Half-duplex gaussian interference channel with transmitter and receiver cooperation

Yong Peng, Dinesh Rajan

Institutions: Southern Methodist University

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Half-Duplex Gaussian Interference Channel with Transmitter and Receiver Cooperation

Yong Peng, *Student Member, IEEE*, and Dinesh Rajan, *Senior Member, IEEE*

***Abstract*—We propose an achievable region and capacity outer bound for half-duplex Gaussian interference channel with both transmitter (TX) and receiver (RX) cooperation. We show the significant improvement in achievable region compared to either TX or RX cooperation alone. Further, we quantify the sum rate increase with respect to the cooperation channel gain.**

***Index Terms*—User cooperation, half-duplex, max-flow-min-cut bound.**

I. Introduction

The capacity of the two-user Gaussian interference channel (IC) is an open problem for many years and is known to within one bit only recently [1]. The capacity region has also been studied under various cooperative strategies, most of which assume that nodes operate in *full-duplex* (FD) mode [2]–[4]. The sum rate capacity with TX cooperation (TXC), RX cooperation (RXC) and both TX and RX cooperation (TXRXC) is studied in [4]. By using decode-and-forward (DF) and dirty paper coding (DPC) at the TX's and Wyner-Ziv compress-and-forward (CF) at the RX's, the proposed schemes are shown to have significant capacity gain over IC. While FD cooperative IC has been significantly studied, only limited results are known in the *half-duplex* (HD) scenario, where each of the nodes can either transmit or receive at one time. In [5], a 2-phase TXC scheme using the so called recycling DPC (RDPC) is introduced: Similar schemes are also proposed in [6], where the TX's have additional flexibility in choosing the order of DPC. In [7], 3-phase transmission schemes for TXC and RXC is constructed. For TXC, the extra phase in addition to the 2-phase RDPC strategy allows joint transmission at the TX's, which results in notable capacity gain. Further, both rates from TXC and RXC increase with cooperation efficiency.

In this letter, we evaluate bounds on the capacity of two user cooperative Gaussian IC with *HD* nodes, which requires simpler and cheaper hardware. Unlike [5]–[7], we assume that the system allows both *TXRXC*. Further, rather than considering a system wide power constraint as in [4], we focus on the more practical per-user power constraint [8]. Our main contributions are: i) We construct an HD transmission scheme for the TXRXC IC and compute its achievable region. ii) We show that there is significant increase in achievable region with TXRXC compared to TXC and RXC. We quantify the sum rate increase by TXRXC with respect to the cooperation channel gain and iii) We develop an single user HD two-relay channel max-flow-min-cut bound, which effectively reduces the gap from the achievable rate to the upper bound (UB).

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The authors are with the Dept. of Electrical Engineering, Southern Methodist University, Dallas, TX (e-mail: {ypeng, rajand}@lyle.smu.edu).

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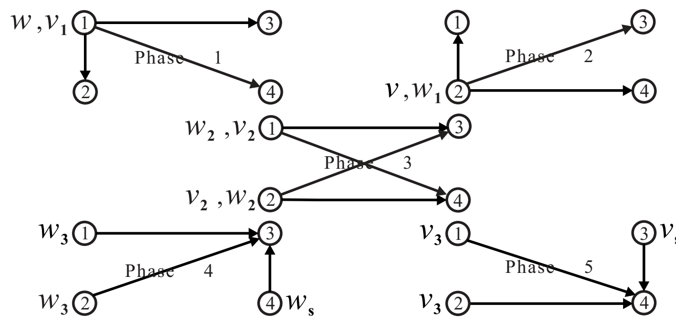

Fig. 1. System model of HD Gaussian cooperative IC.

