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Heat Transfer in the Oscillating Turbulent Boundary Layer

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Measurements of local heat transfer coefficients in the fully established oscillating turbulent boundary layer over a flat plate are reported. In the range of frequencies from 0.1 to 200 cps and amplitudes from 8 to 92 percent of the freestream mean velocity increases in local Nusselt numbers of 3 to 5 percent were found. It is concluded that substantial increases in local coefficients sometimes reported in oscillating flows of low standing wave ratio may be traced to reduced transition Reynolds numbers.

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INTRODUCTION

Convective heat transfer in flow fields incorporating large scale velocity fluctuations has attracted increasing attention recently. This interest is of course motivated by the practical demands of engineering application. Such problems include the effects of unstable operation of rocket motors, gas turbine blade cooling, and the performance of conventional heat exchangers when introduced into a pulsating or vibratory environment.

Analysis of these problems is mainly confined to laminar flows (1-5)[1] for the usual reason, as we shall examine further, that turbulent analysis demands the introduction of considerable empirical information, as yet unavailable. Nevertheless some progress has been made in measurements of heat transfer in oscillating turbulent flows. Largely these measurements have been carried out in ducts in which strong standing wave systems have been established (i.e., resonant accoustic waves) (6-11).

Two investigations of turbulent heat transfer from flat plates immersed in oscillating flows are reported in the literature. Bayley et al. (12) report on measurements made from a small electrically heated flat plate mounted in a 4-in. pipe. Nusselt number increases of as much as 50 percent with pulsating flow are reported. Unfortunately, the range of Reynolds numbers investigated (10^5 to 5×10^5) corresponds to the usual transition regime in such flows. That the authors report steady flow transition as early as 1.8×10^4 is an indication of some flow irregularity or inadequacy of the transition detection method.

Feiler and Yeager (13) performed a similar experiment in the range of Reynolds numbers from 10^4 to 10^5. While the results obtained were similar to those reported in reference (12), apparently the authors recognized that alteration of the transition point constituted a possible mechanism reponsible for the increase in heat transfer rates observed. Subsequently, Feiler (14) altered the original apparatus and conducted a second experiment in which only a single frequency was investigated but the nature of the flow in both the laminar and turbulent regimes were determined from

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[1] Underlined numbers in parentheses designate References at the end of the paper.

NOMENCLATURE

C_p = specific heat (Btu/lb deg R)
h = local film coefficient (Btu/hr sq ft deg R)
k = thermal conductivity (Btu/hr ft deg R)
q = local rate of internal thermal generation (Btu/hr cu ft)
t = temperature (deg R)
T = mean value of temperature (deg R)
t' = fluctuating component of temperature (deg R)
U_i = i^{th} component of velocity (ft/hr)
$\overline{U}_i$ = mean value of the i^{th} component of velocity (ft/hr)
$U_i(N)$ = N^{th} Fourier coefficient of the i^{th} component of velocity, defined by equation (2) (ft/hr)
u'_i = i^{th} component of turbulent fluctuation (ft/hr)
X_i = i^{th} spatial coordinate (ft)
X = distance along the heat transfer surface (ft)
y = distance normal to the heat transfer surface (ft)
N_a = ratio of amplitude of oscillations to mean velocity, $u/\overline{U}$
Nu_x = local Nusselt number, hx/k
N_{Pr} = Prandtl number, $\mu C_p/k$
N_{Rex} = local Reynolds number, $\overline{U} x\rho/\mu$
N_ω = dimensionless frequency parameter, $\omega\mu/\rho\overline{U}$
α = thermal diffusivity, $k/\rho C_p$, (sq ft/hr)
δ = local boundary layer thickness (ft)
μ = coefficient of viscosity (lb/ft hr)
ρ = density (lb/cu ft)
τ = time (hr)
ω = frequency (1/hr)

direct hot-wire measurements of mean velocity profiles. In this case it was found that the superposition of fluctuations did not produce the substantial advances in heat transfer rates previously reported. More recently Miller and Fejer (15) have measured the effects of flow oscillations on transition and have demonstrated that large shifts in both transition Reynolds Number and transition length are produced by fluctuating velocity components.

