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Heat and mass transfer effects on moving vertical plate in the presence of thermal radiation

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Abstract

Thermal radiation effects on moving infinite vertical plate in the presence variable temperature and mass diffusion is considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature and the concentration level near the plate are raised linearly with time. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity and skin-friction are studied for different parameters like thermal Grashof number, mass Grashof number, time and radiation parameter. It is observed that the velocity slightly decreases with increasing value of the radiation parameter.

Keywords: gray, radiation, heat transfer, mass transfer, absorbing, emitting, temperature, velocity, skin-friction.

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Nomenclature

a^*	absorption coefficient
A	constant
C'	concentration
C	dimensionless concentration
C_p	specific heat at constant pressure
D	mass diffusion coefficient
g	accelaration due to gravity
Gr	thermal Grashof number
Gc	mass Grashof number
k	thermal conductivity of the fluid
R	Radiation parameter
Pr	Prandtl number
q_r	radiative heat flux in the y-direction
T	temperature of the fluid near the plate
t'	time
t	dimensionless time
u	velocity of the fluid in the x-direction
u_0	velocity of the plate
U	dimensionless velocity
y	coordinate axis normal to the plate
Y	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	density
σ	Stefan-Boltzmann constant
τ	dimensionless skin-friction
θ	dimensionless temperature
η	similarity parameter

Subscripts

w	conditions on the wall
∞	free stream conditions

1 Introduction

Radiative flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Soundalgekar and Takhar [1] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. For the same gas Takhar *et al* [2] investigated the effects of radiation on the MHD free convection flow past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar [3]. In all above studies, the stationary vertical plate is considered. Raptis and Perdikis [4] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al* [5] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique.

The unsteady flow past a moving infinite vertical plate in the presence of radiation with variable temperature and mass diffusion has not received any attention. It is proposed to study the effects of thermal radiation on flow past an impulsively started infinite vertical plate in the presence of variable temperature and mass diffusion. The governing equations are solved by the Laplace-transform technique. In this paper, the radiation parameter is introduced and the fluid is considered to be a gray, absorbing-emitting radiation but non-scattering medium. The effect of different parameters like the Grashof number, Prandtl number, time and radiation parameter are studied.

2 Analysis

Here the unsteady flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with variable temperature and mass diffusion is considered. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration in a stationary condition. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0, the plate temperature is made to rise linearly with time. Under the above assumptions, the flow governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$\rho\ C_p\ \frac{\partial T}{\partial t'} = k\ \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\ \frac{\partial^2 C'}{\partial y^2}. \tag{3}$$

The initial and boundary conditions are as follows:

$$\begin{array}{ll} t' \leq 0: & u' = 0, \quad T = T_\infty, \quad C' = C'_\infty \ \text{ for all } \ y; \\ t' > 0: & u = u_0, \quad T = T_\infty + (T_w - T_\infty)\ At', \\ & C' = C'_\infty + (C'_w - C'_\infty)At' \quad at \ \ y = 0; \\ & u = 0, \quad T \to T_\infty, \quad C' \to C'_\infty \ \text{ for all } \ y \to \infty; \end{array} \tag{4}$$

where $A = \dfrac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a\ \sigma\left(T_\infty^4 - T^4\right). \tag{5}$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4. \tag{6}$$

By using equations (5) and (6), equation (2) gives

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T). \tag{7}$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, \quad Pr = \frac{\rho \nu C_p}{k},$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \quad Gc = \frac{g\beta^*\nu(C'_w - C'_\infty)}{u_0^3},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Sc = \frac{\nu}{D} \tag{8}$$

in equations (1), (3) and (7), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2}, \tag{9}$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial Y^2} - R\theta, \tag{10}$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial Y^2}. \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$
\begin{array}{lllllll}
 & u=0, & \theta=0, & C=0 & \text{for all} & y,t\leq 0; & \\
t>0: & u=1, & \theta=t, & C=t & \text{at} & y=0; & \\
 & u=0, & \theta\to 0, & C\to 0 & \text{as} & y\to\infty. &
\end{array} \tag{12}
$$