A. System Model

The proposed HD transmission scheme of this channel is shown in Fig. 1, where node 3 is the intended RX of node 1 and node 4 is the intended RX of node 2. The messages transmitted by node i, $i \in \{1, 2, 3, 4\}$ are encoded into N complex symbols $x_i[1], x_i[2], \ldots, x_i[N]$, under the power constraint $\frac{1}{N}\sum_{n=1}^{N} x_i[n]^2 \leq P_i$. The channel gain from node i to node k, is represented by a complex constant $h_{ik} = c_{ik}e^{j\theta_{ik}}$, $k > i$. It is assumed that all nodes have perfect knowledge of the channel gain and all the phase offsets can be perfectly synchronized. The variance of the Gaussian noise at each of the nodes is normalized to 1. It is also assumed that the cooperation nodes are close together, *i.e.*, c_{12} and c_{34} are large compared to the other c_{ik}'s. Further, we define the following non-negative parameters[1] satisfying $\alpha_1 + \alpha_2 = 1$, $\beta_1 + \beta_2 = 1$, $\gamma_1 + \gamma_2 = 1$, $\delta_1 + \delta_2 = 1$, $\sum_{i=1}^{4} \mu_i = 1$, $\sum_{i=1}^{4} \eta_i = 1$, $\sum_{i=1}^{4} \tau_i = 1$ and $\sum_{i=1}^{5} \lambda_i = 1$. Also define $\mathbf{g}_1 = [c_{13}\ c_{23}]$, $\mathbf{g}_2 = [c_{14}\ c_{24}]$, $\mathbf{H} = [c_{13}\ c_{23};\ c_{14}\ c_{24}]$ and $C(x) = \log(1 + x)$. Denote tr$\{\Sigma\}$ as the trace of matrix Σ.

II. Capacity Bounds

A. Achievable Region

Theorem 1: For the HD two-user Gaussian IC where the TX's and RX's can cooperate, all rate pairs (R_1, R_2) satisfying

$$R_i \leq \min\{R_{i,d}, R_{i,1} + R_{i,2} + R_{i,3} + R_{i,4}\}, \ i = 1, 2 \quad (1)$$

are achievable, where $R_{i,d}$ can be found from (2), $R_{i,j}$, $j = 1, 2, 3, 4$ can be found from (6), (3) (5) and (4). □

We construct the HD transmission strategy as shown in Fig. 1. Let w_i's and v_i's be the messages intended for nodes 3 and 4, respectively. The specific messages sent in each phase

[1] In this letter, the capacity bounds are derived by numerically optimizing over these parameters.

are detailed in Fig. 1. The key ideas behind the transmission scheme are as follows: i) The TXC is executed through DF. Nodes 1 and 2 first exchange their information by decoding each other's messages w and v in phases 1 and 2, respectively. Then they serve as relays for each other and forward message w_1 or v_1 to its intended receiver[2]. Further, with the TX's information been exchanged, they can help each other in phases 3, 4 and 5 by jointly encode messages w_2, v_2, w_3 and v_3 to reap the *power gain*. ii) The RXC is executed through CF. Messages w_2 and v_2 are received at nodes 3 and 4 in phase 3. However, nodes 3 and 4 do not decode w_2 and v_2 immediately. In phases 4 and 5, nodes 4 and 3 forward messages w_s and v_s, respectively, which corresponds to a compressed version of the signal received they have received in phase 3. Thus, at the end of phases 4 and 5, each RX receives an additional compressed signal containing information of w_2 or v_2. Finally, each of the RX's can jointly decode both signals it obtained for the message w_2 or v_2 to reap the *power gain*.

Outline of Achievability:

1) *Phase 1:* If $c_{13} > c_{14}$, generate codeword $\mathbf{X}_1(v_1)$ with length $\lambda_1 N$ and power $P_{v_1},\ P_{v_1} = \alpha_2\mu_1 P_1/\lambda_1$. Given $\mathbf{X}_1(v_1)$, use DPC to generate $\mathbf{X}_1(w)$ with length $\lambda_1 N$ and power $P_w,\ P_w = \alpha_1\mu_1 P_1/\lambda_1$. If $c_{13} \le c_{14}$, do DPC in the reverse order. Since v_1 is known to node 2, it can subtract $\mathbf{X}_1(v_1)$ and decode w, if the rate of w satisfies [5]

$$R_{1,d} \le \lambda_1 C\left(c_{12}^2 P_w\right). \tag{2}$$

Node 4 can decode v_1 if the rate of v_1 satisfies

$$R_{2,2} \le \begin{cases} \lambda_1 C\left(c_{14}^2 P_{v_1}/(1+c_{14}^2 P_w)\right), & \text{if } c_{13} > c_{14} \\ \lambda_1 C\left(c_{14}^2 P_{v_1}\right), & \text{otherwise} \end{cases}. \tag{3}$$

