Heat and Mass Transfer Effects on MHD Flow of Viscous Fluid through Non-Homogeneous Porous Medium in Presence of Temperature Dependent Heat Source

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Abstract. In this paper we have investigated the heat and mass transfer effects on MHD flow of viscous incompressible and electrically conducting fluid through a non-homogeneous porous medium in the presence of heat source, oscillatory suction velocity. A uniform transverse magnetic field is applied in the direction of the flow perpendicular to the plates. The equations governing the flow are solved by a simple perturbation technique the effects of various physical parameters viz., magnetic parameter M, modified Grashof number Gm., Schmidt number Sc etc., on primary and secondary velocity distributions. Temperature distribution, skin-friction and rate of heat transfer are discussed through graphs. It is observed that primary velocity increases with an increase in M, where as it shows reverse effect in case of the Gm.

Keywords: MHD, Heat and mass transfer, variable suction, non-homogeneous porous medium

Introduction

Simultaneous heat and mass transfer from different geometries embedded in porous medium has many Engineering and Geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. The science of magneto hydrodynamics (MHD) was concerned with geophysical and astrophysical problem for a number of years. In recent years, the possible use of MHD is to affect a flow stream of an electrically conducting fluid for the purpose of thermal protection, braking, propulsion and control. From the point of applications, model studies on the effect of magnetic field on the convection flow have been made by several investigators. From technological point of view, MHD convection flow problems are also very significant in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Model studies of the above phenomena of MHD convection have been made by many. Ahmed et al. [1] discussed three dimensional free convective flow and heat transfer through a porous medium.The flow is also affected by the difference in concentrations on material constitution. In most of the works, the level of concentration of foreign mass is assumed very low so that the soret effects can be neglected.

However, exceptions are observed therein. The Soret effects, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, H_e) and of medium molecular weight (N_2, air).Chamkha [2] studied Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Chen [3] has studied heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration. Gokhale et al. [4] studied effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux.Israel-Cookey et al.[5] discussed Influence of viscous dissipation on unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time-dependent suction. Kim [6] discussed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. In their study Raju et al. [7] considered unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature. Raptis et al. [8] discussed Magneto hydrodynamics free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Heat transfer effects on flow of viscous fluid through non homogeneous porous medium are studied by Singh et al. [9]. In this paper we made an attempt to study the heat and mass transfer effects on MHD flow of viscous fluid through non homogeneous porous medium.

Formulation of the Problem:

We have considered an oscillatory flow of an incompressible, electrically conducting, and viscous flow through a porous medium past an infinite isothermal, vertical porous plate with constant heat source. A uniform transfer magnetic field is applied perpendicular to the plates let (X,Y,Z) be the Cartesian co-ordinates system, and let us assume that x-axis and y-axis in the plane of the plate and z-axis is normal to the plate with velocity component (u, v, w) in (x, y, z) directions respectively. Both the liquid and the plate are considered in a state of rigid bodyrotation about z-axis with uniform angular velocity Ω. The constant heat source Q is assumed at z=0 and the suction velocity at the plate is $\omega = -\omega_0 (1+ \varepsilon e^{i\alpha t})$, where ω_0 is a real positive constant. In this analysis of the flow, it is assumed that there is no applied voltage which implies the absence of an electric field. It is also assumed that the magnetic Reynolds number is very small and hence the induced magnetic field in negligible in comparison to the applied magnetic field. Viscous dissipation and Joule heating terms are neglected as small velocity usually encountered in free convection flows. Under the above stated assumptions, the governing equations i.e., the momentum, energy conservation and Concentration equations, can be written in a Cartesian frame of reference, as follows.

