

This is a list of more relevant changes/corrections made in the second edition of

HANDBOOK OF BROWNIAN MOTION - FACTS AND FORMULAE.

published year 2002 by Birkhäuser Verlag. These are incorporated into the corrected reprint of the second edition published year 2015.

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1. Chapter II item 10 p. 18 l. -1: “solutions” should be “positive solutions”.
2. Chapter II item 27 p. 32 l. -1:

$$\mathbf{E}_x(A_t; t < \zeta) \quad \text{should be} \quad \mathbf{E}_x(A_t^n; t < \zeta).$$

3. Chapter III: A new item discussing Zvonkin’s condition added after item 22 p. 47.
4. Chapter IV: Two new items discussing perpetual integral functionals added after item 33 p. 70.
5. Chapter IV item 15 p. 59 l. -6: Sentence “The limit is in the sense of weak convergence of measures on the function space $E^+ \cap \{\zeta = l\}$.” changed to “The limit is in the sense of the convergence of finite dimensional distributions.”
6. Chapter VI item 1 p. 103 l. 6: Sentence “Assume also that F and g are bounded.” changed to “Assume that g is bounded and that F is bounded and Hölder continuous.”
7. Appendix 1 item 9 p. 124: Added text at the end: “The expression for the density is valid for all nonnegative values on c and γ ; also when $\gamma c > 1$. We remark that the densities in No. 7 and No. 8 can be obtained from the density above by letting $c \rightarrow 0$ and $\gamma \rightarrow 0$, respectively.”
8. Appendix 1 item 11 p. 125: Added text at the end: “The expression for the density is valid for all nonnegative values on c and γ ; also when $4\gamma c > 1$.”
9. Appendix 1 item 12 p. 126: Following corrections:

Speed measure:

$$m(dx) = \begin{cases} 4\beta dx, & x > 0, \\ 4(1 - \beta) dx, & x < 0. \end{cases}$$

Scale function:

$$s(x) = \begin{cases} \frac{x}{2\beta}, & x \geq 0, \\ \frac{x}{2(1 - \beta)}, & x \leq 0. \end{cases}$$

Green function:

$$G_\alpha(x, y) = \frac{e^{-|x-y|\sqrt{2\alpha}} - e^{-(|x|+|y|)\sqrt{2\alpha}}}{2\sqrt{2\alpha}(1 + (2\beta - 1)\operatorname{sgn}(x \wedge y))} + \frac{e^{-(|x|+|y|)\sqrt{2\alpha}}}{2\sqrt{2\alpha}}.$$

Wronskian: $w_\alpha = 2\sqrt{2\alpha}$.

Transition density w.r.t. m :

$$p(t; x, y) = \frac{e^{-(x-y)^2/2t} - e^{-(|x|+|y|)^2/2t}}{2\sqrt{2\pi t}(1 + (2\beta - 1) \operatorname{sgn}(x \wedge y))} + \frac{e^{-(|x|+|y|)^2/2t}}{2\sqrt{2\pi t}}.$$

10. Appendix 1 item 13 p. 127: “ $\mathcal{T} = \sqrt{1 - 4\gamma c}$ ” should be “ $\mathcal{T} = \sqrt{1 - \gamma c}$ ”. Added text at the end: “The expression for the density is valid for all nonnegative values on c and γ ; also when $\gamma c > 1$.”
11. Appendix 1 item 21 p. 133: Added: “Wronskian: $w_\alpha = 1$.”
12. Appendix 1 item 24 p. 137: the transition density in case $\gamma > 0$ is as follows:

$$p(t; x, y) = \frac{1}{2} \frac{\sqrt{|\gamma|}}{\sqrt{2\pi \operatorname{sh}(|\gamma|t)}} \exp\left(-\frac{|\gamma|t}{2} - \frac{|\gamma|(x^2 + y^2)}{2} - \frac{(x^2 + y^2)|\gamma| \operatorname{ch}(|\gamma|t) - 2xy|\gamma|}{2 \operatorname{sh}(|\gamma|t)}\right).$$