The equations (8) and (9), subject to the boundary conditions (10), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$
\begin{aligned}
\theta = \frac{t}{2}\Big[&\exp(2\eta\sqrt{Rt})erfc(\eta\sqrt{Pr}+\sqrt{at})+ \\
&\exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{Pr}-\sqrt{at})\Big]- \\
\frac{\eta Pr\sqrt{t}}{2\sqrt{R}}\Big[&\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at})- \\
&\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at})\Big]
\end{aligned} \tag{13}
$$

$$
C = t\left[(1+2\eta^2 Sc)\ erfc(\eta\sqrt{Sc}) - 2\ \eta\sqrt{\frac{Sc}{\pi}}\ \exp(-\eta^2\ Sc)\right] \tag{14}
$$

$$
\begin{aligned}
U = &\left(1+\frac{Gr}{b^2(1-Pr)}\right) erfc(\eta)+ \\
&\frac{Gr\ t}{b(1-Pr)}\left[(1+2\eta^2)erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\ \exp(-\eta^2)\right] \\
&-\frac{Gr\ \exp(bt)}{2b^2(1-Pr)}\left[\exp(2\eta\sqrt{bt})erfc(\eta+\sqrt{bt})+\exp(-2\eta\sqrt{bt})erfc(\eta-\sqrt{bt})\right] \\
&-\frac{Gc\ t^2}{6(1-Sc)}\Big[(3+12\eta^2+4\eta^4)erfc(\eta)-\frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2) \\
&-(3+12\eta^2 Sc+4\eta^4 Sc^2)erfc(\eta\sqrt{Sc})+\frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10+4\eta^2 Sc)\exp(-\eta^2 Sc)\Big]
\end{aligned}
$$

$$-\frac{Gr(1+bt)}{2b^2(1-Pr)}\Big[\exp(2\eta\sqrt{a\ Pr\ t})erfc(\eta\sqrt{Pr}+\sqrt{at})$$

$$+\exp(-2\eta\sqrt{a\ Pr\ t})erfc(\eta\sqrt{Pr}-\sqrt{at})\Big]$$

$$+\frac{Gr\ \eta\ Pr\sqrt{t}}{2b(1-Pr)\sqrt{R}}\Big[\exp(-2\eta\sqrt{a\ Pr\ t})erfc(\eta\sqrt{Pr}-\sqrt{at})$$

$$-\exp(2\eta\sqrt{a\ Pr\ t})erfc(\eta\sqrt{Pr}-\sqrt{at})\Big]$$

$$+\frac{Gr\ \exp(bt)}{2b^2(1-Pr)}\Big[\exp(-2\eta\sqrt{Pr(a+b)t})erfc(\eta\sqrt{Pr}-\sqrt{(a+b)t})$$

$$+\exp(2\eta\sqrt{Pr(a+b)t})erfc(\eta\sqrt{Pr}+\sqrt{(a+b)t})\Big], \qquad (15)$$

where $\eta = Y/2\sqrt{t}$, $a = \dfrac{R}{Pr}$ and $b = \dfrac{R}{1-Pr}$.

The numerical values of the velocity and skin-friction are computed for different values of the thermal Grashof number, mass Grashof number,time and radiation parameter. The purpose of the calculations given here is to assess the effects of the parameters Gr, Gc, R and t upon the nature of the flow and transport.

The temperature profiles are calculated from Equation (13) and these are shown in table 1. for air (Pr=0.71) and $t = 0.2$. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. This shows that the heat transfer decreases in the presence of thermal radiation.

The velocity profiles for different values of the radiation parameter, $Gr = 2, Gc = 5, Sc = 0.6, Pr = 0.71$ and $t = 0.2$ are shown in table 2. It is observed that an velocity increases with decreasing values of the radiation parameter. As η increases then the velocity tends to zero.

In table 3, the effect of velocity for different thermal Grashof number, mass Grashof number, $R = 0.2, Sc = 0.6, Pr = 0.71$ and $t = 0.2$

are presented. As expected, the velocity increases with increasing the thermal Grashof number or mass Grashof number.

Knowing the velocity field, we now study the changes in the skin-friction. It is given by

$$\tau = -\left(\frac{dU}{dY}\right)_{Y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0}. \tag{16}$$

Hence, from the equations (15) and (16), the wall shear stress in the presence of thermal radiation field is given by

$$\tau = \frac{1}{\sqrt{\pi\ t}}\left[1 + \frac{Gr(1+2bt)}{b^2(1-Pr)} - \frac{Gr\ \exp(bt)}{b^2(1-Pr)}\left(1+\sqrt{b\pi t}\ erf(\sqrt{bt})\right)\right.$$

$$\left. -\frac{4}{3}\,\frac{Gc\ t^2}{1+\sqrt{Sc}} - \frac{Gr(1+bt)}{b^2(1-Pr)}\left(\sqrt{Pr}+\sqrt{a\pi\ t\ Pr}\ erf(\sqrt{at})\right)\right]$$

$$-\frac{Gr\ \exp(bt)}{b^2(1-Pr)}\left[\sqrt{Pr}+\sqrt{\pi(a+b)\ t\ Pr}\ erf(\sqrt{(a+b)t})\right]$$

$$-\frac{Gr\ Pr\sqrt{\pi t}}{2b(1-Pr)\sqrt{R}}\ erf(\sqrt{at})$$

$$-\frac{Gc(1+bt)}{b^2(1-Sc)}\left(\sqrt{Sc}+\sqrt{\pi\ t\ Sc\ R}\ erf(\sqrt{Rt})\right).$$

