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Hairy planar black holes in higher dimensions

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Abstract: We construct exact hairy planar black holes in D-dimensional AdS gravity. These solutions are regular except at the singularity and have stress-energy that satisfies the null energy condition. We present a detailed analysis of their thermodynamical properties and show that the first law is satisfied. We also discuss these solutions in the context of AdS/CFT duality and construct the associated c-function.

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Contents

1 Introduction **1**

2 General solutions in D dimensions **2**
2.1 General set-up 2
2.2 Solutions 3

3 Thermodynamics **6**

4 Examples and AdS/CFT duality **8**
4.1 Domain walls 9
4.2 Black hole solutions and c-function 9

5 Conclusions **12**

A Details on the equations of motion **13**

B Details on the Weyl tensor and mass calculation **17**

1 Introduction

The existence of scalar fields has been long expected from a theoretical perspective (e.g., they are amongst the basic constituents of string theory). Since the discovery of the Higgs particle [1, 2], it is clear that scalar fields play an important role in physics. They may also play an important role for understanding the black hole physics, in particular for higher dimensional black hole solutions of supergravity, which is the low energy effective theory of string theory.

Motivated by AdS/CFT correspondence of Maldacena [3], extensive has been invested in constructing asymptotically AdS solutions to Einstein gravity. However, analytic hairy black hole solutions in Einstein-Dilaton theories are rare. In [4], we have constructed a family of *exact* hairy black hole solutions with planar horizons in five dimensional AdS gravity. The extension of this analysis to higher dimensions is the goal of this paper. We explicitly write down the solutions in a compact form and discuss their thermodynamical properties.

Indeed, it is well known that hairy asymptotically AdS solutions are relevant in the context of applied holography, e.g. for 'bottom-up models' (see, e.g., [5–7]). However, many of these solutions (for non-trivial scalar potentials) were generated numerically only and so the physics of these solutions in the context of holography can be just partially investigated. Exact solutions in gravity theories with scalar fields (and developing techniques for finding these solutions) are very useful because one can directly study some of their generic features. Since the AdS/CFT correspondence is a weak/strong type of duality, by studying the classical gravity in the bulk one can obtain important information about the strong regime of the dual field theories, which admit gravity duals. We show that the null energy condition (which is the relevant energy condition in AdS) is satisfied and so we expect that the boundary theory is well defined.

The emphasis will be on classical properties of the solutions rather than their implications for the dual field theory. However, even if we do not know all the details of the dual field theory, we provide some computations on the gravity side (for example the c-function) that are important for the interpretation of our solution as an RG flow. The dual gravity theory of a D-dimensional RG flow between two fixed points is a geometry that interpolates between two different AdS spaces (a domain wall) or a black hole at finite temperature [8–11]. In [12] it was shown that there is a deep connection between the c-theorem and null energy condition in the context of domain wall solutions (see, [13–15] for a discussion in the context of black hole solutions). Since the energy condition is satisfied for our solutions, the existence of a c-function that monotonically decreases, moving in from the boundary, is therefore no surprise.

Our paper is structured as follows. In section 2 we construct general D dimensional black hole solutions to the Einstein equations with scalar matter. In section 3 we analyze some of the basic thermodynamics of our solutions and in section 4 we consider specific examples in the context of AdS/CFT duality. We close with some concluding remarks and with two appendices that contain important details for obtaining the solutions and computing their mass.

Note added: after our work was posted in the arXiv, the same self-interaction was studied in the context of spherically symmetric solutions and its embedding in ten and eleven dimensions [16].

2 General solutions in D dimensions

In this section we generalize the black hole solutions of [4]. While exact 5-dimensional planar black hole solutions were presented in [4], we are going to focus on other dimensions here and show that the solutions can be nicely written in a compact form. By carefully studying the asymptotic properties of these solutions, we will obtain the mass of the scalar. In a particular case when a part of the potential vanishes, we will also present a concrete superpotential associated to the scalar potential. This case is particularly interesting to understanding of dynamics in theories with domain walls [17–19].

2.1 General set-up

We begin with the D-dimensional action:

$$I[g_{\mu\nu},\phi]=\frac{1}{2\kappa}\int_M d^Dx\sqrt{-g}\left[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right] \tag{2.1}$$

where $V(\phi)$ is the scalar potential and we use the convention $\kappa = 8\pi G_D$. Since we set $c = 1 = \hbar$, $[\kappa] = M_P^{-(D-2)}$ where M_P is the D-dimensional Planck scale.

The equations of motion for the metric and dilaton are

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}T^{\phi}_{\mu\nu} = 0\ , \qquad \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) - \frac{\partial V}{\partial\phi} = 0 \tag{2.2}$$

where the stress tensors of the matter fields is

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\left(\partial\phi\right)^2 + V(\phi)\right]. \tag{2.3}$$

Next, as in [4, 20], we use the following metric ansatz:

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 dx^2}{f(x)} + d\Sigma^2\right] \tag{2.4}$$

where the parameter η was introduced to obtain a dimensionless radial coordinate x, $\Omega(x)$ is the conformal factor, and

$$d\Sigma^2 = h_{ab}dx^a dx^b, \tag{2.5}$$

where we assume that greek indices run from 0 to $D-1$ and latin indeces from 2 to $D-1$, with $x^0 = t$, $x^1 = x$. We also assume that the $(D-2)$-dimensional metric h_{ab} depends only on the coordinates x^a.

There are three independent (combinations of) equations of motion. One of them in particular implies that h_{ab} is an Einstein metric with constant scalar curvature, that is

$$R[h]_{ab} = \frac{R[h]}{D-2}h_{ab}, \qquad R[h] = constant. \tag{2.6}$$

The relevant (combinations of) equations of motion are then

$$E_t{}^t - E_x{}^x = 0 \Longrightarrow \phi'^2 = \frac{D-2}{2\Omega^2}\left[3\left(\Omega'\right)^2 - 2\Omega\Omega''\right] \tag{2.7}$$

$$E_t{}^t - \frac{1}{D-2}g^{ab}E_{ab} = 0 \Longrightarrow f'' + \frac{D-2}{2\Omega}\Omega' f' + \frac{2\eta^2}{D-2}R[h] = 0 \tag{2.8}$$

$$E_t{}^t + \frac{1}{D-2}g^{ab}E_{ab} = 0 \Longrightarrow V = -\frac{D-2}{2\eta^2\Omega^2}\left[f\Omega'' + \frac{D-4}{2\Omega}f\left(\Omega'\right)^2 + \Omega' f'\right] + \frac{R[h]}{\Omega}. \tag{2.9}$$

where the derivatives are with respect to x. For a more detailed derivation of the equations of motion and its integration see appendix A.

2.2 Solutions

In this section we are going to present the exact solutions obtained for a general class of non-trivial moduli potentials, but before we briefly review some relevant geometric aspects of AdS spacetimes within AdS/CFT duality (see, e.g., [21]).

By choosing different foliations of AdS spacetime, the boundary can be approached in different ways so that the holographic field theory is living on different geometries or topologies. In other words, the choice of 'radial' coordinate is important for defining the family of surfaces that approach the boundary as a limit. In the context of AdS/CFT duality, the radial coordinate plays the role of the *energy scale* in the dual field theory.

JHEP01(2014)153

That is, the radial flow in the bulk geometry can be interpreted as the RG flow of the boundary theory — when we say that the field theory is 'living on the boundary' it means that the processes near the boundary control the short distance (or UV) physics.

