Hadronic Vacuum Polarization Contribution to g-2 from the Lattice

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- **Understanding hadronic contributions to $g_\mu - 2$ from the lattice**
  - systematics: finite volume, non-zero lattice spacing
  - dis-connected contribution
  - improved observables
- **Adlerfunction and $\alpha_{\rm QED}$**
- **A word about light-by-light scattering**

Motivation

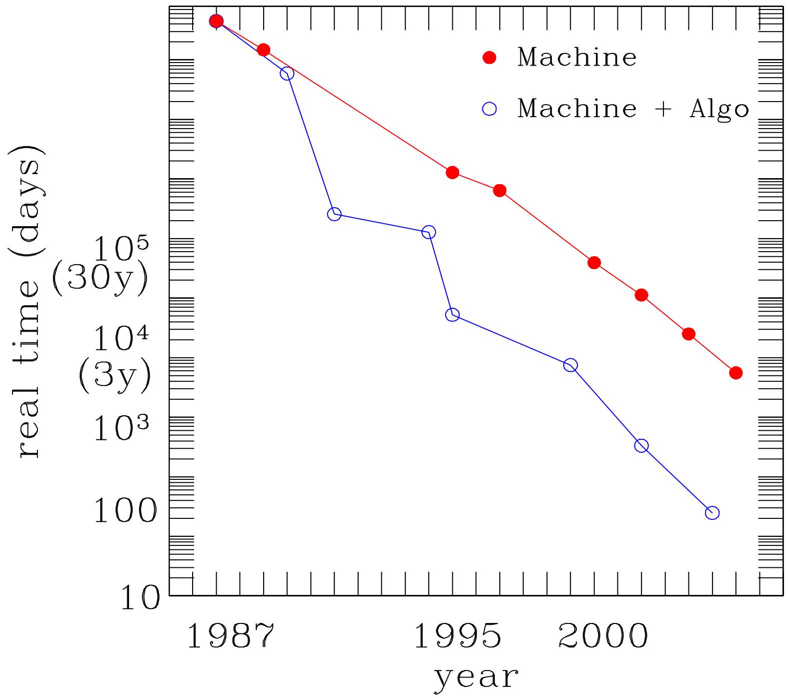
- have a $\approx 3.5\sigma$ discrepancy
- rather constant over time
- τ-data seem to become consistent
  (Jegerlehner, Szafon) $\rho^0 - \gamma$-mixing
- leading order hadronic contribution very important piece

$\Rightarrow$ challenge for the lattice

$\Rightarrow$ unique opportunity for the lattice

Computer and algorithm development over the years

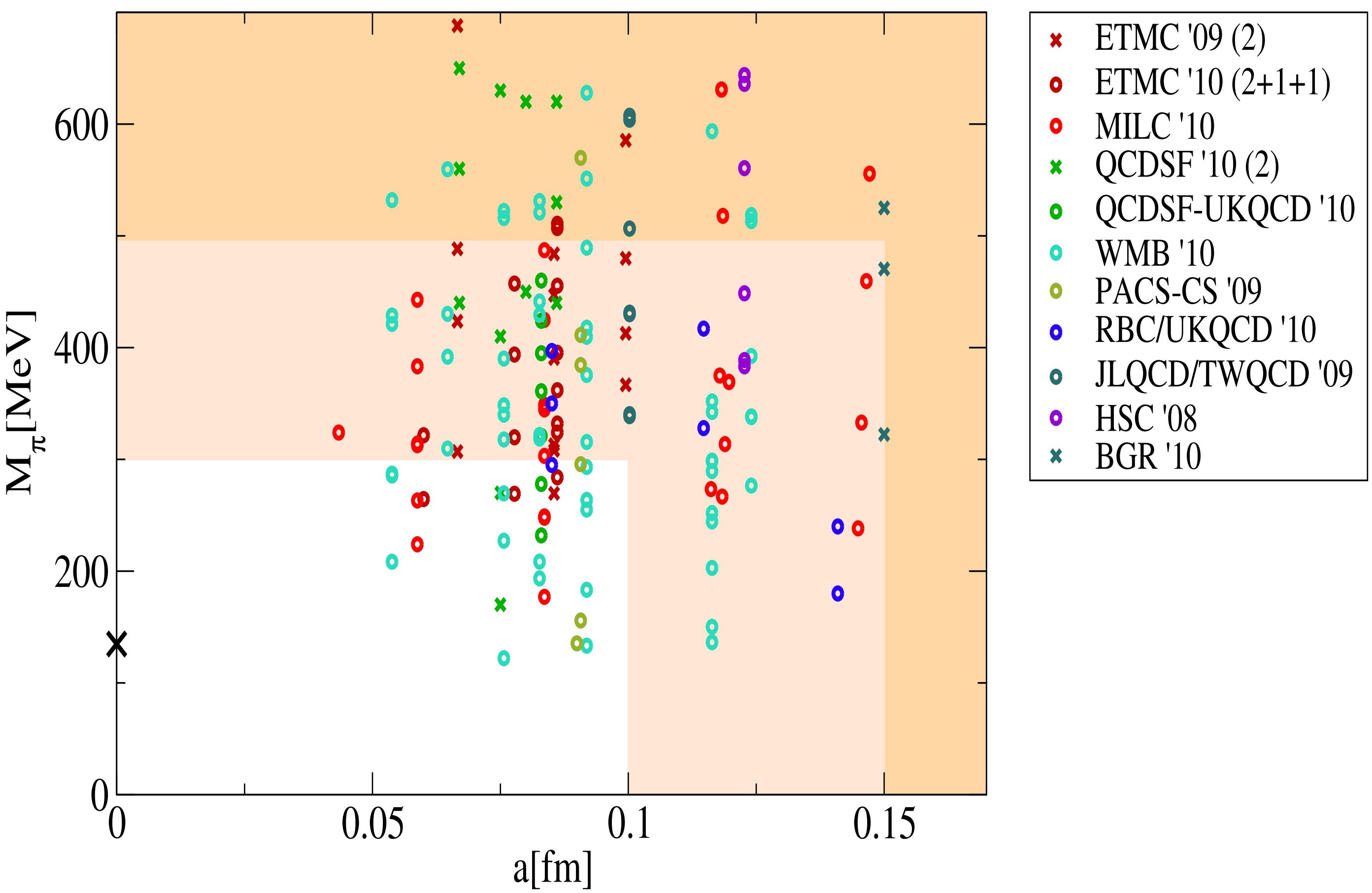
time estimates for simulating $32^3 \cdot 64$ lattice, 5000 configurations



