Hanle effect in transport through quantum dots coupled to ferromagnetic leads

M. Braun[1], J. König[1] and J. Martinek[2,3,4]

[1] *Institut für Theoretische Physik III - Ruhr-Universität Bochum, 44780 Bochum, Germany*

[2] *Institut für Theoretische Festkörperphysik - Universität Karlsruhe, 76128 Karlsruhe, Germany*

[3] *Institute of Molecular Physics - Polish Academy of Science, 60-179 Poznań, Poland*

[4] *Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan*

Abstract. – We suggest a series of transport experiments on spin precession in quantum dots coupled to one or two ferromagnetic leads. Dot spin states are created by spin injection and analyzed via the linear conductance through the dot, while an applied magnetic field gives rise to the Hanle effect. Such a Hanle experiment can be used to determine the spin lifetime in the quantum dot, to measure the spin injection efficiency into the dot, as well as proving the existence of intrinsic spin precession which is driven by the Coulomb interaction.

Recent progress in nanofabrication technology opens the possibility for spintronic devices based on coherent manipulation of single spins in quantum dots [1]. Thereby the spatial confinement of the dot electrons suppresses spin decoherence [2]. While longitudinal spin relaxation times T_1 up to microseconds were measured [3,4], the size of the spin decoherence time T_2 is still an open question.

The spin coherence time T_2 can be accessed in different experiments, for example by ESR techniques [5], or by the Hanle effect, *i.e.*, the decrease of spin accumulation in the quantum dot due to precession about a static magnetic field. The optical realization of such a Hanle experiment involves the measurement of the fluorescent emission of polarized light from semiconductor quantum dots [6]. But this method requires an ensemble of spins, so the total signal varies with the spin dephasing time $T_2^\star$ rather than the decoherence time $T_2 > T_2^\star$.

To void this ensemble averaging, we suggest to measure the Hanle effect in transport through an individual single level quantum dot. For preparation of the initial spin state, electronic spin injection from ferromagnetic leads into the dot can be used. This has been demonstrated in metallic bulk systems [7], quantum dots [8], Zener diodes [9], and metallic grains [10]. For the detection of the accumulated spin, we propose magnetoresistance measurements of the device as already shown for metallic systems [7,10]. So an all electrical

© EDP Sciences

experimental setup can be used to observe the reduction of spin accumulation in a single quantum dot by an external magnetic field, which is a direct measure of the product of precession frequency ω and spin lifetime.

In a recent experiment Zhang $et.$ $al.$ [11] realized this kind of setup but with a whole layer of aluminum dots in a tunnel junction between two Co electrodes with many levels participating in transport in each dot rather than an individual quantum dot with only a single level contributing to the current. Even so the measurements involve averaging over different realizations of the dots, multi levels and local magnetizations, they clearly observe a Hanle resonance in the magnetoresistance of the device.

For our theoretical model we consider only one single-level quantum dot with level energy ε, measured relative to the Fermi energy of the leads and tunable by a gate electrode as sketched in fig.1. The dot level can be empty, singly occupied, or doubly occupied. The Coulomb interaction on the dot is accounted for by the charging energy U for double occupancy. The dot is coupled to source and drain electrodes by tunnel contacts. The coupling strength to the left and right lead is characterized by the intrinsic line width Γ_r with $r = \mathrm{L, R}$. We study sequential-tunneling regime, which yields $k_\mathrm{B}T \gg \Gamma_r$. The electrodes may be nonmagnetic or ferromagnetic, described by their degree of spin polarization p_r with $0 \leq p_r \leq 1$ and magnetization direction $\hat{\boldsymbol{n}}_r$. Further possible realizations of such systems include molecular [12] or carbon nanotube [13] junctions, grains in nano-constrictions [14] and junctions [11], or surface impurities contacted by an STM tip [15].
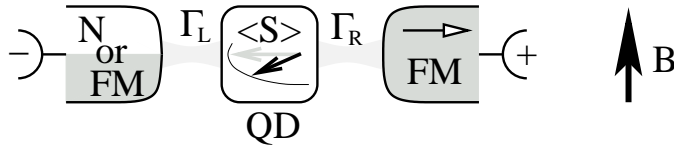

Fig. 1 – Quantum dot connected to one or two ferromagnetic leads. Spin dependent tunnel rates lead to spin accumulation on the dot. This accumulated spin precesses in the external applied magnetic field.