Let us now present the 5-dimensional AdS spacetime with three different foliations,

$$ds^2 = -\left(K + \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{K + \frac{r^2}{l^2}} + r^2 d\Sigma_K^2 \tag{2.10}$$

where $K = \{+1, 0, -1\}$ for the spherical ($d\Sigma_1^2 = d\Omega^2$), toroidal ($d\Sigma_0^2 = \sum_{a=1}^3 dx_a^2$), and hyperbolic ($d\Sigma_{-1}^2 = dH^2$) foliations respectively.[1] The AdS radius l is related to the cosmological constant via $\Lambda = -(D-1)(D-2)/2l^2$.

In these coordinates the boundary is at $r \to \infty$, for which the induced metric is

$$d\gamma^2 = \frac{r^2}{l^2}(-dt^2 + l^2 d\Sigma_K^2) \tag{2.11}$$

This is a conformal boundary and the asymptotic boundary geometry is related to the background geometry on which the dual field theory lives by a conformal transformation (to get rid of the divergent conformal factor r^2/l^2).

The neutral static black holes with different horizon topologies exist in AdS and the corresponding metrics are

$$ds^2 = -f_K(r)dt^2 + \frac{dr^2}{f_K(r)} + r^2 d\Sigma_K^2 \tag{2.12}$$

where $f_K(r) = K - \frac{m}{r^{D-3}} + \frac{r^2}{l^2}$.

Obviously when the mass parameter vanishes ($m = 0$) we obtain AdS spacetime with the corresponding foliation (2.10). It is also worth emphasizing that the different AdS patches are related by diffeomorphisms (in the bulk) that correspond to (singular) conformal transformations in the boundary. However, it is clear that black holes with different horizon topologies are different and cannot be related by a change of coordinates.

Within AdS/CFT duality, a black hole in AdS is described as a thermal state of the dual conformal field theory.[2] Since the dual field theory can live on different topologies, the gravity results 'should know' about the Casimir energy (when there is a non-trivial scale in the boundary, e.g. the $k = 1$ case). In what follows, we focus on the planar case ($k = 0$) for which there is no Casimir energy associated to the dual field theory.

As in [4, 20], we choose the conformal factor

$$\Omega(x) = \frac{\nu^2 x^{\nu-1}}{\eta^2 \left(x^\nu - 1\right)^2} \tag{2.13}$$

so that we can solve (2.7) to get

$$\phi = l_\nu^{-1} \ln(x) \quad , \quad l_\nu^{-1} = \sqrt{\frac{(D-2)(\nu^2-1)}{2}} \tag{2.14}$$

[1] Here, $d\Omega^2$ and dH^2 are the 'unit' metrics on the D-dimensional sphere and hyperboloid, respectively.

[2] Since in the $k = -1$ case the the boundary geometry is conformal to Rindler space, the hyperbolic black hole is a dual description of a thermal Rindler states of the CFT in flat space.

JHEP01(2014)153

Note that with our normalization the scalar field is dimensionless. The parameter ν labels different hairy solutions.

To obtain the metric function $f(x)$ we have to integrate (2.8). As a simplifying assumption we consider from now on $R[h] = 0$. Then

$$f(x) = C_1 + C_2 \sum_{k=0}^{D-2} \binom{D-2}{k} (-1)^{D-k} \frac{x^{\frac{\nu(2k-D+2)+D}{2}} - 1}{\nu(2k-D+2)+D}. \tag{2.15}$$

We are going to rename the constants C_1 and C_2 in order to make explicit the asymptotics of the solution. The first thing to notice is that from (2.13) we have that the conformal boundary is located at $x = 1$. We have $f(x) = C_1$ and as we are interested in AdS asymptotics, we fix $C_1 = 1/l^2$, where the cosmological constant is $\Lambda = -(D-1)(D-2)/(2l^2)$. We denote C_2 by α and so the function $f(x)$ reads

$$f(x) = \frac{1}{l^2} + \alpha \sum_{k=0}^{D-2} \binom{D-2}{k} (-1)^{D-k} \frac{x^{\frac{\nu(2k-D+2)+D}{2}} - 1}{\nu(2k-D+2)+D}. \tag{2.16}$$

The singularities of the metric (and the scalar field) are at $x = 0$ and $x = \infty$, but they are enclosed by an event horizon. We shall concern ourselves with the range $x \in [1, \infty)$, for which the scalar field is positive. In our coordinate system the scalar field is regular at the horizon and diverges at the singularity $x = \infty$. We shall see that this implies $\alpha < 0$ in order that our solutions have physically reasonable thermodynamic behaviour.

It is easy to see that, for any value of Λ, ν, and x_+, there is an α such that $f(x_+) = 0$. Indeed, this simply follows from the fact that $f(x)$ is linear in α. Therefore, there are black holes in an open set of the parameter space. Moreover, a simple inspection of the function (2.15) shows that it is regular for every positive $x \neq 0$ and $x \neq \infty$.

As a point of clarification, we should note that the "-1" in the numerator on the second term of (2.16) is part of the integration constant. We chose to write it in this form because it allows us to separate the constant that determines the asymtotic AdS character of the metric and because it leads to the correct limit when ν is chosen to be $\nu = \frac{D}{D-2-2k}$ for k between 0 and $D-2$, which is

$$\lim_{\nu \to \frac{D}{D-2-2k}} \frac{x^{\frac{\nu(2k-D+2)+D}{2}} - 1}{\nu(2k-D+2)+D} = \frac{1}{2} \ln x. \tag{2.17}$$

As expected, one can also check directly that at the boundary, $x = 1$, where the scalar field vanishes, the metric becomes AdS foliated with planar slices (up to a change of coordinates). Also, it is worth pointing out that for $\nu = 1$ we obtain the Schwarzschild-AdS solution.

JHEP01(2014)153

The dilaton potential is obtained from (2.9) using (2.13), (2.14) and (2.16),

$$V(\phi) = -\frac{D-2}{4\nu^2}\Bigg\{e^{-l_\nu\phi}\left[\frac{1}{l^2}+\alpha\sum_{k=0}^{D-2}\binom{D-2}{k}(-1)^{D-k}\frac{e^{\frac{1}{2}[\nu(2k-D+2)+D]l_\nu\phi}-1}{\nu(2k-D+2)+D}\right] \tag{2.18}$$
$$\times\left[2D(\nu^2-1)+e^{\nu l_\nu\phi}\big(D+(D-2)\nu\big)(\nu+1)-e^{-\nu l_\nu\phi}\big(D-(D-2)\nu\big)(\nu-1)\right]$$
$$+\alpha e^{\frac{D-2}{2}l_\nu\phi}\left(e^{\frac{\nu}{2}l_\nu\phi}-e^{-\frac{\nu}{2}l_\nu\phi}\right)^{D-2}\left[2-e^{\nu l_\nu\phi}(\nu+1)+e^{-\nu l_\nu\phi}(\nu-1)\right]\Bigg\}$$

At the boundary $x=1$, where the scalar field vanishes, the potential and its first two derivatives are

$$V(0)=-\frac{(D-1)(D-2)}{l^2} \qquad \left.\frac{dV}{d\phi}\right|_{\phi=0}=0 \qquad \left.\frac{d^2V}{d\phi^2}\right|_{\phi=0}=-\frac{(D-2)}{l^2}$$

Therefore, the theory has a standard AdS vacuum $V(\phi=0)=2\Lambda$ for $\phi=0$. The second derivative of the potential is in fact the mass of the scalar and for $D=5$, as expected from the previous work [4], we obtain $m^2=-3/l^2$. The rank of the coordinate x can be taken to be either $x\in(0,1]$ or $x\in[1,\infty)$. The scalar field is negative in the first case, but positive in the other range. Since the dilaton potential has no obvious symmetry we therefore cover it completely.

