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HARMONICS AND HOW THEY RELATE TO POWER FACTOR

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Abstract

We are all familiar with power factor, but are we using it to its true potential? In this paper we investigate the effect of harmonics on power factor and show through examples why it is important to use *true* power factor, rather than the conventional 50/60 Hz *displacement* power factor, when describing nonlinear loads.

Introduction

Voltage and current harmonics produced by nonlinear loads increase power losses and, therefore, have a negative impact on electric utility distribution systems and components. While the exact relationship between harmonics and losses is very complex and difficult to generalize, the well-established concept of power factor does provide some measure of the relationship, and it is useful when comparing the relative impacts of nonlinear loads–providing that harmonics are incorporated into the power factor definition.

Power Factor in Sinusoidal Situations

The concept of power factor originated from the need to quantify how efficiently a load utilizes the current that it draws from an AC power system. Consider, for example, the ideal sinusoidal situation shown in Figure 1.

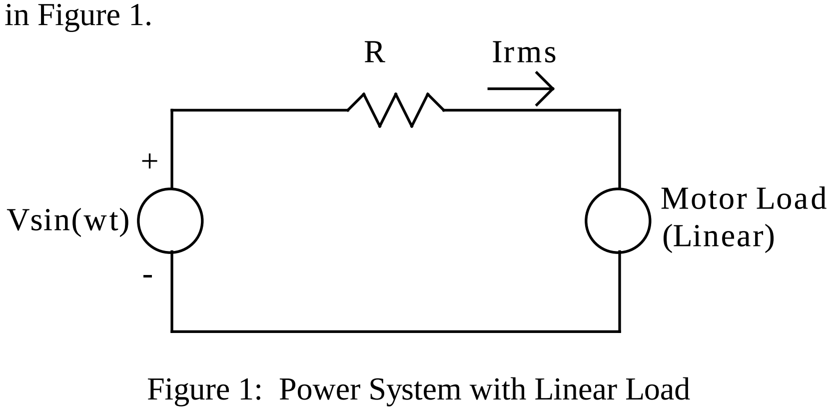

Figure 1: Power System with Linear Load

The voltage and current at the load are

$$v(t) = V_1 sin\ (\omega_0 t + \delta_1) , \quad (1)$$

$$i(t) = I_1 sin\ (\omega_0 t + \theta_1) , \quad (2)$$

where V_1 and I_1 are peak values of the 50/60 Hz voltage and current, and δ_1 and θ_1 are the relative phase angles. The true power factor at the load is defined as the ratio of average power to apparent power, or

$$pf_{true} = \frac{P_{avg}}{S} = \frac{P_{avg}}{V_{rms} I_{rms}} \ . \tag{3}$$

For the purely sinusoidal case, (3) becomes

$$pf_{true} = pf_{disp} = \frac{P_{avg}}{\sqrt{P^2 + Q^2}} = \frac{\frac{V_1}{\sqrt{2}}\frac{I_1}{\sqrt{2}}\cos(\delta_1 - \theta_1)}{\frac{V_1}{\sqrt{2}}\frac{I_1}{\sqrt{2}}} = \cos(\delta_1 - \theta_1) \ , \tag{4}$$

where pf_{disp} is commonly known as the displacement power factor, and where $(\delta_1 - \theta_1)$ is known as the power factor angle. Therefore, in sinusoidal situations, there is only one power factor because true power factor and displacement power factor are equal.

For sinusoidal situations, unity power factor corresponds to zero reactive power Q, and low power factors correspond to high Q. Since most loads consume reactive power, low power factors in sinusoidal systems can be corrected by simply adding shunt capacitors.

Sinusoidal Example

Consider again the case in Figure 1, where a motor is connected to a power system. The losses incurred while delivering the power to the motor are $I_{rms}^2 R$. Now, while holding motor active power P_{avg} and voltage V_{1rms} constant, we vary the displacement power factor of the motor. The variation in losses is shown in Figure 2, where we see that displacement power factor greatly affects losses.

