



# Handbook of Marine Craft Hydrodynamics and Motion Control

Vademecum de Navium Motu Contra Aquas et de Motu Gubernando

Thor I. Fossen

 WILEY



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**Thor I. Fossen**

*Norwegian University of Science and Technology  
Trondheim, Norway*



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*This book is dedicated to my parents Gerd Kristine and Ole Johan Fossen and my family  
Heidi, Sindre and Lone Moa who have always been there for me.*

Thor I. Fossen





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# About the Author

Professor Thor I. Fossen received an MSc degree in Marine Technology in 1987 from the Norwegian University of Science and Technology (NTNU) and a PhD in Engineering Cybernetics from NTNU in 1991. In the period 1989–1990 he pursued postgraduate studies in aerodynamics and flight control as a Fulbright Scholar at the University of Washington, Seattle. His expertise is in the fields of hydrodynamics, naval architecture, robotics, marine and flight control systems, guidance systems, navigation systems and nonlinear control theory. In 1993 he was appointed as a Professor of Guidance and Control at NTNU. He is one of the founders of the company Marine Cybernetics where he was the Vice President R&D in the period 2002–2007. He is the author of *Guidance and Control of Ocean Vehicles* (John Wiley & Sons, Ltd, 1994) and co-author of *New Directions in Nonlinear Observer Design* (Springer Verlag, 1999) and *Parametric Resonance in Dynamical Systems* (Springer Verlag, 2011). Professor Fossen has been instrumental in the development of several industrial autopilot, path-following and dynamic positioning (DP) systems. He has also experience in nonlinear state estimators for marine craft and automotive systems as well as strapdown GNSS/INS navigation systems. He has been involved in the design of the SeaLaunch trim and heel correction systems. He received the Automatica Prize Paper Award in 2002 for a concept for weather optimal positioning control of marine craft. He is currently head of automatic control at the Centre for Ships and Ocean Structures (CESOS), Norwegian Centre of Excellence, and a Professor of Guidance and Control in the Department of Engineering Cybernetics, NTNU.



# Preface

The main motivation for writing this book was to collect new results on hydrodynamic modeling, guidance, navigation and control of marine craft that have been developed since I published my first book:

Fossen, T. I. (1994). *Guidance and Control of Ocean Vehicles*. John Wiley & Sons, Ltd. Chichester, UK. ISBN 0-471-94113-1.

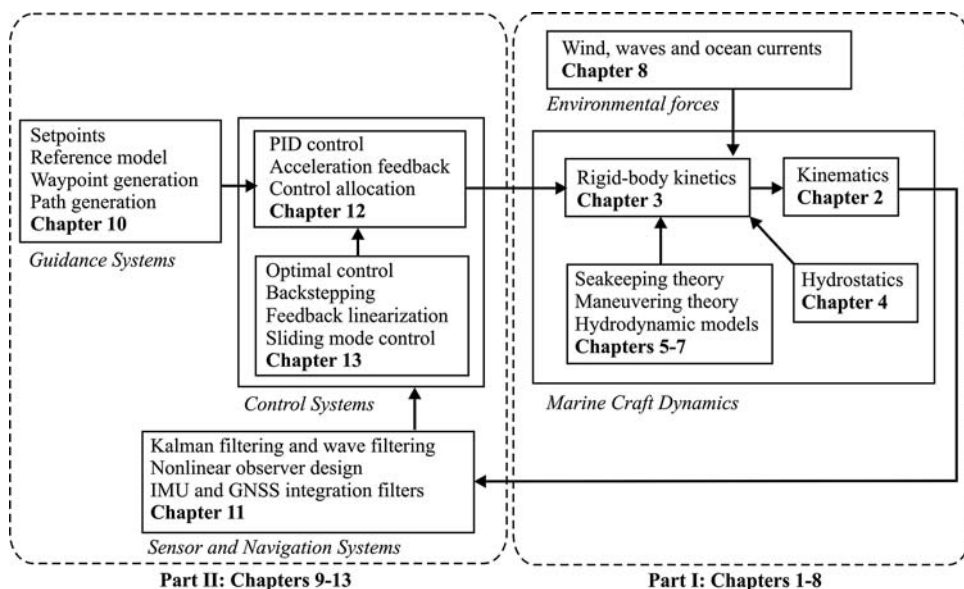
The Wiley book from 1994 was the first attempt to bring hydrodynamic modeling and control system design into a unified notation for modeling, simulation and control. My first book also contains state-of-the-art control design methods for ships and underwater vehicles up to 1994. In the period 1994–2002 a great deal of work was done on nonlinear control of marine craft. This work resulted in many useful results and lecture notes, which have been collected and published in a second book entitled *Marine Control Systems: Guidance, Navigation and Control of Ships and Underwater Vehicles*. The 1st edition was published in 2002 and it was used as the main textbook in my course on Guidance and Control at the Norwegian University of Science and Technology (NTNU). Instead of making a 2nd edition of the book, I decided to write the *Handbook of Marine Craft Hydrodynamics and Motion Control* and merge the most important results from my previous two books with recent results.