Furthermore the correct limit is attained if $\nu=\frac{D}{D-2-2k}$ for k between 0 and $D-2$, where the potential contains a linear term in ϕ, corresponding to the $\ln x$ term in f.

3 Thermodynamics

In what follows, we compute the relevant thermodynamical quantities and show that the first law is indeed satisfied. Since these are planar black holes, the boundary is flat and so there is no Casimir energy associated with the dual field theory. We can therefore calculate the mass using the method of Ashtekar, Das, and Magnon [22, 23] even though this method does not contain information about the Casimir energy. This method resembles the calculation of the Bondi mass and also of the ADM mass[3] used for asymptotically flat spacetimes, but in the context of asymptotically AdS spacetimes. First, the AMD mass resembles the Bondi mass in the sense that a certain integral in a cut of the conformal boundary gives rise to quantities that measure the flux of energy-momentum across the boundary. In particular, if there is no flux of energy-momentum then the mass of the spacetime is conserved and given by the leading order in the asymptotic expansion of the Weyl tensor. As the conformal boundary is time-like, it can be approached not only in null directions but also in space-like directions, which is in particular how the calculation is done. In this sense it resembles the ADM mass, with the difference that now what would be the equivalent of space-like infinity is no longer a point and therefore the mass need not be conserved (though it is since our solutions have a timelike Killing vector).

[3] Note that AMD refers to Ashtekar-Magnon-Das and that ADM refers to Arnowitt-Deser-Minser.

We present here the outline of the calculations and leave the details for appendix B. First we need to consider a conformal metric to $g_{\mu\nu}$ that is regular at $x = 1$. As a straightforward choice we take

$$\bar{g}_{\mu\nu} = \bar{\Omega}^2 g_{\mu\nu}, \tag{3.1}$$

with

$$\bar{\Omega} = \Omega^{-\frac{1}{2}}. \tag{3.2}$$

The mass is then

$$M = \frac{l}{8\pi G_D(D-3)} \oint_\Sigma \bar{\varepsilon}^t{}_t d\bar{\Sigma}_t \bigg|_{x=1}, \tag{3.3}$$

where $\bar{\varepsilon}^\mu{}_\nu$

$$\bar{\varepsilon}^\mu{}_\nu = l^2 \bar{\Omega}^{3-D} \bar{n}^\rho \bar{n}^\sigma \bar{C}^\mu{}_{\rho\nu\sigma} \tag{3.4}$$

is the electric part of the conformal Weyl tensor $\bar{C}^\mu{}_{\nu\lambda\rho}$

$$\bar{C}^\mu{}_{\nu\lambda\rho} = C^\mu{}_{\nu\lambda\rho} = g^{\mu\sigma} W_{\sigma\nu\lambda\rho}, \tag{3.5}$$

and $\bar{n}^\mu$ is the normal vector to the boundary

$$\bar{n}_\mu = \partial_\mu \bar{\Omega}, \tag{3.6}$$

satisfying $\bar{\nabla}_\mu \bar{n}_\nu \big|_{x=1} = 0$. Performing the calculations and using the field equations, we get

$$\bar{\varepsilon}^t{}_t = \frac{l^2 \alpha \nu^{D-2}(D-3)(D-2)}{32\eta^{D+2}(D-1)} \Omega^{-\frac{9}{2}} \Omega'^3 f. \tag{3.7}$$

To calculate $d\bar{\Sigma}_t$ we need the volume element on the hypersurface $x = constant$, which is

$$\text{Vol} = f^{\frac{1}{2}} \sqrt{|h_{ab}|} dt \wedge dx^2 \wedge \ldots \wedge dx^{D-1}, \tag{3.8}$$

so that

$$d\bar{\Sigma}_t = \langle \partial_t, \text{Vol} \rangle = f^{\frac{1}{2}} \sqrt{|h_{ab}|} dx^2 \wedge \ldots \wedge dx^{D-1}. \tag{3.9}$$

The mass is then

$$M = -\frac{1}{32\pi G_D} \frac{\alpha \nu^{D-2}}{\eta^{D-1}} \frac{D-2}{D-1} V_\Sigma \tag{3.10}$$

where V_Σ is the volume of the $(D-2)$-dimensional manifold Σ.

We can also calculate the temperature and entropy of the black brane, which are

$$T = \frac{|f'(x_+)|}{4\pi\eta} = -\frac{\alpha\nu^{D-2}}{8\pi\eta^{D-1}} |\Omega(x_+)|^{-\frac{1}{2}(D-2)} = -\frac{\alpha}{8\pi\eta} \frac{|x_+^\nu - 1|^{D-2}}{x_+^{\frac{1}{2}(\nu-1)(D-2)}}, \tag{3.11}$$

$$S = \frac{A}{4G_D} = \frac{(\Omega(x_+))^{\frac{D-2}{2}} V_\Sigma}{4G_D} = \frac{\nu^{D-2} x_+^{\frac{1}{2}(\nu-1)(D-2)}}{4G_D \eta^{D-2} |x_+^\nu - 1|^{D-2}} V_\Sigma, \tag{3.12}$$

and we see that $\alpha < 0$ in order for M and T to both be positive.

It is straightforward to check that the first law is satisfied. However, there is a subtlety related to the coordinate system we use. Consider the first law for similar coordinates in a simpler example, namely the planar Schwarzschild AdS black hole:

$$ds^2 = -\left(\frac{r^2}{l^2} - \frac{\mu}{r^2}\right)dt^2 + \left(\frac{r^2}{l^2} - \frac{\mu}{r^2}\right)^{-1} dr^2 + r^2 d\Sigma \tag{3.13}$$

where μ is the mass parameter. With the following change of coordinates

$$r = \mu^{\frac{1}{4}} x$$

the metric becomes

$$ds^2 = -\mu^{\frac{1}{2}}\left(\frac{x^2}{l^2} - \frac{1}{x^2}\right)dt^2 + \left(\frac{x^2}{l^2} - \frac{1}{x^2}\right)^{-1} dx^2 \mu^{-\frac{1}{2}} + \mu^{\frac{1}{2}} x^2 d\Sigma$$

In these coordinates the location of the horizon x_+ is independent of the mass. Actually $x_+ = l^{\frac{1}{2}}$ in this example. The thermodynamical quantities in terms of x_+ can be written as:

$$S = \frac{\mu^{\frac{3}{4}} x_+^3 Vol(\Sigma)}{4} \qquad T = \frac{\mu^{\frac{1}{4}}}{2\pi}\left(\frac{x_+}{l^2} + \frac{1}{x_+^3}\right) \qquad M = \frac{3\mu}{16\pi} Vol(\Sigma) \tag{3.14}$$

To check the first law, it is only necessary to make the variation with respect to μ. Then, it is easy to check the first law $T\delta S = \delta M$. Similarly, it is also straightforward to show that the first law is satisfied for our solutions (by considering the variation with respect to η only.)

The AMD mass, as computed, includes no contribution from the scalar field.[4] From the metric expansion we see that the AMD mass coincides with the gravitational mass (see, also, [26])

$$-g_{tt} = \frac{r^2}{l^2} + \frac{\alpha\nu^2}{3\eta^3 r} + O(r^{-2}) \tag{3.15}$$

upon inspection of the subleading term in the large-r expansion.