The numerical values of skin-friction τ are presented in table 4 for different thermal Grashof number, mass Grashof number, radiation parameter, Schmidt number, $Pr = 0.71$ and time. It is inferred from this table, skin-friction increases with decreasing values of the radiation parameter. This shows that the wall shear stress increases with decreasing radiation parameter. It is also observed that the skin-friction increases with increasing values of the Schmidt nmuber. It is interesting to note that the skin-friction decreases with increasing values of the time.

3 Conclusions

Thermal radiation effects on moving infinite vertical plate in the presence of variable temperature and mass diffusion is studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The temperature, velocity and skin-friction values are tabulated for different parameters like thermal Grashof number, mass Grashof number, Schmidt number, radiation parameter. It is observed that, the velocity as well as the temperature increases with decreasing values of the radiation parameter.

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Table 1. Temperature for different R

η	$R = 0.2$	$R = 2$	$R = 5$
0	0.2	0.2	0.2
0.25	0.1203	0.1121	0.1008
0.5	0.0687	0.0603	0.0493
0.75	0.0372	0.0310	0.0233
1	0.0190	0.0151	0.0106
1.25	0.0091	0.0070	0.0046
1.5	0.0041	0.0031	0.0019
1.75	0.0017	0.0013	0.0007
2	0.0007	0.0005	0.0003
2.25	0.0002	0.0002	0.0001
2.5	0.0000	0.0000	0.0000

Table 2. Velocity for different R

η	$R = 0.2$	$R = 0.5$	$R = 5$	$R = 10$
0	1	1	1	1
0.25	0.7679	0.7550	0.7537	0.7527
0.5	0.5114	0.5113	0.5096	0.5083
0.75	0.3118	0.3116	0.3103	0.3093
1	0.1711	0.1710	0.1701	0.1695
1.25	0.0844	0.0844	0.0839	0.0835
1.5	0.0374	0.0374	0.0371	0.0370
1.75	0.0149	0.0149	0.0148	0.0147
2	0.0053	0.0053	0.0053	0.0052
2.25	0.0017	0.0017	0.0017	0.0017

Table 3. Velocity for different Gr and Gc

η	$Gr = 2, Gc = 2$	$Gr = 2, Gc = 5$	$Gr = 5, Gc = 5$
0	1	1	1
0.25	0.7414	0.7679	0.8174
0.5	0.4973	0.5114	0.5201
0.75	0.3105	0.3118	0.3206
1	0.1649	0.1711	0.1762
1.25	0.0811	0.0844	0.0870
1.5	0.0358	0.0374	0.0386
1.75	0.0141	0.0149	0.0154
2	0.0050	0.0053	0.0055
2.25	0.0016	0.0017	0.0018

Table 4. Values of the skin-friction

R	t	Gr	Gc	Sc	τ
2	0.2	5	5	0.16	0.4046
2	0.2	5	5	0.6	0.4553
2	0.2	5	5	2.01	0.5057
2	0.2	2	5	0.6	0.8253
2	0.2	2	2	0.6	0.9390
5	0.2	5	5	0.6	0.3979
10	0.2	5	5	0.6	- 2.0206
2	0.4	5	5	0.6	- 1.3746

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Prenos toplote i mase na pokretnoj vertikalnoj ploči u prisustvu termičke radijacije

UDK 536.7

Razmatraju se efekti termičke radijacije na pokretnoj vertikalnoj ploči u prisustvu promenljive temperature i difuzije mase. Posmatrani fluid je siv, apsorbuje i emituje radijaciju ali ne poseduje raspršivanje. Temperatura ploče i nivo koncentracije blizu ploče rastu linearno sa vremenom. Bezdimenzijske jednačine problema su rešene tehnikom Laplasove transformacije. Brzina i trenje na zidu su proučeni za različite vrenosti parametara kao što su: termički Grashofov broj, maseni Grashofov broj, vreme i radijacioni parametar. Uočeno je da brzina slabo opada sa porastom radijacionog parametra.