The present investigation was undertaken as an extension of the work of Feiler (14) to determine the effects of frequency and amplitude of oscillating flows on the local heat transfer coefficient in the fully established turbulent boundary layer. Consideration is restricted to flows having a negligible standing wave ratio.

ANALYSIS

Postulating a velocity field of the following form:

$$U_i(x_i,\tau) = \bar{U}_i + R\left\{\sum_{N=1}^{\infty} u_i^{(N)}(x_i)\, e^{iN\omega\tau}\right\} + u_i'(x_i,\tau) \tag{1}$$

in which only $u' \neq 0$ for $i = 3$; we restrict ourselves to a two-dimensional mean motion upon which is superimposed a periodic motion expressible in a Fourier expansion and a random turbulent motion represented by u'_i. In general we allow $u_i^{(N)}$ to be complex thus imposing no restrictions on the phase relations of the periodic components. The complex amplitudes are defined by:

$$\begin{aligned} u_i^{(N)} &= 2\,\overline{U_i e^{-iN\omega\tau}} \\ &\equiv 2\lim_{M\to\infty}\frac{1}{M}\sum_{K=1}^{M}\frac{\omega}{2\pi}\int_0^{2\pi} U_i e^{-iN\omega\tau}\,d\tau \end{aligned} \tag{2}$$

in which the process of averaging is carried out over both time and the ensemble. In this way we ensure that the Fourier representation contains the entire periodic component of velocity.

In addition we assume a turbulent temperature field represented by:

$$t = T + t' \tag{3}$$

If the quantities defined in equations (1) to (3) are now introduced into the energy equation for an incompressible, nondissipative, constant property fluid:

$$\frac{\partial t}{\partial \tau} + U_i\frac{\partial t}{\partial x_i} = \alpha\frac{\partial}{\partial x_j}\left(\frac{\partial t}{\partial x_j}\right) + \frac{q}{\rho C_p} \tag{4}$$

and the resulting expression averaged in the conventional way, the following time-average equation results:

$$\begin{aligned} &\frac{\partial T}{\partial \tau} + U_i\frac{\partial T}{\partial x_i} + \overline{R\left\{\sum_{N=1}^{\infty} u_i^{(N)} e^{iN\omega\tau}\right\}\frac{\partial t'}{\partial x_i}} \\ &+ \overline{u_i'\frac{\partial t'}{\partial x_i}} = \alpha\frac{\partial}{\partial x_i}\left\{\frac{\partial T}{\partial x_i}\right\} + \overline{\frac{q}{\rho C_p}} \end{aligned} \tag{5}$$

If we compare equation (5) with the usual time-average turbulent transport equation[2] we find that the oscillations effect the energy equation in a manner similar to the turbulent convective term, through the addition of the third term on the left hand side. Unfortunately the equation is not determinant and indeed even if the nature of u'_i and $u_i^{(N)}$ were known, the nature of t' remains unknown. It becomes clear then that only experimentally can the contributions of the oscillating components be assessed. To this end extensive measurements of local heat transfer coefficients were made in the oscillating turbulent boundary layer over a flat plate at zero angle of attack.

APPARATUS

The flat plate model 2 ft wide and 4.6 ft long was formed of individually heated copper strips 1.5 in. wide separated by plastic thermal insulators. Each section was controlled by a Variac in the heater circuit and instrumented with thermocouples and calibrated heat flux gages of the thermopile type. Surface temperature was maintained at 100 F above the free stream temperature and insulation applied to the back side of the plate reduced unwanted thermal losses to less than 0.5 percent of the total heat flux. In order to ensure a fully established turbulent boundary layer, a conventional cylindrical boundary layer trip of 0.020 in. wire was fastened to the plate 4 in. upstream of the first instrumented strip. The model was aligned with the flow in order to produce a zero pressure gradient by means of static pressure taps in the surface. The experimental work was carried out in a low speed open circuit wind tunnel having a 2 ft-sq test section 18 ft long. A 16:1 contraction ratio coupled with three high-solidity screens produced a free stream turbulence intensity of 0.2 - 0.3 percent in the op-

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[2] See, for example, reference (16) equation (1-28), page 25.