4 Examples and AdS/CFT duality

AdS/CFT duality has stimulated the study of AdS gravity in various dimensions. In particular, AdS black hole solutions in five and seven dimensions are relevant for AdS_5/CFT_4 and AdS_7/CFT_6 dualities. Furthermore domain wall solutions in AdS are important in so-called 'fake supergravity' [27–30].

In this section we present some concrete examples and comment on the significance of our solutions in the context of AdS/CFT duality. First, we are going to obtain analytic domain wall solutions in D-dimensions and explicitly construct the superpotential. We then show that the null energy condition is satisfied for our solutions and construct a general c-function in any dimension. We also present concrete black hole examples in 5 and 7 dimensions.

[4]In some cases there is a contribution from the scalar fields [24], but since the first law is not affected, we are going to present a detailed analysis using different methods existent in the literature, e.g. Hamiltonian mass and counterterm method, in a future work [25].

JHEP01(2014)153

4.1 Domain walls

The scalar potential (2.18) has two parts, one which is controlled by the cosmological constant $\Lambda \sim \frac{1}{l^2}$ and the other by α that is an arbitrary parameter with the same dimension as Λ, namely $[\alpha] = [L^{-2}]$. When α vanishes, the solution becomes a domain wall[5] and the potential can be rewritten as

$$V(\phi) = -\frac{D-2}{4l^2\nu^2}\Big[e^{\phi l_\nu(\nu-1)}(\nu+1)\left((\nu+1)D-2\nu\right) \\ -e^{-\phi l_\nu(\nu+1)}(\nu-1)\left(2\nu-(\nu-1)D\right)+2e^{-\phi l_\nu}(\nu^2-1)D\Big] \tag{4.1}$$

In this particular case, we are able to explicitly write down a superpotential associated with the potential above. We obtain

$$V(\phi) = (D-2)\left(\frac{dP(\phi)}{d\phi}\right)^2 - \frac{(D-1)}{2}P(\phi)^2 \tag{4.2}$$

where

$$P(\phi) = \frac{\sqrt{D-2}}{\sqrt{2}l\nu}\left[(\nu+1)e^{\frac{\phi l_\nu(\nu-1)}{2}} + (\nu-1)e^{-\frac{\phi l_\nu(\nu+1)}{2}}\right] \tag{4.3}$$

$$\frac{dP(\phi)}{d\phi} = \frac{\sqrt{D-2}l_\nu(\nu^2-1)}{2\nu\sqrt{2}l}\left(e^{\frac{\phi l_\nu(\nu-1)}{2}} - e^{-\frac{\phi l_\nu(\nu+1)}{2}}\right) \tag{4.4}$$

In this case, the equations of motion can be rewritten as first order equations, which is the fake supergravity method [27–30]. Since this method requires the general structure of supergravity only to lowest order in fermion fields, it is clear that it is far less restrictive than true supergravity. These concrete solutions that satisfy first order equations may be non-BPS solutions of true supergravity. The resemblance of our theory with supergravity is more than superficial. The theory we study exactly coincides with $D = 4, 5, 7$ gauged supergravity (for a discussion see above eq. (28) in [31]) and with the novel omega-deformed gauged supergravity [32]. It is worth remarking that our pursuit of an integrable model in the Einstein-scalar system leads exactly to the relevant interactions predicted by the embedding tensor formalism [33, 34].

4.2 Black hole solutions and c-function

As a warm up exercise, we begin with a brief review of the 5-dimensional planar black holes presented in [4]. These solutions can be obtained from our general expressions for the following values of the integration constants:

$$C_1 = \frac{1}{l^2}\,, \qquad C_2 = \frac{\alpha}{36\nu^3} \tag{4.5}$$

[5]This can be easily seen from the metric function (2.16), which becomes $1/l^2$ in this limit. Then, the conformal factor $\Omega(x)$ becomes the only relevant metric function that controls the geometry and profile of the scalar.

For concreteness, let us explicitly write down the relevant metric function for $D = 5$:

$$f(x) = -\frac{\Lambda}{6} + \alpha\left[\frac{4}{3\,(\nu^2-25)\,(9\nu^2-25)} + \frac{x^{\frac{5}{2}}}{12\nu^3}\left(\frac{x^{\frac{3\nu}{2}}}{3(3\nu+5)} + \frac{x^{-\frac{3\nu}{2}}}{3(3\nu-5)} - \frac{x^{\frac{\nu}{2}}}{(\nu+5)} - \frac{x^{-\frac{\nu}{2}}}{(\nu-5)}\right)\right] \tag{4.6}$$

Since their thermodynamic and holographic properties were discussed in detail in [4], we do not repeat the analysis here. Instead, we present the asymptotics of these solutions and compare with the results of [24] so that the analysis of these black holes is complete.

We change the coordinates

$$\ln x = \frac{1}{\eta r} - \frac{1}{2\eta^2 r^2} - \frac{\nu^2-9}{24\eta^3 r^3} + \frac{\nu^2-4}{12\eta^4 r^4} \tag{4.7}$$

so that our metric matches the asymptotic form of [24]. We obtain then

$$\begin{aligned} h_{rr} &= -\frac{(\nu^2-1)l^2}{4\eta^2 r^4} + \dots\,, \\ h_{tt} &= -\frac{\alpha + \frac{3(9\nu^2-25)(\nu^2-25)}{10l^2}}{288\eta^4 r^2} + \dots\,, \\ h_{mn} &= \frac{(9\nu^2-25)(\nu^2-25)}{960\eta^4 r^2}\delta_{mn} + \dots \end{aligned} \tag{4.8}$$

where h is the asymptotic departure from locally AdS spacetime as given in (2.10). A second important point that was not discussed in [4] is the limit $\nu = 5$ or $\nu = \frac{5}{3}$. At first sight, it seems that the metric diverges for these special cases. However, based on our general analysis in section 2, we are going to show that, in fact, the solution (and the corresponding potential) is well defined for $\nu = 5$ (similarly, one can prove that the solution is also well defined for $\nu = \frac{5}{3}$). An explicit analysis of $\nu = 5$ case (which corresponds to $k = 1$) shows that by computing (2.16) using (2.17) gives

$$f(x) = \frac{1}{l^2} + \alpha\left(\frac{x^{-5}-1}{10} + \frac{3}{2}\ln x - 3\frac{x^5-1}{10} + \frac{x^{10}-1}{20}\right), \tag{4.9}$$

The same result can be also obtained if we perform directly the integration of (2.8) with this choice of D and ν. In this case, the situation drastically changes in the sense that logarithmic terms appear in the metric. Note that we must have $\alpha < 0$ for f to have a single root (and hence for the solution to have an event horizon). The same is true for the 7D solution below, provided $\nu \geq 1.28$.

