hakin, Phys. Rev. **D22** (1980), 480.
12) F. Karsch and H. Satz, Phys. Rev. **D21** (1980), 1168; **D22** (1980), 480.
13) K. Kajantie and H. I. Mietinen, Z. Phys. **C9** (1981), 341.
14) L. McLerran and B. Svetitsky, Phys. Letters **98B** (1981), 195; Phys. Rev. **D24** (1981), 450.
15) J. Kuti, J. Polónyi and K. Szlachányi, Phys. Letters **98B** (1981), 199.
16) J. Engels, F. Karsch, I. Montvay and H. Satz, Phys. Letters **101B** (1981), 89.
17) A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. **D9** (1974), 3471.
18) T. DeGrand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. **D12** (1975), 2060.
19) W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein and T. M. Yan, Phys. Rev. **D11** (1975), 1094.
20) P. Gnädig, P. Hasenfratz, J. Kuti and A. S. Szalay, Phys. Letters **64B** (1976), 62.
21) R. Friedberg and T. D. Lee, Phys. Rev. **D16** (1977), 1096.
22) P. Hasenfratz and J. Kuti, Phys. Reports **C40** (1978), 77.
23) S. Kagiyama, S. Hirooka, H. Kikukawa, J. Kikukawa and Y. Nishiyama, Prog. Theor. Phys. **62** (1979), 490; **63** (1980), 956.
24) S. Kagiyama, S. Hirooka, H. Kikukawa and J. Kikukawa, Prog. Theor. Phys. **65** (1981), 1684.
25) H. Satz, Phys. Letters **113B** (1982), 245.
26) E. V. Shuryak, Phys. Letters **79B** (1978), 135.
27) C. G. Callan, Jr., R. F. Dashen and D. J. Gross, Phys. Rev. **D19** (1979), 1826.
28) E. V. Shuryak, Phys. Letters **81B** (1979), 65.
29) H. B. Nielsen and P. Olesen, Nucl. Phys. **B61** (1973), 77.
30) A. Yokosawa, Phys. Reports **64** (1980), 47.
31) N. Hoshizaki, Soryushiron Kenkyu (Kyoto) **63** (1981), D87.

32) V. Canut and S. M. Chitre, Phys. Rev. **D9** (1974), 1587.
33) R. G. Palmer and P. W. Anderson, Phys. Rev. **D9** (1974), 3281.
34) V. R. Pandharipande and R. S. Smith, Nucl. Phys. **A237** (1975), 507.
35) V. R. Pandharipande, Nucl. Phys. **A178** (1971), 123.
36) S. A. Chin and J. D. Walecka, Phys. Letters **52B** (1974), 24.