Part I of the book covers both maneuvering and seakeeping theory and it is explained in detail how the equations of motion can be derived for both cases using both frequency- and time-domain formulations. This includes transformations from the frequency to the time domain and the explanation of fluid-memory effects. A great effort has been made in the development of kinematic equations for effective representation of the equations of motion in seakeeping, body, inertial and geographical coordinates. This is very confusing in the existing literature on hydrodynamics and the need to explain this properly motivated me to find a unifying notation for marine and mechanical systems. This was done in the period 2002–2010 and it is inspired by the elegant formulation used in robotics where systems are represented in a vectorial notation. The new results on maneuvering and seakeeping are joint work with *Professor Tristan Perez*, University of Newcastle, Australia. The work with Professor Perez has resulted in several joint publications and I am grateful to him for numerous interesting discussions on hydrodynamic modeling and control. He should also be thanked for proofreading parts of the manuscript.

Part II of the book covers guidance systems, navigation systems, state estimators and control of marine craft. This second part of the book focuses on state-of-the-art methods for feedback control such as PID control design for linear and nonlinear systems as well as control allocation methods. A chapter with more advanced topics, such as optimal control theory, backstepping, feedback linearization and sliding-mode control, is included for the advanced reader. Case studies and applications are treated at the end of each chapter. The control systems based on PID and optimal control theory are designed with a complexity similar to those used in many industrial systems. The more advanced methods using nonlinear theory are included so the user can compare linear and nonlinear design techniques before a final implementation is

made. Many references to existing systems are included so control system vendors can easily find articles describing state-of-the art design methods for marine craft.

The arrangement of the subject matter in major parts can be seen from the following diagram:



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**Thor I. Fossen**

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# Part One

## Marine Craft Hydrodynamics

*De Navium Motu Contra Aquas*





# 1

## Introduction

The subject of this book is *motion control and hydrodynamics of marine craft*. The term marine craft includes ships, high-speed craft, semi-submersibles, floating rigs, submarines, remotely operated and autonomous underwater vehicles, torpedoes, and other propelled and powered structures, for instance a floating air field. Offshore operations involve the use of many marine craft, as shown in Figure 1.1. *Vehicles* that do not travel on land (ocean and flight vehicles) are usually called craft, such as watercraft, sailcraft, aircraft, hovercraft and spacecraft. The term vessel can be defined as follows:

*Vessel*: “hollow structure made to float upon the water for purposes of transportation and navigation; especially, one that is larger than a rowboat.”

The words *vessel*, *ship* and *boat* are often used interchangeably. In *Encyclopedia Britannica*, a ship and a boat are distinguished by their size through the following definition:

*Ship*: “any large floating vessel capable of crossing open waters, as opposed to a boat, which is generally a smaller craft. The term formerly was applied to sailing vessels having three or more masts; in modern times it usually denotes a vessel of more than 500 tons of displacement. Submersible ships are generally called boats regardless of their size.”

Similar definitions are given for submerged vehicles:

*Submarine*: “any naval vessel that is capable of propelling itself beneath the water as well as on the water’s surface. This is a unique capability among warships, and submarines are quite different in design and appearance from surface ships.”