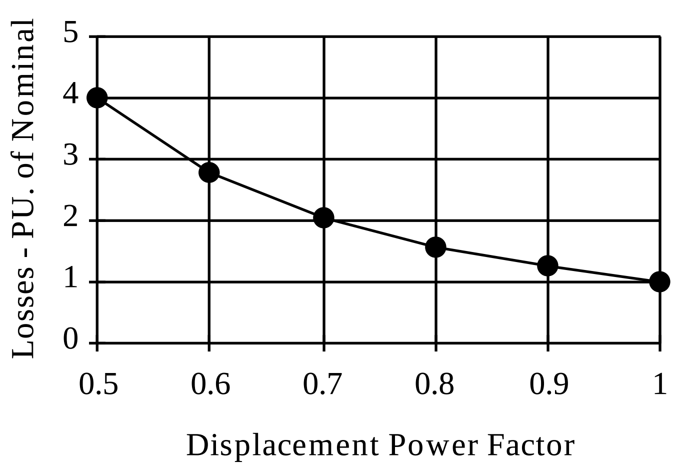

Figure 2: Effect of Displacement Power Factor on Power System Losses for Sinusoidal Example (Note: losses are expressed in per unit of nominal sinusoidal case where $pf_{true} = pf_{disp} = 1.0$)

Power Factor in Nonsinusoidal Situations

Now, consider nonsinusoidal situations, where network voltages and currents contain harmonics. While some harmonics are caused by system nonlinearities such as transformer saturation, most harmonics are produced by power electronic loads such as adjustable-speed drives and diode-bridge rectifiers. The significant harmonics (above the fundamental, i.e., the first harmonic) are usually the 3rd, 5th, and 7th multiples of 50/60 Hz, so that the frequencies of interest in harmonics studies are in the low-audible range.

When steady-state harmonics are present, voltages and currents may be represented by Fourier series of the form

$$v(t) = \sum_{k=1}^{\infty} V_k \sin\left(k\omega_o t + \delta_k\right), \tag{5}$$

$$i(t) = \sum_{k=1}^{\infty} I_k \sin\left(k\omega_o t + \theta_k\right), \tag{6}$$

whose rms values can be shown to be

$$V_{rms} = \sqrt{\sum_{k=1}^{\infty} \frac{V_k^2}{2}} = \sqrt{\sum_{k=1}^{\infty} V_{krms}^2}, \tag{7}$$

$$I_{rms} = \sqrt{\sum_{k=1}^{\infty} \frac{I_k^2}{2}} = \sqrt{\sum_{k=1}^{\infty} I_{krms}^2}. \tag{8}$$

The average power is given by

$$P_{avg} = \sum_{k=1}^{\infty} V_{krms} I_{krms} \cos\left(\delta_k - \theta_k\right) = P_{1avg} + P_{2avg} + P_{3avg} + \cdots, \tag{9}$$

where we see that each harmonic makes a contribution, plus or minus, to the average power.

A frequently-used measure of harmonic levels is total harmonic distortion (or distortion factor), which is the ratio of the rms value of the harmonics (above fundamental) to the rms value of the fundamental, times 100%, or

$$THD_V = \frac{\sqrt{\sum_{k=2}^{\infty} V_{krms}^2}}{V_{1rms}} \bullet 100\% = \frac{\sqrt{\sum_{k=2}^{\infty} V_k^2}}{V_1} \bullet 100\%, \tag{10}$$

$$THD_I = \frac{\sqrt{\sum_{k=2}^{\infty} I_{krms}^2}}{I_{1rms}} \bullet 100\% = \frac{\sqrt{\sum_{k=2}^{\infty} I_k^2}}{I_1} \bullet 100\% \ . \qquad (11)$$

Obviously, if no harmonics are present, then the *THD*s are zero. If we substitute (10) into (7), and (11) into (8), we find that

$$V_{rms} = V_{1rms}\sqrt{1+\left(THD_V\ /\ 100\right)^2}\ , \qquad (12)$$

$$I_{rms} = I_{1rms}\sqrt{1+\left(THD_I\ /\ 100\right)^2}\ . \qquad (13)$$

Now, substituting (12) and (13) into (3) yields the following exact form of true power factor, valid for both sinusoidal and nonsinusoidal situations:

$$pf_{true} = \frac{P_{avg}}{V_{1rms}I_{1rms}\sqrt{1+\left(THD_V\ /\ 100\right)^2}\ \sqrt{1+\left(THD_I\ /\ 100\right)^2}}\ . \qquad (14)$$

A useful simplification can be made by expressing (14) as a product of two components,

$$pf_{true} = \frac{P_{avg}}{V_{1rms}I_{1rms}} \bullet \frac{1}{\sqrt{1+\left(THD_V\ /\ 100\right)^2}\ \sqrt{1+\left(THD_I\ /\ 100\right)^2}}\ , \qquad (15)$$

and by making the following two assumptions:

1. In most cases, the contributions of harmonics above the fundamental to average power in (9) are small, so that $P_{avg} \approx P_{1avg}$.
2. Since THD_V is usually less than 10%, then from (12) we see that $V_{rms} \approx V_{1rms}$.