$$\frac{\partial u^*}{\partial t^*} - \left(1 + \varepsilon e^{iw^*t^*}\right)\frac{\partial u^*}{\partial z^*} - 2\Omega v^* = v\frac{\partial^2 u^*}{\partial z^{*2}} + g\beta_T\left(T^* - T_\infty^*\right) + g\beta_c\left(C^* - C_\infty^*\right) - \frac{1}{k_0^*\left(1 + \varepsilon e^{iw^*t^*}\right)}u^* - \frac{\sigma B_0^2}{\rho}u^* \tag{1}$$

$$\frac{\partial v^*}{\partial t^*} - \left(1 + \varepsilon e^{iw^*t^*}\right)\frac{\partial v^*}{\partial z^*} + 2\Omega u^* = v\frac{\partial^2 u^*}{\partial z^{*2}} - \frac{1}{k_0^*\left(1 + \varepsilon e^{iw^*t^*}\right)}v^* - \frac{\sigma B_0^2}{\rho}v^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} - \left(1 + \varepsilon e^{iw^*t^*}\right)\frac{\partial T^*}{\partial z^*} = \frac{\alpha}{\rho C_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{q}{\rho C_p}\left(T^* - T_\infty^*\right) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} - \left(1 + \varepsilon e^{iw^*t^*}\right)\frac{\partial C^*}{\partial z^*} = D\frac{\partial^2 C^*}{\partial z^{*2}} \tag{4}$$

The boundary conditions of the problem are

$$u^* = 0, \ v^* = 0, \ T^* = T_w^* + \varepsilon\left(T_w^* - T_\infty^*\right)e^{iw^*t^*}, \ C^* = C_w^* + \varepsilon\left(C_w^* - C_\infty^*\right)e^{iw^*t^*} \ at \ z^* = 0$$

$$u^* \to 0, \ v^* \to 0, \ T^* \to 0, \ C^* \to 0 \ as \ z^* \to \infty \tag{5}$$

Now using the non-dimensional parameters and variables reported by Singh et al. [9] the governing equations in the non-dimensional form are given by

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon e^{iwt}\right)\frac{\partial u}{\partial z} - 2Ev = \frac{\partial^2 u}{\partial z^2} + Gr\theta + Gm\phi - \frac{1}{k_0\left(1 + \varepsilon e^{iwt}\right)}u - M^2 u \tag{6}$$

$$\frac{\partial v}{\partial t} - \left(1 + \varepsilon e^{iwt}\right)\frac{\partial v}{\partial z} + 2Eu = \frac{\partial^2 u}{\partial z^2} - \frac{1}{k_0\left(1 + \varepsilon e^{iwt}\right)}v - M^2 v \tag{7}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon\, e^{iwt}\right)\frac{\partial \theta}{\partial z} = \frac{1}{pr}\frac{\partial^2 \theta}{\partial z^2} - \frac{H}{pr}\theta \tag{8}$$

$$\frac{\partial \phi}{\partial t} - \left(1 + \varepsilon\, e^{iwt}\right)\frac{\partial \phi}{\partial z} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial z^2} \tag{9}$$

The corresponding boundary conditions in non-dimensional form are given by

$q=0$, $\quad$ $T=\left(1+\varepsilon e^{iwt}\right)$, $\quad$ $C=\left(1+\varepsilon e^{iwt}\right)$ $\quad$ at Z=0

$q\rightarrow 0$, $\quad$ $T\rightarrow 0$, $\quad\quad$ $C\rightarrow 0$ $\quad\quad$ at Z=∞ $\tag{10}$

Solution of the problem: In order to solve the equations we assume the velocity q (z, t) and Temperature T (z, t) and concentration C (z, t) as

$$\left.\begin{aligned}
q(z,t) &= q_0(z) + \varepsilon\, q_1(z)\, e^{iwt} \\
T(z,t) &= T_0(z) + \varepsilon\, T_1(z)\, e^{iwt} \\
c(z,t) &= c_0(z) + \varepsilon\, c_1(z)\, e^{iwt}
\end{aligned}\right\} \tag{11}$$