*Underwater Vehicle*: “small vehicle that is capable of propelling itself beneath the water surface as well as on the water’s surface. This includes unmanned underwater vehicles (UUV), remotely operated vehicles (ROV), autonomous underwater vehicles (AUV) and underwater robotic vehicles (URV). Underwater vehicles are used both commercially and by the navy.”

From a hydrodynamic point of view, marine craft can be classified according to their maximum operating speed. For this purpose it is common to use the *Froude number*:

$$Fn := \frac{U}{\sqrt{gL}} \quad (1.1)$$

where  $U$  is the craft speed,  $L$  is the overall submerged length of the craft and  $g$  is the acceleration of gravity. The pressure carrying the craft can be divided into *hydrostatic* and *hydrodynamic* pressure. The corresponding forces are:

- Buoyancy force due to the hydrostatic pressure (proportional to the displacement of the ship).
- Hydrodynamic force due to the hydrodynamic pressure (approximately proportional to the square of the relative speed to the water).

For a marine craft sailing at constant speed  $U$ , the following classifications can be made (Faltinsen, 2005):

**Displacement Vessels ( $Fn < 0.4$ ):** The buoyancy force (restoring terms) dominates relative to the hydrodynamic forces (added mass and damping).

**Semi-displacement Vessel ( $0.4-0.5 < Fn < 1.0-1.2$ ):** The buoyancy force is not dominant at the maximum operating speed for a high-speed submerged hull type of craft.

**Planing Vessel ( $Fn > 1.0-1.2$ ):** The hydrodynamic force mainly carries the weight. There will be strong flow separation and the aerodynamic lift and drag forces start playing a role.

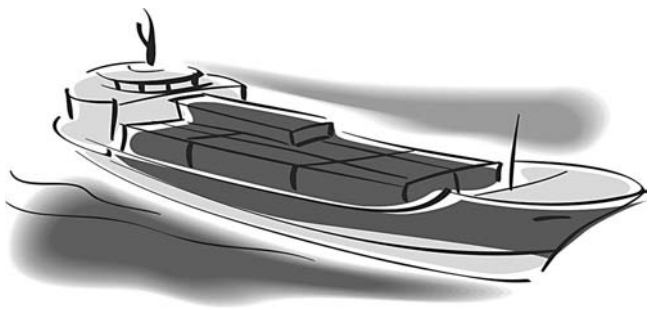
In this book only displacement vessels are covered; see Figure 1.2.

The Froude number has influence on the hydrodynamic analysis. For displacement vessels, the waves radiated by different parts of the hull do not influence other parts of the hull. For semi-displacement vessels, waves generated at the bow influence the hydrodynamic pressure along the hull towards the stern. These characteristics give rise to different modeling hypotheses, which lead to different hydrodynamic theories.

For displacement ships it is widely accepted to use two- and three-dimensional potential theory programs to compute the potential coefficients and wave loads; see Section 5.1. For semi-displacement



**Figure 1.1** Marine craft in operation. Illustration Bjarne Stenberg/Department of Marine Technology, NTNU.

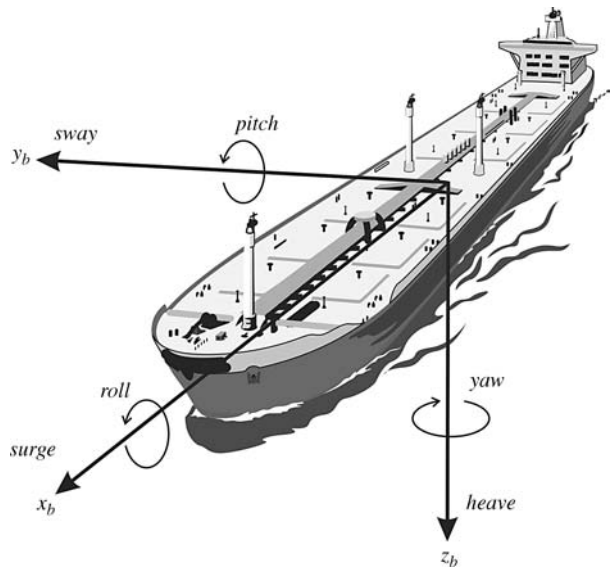


**Figure 1.2** Displacement vessel.

vessels and planing vessels it is important to include the lift and drag forces in the computations (Faltinsen, 2005).