2) *Phase 2:* If $c_{24} > c_{23}$, generate codeword $\mathbf{X}_1(w_1)$ with length $\lambda_2 N$ and power $P_{w_1},\ P_{w_1} = \beta_2\eta_1 P_2/\lambda_2$. Given $\mathbf{X}_1(w_1)$, use DPC to generate $\mathbf{X}_2(v)$ with length $\lambda_2 N$ and power $P_v,\ P_v = \beta_1\eta_1 P_2/\lambda_2$. If $c_{24} \le c_{23}$, do DPC in the reverse order. The rates $R_{2,d}$ for node 1 to decode v, and $R_{1,2}$ for node 4 to decode w_1 can be computed similarly as $R_{1,d}$ and $R_{2,2}$ in phase 1, respectively, we omit the detailed expressions for simplicity.

3) *Phase 3:* After phases 1 and 2, messages, w and v, have been exchanged through TXC. The TX's can then send messages w_2 and v_2 jointly in phase 3. Further, the RX's can take advantage of RXC by decoding w_2 and v_2 in phases 4 and 5, We illustrate the generations of codes and their corresponding rates after the discussion of phase 5.

4) *Phase 4:* Generate codewords $\mathbf{X}(w_3)$ with length $\lambda_4 N$ and powers $P_{w_3,1} = \mu_3 P_1/\lambda_4$ and $P_{w_3,2} = \eta_3 P_2/\lambda_4$ at nodes 1 and 2, respectively. Node 4 applies CF to the signal it received in phase 3. At node 4, generate $\lambda_4 N$ length codeword $\mathbf{X}_4(w_s)$ with power $P_{w_s} = P_4/\lambda_4$. Node 3 first decodes w_3, if the rate of w_3 satisfies

$$R_{1,4} \le \lambda_4 C\left(\left(\sqrt{c_{13}^2 P_{w_3,1}} + \sqrt{c_{23}^2 P_{w_3,2}}\right)^2\right). \tag{4}$$

Node 3 can then decode w_s, if the rate of w_s satisfies

$$R_{1,s} \le \lambda_4 C\left(c_{34}^2 P_{w_s}/\left(1+\left(\sqrt{c_{13}^2 P_{w_3,1}} + \sqrt{c_{23}^2 P_{w_3,2}}\right)^2\right)\right).$$

[2]For the transmission order given in Fig. 1, v_1 is known to node 1 by decoding v in the previous block, please refer to [7] for more details.

5) *Phase 5:* Generate codewords $\mathbf{X}(v_3)$ with length $\lambda_5 N$ and powers $P_{v_3,1} = \mu_4 P_1/\lambda_5$ and $P_{v_3,2} = \eta_4 P_2/\lambda_5$, respectively, at nodes 1 and 2. Node 3 applies CF to the signal it received in phase 3. At node 3, generate $\lambda_5 N$ length codeword $\mathbf{X}_3(v_s)$ with power $P_{v_s} = P_3/\lambda_5$. The rates $R_{2,4}$ for node 4 to decode v_3, and $R_{2,s}$ for node 4 to decode v_s can be computed similarly as $R_{1,4}$ and $R_{1,s}$ in phase 4, respectively,