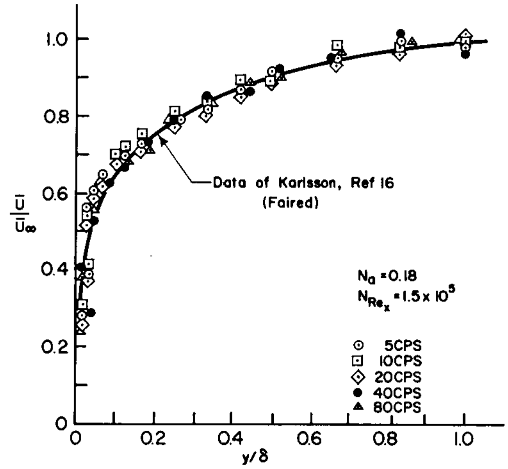

Fig.1 Mean velocity distribution in the oscillating turbulent boundary layer- 18 percent fluctuation amplitude

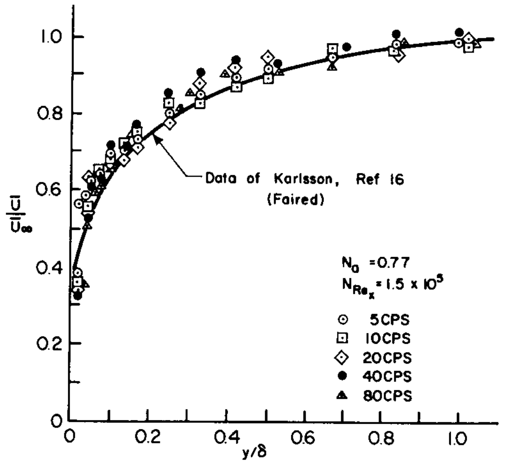

Fig.2 Mean velocity distribution in the oscillating turbulent boundary layer- 77 percent fluctuation amplitude

erating range of 10-225 fps.

Oscillations were produced by a rotating shutter valve similar to that reported in reference (15). With this device fluctuations of very nearly sinusoidal configuration may be superimposed on the freestream in the frequency range 0.1 - 250 cps. By altering the width of the blades employed fluctuation amplitudes of 8 - 92 percent of the freestream velocity may be produced.

Instantaneous velocity was measured with Security Associates constant temperature hot-wire anemometers having a linearized output. Hot-wire signals were monitored by both an oscilloscope and a Ballantine true R.M.S. meter. Mean velocity was also measured with a Prandtl probe and micromanometer.

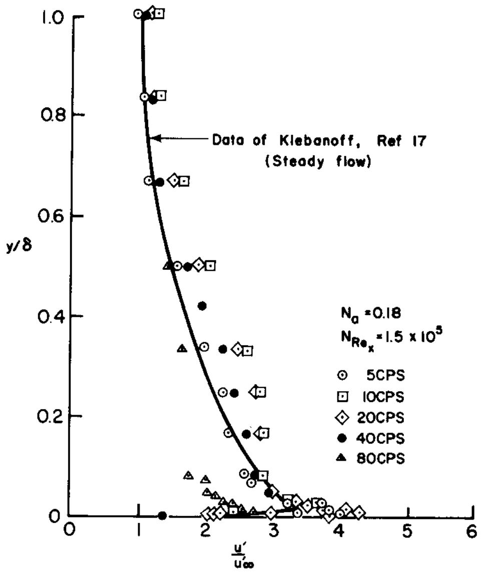

Fig.3 Distribution of turbulent intensity in the oscillating turbulent boundary layer- 18 percent fluctuation amplitude

CHARACTERISTICS OF THE FLOW

In order to ensure that the experimental apparatus and instrumentation would produce results consistent with previous work, extensive measurements of both steady and oscillating flow characteristics were made. In particular the velocity profiles and turbulent intensity distribution were measured at amplitudes of 18 and 77 percent and for frequencies in the range of 5 - 80 cps. The velocity profiles are shown in Figs.1 and 2. The results indicate excellent agreement with those of Karlsson (16) who investigated oscillating turbulent boundary layers produced in a very similar manner to that reported here. A somewhat surprising aspect of these results is that the mean turbulent profile is not appreciably altered by the superposition of the fluctuating component.