**Degrees of Freedom and Motion of a Marine Craft**

In maneuvering, a marine craft experiences motion in 6 degrees of freedom (DOFs); see Section 9.4. The DOFs are the set of independent displacements and rotations that specify completely the displaced position and orientation of the craft. The motion in the horizontal plane is referred to as *surge* (longitudinal motion, usually superimposed on the steady propulsive motion) and *sway* (sideways motion). *Yaw* (rotation about the vertical axis) describes the heading of the craft. The remaining three DOFs are *roll* (rotation about the longitudinal axis), *pitch* (rotation about the transverse axis) and *heave* (vertical motion); see Figure 1.3.



**Figure 1.3** Motion in 6 degrees of freedom (DOF).

Roll motion is probably the most influential DOF with regards to human performance, since it produces the highest accelerations and, hence, is the principal villain in seasickness. Similarly, pitching and heaving feel uncomfortable to people. When designing ship autopilots, yaw is the primary mode for feedback control. Stationkeeping of a marine craft implies stabilization of the surge, sway and yaw motions.

When designing feedback control systems for marine craft, reduced-order models are often used since most craft do not have actuation in all DOF. This is usually done by decoupling the motions of the craft according to:

**1 DOF** models can be used to design forward speed controllers (*surge*), heading autopilots (*yaw*) and roll damping systems (*roll*).

**3 DOF** models are usually:

- Horizontal plane models (*surge*, *sway* and *yaw*) for ships, semi-submersibles and underwater vehicles that are used in dynamic positioning systems, trajectory-tracking control systems and path-following systems. For slender bodies such as submarines, it is also common to assume that the motions can be decoupled into *longitudinal* and *lateral* motions.
- Longitudinal models (*surge*, *heave* and *pitch*) for forward speed, diving and pitch control.
- Lateral models (*sway*, *roll* and *yaw*) for turning and heading control.

**4 DOF** models (*surge*, *sway*, *roll* and *yaw*) are usually formed by adding the roll equation to the 3 DOF horizontal plane model. These models are used in maneuvering situations where it is important to include the rolling motion, usually in order to reduce roll by active control of fins, rudders or stabilizing liquid tanks.

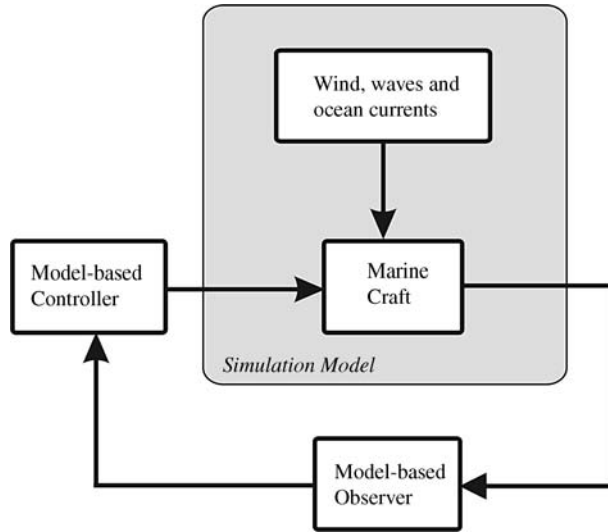
**6 DOF** models (*surge*, *sway*, *heave*, *roll*, *pitch* and *yaw*) are fully coupled equations of motion used for simulation and prediction of coupled vehicle motions. These models can also be used in advanced control systems for underwater vehicles that are actuated in all DOF.