A full theoretical description of the spin dynamics and its implication on sequential-tunneling transport has been derived in ref. [16] within a diagrammatic transport theory. Here, we use this theory to analyze the Hanle effect. To keep the discussion transparent, we assume symmetric coupling constants, $\Gamma_\mathrm{L} = \Gamma_\mathrm{R} = \Gamma/2$ and consider only the regime of linear transport, $i.e.$ bias voltages smaller than temperature $eV \ll k_\mathrm{B}T$. The spin accumulation on the quantum dot is described by the quantum statistical value $\boldsymbol{S}$, which has the co-domain $0 < |\boldsymbol{S}| < \hbar/2$. The dynamics of the dot spin $\boldsymbol{S}$ is governed by a Bloch-like equation

$$\frac{d\boldsymbol{S}}{dt} = \sum_{r=\mathrm{L/R}} \left(\frac{\hbar}{2e} I_r p_r \hat{\boldsymbol{n}}_r - \frac{\boldsymbol{S} - p^2(\hat{\boldsymbol{n}}_r \cdot \boldsymbol{S})\hat{\boldsymbol{n}}_r}{\tau_{c,r}} \right) + \boldsymbol{S} \times \boldsymbol{\omega} - \frac{\boldsymbol{S}}{\tau_\mathrm{rel}} . \tag{1}$$

The first term describes spin accumulation due to spin-polarized currents from or into ferromagnetic leads. In the steady state, $I_\mathrm{L} = -I_\mathrm{R}$, so the spin does accumulate in the direction $p_\mathrm{L}\hat{\boldsymbol{n}}_\mathrm{L} - p_\mathrm{R}\hat{\boldsymbol{n}}_\mathrm{R}$.

The second term describes the relaxation of the dot spin due to coupling to the leads. Since neither an empty nor a doubly-occupied dot can bear a net spin, the relaxation time is exactly the life time of the single-occupation dot state. The time scale for tunneling of

an electron to or from the electrode r is given by $\tau_{c,r}^{-1} = \Gamma_r/\hbar(1 - f_r(\varepsilon) + f_r(\varepsilon + U))$, where $f_r(\varepsilon)$ is the Fermi function of the lead. The total life time of the single-occupation state for a weak bias voltage is given by $\tau_c^{-1} = \sum_r \tau_{c,r}^{-1} = \Gamma/\hbar(1 - f(\varepsilon) + f(\varepsilon + U))$. Together with the phenomenological spin-relaxation rate τ_{rel}^{-1}, describing decoherence *e.g.*, due to spin-orbit coupling or hyperfine interaction within the quantum dot, the total spin coherence time of the dot spin is

$$(\tau_s)^{-1} = (\tau_{rel})^{-1} + (\tau_c)^{-1} . \tag{2}$$

Quantum information processing with quantum-dot spins is anticipated to be operated in the Coulomb-blockade regime, where τ_c is large and T_2 is limited by τ_{rel}. Below we will propose a scheme to measure τ_s in the opposite, the sequential-tunneling regime. Therefore, in order to estimate the relevant T_2 for quantum-computing applications from the measured τ_s, one has to subtract the influence of τ_c.

The third term in eq. (1) describes precession of the quantum-dot spin about an effective magnetic field. This includes an externally-applied magnetic field $\boldsymbol{B}$ and the exchange interaction to the ferromagnetic leads. In the presence of strong Coulomb interaction on the dot, the tunnel coupling to ferromagnetic electrodes renormalizes the energy levels spin dependent [16,17]. This renormalization leads also to spin precession which can be described by a magentic-like exchange field. In total, we get

$$\boldsymbol{\omega} = \boldsymbol{\omega}_B + \boldsymbol{\omega}_x , \tag{3}$$

with $\hbar\boldsymbol{\omega}_B = g\mu_B\boldsymbol{B}$ and the exchange contribution $\boldsymbol{\omega}_x = A\Gamma[p_L\hat{\boldsymbol{n}}_L + p_R\hat{\boldsymbol{n}}_R]$. In the linear-response regime, the ε dependent numerical factor A is determined by the integral $A = (1/2\pi)\int' dE[(E - \varepsilon - U)^{-1} - (E - \varepsilon)^{-1}]f(E)$.