Incorporating these two assumptions into (15) yields the following approximate form for true power factor:

$$pf_{true} \approx \frac{P_{avg1}}{V_{1rms}I_{1rms}} \bullet \frac{1}{\sqrt{1+\left(THD_I\ /\ 100\right)^2}} = pf_{disp} \bullet pf_{dist}\ . \qquad (16)$$

Because displacement power factor pf_{disp} can never be greater than unity, (16) shows that the true power factor in nonsinusoidal situations has the upper bound

$$pf_{true} \le pf_{dist} = \frac{1}{\sqrt{1+\left(THD_I\ /\ 100\right)^2}}\ . \qquad (17)$$

Equation (17), which is plotted in Figure 3, provides insight into the nature of the true power factors of power electronic loads, especially single-phase loads. Single-phase power electronic loads such as desktop computers and home entertainment equipment tend to have high current distortions, near 100%. Therefore, their *true* power factors are generally less than 0.707, even though their *displacement* power factors are near unity.

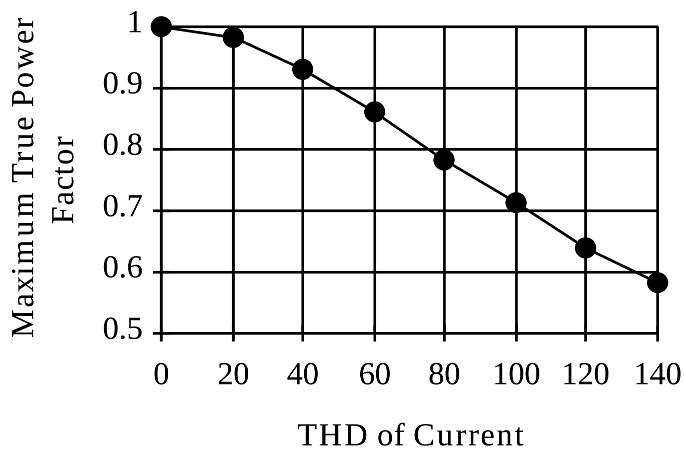

Figure 3: Maximum True Power Factor pf_{true} Versus THD_I

On the other hand, three-phase power electronic loads inherently have lower current distortions than single-phase loads and, thus, higher distortion power factors. However, if three-phase loads employ phase control, their true power factors may be poor at reduced load levels due to low displacement power factors.

It is important to point out that one cannot, in general, compensate for poor *distortion* power factor by adding shunt capacitors. Only the *displacement* power factor can be improved with capacitors. This fact is especially important in load areas that are dominated by single-phase power electronic loads, which tend to have high displacement power factors but low distortion power factors. In these instances, the addition of shunt capacitors will likely worsen the power factor by inducing resonances and higher harmonic levels. A better solution is to add passive or active filters to remove the harmonics produced by the nonlinear loads, or to utilize low-distortion power electronic loads.

Power factor measurements for some common single-phase residential loads are given in Table 1, where it is seen that their current distortion levels tend to fall into the following three categories: low ($THD_I \leq 20\%$), medium ($20\% < THD_I \leq 50\%$), high ($THD_I > 50\%$).

Table 1: Power Factor and Current Distortion Measurements for Common Single-Phase Residential Loads

Load Type	pf_{disp}	THD_I	pf_{dist}	pf_{true}
Ceiling Fan	0.999	1.8	1.000	0.999
Refrigerator	0.875	13.4	0.991	0.867
Microwave Oven	0.998	18.2	0.984	0.982
Vacuum Cleaner	0.951	26.0	0.968	0.921
Fluorescent Ceiling Lamp	0.956 *	39.5	0.930	0.889
Television	0.988 *	121.0	0.637	0.629
Desktop Computer and Printer	0.999 *	140.0	0.581	0.580

* Leading displacement power factor

Nonsinusoidal Example

Now, consider the situation shown in Figure 4, where the motor load of Figure 1 is replaced by a nonlinear load with the same P_{avg}.