By decoding w_s and v_s in phases 4 and 5, compressed versions of the signals received in phase 3 are exchanged between the RX's. Let σ_1^2 and σ_2^2 be the compressing noise of the received signal in phase 3 at nodes 4 and 3, respectively. Using similar derivations as in [4], σ_1^2 and σ_2^2 are given by, $\sigma_i^2 = \frac{(1+\mathbf{g}_i\Sigma_x\mathbf{g}_i^\dagger)(1+\mathbf{g}_j\Sigma_x\mathbf{g}_j^\dagger)-(\mathbf{g}_i\Sigma_x\mathbf{g}_j^\dagger)^2}{(2^{R_{i,s}/\lambda_3}-1)(1+\mathbf{g}_j\Sigma_x\mathbf{g}_j^\dagger)}$, where $i = 1,2,\ j \in \{1,2\},\ j \ne i$ and $\Sigma_x = \Sigma_1 + \Sigma_2$ is the covariance matrix of the transmit signal at phase 3, where Σ_1 and Σ_2 are the covariance matrices of the signals bearing messages w_2 and v_2, respectively. As discussed in [4], since each RX now has a noisy version of the received signal at the other RX and a directly received signal at phase 3, the network is equivalent to a 2 user BC with 2-transmit-2-receive antennas. Since the total noises of decoding the compressed signals at nodes 4 and 3 are, respectively, $1+\sigma_1^2$ and $1+\sigma_2^2$, after normalizing the noise power to 1 for all receive "antennas", the equivalent channel gains between the transmit and receive node pairs are given as $\mathbf{H}_1 = [c_{13}\ c_{23};\ \kappa_1 c_{14}\ \kappa_1 c_{24}]$ and $\mathbf{H}_2 = [\kappa_2 c_{13}\ \kappa_2 c_{23};\ c_{14}\ c_{24}]$, where $\kappa_i = \sqrt{1/(1+\sigma_i^2)}$. Thus, in phase 3, nodes 1 and 2 can use DPC to generate codewords for a BC with channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$. If $\mathrm{tr}\{\mathbf{H}_1\mathbf{H}_1^\dagger\} > \mathrm{tr}\{\mathbf{H}_2\mathbf{H}_2^\dagger\}$, generate length $\lambda_3 N$ codewords $\mathbf{X}_1(v_2)$ and $\mathbf{X}_2(v_2)$ at nodes 1 and 2, respectively, with covariance matrix Σ_2. Then, use DPC to generate length $\lambda_3 N$ codewords $\mathbf{X}_1(w_2)$ and $\mathbf{X}_2(w_2)$ at nodes 1 and 2, respectively, with covariance matrix Σ_1, where Σ_1 and Σ_2 are both positive semi-definite matrices, and $\mathrm{diag}\{\Sigma_1\} = [P_{w_2,1}\ P_{w_2,2}]$, $\mathrm{diag}\{\Sigma_2\} = [P_{v_2,1}\ P_{v_2,2}]$. Further, the transmitting power at each of the equivalent transmit antennas (nodes 1 and 2) satisfies $P_{w_2,1}+P_{v_2,1} \le \mu_2 P_1/\lambda_3$ and $P_{w_2,2}+P_{v_2,2} \le \eta_2 P_2/\lambda_3$. If $\mathrm{tr}\{\mathbf{H}_1\mathbf{H}_1^\dagger\} \le \mathrm{tr}\{\mathbf{H}_2\mathbf{H}_2^\dagger\}$, do DPC in the reverse order. Instead of numerically enumerating all possible power allocations and correlation coefficients, the calculation of the optimal Σ_1 and Σ_2 with per-antenna power constraint can be transformed into a simpler dual MIMO multiple access channel (MAC) problem with uncertain noise ([8], eq. (27)). Due to limited space, we omit the detailed derivations. Node 3 can decode w_2 in phase 4, node 4 can decode v_2 in phase 5, if the rate of w_2, $R_{1,3}$, and the rate of v_2, $R_{2,3}$, satisfies

$$R_{i,3} \le \begin{cases} \lambda_3 \log\left|\mathbf{I}+\mathbf{H}_i\Sigma_i\mathbf{H}_i^\dagger\right|, & \text{if } \mathrm{tr}\{\mathbf{H}_i\mathbf{H}_i^\dagger\} > \mathrm{tr}\{\mathbf{H}_j\mathbf{H}_j^\dagger\} \\ \lambda_3 \log\left(\dfrac{\left|\mathbf{I}+\mathbf{H}_i\Sigma_i\mathbf{H}_i^\dagger\right|}{\left|\mathbf{I}+\mathbf{H}_i\Sigma_i\mathbf{H}_i^\dagger+\mathbf{H}_j\Sigma_j\mathbf{H}_j^\dagger\right|}\right), & \text{otherwise} \end{cases} \tag{5}$$

where $i = 1,2,\ j \in \{1,2\},\ j \ne i$ and $\mathbf{I}$ is a 2×2 identity matrix. After decoding w_1, w_2 and w_3, node 3 can finally decode w if the rate of w satisfies $R_{1,w} \le \sum_{i=1}^4 R_{1,i}$, where

$$R_{1,1} \le \begin{cases} \lambda_1 C\left(c_{13}^2 P_w\right), & \text{if } c_{13} > c_{14} \\ \lambda_1 C\left(c_{13}^2 P_w/(1+c_{13}^2 P_{v_1})\right), & \text{otherwise} \end{cases}. \tag{6}$$