## 1.1 Classification of Models

The models in this book can be used for prediction, real-time simulation and controller-observer design. The complexity and number of differential equations needed for the various purposes will vary. Consequently, one can distinguish between three types of models (see Figure 1.4):

**Simulation Model:** This model is the most accurate description of a system, for instance a 6 DOF *high-fidelity model* for simulation of coupled motions in the time domain. It includes the marine craft dynamics, propulsion system, measurement system and the environmental forces due to wind, waves and ocean currents. It also includes other features not used for control and observer design that have a direct impact on model accuracy. The simulation model should be able to reconstruct the time responses of the real system and it should also be possible to trigger failure modes to simulate events such as accidents and erroneous signals. Simulation models where the fluid-memory effects are included due to frequency-dependent added mass and potential damping typically consist of 50–200 ordinary differential equations (ODEs) while a maneuvering model can be represented in 6 DOF with 12 ODEs for generalized position and velocity. In addition, some states are needed to describe the environmental forces and actuators, but still the number of states will be less than 50 for a marine craft.

**Control Design Model:** The controller model is a reduced-order or simplified version of the simulation model that is used to design the *motion control system*. In its simplest form, this model is used to compute a set of constant gains for a proportional, integral, derivative (PID) controller. More



**Figure 1.4** Models used in guidance, navigation and control.

sophisticated control systems use a dynamic model to generate feedforward and feedback signals. This is referred to as *model-based control*. The number of ODEs used in conventional model-based ship control systems is usually less than 20. A PID controller typically requires two states: one for the integrator and one for the low-pass filter used to limit noise amplification. Consequently, setpoint regulation in 6 DOF can be implemented by using 12 ODEs. However, trajectory-tracking controllers require additional states for feedforward as well as filtering so higher-order control laws are not uncommon.

**Observer Design Model:** The observer model will in general be different from the model used in the controller since the purpose is to capture the additional dynamics associated with the sensors and navigation systems as well as disturbances. It is a simplified version of the simulation model where attention is given to accurate modeling of measurement noise, failure situations including dead-reckoning capabilities, filtering and motion prediction. For marine craft, the *model-based observer* often includes a disturbance model where the goal is to estimate wave, wind and ocean current forces by treating these as colored noise. For marine craft the number of ODEs in the state estimator will typically be 20 for a dynamic positioning (DP) system while a basic heading autopilot is implemented with less than five states.

## 1.2 The Classical Models in Naval Architecture

The motions of a marine craft exposed to wind, waves and ocean currents takes place in 6 DOF. The equations of motion can be derived using the Newton–Euler or Lagrange equations. The equations of motion are used to simulate ships, high-speed craft, underwater vehicles and floating structures operating under or on the water surface, as shown in Figure 1.5. In Section 3.3 it is shown that a rigid body with



**Figure 1.5** Ship and semi-submersibles operating offshore. Illustration Bjarne Stenberg/MARINTEK.

constant mass  $m$  and center of gravity  $(x_g, y_g, z_g)$  relative to a fixed point on the hull can be described by the following coupled differential equations:

$$\begin{aligned}
 m [\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})] &= X \\
 m [\dot{v} - wp + ur - y_g(r^2 + p^2) + z_g(qr - \dot{p}) + x_g(qp + \dot{r})] &= Y \\
 m [\dot{w} - uq + vp - z_g(p^2 + q^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p})] &= Z \\
 I_x \dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \\
 + m [y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)] &= K \\
 I_y \dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} \\
 + m [z_g(\dot{u} - vr + wq) - x_g(\dot{w} - uq + vp)] &= M \\
 I_z \dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} \\
 + m [x_g(\dot{v} - wp + ur) - y_g(\dot{u} - vr + wq)] &= N
 \end{aligned} \tag{1.2}$$

where  $X, Y, Z, K, M$  and  $N$  denote the external forces and moments. This model is the basis for time-domain simulation of marine craft. The external forces and moments acting on a marine craft are usually modeled by using:

**Maneuvering Theory:** The study of a ship moving at constant positive speed  $U$  in calm water within the framework of maneuvering theory is based on the assumption that the maneuvering (hydrodynamic) coefficients are *frequency independent* (no wave excitation). The maneuvering model will in its simplest representation be linear while nonlinear representations can be derived using methods such as cross-flow drag, quadratic damping or Taylor-series expansions; see Chapter 6.

**Seakeeping Theory:** The motions of ships at zero or constant speed in waves can be analyzed using seakeeping theory where the hydrodynamic coefficients and wave forces are computed as a function of the wave excitation frequency using the hull geometry and mass distribution. The seakeeping models