The 7-dimensional class of black hole solutions have the metric function

$$f(x) = \frac{1}{l^2} + \alpha\left(\frac{x^{\frac{5\nu+7}{2}}-1}{5\nu+7} - 5\frac{x^{\frac{3\nu+7}{2}}-1}{3\nu+7} + 10\frac{x^{\frac{\nu+7}{2}}-1}{\nu+7} - 10\frac{x^{\frac{-\nu+7}{2}}-1}{-\nu+7} + 5\frac{x^{\frac{-3\nu+7}{2}}-1}{-3\nu+7} - \frac{x^{\frac{-5\nu+7}{2}}-1}{-5\nu+7}\right) \tag{4.10}$$

JHEP01(2014)153

The scalar potential is again a combination of exponentials involving the scalar:

$$V(\phi) = -\frac{5}{4\nu^2}\Bigg\{e^{-l_\nu\phi}\Bigg[\frac{1}{l^2}+\alpha\Bigg(\frac{e^{\frac{1}{2}(5\nu+7)l_\nu\phi}-1}{5\nu+7}-5\frac{e^{\frac{1}{2}(3\nu+7)l_\nu\phi}-1}{3\nu+7}+10\frac{e^{\frac{1}{2}(\nu+7)l_\nu\phi}-1}{\nu+7}$$
$$-10\frac{e^{\frac{1}{2}(-\nu+7)l_\nu\phi}-1}{-\nu+7}+5\frac{e^{\frac{1}{2}(-3\nu+7)l_\nu\phi}-1}{-3\nu+7}-\frac{e^{\frac{1}{2}(-5\nu+7)l_\nu\phi}-1}{-5\nu+7}\Bigg)\Bigg]$$
$$\Big[14(\nu^2-1)+e^{\nu l_\nu\phi}(7+5\nu)(\nu+1)-e^{-\nu l_\nu\phi}(7-5\nu)(\nu-1)\Big]$$
$$+\alpha e^{\frac{5}{2}l_\nu\phi}\Big(e^{\frac{\nu}{2}l_\nu\phi}-e^{-\frac{\nu}{2}l_\nu\phi}\Big)^5\Big[2-e^{\nu l_\nu\phi}(\nu+1)+e^{-\nu l_\nu\phi}(\nu-1)\Big]\Bigg\} \quad (4.11)$$

A similar analysis as in the 5-dimensional case can be done, but we do not want to repeat such an analysis here. Instead, we are going to compute a c-function on the gravity side that has the right properties. As in the famous AdS_5/CFT_4 correspondence, the 7-dimensional black hole solution is expected to describe the strong coupling behaviour of certain finite temperature 6-dimensional QFTs as well as the holographic RG flows between different 6-dimensional CFTs. The conformal invariance in the bulk is broken due to the non-trivial profile of the scalar.

First we would like to verify if the null energy condition (in the bulk) is satisfied for our solutions. It is by now well known that violations of the null energy condition will lead to superluminal propagation and instabilities in the bulk [35–37] with corresponding violations in the holographic dual theory (see, e.g., [38]). The analysis can be done in general and we show the validity of the null energy condition and present the c-function in any dimension.

When formulated in asymptotically flat spacetimes the no hair theorems are a statement of the fact that for theories with a scalar field potential satisfying the condition $V(\phi)'' \geq 0$, the stationary and axisymmetric solutions are Ricci flat. Since this very condition can be violated in AdS spacetime, a relevant question is what kind of energy condition is satisfied by a generic scalar field theory. This question can be easily determined for metrics of the form (2.4).

The null energy condition demands that for any null vector $n^\alpha n_\alpha = 0$, the energy-momentum tensor must satisfy the inequality $T_{\alpha\beta}n^\alpha n^\beta \geq 0$. We work with an orthonormal frame e^A_μ for which the energy density and pressure are $\text{diag}(\rho,\, p_x,\, p_a) = e^A_\mu e^B_\nu T^{\mu\nu}$, namely

$$\rho = \frac{g^{xx}\phi'^2}{2}+V, \qquad p_x = \phi'^2-\rho, \qquad p_a = -\rho. \quad (4.12)$$

It easily follows that the null energy condition it is satisfied: $\forall p,\ \ \rho+p\geq 0$.

Let us know turn to the computation of the c-function — here we follow closely [12, 15]. The idea of an RG flow for quantum field theory is based on the intuition that a coarse graining removes the information about the small scales. In the context of holography, since there is a gradual loss of non-scale invariant degrees of freedom, there should exist a c-function that is decreasing monotonically from the UV regime (large radii in the dual AdS space) to the IR regime (small radii in the gravity dual). A function with this property can be built on the gravity side as follows.

A D-dimensional metric of the form

$$ds^2 = -a^2(x)dt^2 + b^{-2}(x)dx^2 + c^2(x)d\Sigma \tag{4.13}$$

has an Einstein tensor, G_μ^ν, such that

$$\phi'^2 = \left(G_x^x - G_t^t\right) b^{-2}(x) = (D-2)\left[-c'' + c'\left(\ln\left(\frac{a}{b}\right)\right)'\right] = (D-2)\frac{c'}{c}\left[\ln\left(\frac{a}{bc'}\right)\right]' \tag{4.14}$$

It is then straightforward to see that

$$C(x) = C_0\left(\frac{a}{bc'}\right)^{(D-2)} \tag{4.15}$$

is a monotonically increasing function of r for any positive constant C_0. The power was chosen so that the c-function matches the entropy of the black hole at the horizon — since $c'(r)$ acts as the conformal radius of the surface Σ, the c-function should indeed scale as $(c'(r))^{D-2}$. One can also check that this is the right power by taking the domain wall limit and compare with the c-function of [12] (there, the c-function was fixed by comparing it with the two-point function of the stress tensor). When evaluated for our solution the c-function (4.15) becomes

$$C(x) = C_0\left(\frac{2\eta\Omega^{3/2}}{\Omega'}\right)^{(D-2)} = \mathcal{C}_0\left[\frac{x^{\frac{\nu+1}{2}}}{x^\nu(\nu+1)+(\nu-1)}\right]^{(D-2)} \tag{4.16}$$

where we have absorbed some constants into $\mathcal{C}_0$.

Let us conclude this section with a discussion of the c-function for the hairy and regular planar black hole. First, let's work with the general expression (4.15) and observe that for the usual planar AdS black hole, $a = b$ and $c' = 1$ and so the c-function is a constant. This is what we expect because the flow is trivial in this case, the black hole is a thermal state in the dual theory. On the other hand, for the hairy black hole the flow is non-trivial. This can be seen from (4.16) because the c-function is constant just for $\nu = 1$, which corresponds to the no-hair case.

5 Conclusions

In this paper, we have constructed a family of higher-dimensional asymptotically AdS hairy black branes. These are the first explicit analytical examples of *neutral regular* hairy planar black holes in dimensions higher than five.[6] Our solutions are regular outside of the horizon and have stress-energy that satisfies the null energy condition. They can be straightforwardly generalized to solutions with spherical or hyperbolic compact sections.

Settings where there is no supersymmetry or conformal symmetry to constrain the dynamics could play an important role to studying real world systems where these symmetries are absent. Therefore, studying the generic properties of exact hairy solutions can be useful even if the underlying dynamics of the dual field theory is poorly understood.

[6] Using the techniques presented in [39–41], one could generalize these solutions by including gauge fields and obtain new extremal and non-extremal black hole solutions [42].

JHEP01(2014)153

The scalar potential has two parts, one that vanishes at the boundary where the scalar field also vanishes and another part that, as expected, becomes the cosmological constant at the boundary (so that our solutions are asymptotically AdS). The part that vanishes at the boundary is controlled by a parameter α, and when $\alpha = 0$ the solution is a naked singularity/domain wall. From this point of view, our solutions are also important because they provide concrete examples of naked singularities that can be dressed with a horizon due to the new non-trivial (α-)terms/corrections in the potential.