Since both, the accumulation and the relaxation term in eq.(1) transfer spin through the tunnel barrier, the partition in accumulation and damping is to some extend arbitrary. The current choice has the advantage, that the accumulation term is direct proportional to the electrical current. On the other hand the interpretation in ref. [16] gives a more intuitive isotropic damping term proportional to $\boldsymbol{S}$.

In the following calculation we consider the parameter regime $\Gamma \approx \hbar\omega_B \ll eV \ll k_BT$. The magnetic field $\hbar\omega_B$ must be comparable to Γ, to observe coherent rotation of the dot spin rather than incoherent transport through Zeeman-splitted levels. Furthermore, if $\hbar\boldsymbol{\omega}_B \lesssim \Gamma$, we can neglect the influence of the Zeeman splitting on the transition rates, since these corrections would be of the order $\Gamma B \approx \Gamma^2$, comparable to cotunneling. The condition $\hbar\omega_B \ll eV$ ensures that the electrical spin injection dominates the spin dynamics rather than the equilibrium spin due to the Zeeman splitting. We emphasize that, although the latter condition requires a finite transport voltage, we can calculate the conductance in linear response as long as $eV \ll k_BT$.

In the rest of the paper we apply our formalism to three different setups. The first one consists of a quantum dot attached to one nonmagnetic and one ferromagnetic electrode (fig. 2), a geometry already realized experimentally [14].

When a transport voltage is applied, the quantum dot becomes spin polarized. The direction of the accumulated spin depends on the direction of current flow (the linear conductance, though, does not). In case the electrons are flowing from the unmagnetized source lead (left) to the ferromagetic drain (right), the dot spin will accumulate anti-parallel to the magnetization direction $\hat{\boldsymbol{n}}_R$ since the tunnel barrier to the drain is more transparent for electrons polarized along $\hat{\boldsymbol{n}}_R$ than for those along $-\hat{\boldsymbol{n}}_R$. A transverse magnetic field rotates the dot spin away from this anti-parallel position. The rotated spin has an increased component along $\hat{\boldsymbol{n}}_R$, *i.e.*, the electron can more easily tunnel into the ferromagnetic drain electrode. As a consequence,
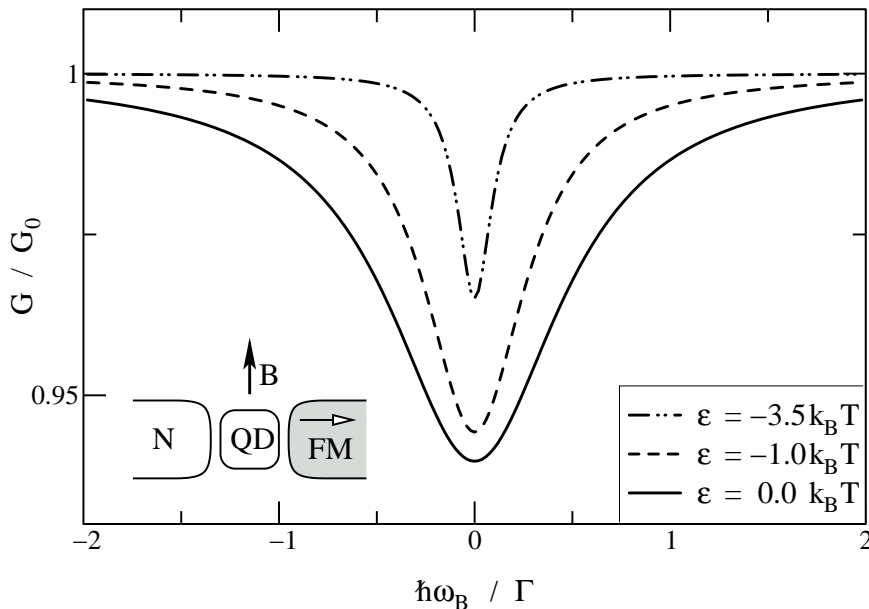
Fig. 2 – The differential conductance as function of the magnetic field applied transverse to the ferromagnetic lead fior different gate voltages. By shifting the dot into Coulomb blockade, the electron dwell time of the electrons increase. Since the electrons have then more time to precess and to relax inside the dot, width of the Hanle resonance as well as the depth decreases. The parameters are $p = 0.5$, $U = 7k_BT$, $\tau_{\rm rel} = 20\hbar/\Gamma$, and level positions $\varepsilon/k_BT = -3.5$ (dot-dashed), -1.5 (dashed), and 0 (solid).