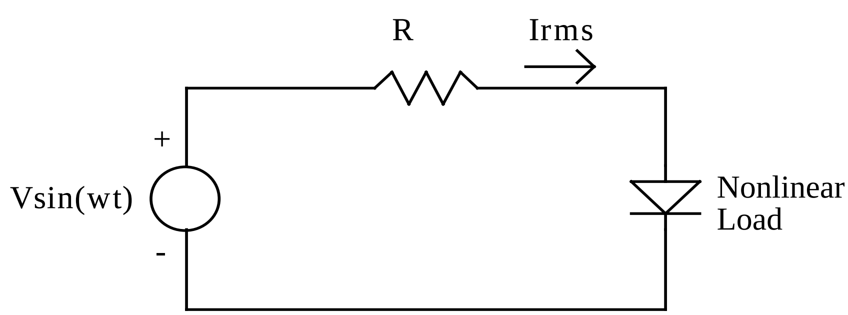

Figure 4: Power System with Nonlinear Load

Assuming that P_{avg} is constant, we vary the displacement power factor and compute the impact on system losses. The results are plotted in Figure 5, where it is seen that THD_I has a significant impact on system efficiency and that the efficiency is considerably less than in the sinusoidal case of Figure 2.

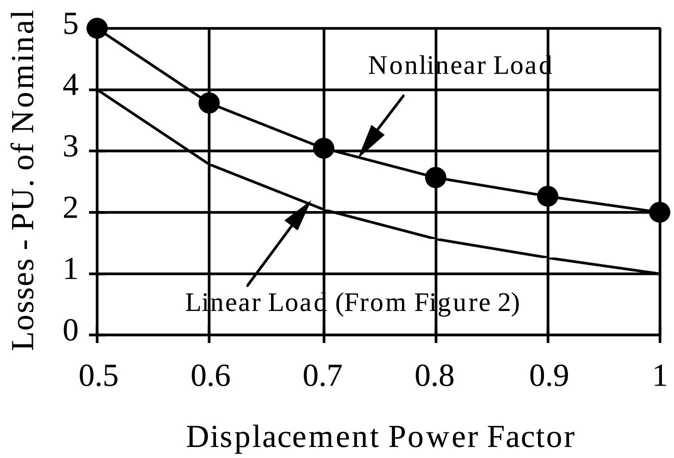

Figure 5: Effect of Displacement Power Factor on Power System Losses for Nonsinusoidal Example (Note: harmonic amperes held constant at the level corresponding to the following: $THD_I = 100\%$, $pf_{disp} = 1.0$. Losses are expressed in per unit of nominal sinusoidal case where $pf_{true} = 1.0$.)

Other Considerations

In the previous examples, we assumed that the resistance of the power system does not vary with frequency, so the losses are simply

$$P_{loss} = \sum_{k=1}^{\infty} I_{krms}^2 R_k = R \sum_{k=1}^{\infty} I_{krms}^2 = I_{rms}^2 R \quad . \tag{18}$$

In an actual system, however, resistance increases with frequency because of the resistive skin effect, so an ampere of harmonic current (above the fundamental) produces more loss than does an ampere of fundamental current. For typical wire sizes found in distribution systems, the resistance at the 25th harmonic may be 2 - 4 times greater than the 50/60 Hz resistance. Generally speaking, the larger the diameter of a wire, the greater the impact. This resistance increase is especially important in transformers, and it forms the basis upon which transformer derating calculations are made [1].

Another consideration is the affect of voltage harmonics on losses, which is even more complex than that of current. Studies by Fuchs, et al., [2] show that voltage harmonics can either increase or decrease losses in equipment, depending on their phase angles.

Because of the belief that harmonic voltages and currents should be weighted according to frequency, McEachern [3] proposed the following generalized harmonic-adjusted power factor definition:

$$hpf = \frac{P_{avg}}{\sqrt{\sum_{k=1}^{\infty} (C_k V_{krms})^2} \sqrt{\sum_{k=1}^{\infty} (D_k I_{krms})^2}} \quad . \tag{19}$$

He proposed several sets of C_k and D_k weighting coefficients, but there is not yet a consensus of opinion on which set is most appropriate.

Conclusions

Harmonics and power factor are closely related. In fact, they are so tightly coupled that one can place limitations on the current harmonics produced by nonlinear loads by using the widely-accepted concept of power factor, providing that *true* power factor is used rather than *displacement* power factor.

Equation (17) gives the limit on true power factor due to harmonic current distortion. Each THD_I corresponds to a maximum true power factor, so a limit on maximum true power factor automatically invokes a limitation on THD_I. Some examples are

Desired Limit on THD_I - %	Corresponding Limit on pf_{true}
20	0.981
50	0.894
100	0.707

Efforts are presently underway to develop new power factor definitions, such as harmonic-adjusted power factor, that take into account the frequency-dependent impacts of voltage and current harmonics.

In conclusion, even though power factor is an old and at first glance uninteresting concept, it is worthy of being "re-visited" because it has, in a relatively simple way, the potential of being very useful in limiting the harmonics produced by modern-day distorting loads.

Acknowledgments

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