We have also constructed a c-function (4.16) for this class of solutions. This result relies only on the structure of the field equations. For the usual AdS planar black hole this quantity is constant, whereas for the class of solutions we construct it is a non-trivial monotonically increasing function of radial distance (equivalently, it is monotonically decreasing as the scalar flows from the asymptotic AdS boundary to the interior). Given that the null energy condition is satisfied for these solutions, this is no surprise; however it does indicate that our solutions have an interpretation in the dual theory as an RG flow.

When $\alpha = 0$ we can explicitly write down the superpotential and study the RG flow. However, in this case, since the solutions are domain walls, the singularity is not hidden by a horizon and the scalar flow is from $\phi = 0$ at the boundary $x = 1$ to $\phi \to \infty$ at the singularity $x = +\infty$. One resolution of this problem is to consider star solutions in AdS [43] which do not have a horizon and their entropy is much smaller than the black hole entropy (similar solutions with different scalar field configurations are the AdS boson stars [44, 45]).

For future work, it will be interesting to find the phase diagram of these solutions (in fact there are both, first- and second-order phase transitions), to obtain similar solutions but in spacetimes with positive cosmological constant, and to generalize these solutions by including gauge fields.

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A Details on the equations of motion

For the metric we have the ansatz (2.4) and therefore the Christoffel symbols are given by

$$\Gamma^{t}{}_{\nu\lambda} = \delta^{t}_{(\nu}\delta^{x}_{\lambda)}\frac{1}{\eta}\left(\frac{\Omega'}{\Omega} + \frac{f'}{f}\right), \tag{A.1}$$

$$\Gamma^{x}{}_{\nu\lambda} = \delta^{t}_{\nu}\delta^{t}_{\lambda}\frac{f^2}{2\eta^2}\left(\frac{\Omega'}{\Omega} + \frac{f'}{f}\right) + \delta^{x}_{\nu}\delta^{x}_{\lambda}\frac{1}{2\eta^2}\left(\frac{\Omega'}{\Omega} - \frac{f'}{f}\right) - \delta^{a}_{\nu}\delta^{b}_{\lambda}h_{AB}\frac{f}{2\eta^2}\frac{\Omega'}{\Omega}, \tag{A.2}$$

$$\Gamma^{a}{}_{\nu\lambda} = \delta^{b}_{\nu}\delta^{c}_{\lambda}\Gamma[h]^{a}{}_{bc} + \delta^{x}_{(\nu}\delta^{a}_{\lambda)}\frac{1}{\eta}\frac{\Omega'}{\Omega}, \tag{A.3}$$

where the prime denotes the derivative with respect to x and $\Gamma[h]^a{}_{bc}$ are the Christoffel symbols of h_{ab}.

The Riemann tensor is given by

$$R^t{}_{\nu\lambda\rho} = \delta^x_\nu \delta^x_{[\lambda}\delta^t_{\rho]} \frac{1}{\eta^2}\left(\frac{\Omega''}{\Omega}+\frac{f''}{f}-\frac{\Omega'^2}{\Omega^2}+\frac{\Omega' f'}{\Omega f}\right)+h_{ab}\delta^a_\nu\delta^b_{[\lambda}\delta^t_{\rho]}\frac{f}{2\eta^2}\frac{\Omega'}{\Omega}\left(\frac{\Omega'}{\Omega}+\frac{f'}{f}\right), \quad \text{(A.4)}$$

$$R^x{}_{\nu\lambda\rho} = -\delta^t_\nu \delta^t_{[\lambda}\delta^x_{\rho]} \frac{f^2}{\eta^3}\left(\frac{\Omega''}{\Omega}+\frac{f''}{f}-\frac{\Omega'^2}{\Omega^2}+\frac{\Omega' f'}{\Omega f}\right)+h_{ab}\delta^a_\nu\delta^b_{[\lambda}\delta^x_{\rho]}\frac{f}{\eta^3}\left(\frac{\Omega''}{\Omega}-\frac{\Omega'^2}{\Omega^2}+\frac{\Omega' f'}{2\Omega f}\right) \quad \text{(A.5)}$$

$$R^a{}_{\nu\lambda\rho} = R[h]^a{}_{bcd}\delta^b_\nu\delta^c_{[\lambda}\delta^d_{\rho]} - \delta^t_\nu\delta^t_{[\lambda}\delta^a_{\rho]}\frac{f^2}{2\eta^2}\frac{\Omega'}{\Omega}\left(\frac{\Omega'}{\Omega}+\frac{f'}{f}\right) \quad \text{(A.6)}$$

$$+\delta^x_\nu\delta^x_{[\lambda}\delta^a_{\rho]}\frac{1}{\eta^2}\left(\frac{\Omega''}{\Omega}-\frac{\Omega'^2}{\Omega^2}+\frac{\Omega' f'}{2\Omega f}\right)+h_{bc}\delta^b_\nu\delta^c_{[\lambda}\delta^a_{\rho]}\frac{f}{2\eta^2}\frac{\Omega'^2}{\Omega^2}, \quad \text{(A.7)}$$

where $R[h]^a{}_{bcd}$ is the Riemann tensor of h_{ab}. The Ricci tensor and Ricci scalar are

$$\begin{aligned} R_{\nu\rho} &= \delta^t_\nu\delta^t_\rho\frac{f^2}{2\eta^2}\left[\frac{\Omega''}{\Omega}+\frac{f''}{f}+\frac{D-4}{2}\frac{\Omega'^2}{\Omega^2}+\frac{D}{2}\frac{\Omega' f'}{\Omega f}\right] \\ &\quad -\delta^x_\nu\delta^x_\rho\frac{1}{2}\left[(D-1)\frac{\Omega''}{\Omega}+\frac{f''}{f}-(D-1)\frac{\Omega'^2}{\Omega^2}+\frac{D}{2}\frac{\Omega' f'}{\Omega f}\right] \\ &\quad -h_{ab}\delta^a_\nu\delta^b_\rho\frac{f}{2\eta^2}\left[\frac{\Omega''}{\Omega}+\frac{D-4}{2}\frac{\Omega'^2}{\Omega^2}+\frac{\Omega' f'}{\Omega f}\right]+R[h]_{ab}\delta^a_\nu\delta^b_\rho, \end{aligned} \quad \text{(A.8)}$$

$$R = \frac{R[h]}{\Omega}-\frac{1}{\eta^2}\frac{f}{\Omega}\left[(D-2)\frac{\Omega''}{\Omega}+\frac{f''}{f}+\frac{1}{4}(D-1)(D-6)\frac{\Omega'^2}{\Omega^2}+(D-1)\frac{\Omega' f'}{\Omega f}\right] \quad \text{(A.9)}$$

where $R[h]_{ab} = R[h]^c{}_{acb}$ and $R[h] = h^{ab}R[h]_{ab}$.