the conductance increases, which defines the Hanle effect in transport. Based on the transport theory in ref. [16], we derive the the linear conductance of this setup to be

$$
\frac{G}{G_0} = 2 - \left[1 - \frac{p^2}{4} \frac{\tau_{\rm s}}{\tau_{\rm c}} \frac{1 + [\hat{\boldsymbol{n}}_{\rm R} \cdot (\boldsymbol{\omega}_{\rm B} + \boldsymbol{\omega}_{\rm x}) \tau_{\rm s}]^2}{1 + (\boldsymbol{\omega}_{\rm B} + \boldsymbol{\omega}_{\rm x})^2 \tau_{\rm s}^2} \right]^{-1} , \tag{4}
$$

where $G_0 = e^2 \, P_1/\tau_c k_BT$ is the asymptotic value of the conductance for large magnetic field, $B \to \infty$, for which the spin accumulation is completely destroyed. The latter is proportional to P_1, the equilibrium probability to find the dot occupied by a single electron, and τ_c is given by $\tau_c^{-1} = (\Gamma/\hbar) [1 - f(\varepsilon) + f(\varepsilon + U)]$. Note that the dependence of the expression in eq. (4) on the magnetic field differs from the optically-measured Hanle signal [9], as a consequence of the different way to probe the spin.

Results are shown in fig. 2, where $\boldsymbol{\omega}_{\rm B} = g\mu_B \boldsymbol{B}/\hbar$ characterizes an external magnetic field applied perpendicular to the direction of lead magnetization, and $\boldsymbol{\omega}_{\rm x}$ the exchange field (aligned parallel), such that $\boldsymbol{\omega} = \boldsymbol{\omega}_{\rm B} + \boldsymbol{\omega}_{\rm x}$. For not too large values of the spin polarization p in the ferromagnet, $(p^2/4)\tau_{\rm s}/\tau_{\rm c}$ is small, and eq. (4) can be expanded in the latter quantity. The dip in the relative conductance at zero magnetic field is, thus, approximatively $(p^2/4)\tau_{\rm s}/\tau_{\rm c}$, which provides an estimate of the degree p of spin polarization in the ferromagnet. To maximize $\tau_{\rm s}/\tau_{\rm c} \leq 1$ one can tune the quantum-dot level on resonance. The widths $\Delta\omega_{\rm B}$ of the dip as a function of the applied field is determined by the condition $\Delta\omega_{\rm B}\tau_{\rm s} = \sqrt{1 + 2(\omega_{\rm x}\tau_{\rm s})^2}$. The inverse of the measured width, $1/\Delta\omega_{\rm B}$, does therefore only provide a lower limit for the spin lifetime $\tau_{\rm s}$, since the exchange field modifies the line width.