→ O(few months) nowadays with a typical collaboration supercomputer contingent

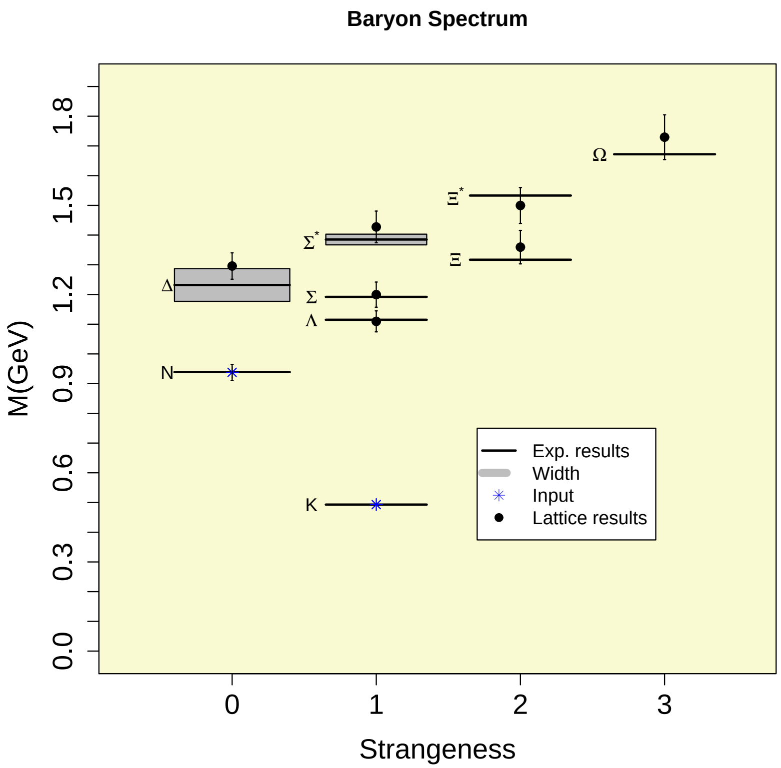
Todays landscape of lattice simulations worldwide

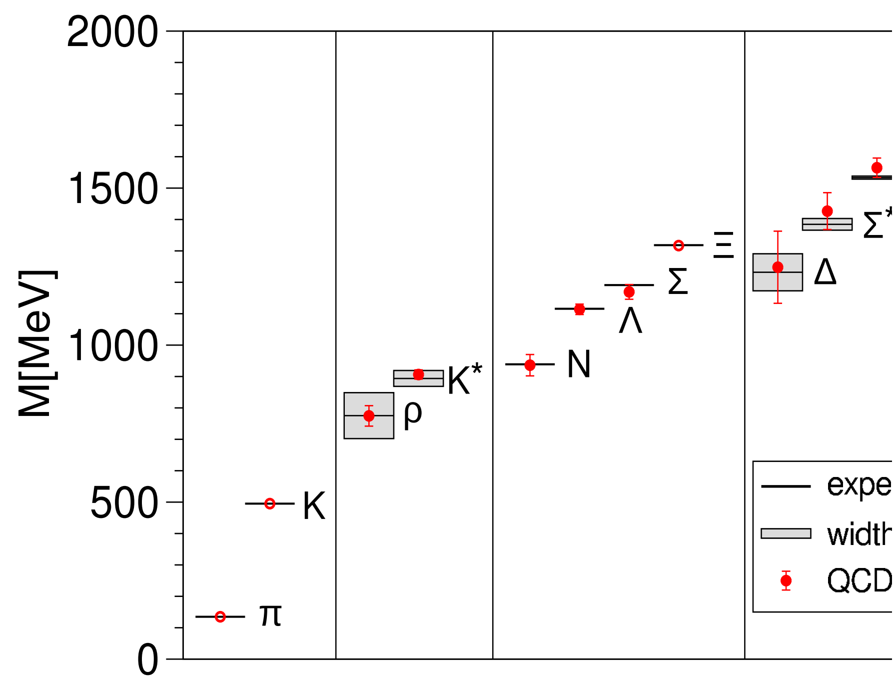
(from C. Hoelbling, Lattice 2010)



The lattice QCD benchmark calculation: the spectrum

ETMC ($N_f = 2$), BMW ($N_f = 2+1$)





$N_f = 2$

$N_f = 2 + 1$

Our fermion discretization: twisted mass fermions

(Frezzotti, Grassi, Weisz, Sint; Frezzotti, Rossi)

$$D_{\rm tm} = m_q + i\mu\tau_3\gamma_5 + \frac{1}{2}\gamma_\mu\left[\nabla_\mu + \nabla_\mu^*\right] - a\frac{1}{2}\nabla_\mu^*\nabla_\mu$$

quark mass parameter m_q , twisted mass parameter μ, lattice spacing a

- $m_q = m_{\rm crit} \leftrightarrow m_{\rm PCAC} = 0 \rightarrow$ O(a) improvement for *hadron masses, matrix elements, form factors, decay constants,* $\cdots$, $g_\mu - 2$

- this means: $(g_\mu - 2)_{\rm latt} = (g_\mu - 2)_{\rm cont} + O(a^2)$

  $\star$ **based on symmetry arguments**

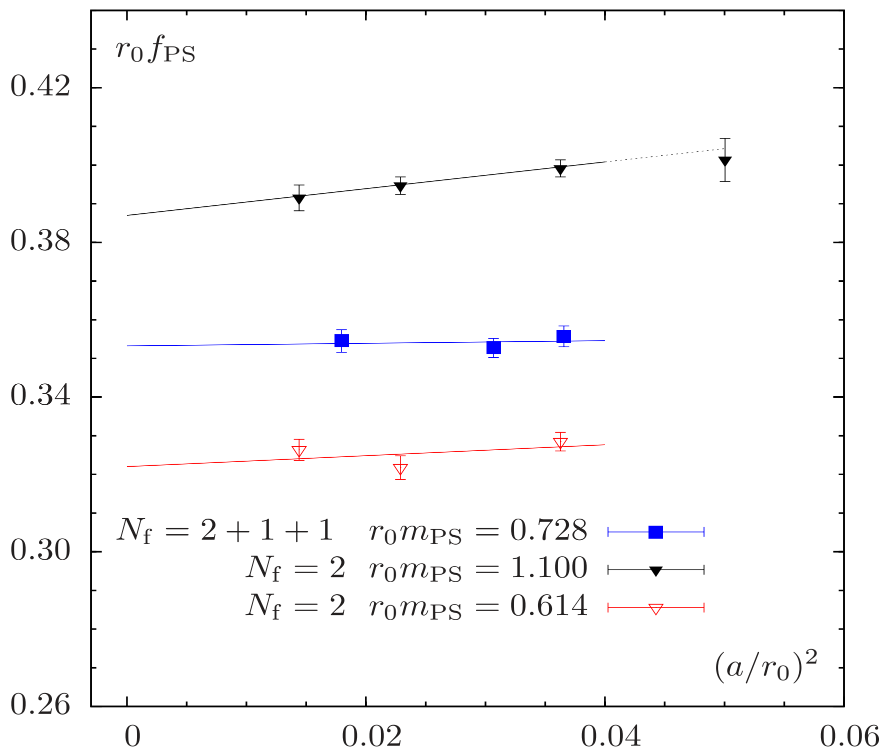
- no need for further operator specific improvement coefficients