Measurements of the streamwise component of turbulent intensity for the same operating conditions discussed above are shown in Figs.3 and 4. Again the interesting aspect of these data is that comparison with the steady flow measurements of Klebanoff (17) indicates that the turbulent intensity structure is not materially altered by the superposition of the fluctuating flow. However, such results are consistent with the 4 percent increase in local skin friction reported by Karlsson (16).

Harmonic analysis of the hotwire traces indicated that the fluctuations were nearly sinusoidal in nature and could be represented to within 1 percent by the first four Fourier coefficients.

Transverse and longitudinal surveys with

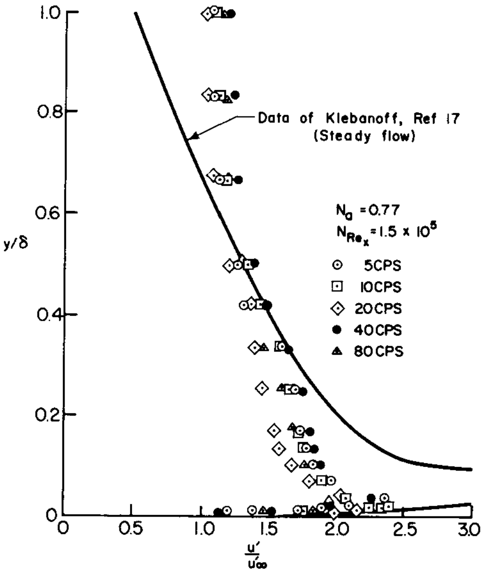

Fig.4 Distribution of turbulent intensity in the oscillating turbulent boundary layer- 77 percent fluctuation amplitude

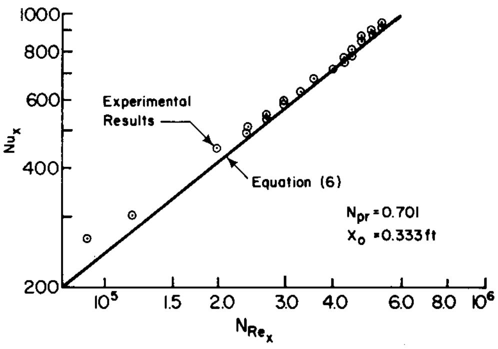

Fig.5 Heat transfer in the steady turbulent boundary layer

two identical hot-wires failed to indicate standing wave ratios over the heated flat plate in excess of 2 percent. Mean velocity profiles were found to be flat within 1 percent over the working section and typical wall boundary layer thicknesses were found to be less than 2 in. in the test section.

HEAT TRANSFER MEASUREMENTS

In order to provide a basis for comparison for the oscillating flow data a series of steady flow runs were made at a constant surface temperature 100 F in excess of the fluid free stream temperature. Data collected from these measurements are reported in Fig.5. It was found that it could be correlated within 5 percent by the well-known (18) semi-empirical correlation for turbulent heat transfer from an isothermal plate with an unheated starting length Xo:

$$Nu_x = \frac{0.0295\, N_{Re_x}^{0.8}\, N_{Pr}^{0.6}}{\left[1 - (X_o/x)^{0.9}\right]^{1/9}} \tag{6}$$

Oscillating flow data were subsequently collected for oscillation amplitudes extending from 11.5 percent to 71.5 percent of the freestream mean velocity. Frequencies from 3 to 200 cps were investigated. In each case the plate was maintained at an isothermal level of 100 F in excess of the fluid freestream temperature. These results are plotted in Figs.6 to 12 in terms of the measured local Nusselt number, $(nux)_{osc}$, normalized with the measured value of the local Nusselt number, $(Nux)_{stdy}$, in steady flow corresponding to the same mean Reynolds number. The data would seem to indicate a very slight increase in local heat transfer coefficient due to the superposition of oscillations on the flow field; perhaps 3 to 5 percent. These results show excellent quantitative agreement, by analogy, with those of Karlsson (16) who measured a 4 percent increase in local skin friction for a similar flow.