External field components parallel to the magnetization directions influence the result in exactly the same way as the exchange field $\boldsymbol{\omega}_x$ does, leading to a broadening of the Hanle resonance. In the experiment by Zhang *et. al.* [11], the authors propose that stray fields leading to this kind of broadening mechanism are the major source of error for measuring T_2. To reduce the influence of the stray fields, the dot-lead coupling can be increased. Since τ_s is inverse to Γ, the accumulated spin becomes less sensitive to the magnetic fields for stronger couplings, making small stray fields insignificant. Typical transport experiments [12,14] show current plateaus of the order 10pA to 10nA in the I-V characteristic, which yields $\Gamma/\hbar$ to be of the order of $10^8 s^{-1}$ to $10^{11} s^{-1}$. The typical magnetic field strength required for the observation of the precession is then of the order $100\mu T$ to $100mT$, which can exceed the stray fields of the ferromagnetic leads in appropriate probe designs [10, 11]. It is worth to mention, that since the exchange field is also a linear function of Γ, its influence, which is proportional to $\omega_x \tau_s$, does not depend on the coupling strength.

The structure discussed so far has the advantage that only one lead is ferromagnetic, which might simplify the manufacturing procedure. It is suitable to prove the existence of the spin precession analogous to the optical Hanle effect. The influence of the exchange field, though, makes it difficult to directly determine τ_s. We, therefore, turn now to a second setup which involves two ferromagentic leads with magnetization directions anti-parallel to each other, see fig.3.
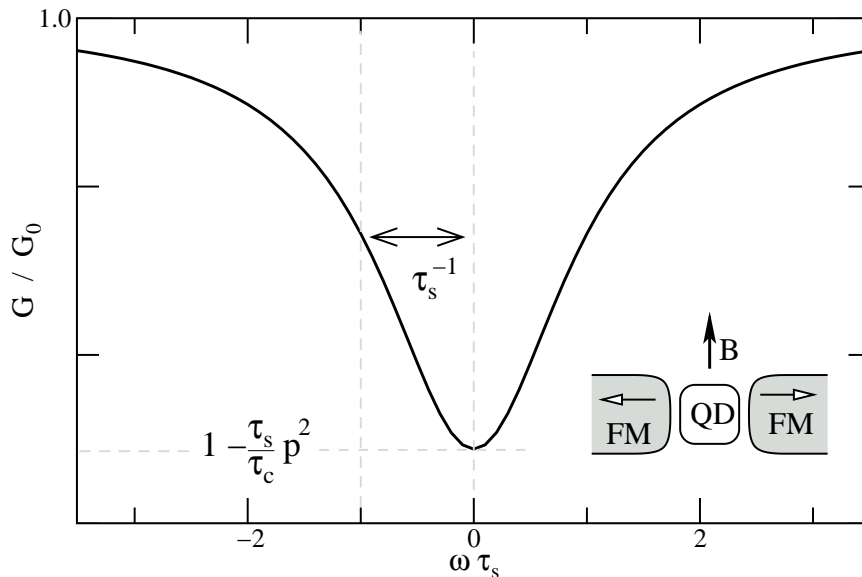

Fig. 3 – Differential conductance, for ferromagentic leads with anti-parallel magnetization, as a function of the magnetic field ω applied perpendicular to the accumulated spin. The half line width of the Hanle resonance directly determines the spin coherence time τ_s.

For symmetric coupling $\Gamma_L = \Gamma_R$, equal degree of polarization $p_L = p_R = p$ and in the linear-response regime, the exchange field originating from the left and the right tunnel barrier cancel out each other so $\omega_x = 0$, and the dot spin precesses only due to the external magnetic

field. The linear conductance, then, is

$$\frac{G}{G_0} \;=\; 1 - p^2 \frac{\tau_s}{\tau_c} \frac{1 + (\frac{\hat{\boldsymbol{n}}_L - \hat{\boldsymbol{n}}_R}{2} \boldsymbol{\omega}_B \tau_s)^2}{1 + (\boldsymbol{\omega}_B \tau_s)^2} \;. \tag{5}$$

If we assume the field to be aligned perpendicular to the lead magnetizations (see fig. 3), we find the Lorentzian dependence on the external magnetic field that familiar from the optical Hanle effect. The depth of the dip is given by $p^2 \tau_s / \tau_c$ while the width of the dip in fig. 3 provides a direct access to the spin lifetime τ_s. Of course, the conversion of applied magnetic field to frequency requires the knowledge of the Lande factor g, which must be determined separately like in ref. [14].

