

# CONTEMPORARY MATHEMATICS

505

## Harmonic Analysis and Partial Differential Equations

8th International Conference on  
Harmonic Analysis and Partial Differential Equations  
June 16–20, 2008  
El Escorial, Madrid, Spain

Patricio Cifuentes  
José García-Cuerva  
Gustavo Garrigós  
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Javier Parcet  
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**American Mathematical Society**  
Providence, Rhode Island

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## Introduction

This volume contains the Proceedings of the 8th International Conference on Harmonic Analysis and Partial Differential Equations, which took place in El Escorial, Madrid (Spain) during the week of June 16-20, 2008.

The celebration of this El Escorial 2008 Conference continues a tradition initiated in 1979 thanks to the leadership and enthusiasm of Professor Miguel de Guzmán. The purpose of that first Conference was to bring together the best mathematicians in the field and let them show the progress in the area to a wide audience of senior and – more importantly – young researchers. The success of El Escorial 1979 Conference, which counted among the main speakers Alberto Calderón, Ronald Coifman, Yves Meyer and Peter Jones, inspired a group of people in the Department of Mathematics of the Universidad Autónoma de Madrid to continue to hold an International Conference with the same aim, (almost) every four years. From that first Conference of 1979, another seven have been held in 1983, 1987, 1992, 1996, 2000, 2004 and 2008. The El Escorial Conferences have kept growing in size and impact and are by now a very valuable fixed point on the mathematical calendar taking place every olympic year.

The format of these Conferences has always been essentially the same, and it has established a model that counts with general approval within the mathematical community and has been adopted by many conferences in different fields of Mathematics. Four outstanding mathematicians in the field of Harmonic Analysis and Partial Differential Equations are invited to teach one mini-course each. These mini-courses are intended to present in three or four one-hour sessions the state of the art in some topic of current interest, assuming minimal background from the audience and reaching the level of present-day research in such a way as to be useful for young researchers seeking to join teams doing high quality, original work. Another important part of the Conference consists of some 15 to 20 invited one-hour lectures, which can be of a more specialized nature. Finally, a few sessions of short talks are scheduled to provide an opportunity for those participants who want to present their latest results.

These Proceedings contain the written versions of two of the four mini-courses given this time at the Conference, namely, that of Steve Hofmann on “Local T(b) Theorems and Applications in Partial Differential Equations” and the survey of Carlos E. Kenig about “The global behavior of solutions to critical nonlinear dispersive and wave equations”. R. DeVore, who gave a mini-course at El Escorial 2008 on “The Mathematical Foundations of Compressed Sensing”, has chosen to present

in these Proceedings his paper on “Instance Optimal Decoding by Thresholding in Compressed Sensing”, written in collaboration with Albert Cohen and Wolfgang Dahmen. Also in these Proceedings one can find the contributions of most of the other invited speakers. The topics of these Contributed Lectures cover a wide range of areas within Harmonic Analysis and Partial Differential Equations and illustrate well the fruitful interplay between the two subfields.

The Proceedings of all the El Escorial Conferences have been published in different Mathematical journals of wide circulation. We consider the publication of the Proceedings an essential part of the Conference, the very final act and the starting point of the process to prepare the next El Escorial Conference. In this occasion, we want to thank the American Mathematical Society for its help in publishing the Proceedings of El Escorial 2008 in its “Contemporary Mathematics” series. It is also proper to thank the institutions that have helped financially with the organization of El Escorial 2008, namely, the Spanish Ministries of Education and Science, the Universidad Autónoma de Madrid, the Consejo Superior de Investigaciones Científicas, the project Consolider I-Math and the Real Sociedad Matemática Española. The next El Escorial Conference will be held in 2012.

The Organizing Committee  
Madrid, July 2009

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This volume contains the Proceedings of the 8th International Conference on Harmonic Analysis and Partial Differential Equations, held in El Escorial, Madrid, Spain, on June 16–20, 2008.

Featured in this book are papers by Steve Hoffmann and Carlos Kenig, which are based on two mini-courses given at the conference. These papers present topics of current interest, which assume minimal background from the reader, and represent state-of-the-art research in a useful way for young researchers. Other papers in this volume cover a range of fields in Harmonic Analysis and Partial Differential Equations and, in particular, illustrate well the fruitful interplay between these two fields.

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