- expected to simplify mixing problems for renormalization

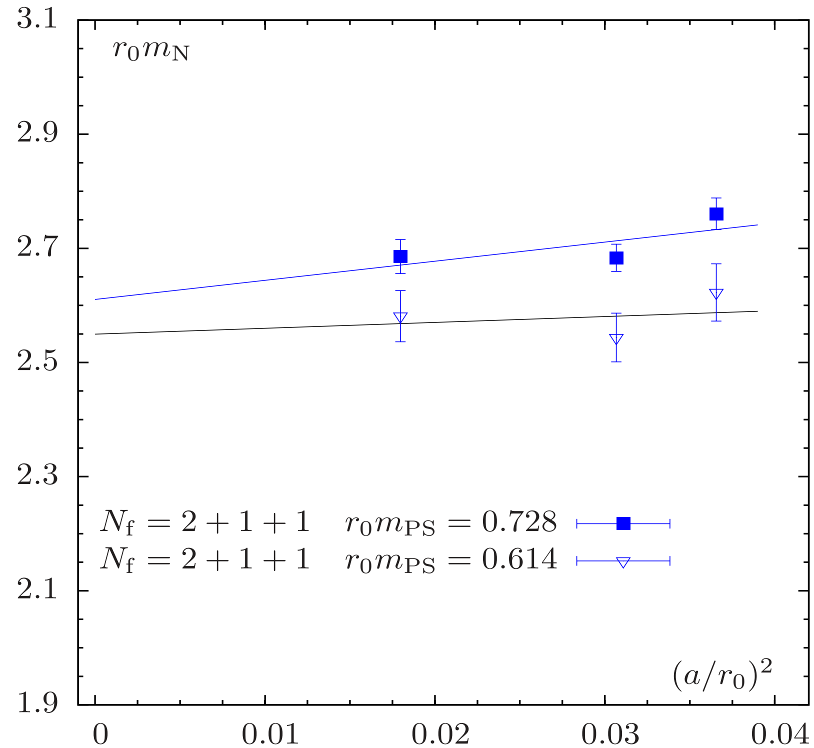
- drawback: explicit violation of isospin $\rightarrow O(a^2)$ effect
  $\rightarrow$ seems to affect only neutral pion sector

- **Cyprus (Nicosia)**
- **France (Orsay, Grenoble)**
- **Italy (Rome I,II,III, Trento)**
- **Netherlands (Groningen)**
- **Poland (Poznan)**
- **Spain (Huelva, Madrid, Valencia)**
- **Switzerland (Bern)**
- **United Kingdom (Glasgow, Liverpool)**
- **Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg)**

$N_f = 2+1+1$ light quark sector: scaling



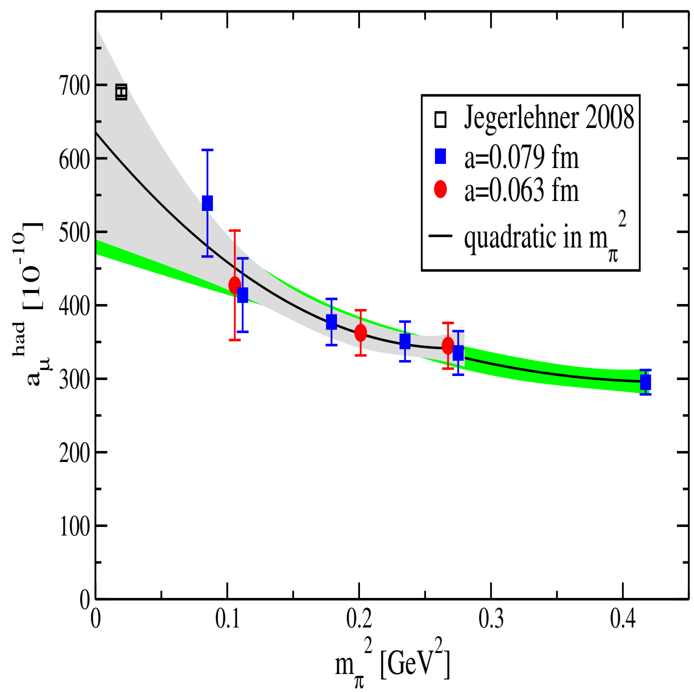
pseudoscalar decay constant f_{PS}



nucleon mass

Do we control hadronic vacuum polarisation?

(Xu Feng, Dru Renner, Marcus Petschlies, K.J.; Lattice 2010)



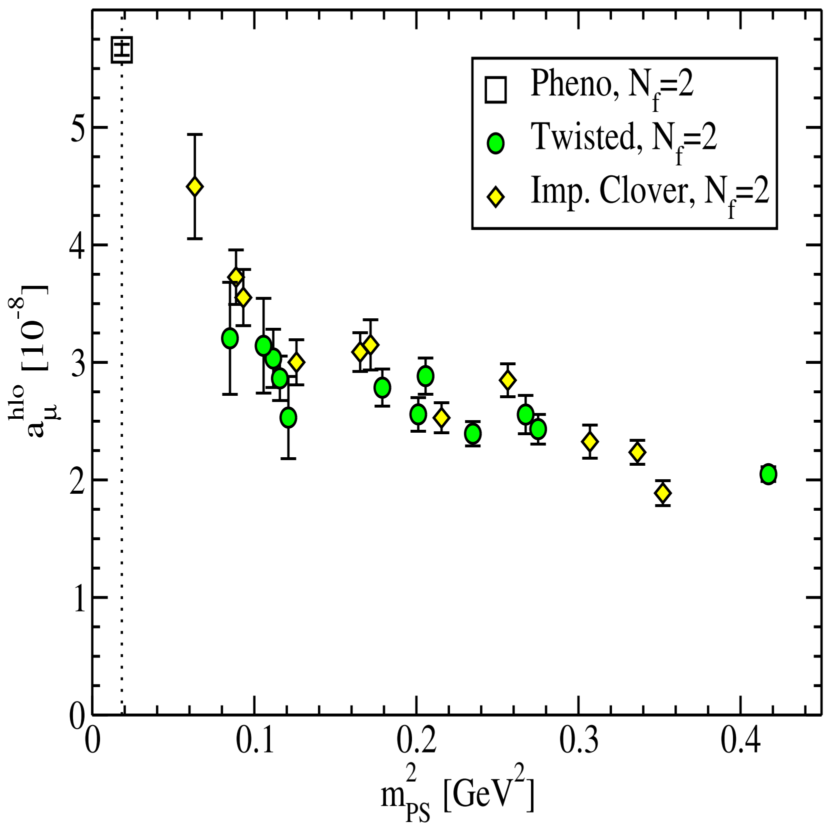
- experiment: $a_{\mu,N_f=2}^{\mathrm{hvp,exp}} = 5.66(05)10^{-8}$
- lattice: $a_{\mu,N_f=2}^{\mathrm{hvp,old}} = 2.95(45)10^{-8}$

  (numbers are scaled to $N_f = 4$ in plot)

$\rightarrow$ misses the experimental value
$\rightarrow$ order of magnitude larger error