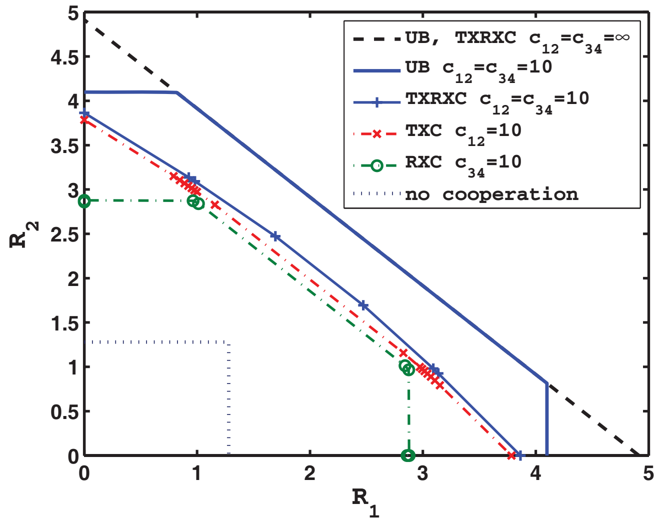

Fig. 2. Capacity regions with nodes cooperation ($c_{13} = c_{24} = 1$, $c_{14} = c_{23} = \sqrt{1/2}$ and $P_i = 5,\ i = 1, 2, 3, 4$).

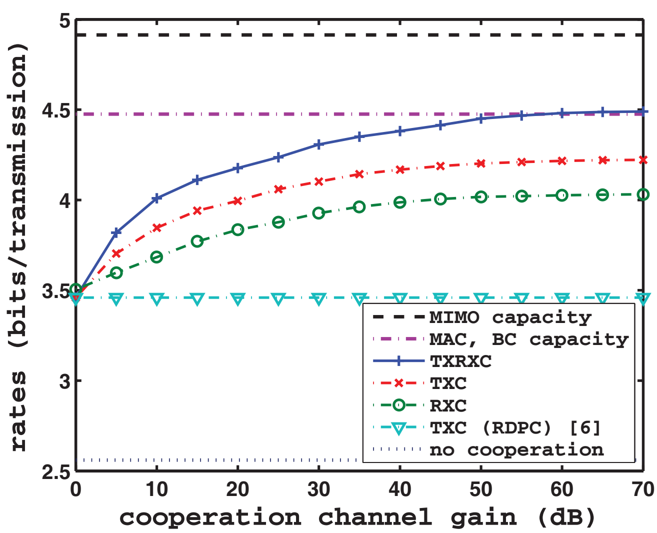

Fig. 3. Sum rate with different c_{12}^2 and c_{34}^2 ($c_{12}^2 = c_{34}^2$).

Similarly, node 4 can decode v if the rate of v satisfies[3] $R_{2,v} \leq \sum_{i=1}^{4} R_{2,i}$, where $R_{2,1}$ can be computed similarly to $R_{1,1}$.

B. Upper Bound

We derive max-flow-min-cut UB's of the system. For the sum rate UB, the only cut is $U = \{\text{nodes } 1, 2\}$. Hence the UB is given by $(R_1 + R_2)^{UB} = \max_{0 \leq \varepsilon \leq 1} \log |\mathbf{I} + \mathbf{HPH}^{\dagger}|$, where $\mathbf{P} = [P_1\ \varepsilon\sqrt{P_1 P_2};\ \varepsilon\sqrt{P_1 P_2}\ P_2]$. Note that this bound also equals the capacity of a 2×2 MIMO channel with per-antenna power constraint [8]. For the single user UB, take user 1 for example, phases 1, 3, 4 in Fig. 1 and another phase where only nodes 1 and 4 transmit form the single user two relay HD transmission scheme. Let $U_i,\ i = 1, 2, 3, 4$ denote the four cuts between nodes 1 and 3, then $U_1 = \{\text{node } 1\}$, $U_2 = \{\text{nodes } 1, 2\}$, $U_3 = \{\text{nodes } 1, 2, 4\}$ and $U_4 = \{\text{nodes } 1, 4\}$, with $F_{1,1}$, $F_{1,2}$ $F_{1,3}$ and $F_{1,4}$ being the flows of each cut, respectively. User 1's rate is bounded by