13. Appendix 1 item 25 p. 139 1.12: the sentence “For $\nu = 0$ this is ... of freedom.” should be “For $\nu = 0$ and $\gamma = 1/2$ this is the so-called Rayleigh distribution, for $\nu = 1/2$ and $\gamma = 1/2$ we have the Maxwell distribution and, in general, for $\nu = n/2 - 1$, $n = 1, 2, \dots$, and $\gamma = 1/2$ the distribution of the square root of the χ^2 -distributed random variable with n degrees of freedom.”
14. Appendix 1 item 25 p. 139 1.17: θ should be γ
15. Appendix 1 item 25 p. 140 1.6,10:

$$\exp\left(\frac{|\gamma|x^2}{2}\right) \quad \text{should be} \quad \exp\left(-\frac{|\gamma|x^2}{2}\right)$$

16. Appendix 1 item 25 p. 140 1.7,11:

$$\exp\left(\frac{|\gamma|y^2}{2}\right) \quad \text{should be} \quad \exp\left(-\frac{|\gamma|y^2}{2}\right)$$

17. Appendix 1 item 25 p. 140 1.12: θ should be γ
18. Appendix 1 item 26 p. 141 1.-2: θ should be γ
19. Appendix 1 item 26 p. 142 1.-8,-4,;

$$\exp\left(-\frac{|\gamma|x}{2}\right) \quad \text{should be} \quad \exp\left(\frac{|\gamma|x}{2}\right)$$

20. Appendix 1 item 26 p. 142 1.-7,-3,;

$$\exp\left(-\frac{|\gamma|y}{2}\right) \quad \text{should be} \quad \exp\left(\frac{|\gamma|y}{2}\right)$$

21. Appendix 1 item 26 p. 142 1.-2: θ should be γ
22. Appendix 1: added a section on CEV processes.
23. Part II: right hand sides of the following formulas should be divided by $|p - q|$:
- a. 1.1.6.4, 1.1.6.8, 1.1.27.4, 1.1.27.8, 1.1.29.8,

- b. 2.1.6.4, 2.1.6.8, 2.1.27.4, 2.1.27.8, 2.1.29.8,
- c. 3.1.6.4, 3.1.6.8, 3.1.27.4, 3.1.27.8, 3.1.29.8,
- d. 5.1.6.4, 5.1.6.8, 5.1.27.4, 5.1.27.8,
- e. 9.1.6.4.

The right hand side of the inverse Laplace transform formula k. in Appendix 3 p. 649 should also be divided by $|p - q|$. For Formula 5.1.29.8 see below.

- 24. Part II, Formula 3.1.13.3 p. 344: $\star sc_t(y)$ should be $\star sc_t(x, y)$
- 25. Part II, Formula 3.1.13.4 p. 345: $\star sc_{t-v}(y)$ should be $\star sc_{t-v}(x, y)$
- 26. Part II, Formula 5.1.29.6, p. 461: R.H.S. should be:

$$\frac{\lambda z e^{-\lambda(u+v)}}{x} B_x^{(29)}(u, v, y, z) du dv dy dz \quad \text{for} \quad B_x^{(29)}(u, v, y, z) \quad \text{see 1.1.29.6.}$$

- 27. Part II, Formula 5.1.29.8, p. 461: R.H.S. should be:

$$\frac{z \mathbb{I}_{((q \wedge p)t, (q \vee p)t)}(v)}{x|p - q|} B_x^{(29)}\left(\frac{|qt - v|}{|p - q|}, \frac{|pt - v|}{|p - q|}, y, z\right) dv dy dz \quad \text{for} \quad B_x^{(29)}(u, v, y, z) \quad \text{see 1.1.29.6}$$

- 28. Part II, Formula 7.1.9.8 (1) p. 526 is replaced by the following formula for the density

$$\begin{aligned} \mathbf{P}_0\left(\int_0^t U_s^2 ds \in dy \mid U_t = 0\right) e^{-\theta t/2 + \theta y/4\sigma^2} \\ = \frac{\sqrt{\sigma} \sqrt{1 - e^{-2\theta t}}}{(2\theta)^{1/4} \pi y^{5/4}} \sum_{k=0}^{\infty} \frac{\Gamma(k + 1/2)}{k!} e^{-(4k+1)^2 t^2 \sigma^2 \theta/8y} D_{3/2}\left(\frac{(4k+1)t\sigma\sqrt{\theta}}{\sqrt{2y}}\right) dy \end{aligned}$$

- 29. Part II, Formula 8.1.28.1 p. 578: divide RHS by θ .
- 30. Appendix 2 p. 637: Expanded; in particular, a section on hypergeometric functions is included
- 31. Appendix 4 p. 652: Expanded with 8 equations.