- have used different volumes
- have used different values of lattice spacing

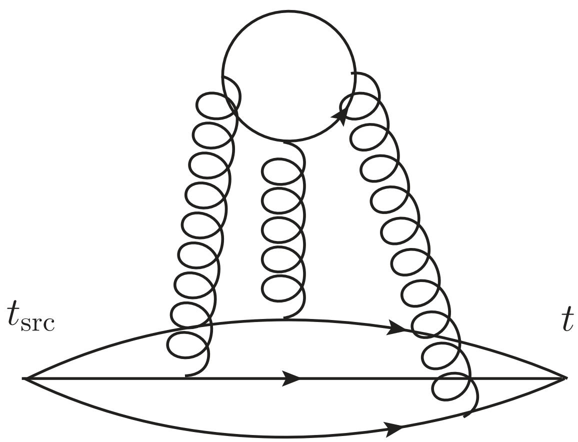
it's not only our problem

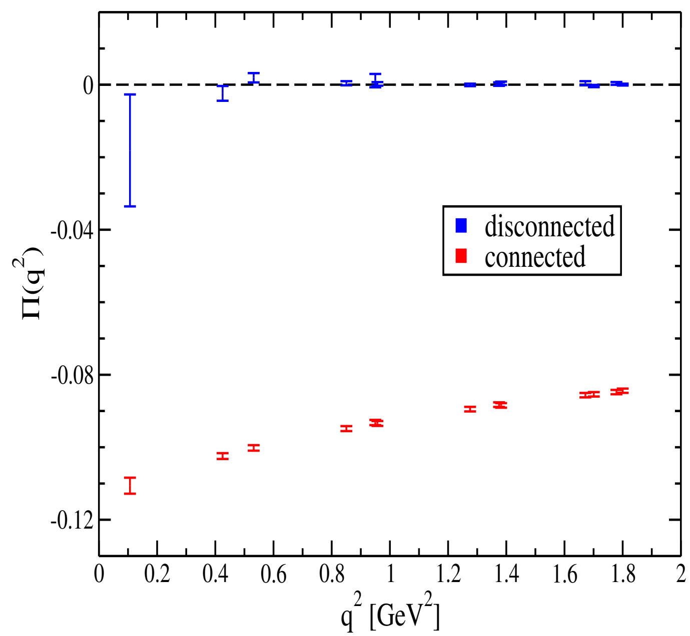


- twisted mass: us
- Imp. clover: Mainz
- data are fully consistent

Can it be the dis-connected (singlet) contribution?

(Xu Feng, Dru Renner, Marcus Petschlies, K.J.)





- dedicated effort
- have included dis-connected contributions for first time
- smallness consistent with chiral perturbation theory (Della Morte, Jüttner)

Different extrapolation to the physical point

lattice: simulations at unphysical quark masses, demand only

$$\lim_{m_{\mathrm{PS}}\to m_\pi} a_l^{\mathrm{hvp,latt}} = a_l^{\mathrm{hvp,phys}}$$

$\Rightarrow$ flexibility to define $a_l^{\mathrm{hvp,latt}}$

standard definitions in the continuum

$$a_l^{\mathrm{hvp}} = \alpha^2 \int_0^\infty dQ^2 \frac{1}{Q^2}\omega(r)\Pi_R(Q^2)$$

$$\Pi_R(Q^2) = \Pi(Q^2) - \Pi(0)$$

$$\omega(r) = \frac{64}{r^2\left(1+\sqrt{1+4/r}\right)^4\sqrt{1+4/r}}$$

with $r = Q^2/m_l^2$

Redefinition of $a_l^{\mathrm{hvp,latt}}$

redefinition of r for lattice computations

$$r_{\mathrm{latt}} = Q^2 \cdot \frac{H^{\mathrm{phys}}}{H}$$

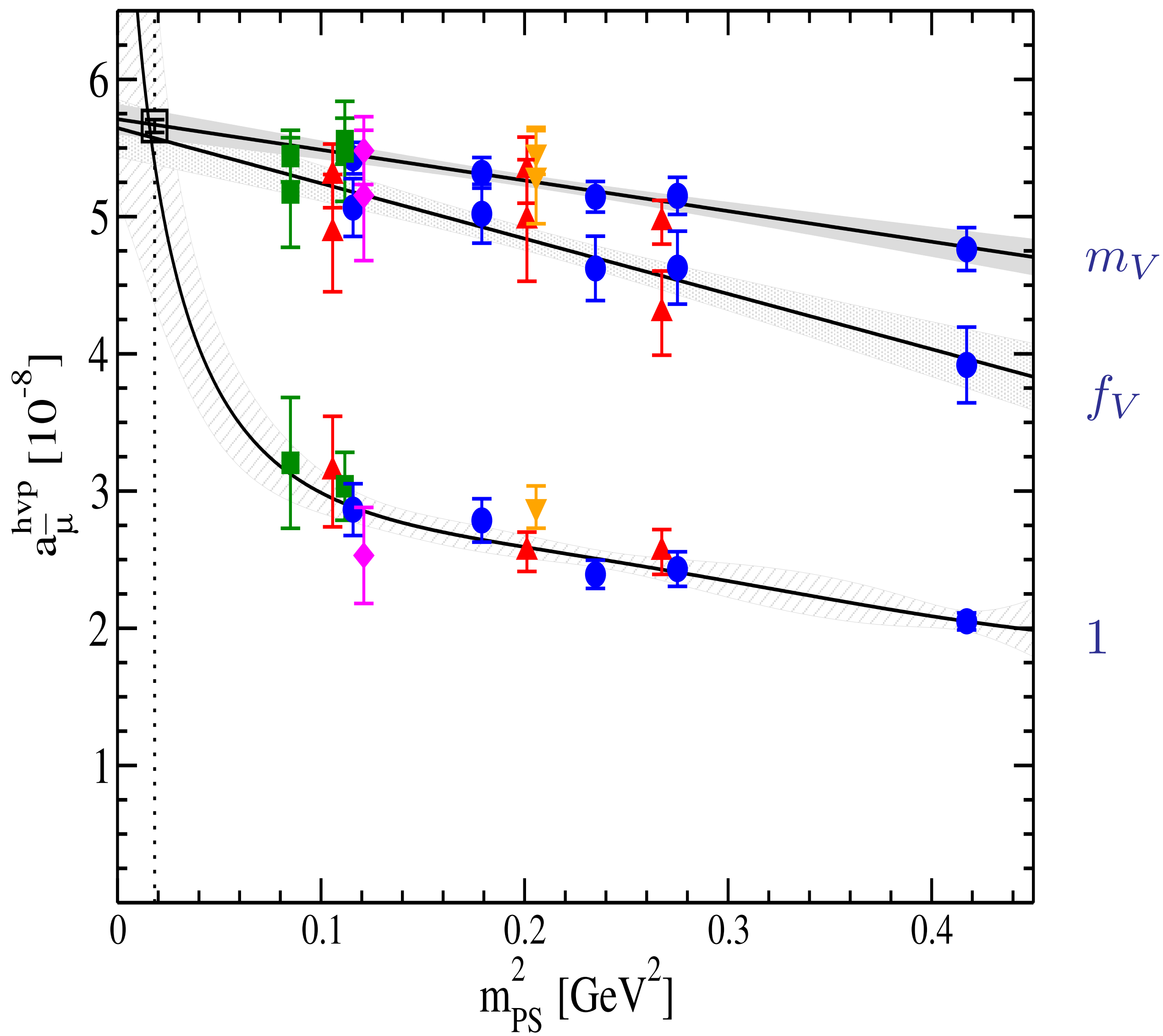
choices

- r_1: $H = 1$; $H^{\mathrm{phys}} = 1/m_l^2$
- r_2: $H = m_V^2(m_{\mathrm{PS}})$; $H^{\mathrm{phys}} = m_\rho^2/m_l^2$
- r_3: $H = f_V^2(m_{\mathrm{PS}})$; $H^{\mathrm{phys}} = f_\rho^2/m_l^2$