Using equation (11) into (6) to (9) we get the following set of equations

$$\frac{\partial^2 T_0}{\partial z^2} + p_r\frac{\partial T_0}{\partial z} - \alpha_0 T_0 = 0 \tag{12}$$

$$\frac{\partial^2 T_1}{\partial z^2} + p_r\frac{\partial T_1}{\partial z} - (iwp_r + \alpha_0)T_1 = -p_r\frac{\partial T_0}{\partial z} \tag{13}$$

$$\frac{\partial^2 c_0}{\partial z^2} + S_c\frac{\partial c_0}{\partial z} = 0 \tag{14}$$

$$\frac{\partial^2 c_1}{\partial z^2} + S_c\frac{\partial c_1}{\partial z} - iw\, S_c\, c_1(z) = -S_c\frac{\partial c_0}{\partial z} \tag{15}$$

$$\frac{\partial^2 q_0}{\partial z^2} + \frac{\partial q_0}{\partial z} - \left(2iE + \frac{1}{k_0} + M^2\right)q_0 = -G_r T_0(Z) - G_m c_0(z) \tag{16}$$

$$\frac{\partial^2 q_1}{\partial z^2} + \frac{\partial q_1}{\partial z} - \left(\frac{1}{k_0} + M^2 + i(w + 2E)\right)q_1 = -\frac{1}{k_0}q_0$$

$$- \frac{\partial q_0}{\partial z} - G_r T_1(z) - G_m c_1(z) \tag{17}$$

The corresponding boundary conditions are given by

$q_0 = q_1 = 0,\ T_0 = T_1 = 1,\quad c_0 = c_1 = 1\quad at\quad z = 0$

$q_0 = q_1 = 0,\quad T_0 = T_1 = 0,\quad c_0 = c_1 = 0\quad at\quad z = \infty$ $\tag{18}$

Solving the equations (12) to (17) under the above boundary conditions (18), we have

$$q_0 = k_2 e^{-m_8 z} + k_3 e^{-m_8 z} - k_2 e^{-m_2 z} - k_3 e^{-S_c z} \tag{19}$$

$$T_0 = e^{-m_2 z} \tag{20}$$

$$c_0 = e^{-Sc\,z} \tag{21}$$

$$q_1 = k_{18} e^{-m_{10} z} - k_4 e^{-m_8 z} - k_5 e^{-m_8 z} + k_6 e^{-m_2 z} + k_7 e^{-S_c z} + k_8 e^{-m_8 z} + k_9 e^{-m_8 z} - k_{10} e^{-m_2 z}$$
$$- k_{11} e^{-S_c z} - k_{12} e^{-m_4 z} + k_{13} e^{-m_4 z} - k_{14} e^{-m_2 z} - k_{15} e^{-m_6 z} + k_{16} e^{-m_6 z} - k_{17} e^{-S_c z} \tag{22}$$

$$T_1 = (1 - k_1) e^{-m_4 z} + k_1 e^{-m_2 z} \tag{23}$$

$$c_1 = e^{-m_6 z} + \frac{S_c}{iw} e^{-m_6 z} - \frac{S_c}{iw} e^{-S_c z} \tag{24}$$

Skin-friction: The Skin-friction (τ_p) due to primary velocity and skin-friction (τ_s) due to secondary velocity at the plate are obtained as follows:

$$\tau_p = \left(\frac{\partial u}{\partial z}\right)_{z=0} = N_{93} + \varepsilon(N_{94} \cos wt - N_{95} \sin wt) \tag{25}$$

$$\tau_s = \left(\frac{\partial v}{\partial z}\right)_{z=0} = N_{96} + \varepsilon(N_{94} \sin wt + N_{95} \cos wt) \tag{26}$$

Rate of Heat Transfer: The rate of heat transfer in terms of Nusselt number N_u is given by $N_u = \left(\frac{\partial T}{\partial z}\right)_{z=0} = -m_2 + \varepsilon(N_{97} \cos wt - N_{98} \sin wt) \tag{27}$

Rate of Mass Transfer: Mass Transfer coefficient (S_h) at the plate in terms of amplitude and phase is given by

$$S_h = \left(\frac{\partial c}{\partial z}\right)_{z=0} = -S_c + \varepsilon(N_{99} \cos wt - N_{100} \sin wt) \tag{28}$$

Results and discussion

The formulation of the effect of magnetic fields on the oscillatory flow of incompressible, viscous, electrically conducting fluid through a non-homogeneous porous medium fast an infinite isothermal vertical plate with constant heat source has been carried out in the preceding sections. This enables to carryout numerical computation for the velocity, Temperature and Concentration, also Skin-friction, Nusselt number and Sherwood number for various values of the material parameters. In this steady the boundary condition for z→∞ is replaced by z_{max} is a sufficiently large value of z, where the velocity profiles u and v can be approached to the relevant free stream velocity. A span wise step distance Δz of 0.001 is used with z_{max} =2.5.