Finally, we discuss the case of a non-collinear configuration of the leads' magnetizations with a magnetic field applied along the direction $\hat{\boldsymbol{n}}_L + \hat{\boldsymbol{n}}_R$ as shown in fig. 4.
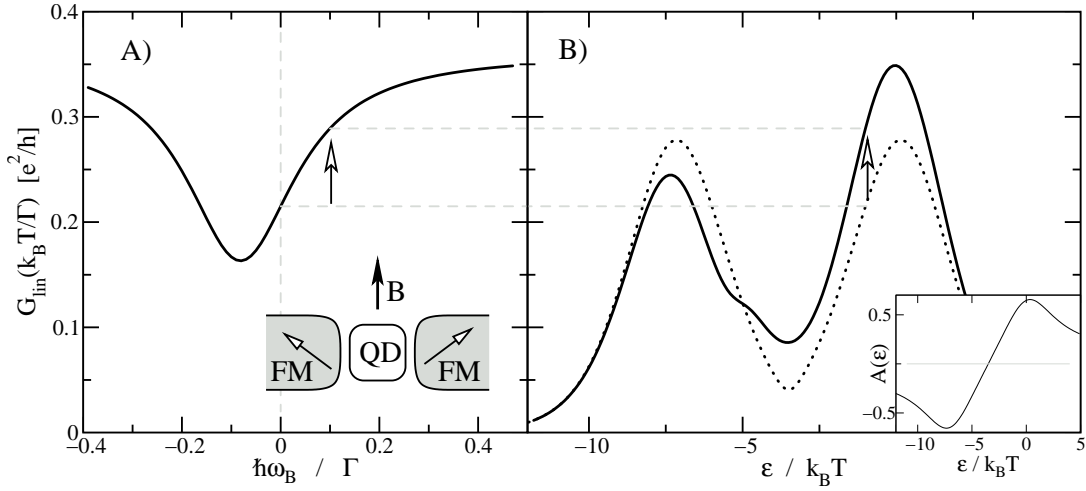

Fig. 4 – Linear conductance of the dot for an applied external magnetic field $\boldsymbol{B}$ along $\hat{\boldsymbol{n}}_L + \hat{\boldsymbol{n}}_R$. A) Linear conductance as a function of the applied field for $\varepsilon = 0$. B) Linear conductance as a function of the level position ε without external field (dashed-dot-dot) and for the applied field $\omega_B = 0.1\Gamma/\hbar$ (solid). Further parameters are $\phi = 3\pi/4$, $p = 0.8$, $U = 7k_B T$, and $\tau_{rel} = 0$. The vertical lines relate the Conductance increase of the dot at $\varepsilon = 0$ for a magnetic field $\hbar\omega_B = 0.1\Gamma$.

In this case, both the exchange field and the external magnetic field are pointing along $\hat{\boldsymbol{n}}_L + \hat{\boldsymbol{n}}_R$, perpendicular to the accumulated spin. The linear conductance is, then,

$$\frac{G}{G_0} \;=\; 1 - p^2 \frac{\tau_s}{\tau_c} \frac{\sin^2 \frac{\phi}{2}}{1 + (\boldsymbol{\omega}_B + \boldsymbol{\omega}_x)^2 \tau_s^2} \;, \tag{6}$$

where $\phi = \sphericalangle(\hat{\boldsymbol{n}}_L; \hat{\boldsymbol{n}}_R)$ is the angle enclosed by the leads' magnetization directions.

This setup allows for a stringent experimental verification of spin precession due to the exchange field. The conductance reaches its minimal value when the external magnetic field has opposite direction and equal magnitude as the exchange field. The shift of the minimum's position relative to $B = 0$, thus, measures the exchange field, see fig. 4A. One can clearly separate the exchange field from possible stray fields of the leads by varying the gate voltage of the quantum dot. While the stray fields does not depend on the gate voltage, the exchange interaction does as plotted in the inset of fig. 4B.