each definition of r will show a different dependence on m_{PS} but agree *by construction* at the physical point

remark: strategy often used in continuum limit extrapolations, e.g. charm quark mass determination

comparison using r_1, r_2, r_3



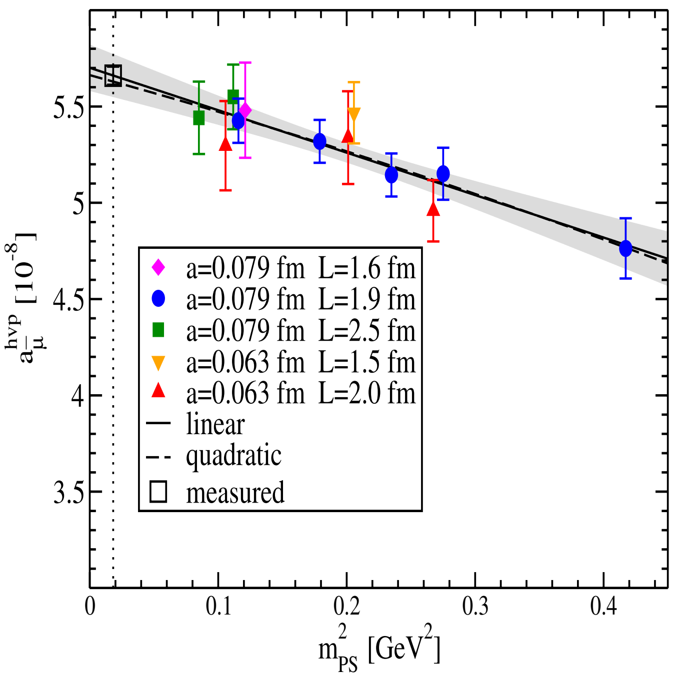
Some numbers

- experimental value: $a_{\mu,N_f=2}^{\mathrm{hvp,exp}} = 5.66(05)10^{-8}$
- from our old analysis: $a_{\mu,N_f=2}^{\mathrm{hvp,old}} = 2.95(45)10^{-8}$

$\rightarrow$ misses the experimental value
$\rightarrow$ order of magnitude larger error

- from our new analysis: $a_{\mu,N_f=2}^{\mathrm{hvp,new}} = 5.66(11)10^{-8}$

$\rightarrow$ error (including systematics) almost matching experiment

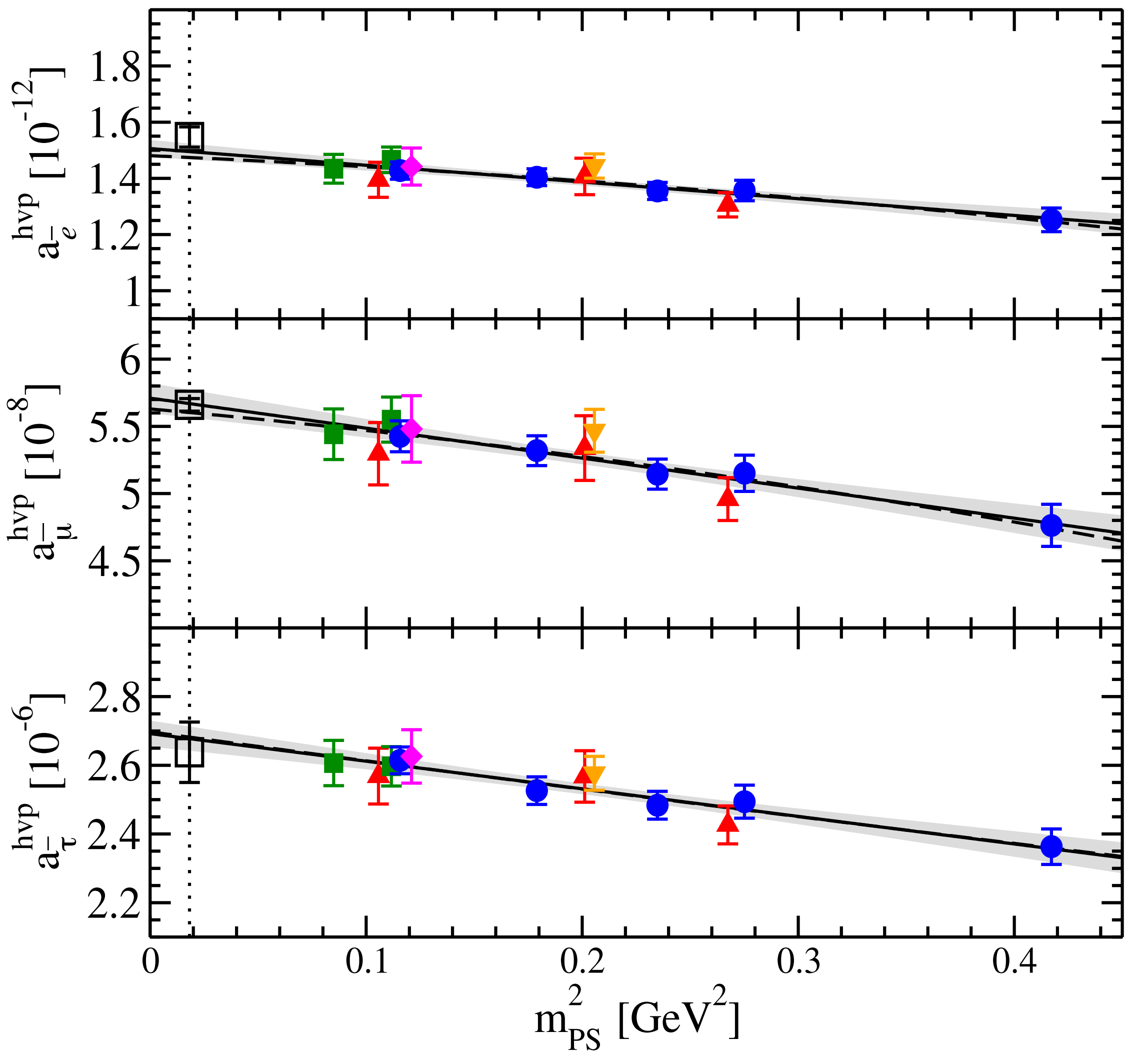


– different volumes

– different values of lattice spacing

– included dis-connected contributions

Anomalous magnetic moments, a check



$$a_e^{\rm hlo} = 1.513\,(43)\cdot 10^{-12} \quad \text{LQCD}$$
$$a_e^{\rm hlo} = 1.527\,(12)\cdot 10^{-12} \quad \text{PHENO}$$

$$a_\mu^{\rm hlo} = 5.72\,(16)\cdot 10^{-8} \quad \text{LQCD}$$
$$a_\mu^{\rm hlo} = 5.67\,(05)\cdot 10^{-8} \quad \text{PHENO}$$

$$a_\tau^{\rm hlo} = 2.650\,(54)\cdot 10^{-6} \quad \text{LQCD}$$
$$a_\tau^{\rm hlo} = 2.638\,(88)\cdot 10^{-6} \quad \text{PHENO}$$