The energy-momentum tensor is given by (2.3), and using the metric expression we have

$$T^\phi_{\mu\nu} = \delta^t_\mu\delta^t_\nu\frac{f^2}{2}\left(\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f}\right)+\delta^x_\mu\delta^x_\nu\frac{\eta^2}{2}\left(\frac{\phi'^2}{\eta^2}-2V\frac{\Omega}{f}\right)-h_{ab}\delta^a_\mu\delta^b_\nu\frac{f}{2}\left(\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f}\right). \quad \text{(A.10)}$$

The Einstein equations are then

$$E_{tt} = \frac{f}{2}R[h]-\frac{f^2}{2\eta^2}(D-2)\left(\frac{\Omega''}{\Omega}+\frac{1}{4}(D-7)\frac{\Omega'^2}{\Omega^2}+\frac{1}{2}\frac{\Omega'}{\Omega}\frac{f'}{f}\right)-\frac{f^2}{4}\left(\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f}\right)=0, \quad \text{(A.11)}$$

$$E_{xx} = -\frac{\eta^2}{2f}R[h]+\frac{D-2}{8}\frac{\Omega'}{\Omega}\left((D-1)\frac{\Omega'}{\Omega}+2\frac{f'}{f}\right)-\frac{\eta^2}{4}\left(\frac{\phi'^2}{\eta^2}-2V\frac{\Omega}{F}\right)=0, \quad \text{(A.12)}$$

$$\begin{aligned} E_{ab} &= R[h]_{ab}-\frac{1}{2}h_{ab}R[h]+h_{ab}\frac{f}{2\eta^2}\left[(D-2)\frac{\Omega''}{\Omega}+\frac{f''}{f}\right. \\ &\quad \left.+\frac{1}{4}(D-2)(D-7)\frac{\Omega'^2}{\Omega^2}+(D-2)\frac{\Omega'}{\Omega}\frac{f'}{f}\right]+h_{ab}\frac{f}{4}\left(\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f}\right)=0. \end{aligned} \quad \text{(A.13)}$$

JHEP01(2014)153

The last equation is equivalent to

$$R[h]_{ab} = h_{ab}\frac{f}{(D-4)\eta^2}\left[\frac{\eta^2}{2}\left(\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f}\right)+(D-2)\frac{\Omega''}{\Omega}+\frac{f''}{f}\right.$$
$$\left.+\frac{1}{4}(D-2)(D-7)\frac{\Omega'^2}{\Omega^2}+(D-2)\frac{\Omega'}{\Omega}\frac{f'}{f}\right] \tag{A.14}$$

and directly from this

$$R[h] = \frac{(D-2)f}{(D-4)\eta^2}\left[\frac{\eta^2}{2}\left(\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f}\right)+(D-2)\frac{\Omega''}{\Omega}+\frac{f''}{f}\right.$$
$$\left.+\frac{1}{4}(D-2)(D-7)\frac{\Omega'^2}{\Omega^2}+(D-2)\frac{\Omega'}{\Omega}\frac{f'}{f}\right]. \tag{A.15}$$

The l.h.s. of this equation depends only on x^a and the r.h.s. depends only on x, therefore $R[h]$ must be a constant. The same can be deduced from the other two equations. If we just continue to denote that constant by $R[h]$ we have that

$$R[h]_{ab} = \frac{R[h]}{D-2}h_{ab}. \tag{A.16}$$

So rearanging (A.11), (A.12) and (A.15) gives

$$\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f} = \frac{2}{f}R[h]-\frac{2(D-2)}{\eta^2}\left(\frac{\Omega''}{\Omega}+\frac{1}{4}(D-7)\frac{\Omega'^2}{\Omega^2}+\frac{1}{2}\frac{\Omega'}{\Omega}\frac{f'}{f}\right), \tag{A.17}$$

$$\frac{\phi'^2}{\eta^2}-2V\frac{\Omega}{f} = -\frac{2}{f}R[h]+\frac{D-2}{2\eta^2}\frac{\Omega'}{\Omega}\left((D-1)\frac{\Omega'}{\Omega}+2\frac{f'}{f}\right), \tag{A.18}$$

$$\frac{\phi'^2}{\eta^2}+2V\frac{\Omega}{f} = \frac{2(D-4)}{D-2}\frac{R[h]}{f}-\frac{1}{\eta^2}\left((D-2)\frac{\Omega''}{\Omega}+\frac{f''}{f}\right.$$
$$\left.+\frac{1}{4}(D-2)(D-7)\frac{\Omega'^2}{\Omega^2}+(D-2)\frac{\Omega'}{\Omega}\frac{f'}{f}\right). \tag{A.19}$$

If we add (A.17) and (A.18) and solve for ϕ' we obtain the field equation for ϕ, (2.7), namely

$$\phi'^2 = \frac{D-2}{2\Omega^2}(3\Omega'^2-2\Omega\Omega''). \tag{A.20}$$

If we substract (A.17) from (A.19) we obtain (2.8) for f,

$$f''+\frac{D-2}{2\Omega}\Omega' f'+\frac{2\eta^2}{D-2}R[h]=0. \tag{A.21}$$

Finally, adding (A.17) and (A.19) we have the equation for V, (2.9),

$$V = -\frac{D-2}{2\eta^2\Omega^2}\left(f\Omega''+\frac{D-4}{2\Omega}f\Omega'^2+\Omega' f'\right)+\frac{R[h]}{\Omega}. \tag{A.22}$$

As stated we use now the conformal factor (2.13) and then it is straightforward to solve (A.20),

$$\phi = \sqrt{\frac{(D-2)(\nu^2-1)}{2}} \ln x, \tag{A.23}$$

where the integration constant was chosen as to be $\phi = 0$ at the conformal boundary $x = 1$. To obtain f we notice that (A.21) can be written as

$$(\Omega^{\frac{D-2}{2}} f')' = -\frac{2\eta^2 R[h]}{D-2} \Omega^{\frac{D-2}{2}} \tag{A.24}$$

and therefore

$$f = -\frac{2\eta^2 R[h]}{D-2} \int^x \Omega(\tilde{x})^{-\frac{D-2}{2}} \left(\int^{\tilde{x}} \Omega(\tilde{\tilde{x}})^{\frac{D-2}{2}} d\tilde{\tilde{x}} \right) d\tilde{x} + c_2 \int^x \Omega(\tilde{x})^{-\frac{D-2}{2}} d\tilde{x} + c_1. \tag{A.25}$$

At this point it is difficult to perform the double integral, therefore we restrict ourselves to the case $R[h] = 0$. Then we obtain (2.15),

$$\begin{aligned} f &= c_1 + c_2 \frac{\eta^{D-2}}{\nu^{D-2}} \sum_{k=0}^{D-2} \binom{D-2}{k} (-1)^{D-k} \int^x \tilde{x}^{\nu k - \frac{1}{2}(\nu-1)(D-2)} d\tilde{x} & \text{(A.26)} \\ &= C_1 + C_2 \sum_{k=0}^{D-2} \binom{D-2}{k} (-1)^{D-k} \frac{x^{\frac{\nu(2k-D+2)+D}{2}} - 1}{\nu(2k-D+2)+D}, & \text{(A.27)} \end{aligned}$$

where the integration constants are

$$C_1 = c_1 + c_2 \frac{2\eta^{D-2}}{\nu^{D-2}} \sum_{k=0}^{D-2} \binom{D-2}{k} \frac{(-1)^{D-k}}{\nu(2k-D+2)+D}, \qquad C_2 = c_2 \frac{2\eta^{D-2}}{\nu^{D-2}}, \tag{A.28}$$

and have been written in this form to make explicit the limit of the function f in case ν has been chosen as $\nu = \frac{D}{D-2-2k}$ for k between 0 and $D-2$, which is

$$\lim_{\nu \to \frac{D}{D-2-2k}} \frac{x^{\frac{\nu(2k-D+2)+D}{2}} - 1}{\nu(2k-D+2)+D} = \frac{1}{2} \ln x. \tag{A.29}$$

