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Handbook of Elliptic Integrals for Engineers and Scientists

Second Edition, Revised

With 22 Figures



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