Why it works: fitting the Q^2 dependence

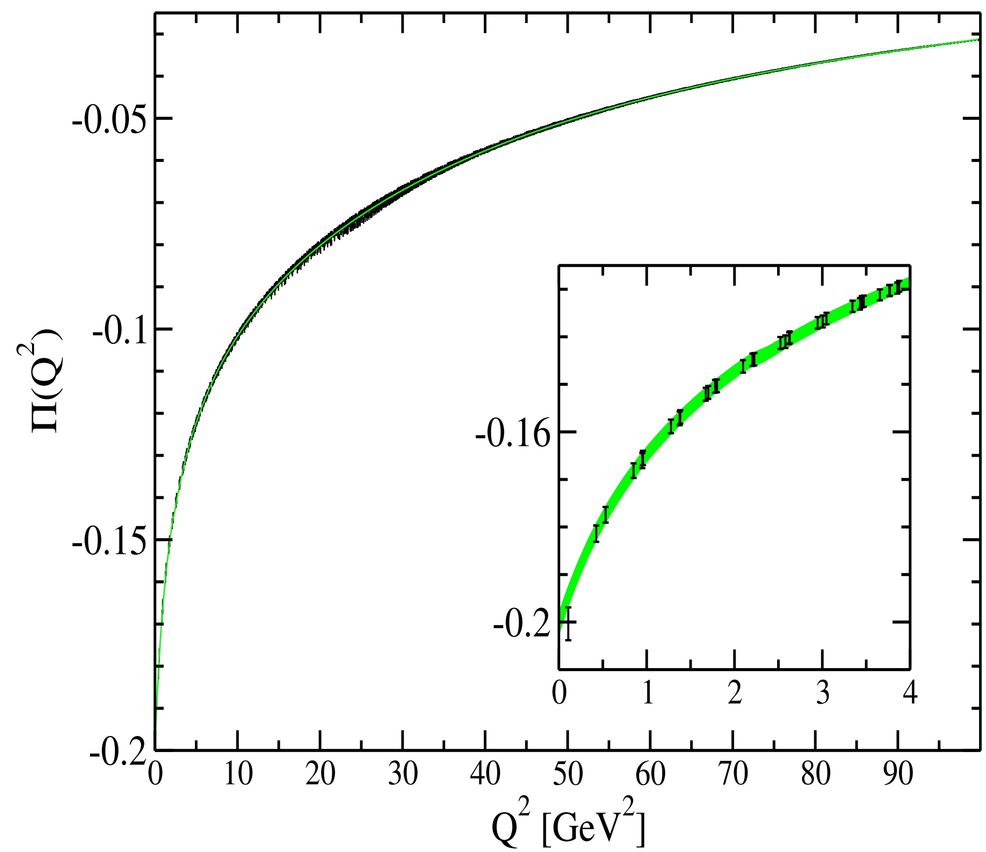
Fit function

$$\Pi_{M,N}(Q^2) = -\frac{5}{9}\sum_{i=1}^{M}\frac{m_i^2}{Q^2+m_i^2} + \sum_{n=0}^{N} a_n (Q^2)^n$$

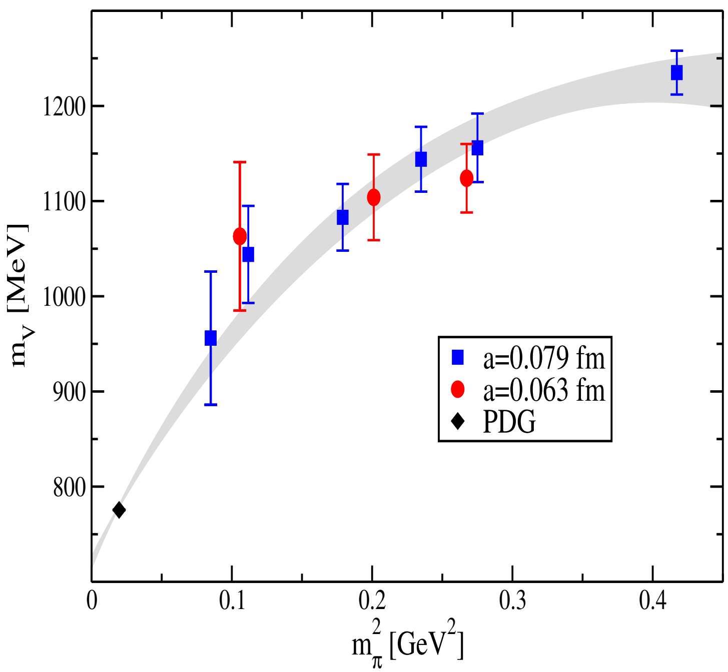
$i = 1$: ρ-meson $\rightarrow$ dominant contribution $\propto 5.010^{-8}$

$i = 2$: ω-meson $\propto 3.710^{-9}$

$i = 3$: ϕ-meson $\propto 3.410^{-9}$



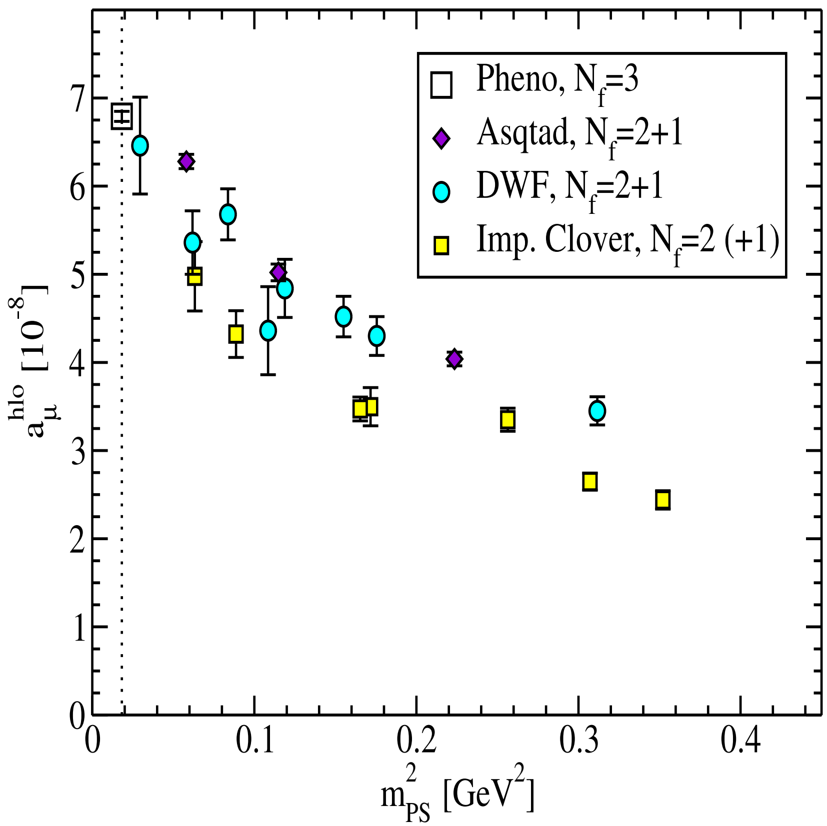
Why it works



- m_V consistent with resonance analysis (Feng, Renner, K.J.)
- strong dependence on $m_{\rm PS}$