In order to investigate Reynolds number effects the data taken at 100 cps were cross-plotted against Reynolds number in Fig.13. These results indicate a slight trend toward an augmentation of heat transfer with increasing Reynolds number. They are also in excellent quantitative agreement with the 100 cps results of Feiler (14) who reported results correlated by:

$$Nu_x = \frac{0.0296\, N_{Re_x}^{0.8}\, N_{Pr}^{0.4}}{\left[1 - (X_o/x)^{\frac{39}{40}}\right]^{7/39}} \tag{7}$$

which differs from equation (6) only very slightly and then only in the effects of the starting length, X_o.

CONCLUDING REMARKS

The present work clearly indicates that in the absence of a strong standing wave structure that the effect on local turbulent heat transfer of the superposition of large scale flow oscillations is less than 5 percent. Since it has been demonstrated (3, 13, 14) that similar conclusions may be drawn in the laminar regime, and that reduction of transition Reynolds numbers is a consequence of flow oscillations (15) one concludes that the increased heat transfer results occasionally reported (13) are a result of early transition and not of alteration of the local heat transfer coefficient.

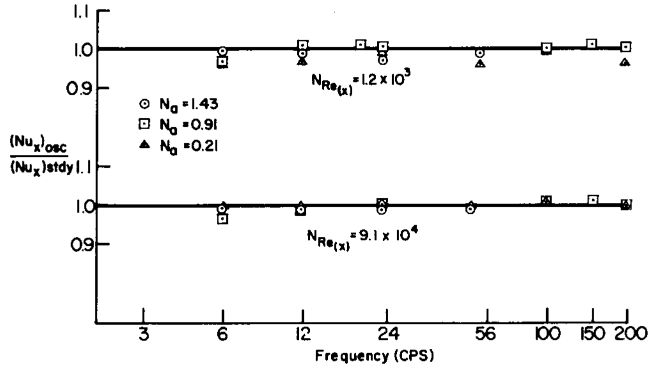

Fig.6 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

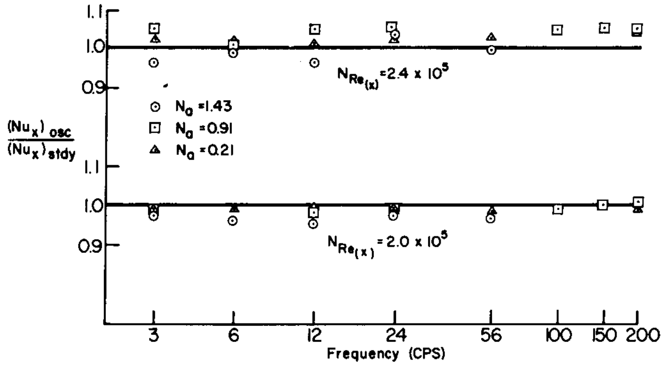

Fig.7 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

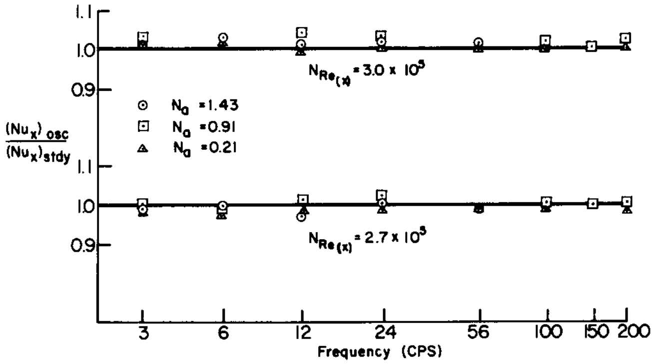

Fig.8 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

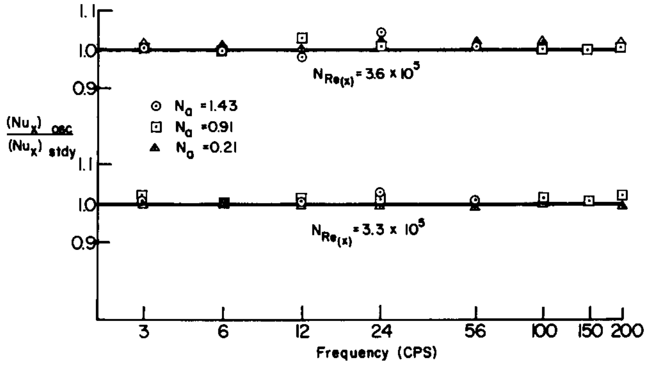

Fig.9 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

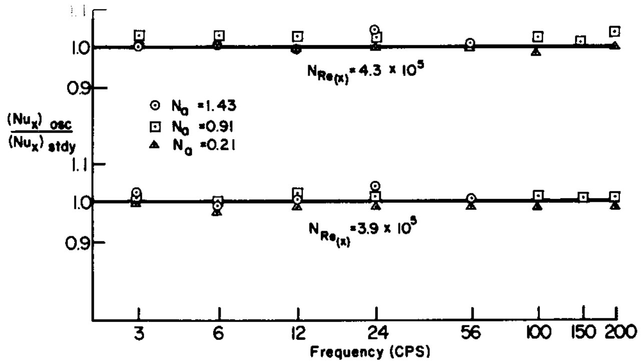

Fig.10 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

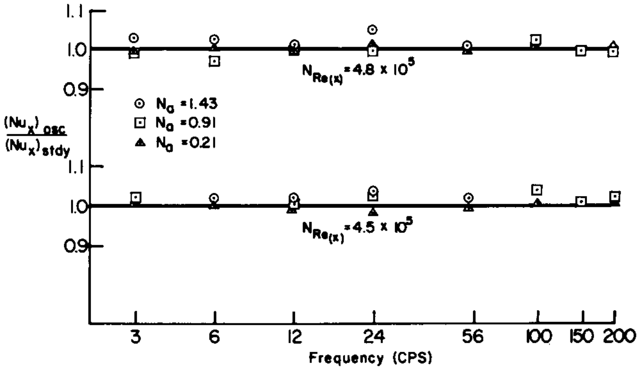

Fig.11 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

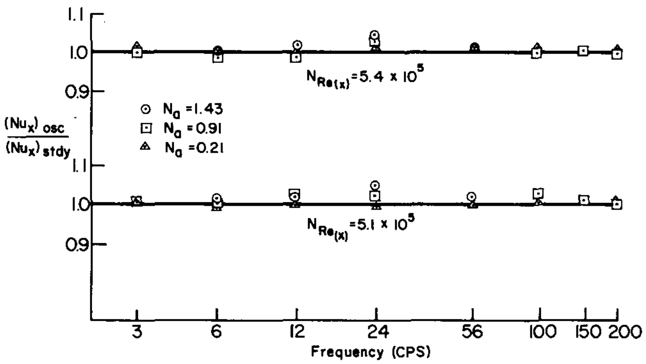

Fig.12 Effect of frequency and amplitude of flow oscillations on heat transfer through the turbulent boundary layer

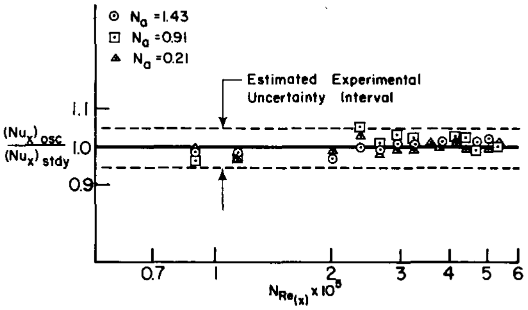

Fig.13 Effect of Reynolds Number and amplitude of flow oscillations on heat transfer through the turbulent boundary layer at 1000 cps

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