Numerical evaluation of the analytical results is reported graphically in figures 1-4. These result are obtain to illustrate the influence of the magnetic parameter M, modified Grashof number Gm and Schmidt number Sc on the primary velocity, secondary velocity, Concentration, Skin-friction and Sherwood number profiles.
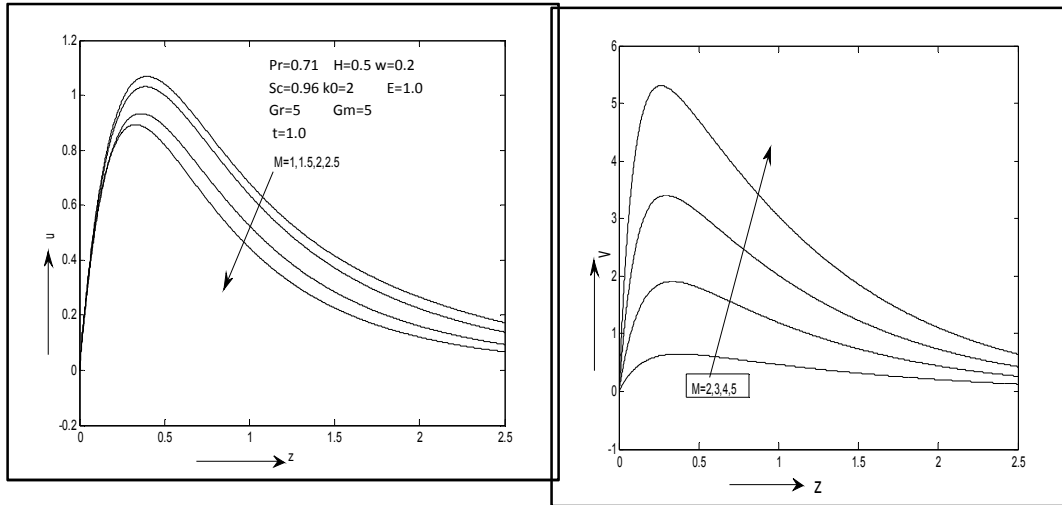

Fig.1. Primary and secondary Velocity profile against span wise coordinates z for different values of M
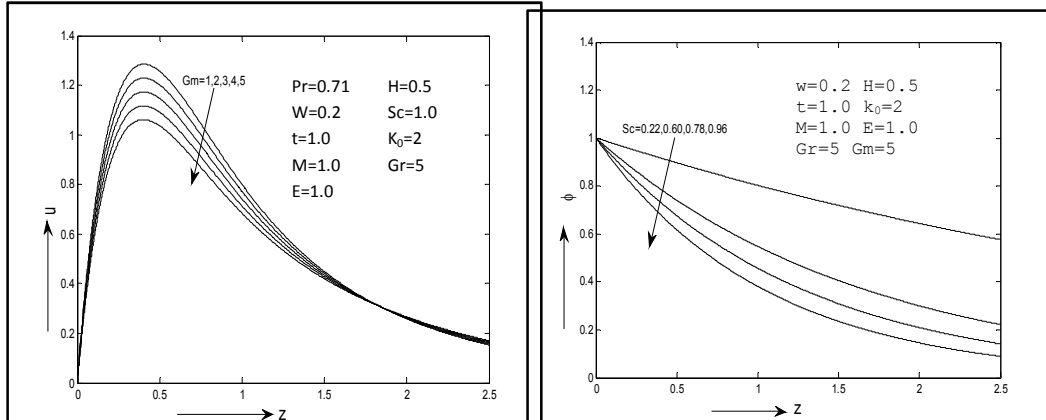

Fig.2. Primary Velocity profile against span wise coordinates z for different values of Gm and Concentration profile against span wise co-ordinates z for Schmidt number Sc.