$$R_1^{UB} = \max_{\gamma_1, \delta_1, \tau_1, \tau_2, \tau_3, \psi} \min\{F_{1,1}, F_{1,2}, F_{1,3}, F_{1,4}\}. \quad (7)$$

Let $x_i^{(j)}, y_i^{(j)}, P_i^{(j)}, i, j = 1, 2, 3, 4$ denote the transmit, receive signal and transmit power of node i in phase j, respectively. The variable τ_i denote the time portion, γ_i, δ_i and ρ_i denote the correlation factors of the transmit signals satisfying $E[x_1^{(2)} x_2^{(2)}] = \sqrt{\gamma_2 P_1^{(2)} P_2^{(2)}}$, $E[x_1^{(3)} x_2^{(3)}] = \sqrt{\rho_1 P_1^{(3)} P_2^{(3)}}$, $E[x_1^{(3)} x_4^{(3)}] = \sqrt{\rho_2 P_1^{(3)} P_4^{(3)}}$, $E[x_2^{(3)} x_4^{(3)}] = \sqrt{\rho_3 P_2^{(3)} P_4^{(3)}}$, $E[x_1^{(4)} x_4^{(4)}] = \sqrt{\delta_2 P_1^{(4)} P_4^{(4)}}$ and $\psi = 1 - \sum_{i=1}^{3} \rho_i + 2\sqrt{\prod_{i=1}^{3} \rho_i} \geq 0$. The computation of the flows follows the standard cut-set bound derivation process. Thus, for simplicity, we only show the calculation of $F_{1,1}$ as an example. $F_{1,1} = \tau_1 I(x_1^{(1)}; y_2^{(1)}, y_3^{(1)}, y_4^{(1)}) + \tau_2 I(x_1^{(2)}; y_3^{(2)}, y_4^{(2)} | \ x_2^{(2)}) + \tau_3 I(x_1^{(3)}; y_3^{(3)} | \ \ x_2^{(3)}, x_4^{(3)}) + \tau_4 I(x_1^{(4)}, x_4^{(4)}; y_2^{(4)}, y_3^{(4)}) = \tau_1 C((c_{12}^2 + c_{13}^2 + c_{14}^2) P_1^{(1)}) + \tau_2 C(\gamma_1 (c_{13}^2 + c_{14}^2)\ P_1^{(2)}) + \tau_3 C(\psi c_{13}^2 P_1^{(3)} / (1 - \rho_3)) + \tau_4 C(\delta_1 (c_{12}^2 + c_{13}^2) P_1^{(4)})$. The single user UB for user 2 can be derived in similar fashion.

III. Numerical Results

Fig. 2 compares the achievable regions of the proposed HD TXRXC scheme with TXC/RXC alone [7] and IC with no cooperation. It is shown that with TXRXC, extra gain in achievable region is obtained. Further, as c_{12} and c_{34} increase from 10 dB to ∞, the achievable region meets the UB.

Fig. 3 compares the variations of sum rates with respect to c_{12}^2 and c_{34}^2 ($c_{12}^2 = c_{34}^2$) for TXRXC, TXC, RXC and TXC with RDPC [5]. This figure shows again the significant capacity gain using TXRXC over TXC and RXC. The sum rates with TXRXC, TXC and RXC all increase with the cooperation channel gain. However, the sum rate of TXC using RDPC does not change with the cooperation channel gain due to the absence of a joint transmission phase [7]. Further, the sum rates with TXRXC, TXC and RXC gradually approach their respective UB's, which are, respectively, the 2×2 MIMO channel, 2 user 2-transmit-1-receive antenna BC and 1-transmit-2-receive antenna MAC capacity [4].

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[3]The RX can decode v only after v_1, v_2 and v_3 have been decoded at phases 1, 3 and 5, respectively, in the previous transmission block.