$\Rightarrow$ modified definition takes out the ρ-meson dependence

anomalous magnetic moment of muon including starnge quark



- Asqtad $\rightarrow$ **Aubin and Blum**
- DW $\rightarrow$ **Edinburg**
- Imp. clover $\rightarrow$ **QCDSF**

$\Rightarrow$ need analysis with our improved observables

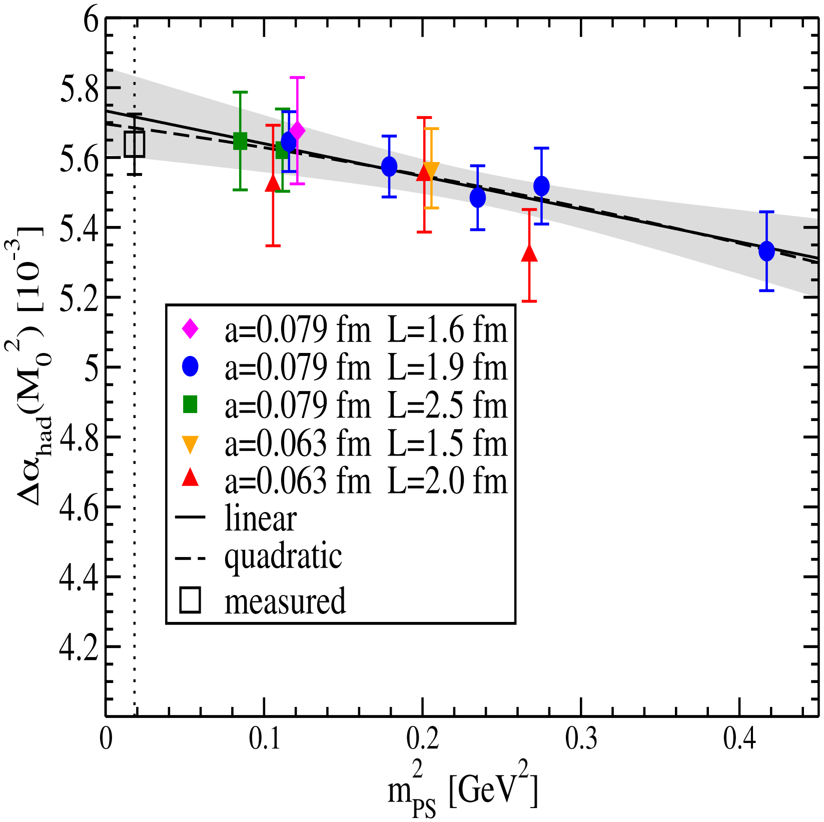
Running of QED coupling (Preliminary)

$$\alpha(Q^2) = \frac{\alpha}{1-\Delta\alpha(Q^2)}$$

$$\Delta\alpha^{\rm had}(Q^2) = 4\pi\alpha\Pi_R(Q^2)$$

apply same idea:

$$\Delta\overline{\alpha}_{\rm had}(Q^2) = 4\pi\alpha\pi_R\left(Q^2/H^2_{\rm phys}\cdot H^2\right)$$



$$\Delta\alpha(M_0^2) = 5.72\,(12)\cdot 10^{-3} \quad \text{LQCD}$$

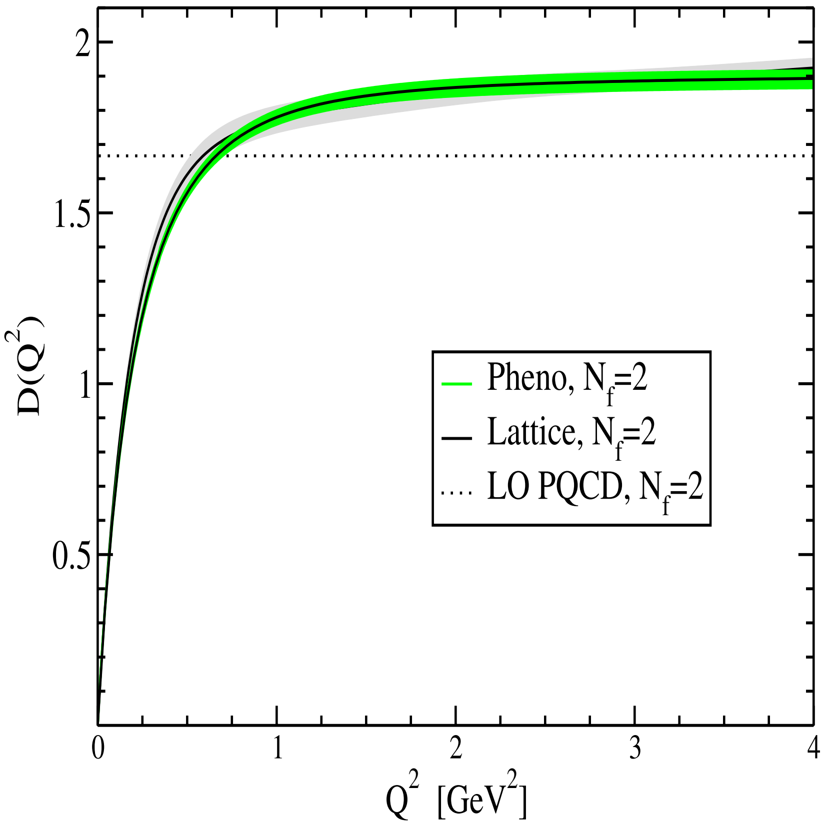
$$\Delta\alpha(M_0^2) = 5.60\,(06)\cdot 10^{-3} \quad \text{PHENO}$$

$$M_0 = 2.5\text{GeV}$$

Adler function (preliminary)

$$D(Q^2) = 12\pi^2 Q^2 \frac{d\Pi_R}{dQ^2} \quad \rightarrow \quad \overline{D}(Q^2) = D(Q^2/H^2_{\rm phys} \cdot H^2)$$

divergence cancelled by derivative



$$\alpha_s^{(2)}(2\ \mathrm{GeV}^2) = 0.263\,(16)$$

$$\Lambda^{(2)} = 222\,(27)\ \mathrm{MeV}$$

The accuracy question

We need a precision $< 1\%$

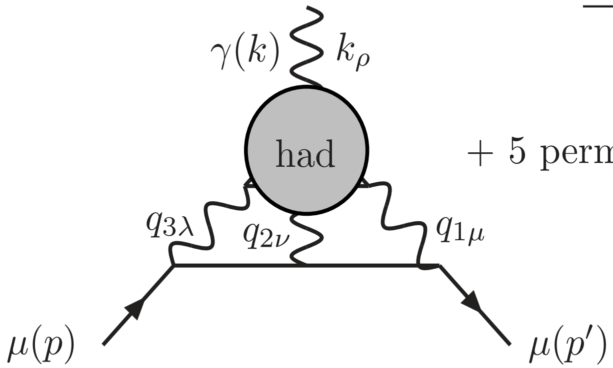
- include up, down, strange and charm quarks
- include explicit isospin breaking
- include electromagnetism
- need computation of light-by-light contribution
- reach small quark mass $\rightarrow$ physical point