We can now calculate $V(\phi(x))$ from (A.22), remembering that in our case $R[h] = 0$, and that from (A.26)

$$f' = c_2 \Omega^{-\frac{D-2}{2}} = \frac{C_2 (x^\nu - 1)^{D-2}}{2x^{\frac{1}{2}(\nu-1)(D-2)}}, \tag{A.30}$$

we have

$$\begin{aligned} V(\phi(x)) = -\frac{D-2}{4\nu^2} \Bigg\{ & x^{-1} \left[C_1 + C_2 \sum_{k=0}^{D-2} \binom{D-2}{k} (-1)^{D-k} \frac{x^{\frac{\nu(2k-D+2)+D}{2}} - 1}{\nu(2k-D+2)+D} \right] \\ & \left[2D(\nu^2-1) + x^\nu \big(D + (D-2)\nu\big)(\nu+1) - x^{-\nu} \big(D - (D-2)\nu\big)(\nu-1) \right] \\ & + C_2 x^{-\frac{1}{2}(\nu-1)(D-2)} (x^\nu - 1)^{D-2} \left[2 - x^\nu(\nu+1) + x^{-\nu}(\nu-1) \right] \Bigg\}. \end{aligned} \tag{A.31}$$

Now we use (2.14) to explicitly obtain $V(\phi)$ as a function of ϕ, which, after substituting C_1 and C_2 by $1/l^2$ and α respectively, gives (2.18).

B Details on the Weyl tensor and mass calculation

In this appendix we present the calculation of the mass of the solution calculated following the ADM procedure [22, 23]. For this we need the Weyl tensor, which is defined as

$$W_{\mu\nu\lambda\rho} = R_{\mu\nu\lambda\rho} - \frac{2}{D-2}(g_{\mu[\nu}R_{\rho]\nu} - g_{\nu[\lambda}R_{\rho]\mu}) + \frac{2}{(D-2)(D-1)}Rg_{\mu[\lambda}g_{\rho]\nu}. \tag{B.1}$$

For the metric (2.4) the expressions obtained in the previous appendix lead to

$$\begin{aligned} W_{\mu\nu\lambda\rho} = {} & \delta^t_{[\mu}\delta^x_{\nu]}\delta^t_{[\lambda}\delta^x_{\rho]}\frac{2}{D-1}\Omega\left[(D-3)f'' - \frac{2\eta^2}{D-2}R[h]\right] \\ & -\delta^t_{[\mu}\delta^a_{\nu]}\delta^t_{[\lambda}\delta^b_{\rho]}\frac{2}{D-2}\Omega f\left[\frac{D-3}{D-1}\frac{f''}{\eta^2}h_{ab} + \frac{2}{D-1}R[h]h_{ab} - 2R[h]_{ab}\right] \\ & +\delta^x_{[\mu}\delta^a_{\nu]}\delta^x_{[\lambda}\delta^b_{\rho]}\frac{2}{D-2}\frac{\Omega}{f}\left[\frac{D-3}{D-1}f''h_{ab} + \frac{2\eta^2}{D-1}R[h]h_{ab} - 2\eta^2 R[h]_{ab}\right] \\ & +\delta^a_{[\mu}\delta^b_{\nu]}\delta^c_{[\lambda}\delta^d_{\rho]}\frac{2}{D-2}\Omega\left[\frac{1}{D-1}\frac{f''}{\eta^2}h_{ad}h_{bc} + \frac{D-2}{2}W[h]_{abcd}\right. \\ & \left. -\frac{4}{D-4}h_{ad}R[h]_{bc} + \frac{2(2D-5)}{(D-1)(D-3)(D-4)}R[h]h_{ad}h_{bc}\right], \end{aligned} \tag{B.2}$$

where $W[h]_{abcd}$ is the Weyl tensor of h_{ab}.

To calculate the mass we need a conformal metric which is regular at the boundary $x = 1$. As the divergence of $g_{\mu\nu}$ is only due to the behaviour of the conformal factor we consider

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = \bar{\Omega}^2 g_{\mu\nu}dx^\mu dx^\nu = -f(x)dt^2 + \frac{\eta^2}{f(x)}dx^2 + h_{ab}dx^a dx^b, \tag{B.3}$$

where

$$\bar{\Omega} = \Omega^{-\frac{1}{2}}. \tag{B.4}$$

The ADM-mass is then

$$M = \frac{l}{8\pi G_D(D-3)}\oint_\Sigma \bar{\varepsilon}^t{}_t d\bar{\Sigma}_t\bigg|_{x=1}, \tag{B.5}$$

where $\bar{\varepsilon}^t{}_t$ is the tt-component of the electric part of the Weyl tensor,

$$\bar{\varepsilon}^\mu{}_\nu = l^2\bar{\Omega}^{3-D}\bar{n}^\rho\bar{n}^\sigma\bar{C}^\mu{}_{\rho\nu\sigma}, \tag{B.6}$$

the normal vector is obtained from

$$\bar{n}_\mu = \partial_\mu\bar{\Omega} \tag{B.7}$$

and the conformal Weyl tensor is

$$\bar{C}^\mu{}_{\nu\lambda\rho} = C^\mu{}_{\nu\lambda\rho} = g^{\mu\sigma}W_{\sigma\nu\lambda\rho}. \tag{B.8}$$

One of the requirements to calculate the ADM-mass is that $\bar{\nabla}_\mu\bar{n}_\nu\big|_{x=1} = 0$, which is automatically satisfied by our choice of conformal factor. We have then

$$\bar{\varepsilon}^\mu{}_\nu = \frac{l^2}{4\eta^4}\Omega^{\frac{1}{2}(D-9)}\Omega' f^2 g^{\mu\lambda}W_{\lambda x\nu x}, \tag{B.9}$$

JHEP01(2014)153

and

$$\bar{\varepsilon}^t{}_t = -\frac{l^2}{4\eta^4}\Omega^{\frac{1}{2}(D-11)}\Omega'^2 f W_{txtx} = -\frac{l^2(D-3)}{8\eta^4(D-1)}\Omega^{\frac{1}{2}(D-9)}\Omega'^2 f f''. \tag{B.10}$$

We use (A.21) and that with our choice of constants

$$f' = \frac{\alpha\nu^{D-2}}{2\eta^{D-2}}\Omega^{-\frac{D-2}{2}}, \tag{B.11}$$

to finally get

$$\bar{\varepsilon}^t{}_t = \frac{l^2\alpha\nu^{D-2}(D-3)(D-2)}{32\eta^{D+2}(D-1)}\Omega^{-\frac{9}{2}}\Omega'^3 f. \tag{B.12}$$

To calculate $d\bar{\Sigma}_t$ we need the volume element on a hypersurface $x = constant$, which is

$$\mathrm{Vol} = f^{\frac{1}{2}}\sqrt{|h_{ab}|}dt \wedge dx^2 \wedge \ldots \wedge dx^{D-1}, \tag{B.13}$$

and then

$$d\bar{\Sigma}_t = \langle \partial_t, \mathrm{Vol}\rangle = f^{\frac{1}{2}}\sqrt{|h_{ab}|}dx^2 \wedge \ldots \wedge dx^{D-1}. \tag{B.14}$$

Now we can perform the integration and obtain the mass

$$M = \frac{l^3\alpha\nu^{D-2}(D-2)}{256\pi\eta^{D+2}(D-1)}\Omega^{-\frac{9}{2}}\Omega'^3 f^{\frac{3}{2}} V_\Sigma\bigg|_{x=1} = -\frac{1}{32\pi G_D}\frac{\alpha\nu^{D-2}}{\eta^{D-1}}\frac{D-2}{D-1}V_\Sigma. \tag{B.15}$$

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JHEP01(2014)153

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JHEP01(2014)153