In the flat band limit, the exchange field even changes its sign as a function of gate voltage, so by plotting the conductance as function of the gate voltage in Rig. 4B, we observe an increased conductance for one resonance peak, and a decreased for the other one. At the intersection point of the conduction curves with and without external field, the absolute value of ω remains unchanged but has opposite sign, *i.e.*, the exchange field at this point is $-g\mu_\mathrm{B}B/2$.

In summary, we suggest to measure the Hanle effect in transport through quantum dots coupled to one or two ferromagnetic leads. We propose schemes how to observe non-equilibrium spin accumulation, determine the dot-spin lifetime and verify the existence of an intrinsic spin precession caused by Coulomb interaction.

REFERENCES

[1] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[2] A. V. Khaetskii, D. Loss, and L. Glazman, Phys. Rev. Lett. 88, 186802 (2002); A. V. Khaetskii and Y. V. Nazarov, Phys. Rev. B **64**, 125316 (2001).
[3] T. Fujisawa, D. G. Austing, Y. Tokura, Y. Hirayama, and S. Tarucha, Phys. Rev. Lett. **88**, 236802 (2002).
[4] J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, Nature **430**, 431 (2004).
[5] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. Lett. **85**, 1962 (2000); H. Engel and D. Loss, Phys. Rev. Lett. **86**, 4648 (2001); Phys. Rev. B **65**, 195321 (2002).
[6] R. J. Epstein, D. T. Fuchs, W. V. Schoenfeld, P. M. Petroff, and D. D. Awschalom, Appl. Phys. Lett. **78**, 733 (2001).
[7] M. Johnson and R. H. Silsbee, Phys. Rev. Lett. **55**, 1790 (1985); Phys. Rev. B. **37**, 5326 (1988).
[8] Y. Chye, M. E. White, E. Johnston-Halperin, B. D. Gerardot, D. D. Awschalom, and P. M. Petroff, Phys. Rev. B **66**, 201301(R) (2002).
[9] P. Van Dorpe, Z. Liu, W. Van Roy, V. F. Motsnyi, M. Sawicki, G. Borghs and J. De Boeck, Appl. Phys. Lett. **84**, 3495 (2004); V. F. Motsnyi, P. Van Dorpe, W. Van Roy, E. Goovaerts, V. I. Safarov, G. Borghs and J. De Boeck, Phys. Rev. B **68**, 245319 (2003); Appl. Phys. Lett. **81**, 265 (2002).
[10] M. Zaffalon and B. J. van Wees, Phys. Rev. Lett. **91**, 186601 (2003).
[11] L. Y. Zhang, C. Y. Wang, Y. G. Wei, X. Y. Liu, D. Davidović, *cond-mat/0502181*.
[12] J. Park, A. N. Pasupathy, J. I. Goldsmith, C. Chang, Y. Yaish, J. R. Petta, M. Rinkoski, J. P. Sethna, H. D. Abruña, P. M. McEuen, and D. C. Ralph, Nature **417**, 722 (2002); A. N. Pasupathy, R. C. Bialczak, J. Martinek, J. E. Grose, L. A. K. Donev, P. L. McEuen, and D. C. Ralph, Science **306**, 86 (2004).
[13] B. Zhao, I. Mönch, H. Vinzelberg, T. Mühl, and C. M. Schneider, Appl. Phys. Lett. **80**, 3144 (2002).
[14] M. M. Deshmukh and D. C. Ralph, Phys. Rev. Lett. **89**, 266803 (2002).
[15] C. Durkan and M. E. Welland, Appl. Phys. Lett. **80**, 458 (2002).
[16] J. König and J. Martinek, Phys. Rev. Lett. **90**, 166602 (2003); M. Braun, J. König, and J. Martinek, Phys. Rev. B **70**, 195345 (2004).
[17] J. Martinek, Y. Utsumi, H. Imamura, J. Barnaś, S. Maekawa, J. König, and G. Schön, Phys. Rev. Lett. **91**, 127203 (2003).