The effects of the other physical parameter on the solutions were reported previously by Singh et al. [9] and therefore will not be reported herein. In order to assess the accuracy of our results, we have compared our results with data sets for the velocity,
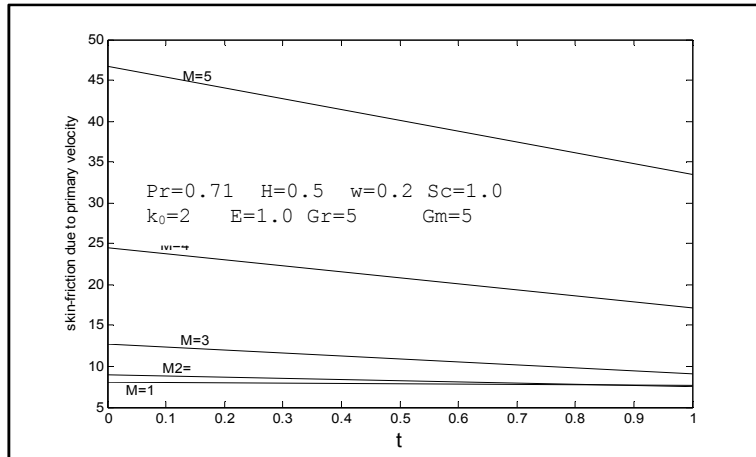
Fig.3. Coefficient of skin-friction profiles for various values of M.

temperature and skin friction profiles computed by Singh et al. [9]. The results of this comparison are found to be in very good agreement.Fig.1 illustrates typical velocity profiles against span wise distance in the boundary layer for different values of the magnetic parameter M. The velocity distribution attains a distinctive Maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. As expected the primary velocity decreases due to an increase in the magnetic parameter M and it shows the reverse effect in the case of secondary velocity.
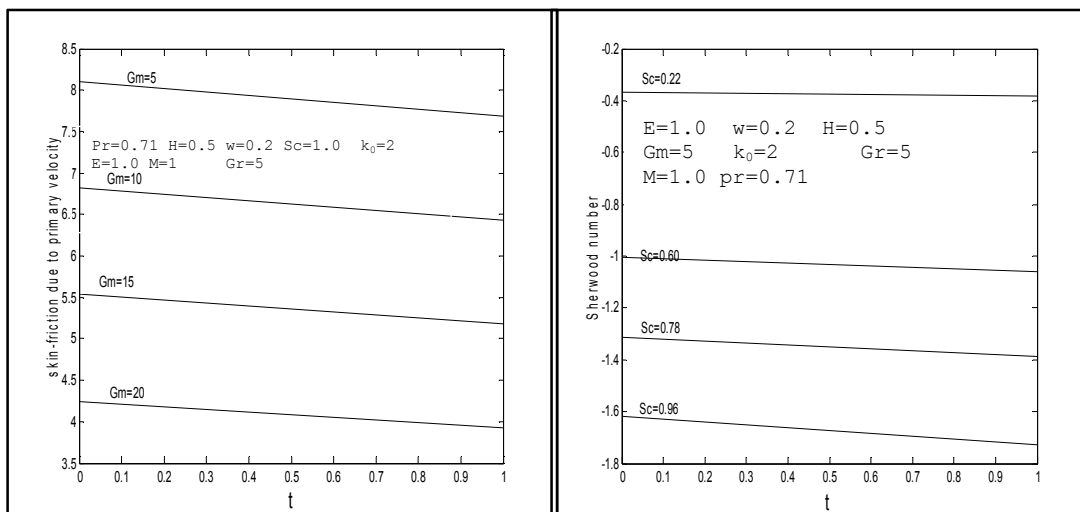

Fig.4. Coefficient of skin-friction profiles for various values of Gm and Sc

Fig. 2 disciples the effects of modified Grashof number Gm on the velocity and the concentration profile with span wise co-ordinate z for various values of Schmidt number Sc. As Gm increases, the velocity decreases and the effect of increasing values of Sc results a decrease in concentration.The effects of M, Gm and Sc are shown on the coefficient of skin-friction τ_p in figs 3-4, due to primary velocity.τ_p increases with an increase in M where as it shown reverse effects in case of Gm, interestingly the Sherwood number on the porous plate, decreases by increasing the strength of the Schmidt number Sc.

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