Simulation setup for $N_f = 2 + 1 + 1$
Configurations available through ILDG

β	a[fm]	L^3T/a^4	m_π[MeV]	status
1.9	≈ 0.085	$24^3 48$	300 – 500	ready
1.95	≈ 0.075	$32^3 64$	300 –500	ready
2.0	≈ 0.065	$32^3 64$	300	ready
2.1	≈ 0.055	$48^3 96$	300 – 500	running/ready
		$64^3 128$	200	thermalizing

- trajectory length always one
- 1000 trajectores for thermalization
- $\geq$ 5000 trajectores for measurements

Next, α_s^3, contribution



$+$ 5 permutations of the q_i

light-by-light scattering

involves 4-point function

$$\Pi_{\mu\nu\alpha\beta}(q_1,q_2,q_3) = \int_{xyz} e^{iq_1\cdot x+iq_2\cdot y+iq_3\cdot z} \langle j_\mu(0)j_\nu(x)j_\alpha(y)j_\beta(z)\rangle$$

j_μ electromagnetic quark current

$$j_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \tfrac{2}{3}\bar{c}\gamma_\mu c$$

Momentum sources

(Alexandrou, Constantinou, Korzec, Panagopoulos, Stylianou)

$\leftarrow$ following Göckeler et.al.

for renormalization: need Green function in momentum space

$$G(p) = \frac{a^{12}}{V} \sum_{x,y,z,z'} e^{-ip(x-y)} \langle u(x)\overline{u}(z)\mathcal{J}(z,z')d(z')\overline{d}(y)\rangle$$

e.g. $\mathcal{J}(z,z'){=}\delta_{z,z'}\gamma_\mu$ corresponds to local vector current

sources:

$$b^a_\alpha(x) = e^{ipx}\delta_{\alpha\beta}\delta_{ab}$$

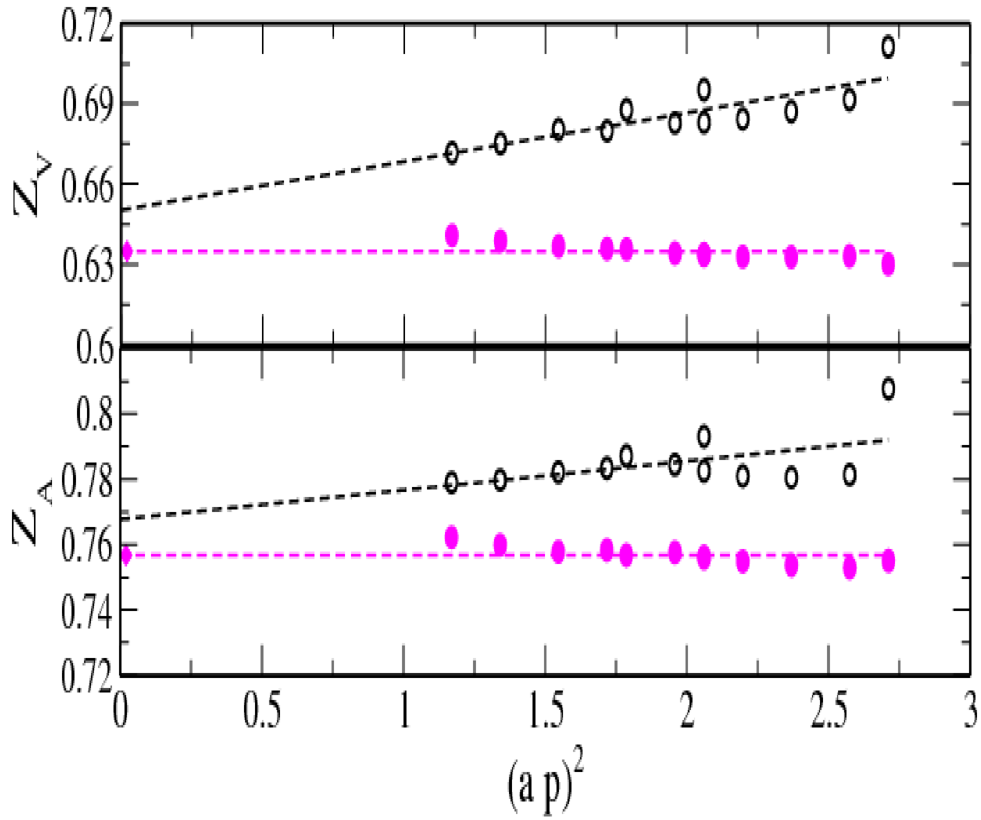
solve for

$$D_{\mathrm{latt}}G(p) = b$$

Advantage: very high, sub-percent precision data (only moderate statistics)

Disadvantage: need inversion for each momentum separately

Illustration of precision



with $O(a^2)$ effects

$O(a^2)$ effects pert. subtracted

with $O(a^2)$ effects

$O(a^2)$ effects pert. subtracted

use the momentum source method to attack the 4-point function
as needed for light-by-light scattering (P. Rakow et.al., lattice'08)

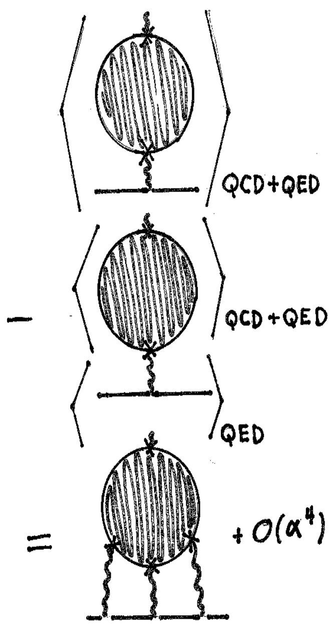
Alternative approach

(Aubin, Blum, Chowdhury, Hayakawa, Izubuchi, Yamada, Yamazaki)

include both, QCD and QED $\Rightarrow$ easier calculation

But: need cancelation of large terms

Subtraction of lowest order piece:



Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

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(taken from talk of T. Blum at Seattle workshop on light-by-light scattering)

Summary

- **lattice is catching up for hadronic vacuum polarization**
- **a number of collaborations working on problem**
  - Feng, Petschlies, Renner, K.J. et.al. $\leftarrow$ (this talk)
  - Boyle, Del Debbio, Kerrane, Zanotti (Edingburg)
  - Della Morte, Jäger, Jüttner, Wittig (Mainz)
  - Aubin and Blum (Riken)
- **new lattice method for** $a_\mu^{\rm had}$
  $\rightarrow$ **prospect to match experimental precision, i.e.** $<$**0.5%**
- **can be applied to further quantities**
  - $\Delta_{\alpha_{\rm QED}}^{\rm had}$, Adlerfunction, $\Lambda_{\rm QCD}$
- **On the way:**
  - four flavour calculation
  - inclusion of isospin splitting and electromagnetism
- **Challenge: light-by-light scattering**

But it will take a while ...