

Handbook of Radiation and Scattering of Waves:

- Acoustic Waves in Fluids
- Elastic Waves in Solids
- Electromagnetic Waves

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