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Heat Transfer in the Steady Flow of a Second-Order Fluid between Two Enclosed Discs Rotating With Different Angular Velocities

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Abstract: *The problem of heat transfer in the steady flow of an incompressible second-order fluid between two enclosed discs rotating with different angular velocities has been discussed. The effect of elastico-viscosity, cross-viscosity parameters and ratio of the angular velocity on the temperature profile and Nusselt number for the cases of radial outflow and inflow have been investigated in the regions of no-recirculation and recirculation.*
Key words : *Heat transfer, steady flow, second-order fluid, two enclosed discs, rotating with different angular velocities.*

I. Introduction

The phenomenon of flow and heat transfer of the fluid between two enclosed rotating discs (enclosed in a cylindrical casing) or shrouded discs has important engineering application as its generalization could be helpful in the study of heat transfer analysis of air cooling of turbine discs[1] and the determination of oil film temperature of pedestal bearing with central feeding of lubricant[2]. Soo et al[3] has investigated the nature of heat transfer from an enclosed rotating disc for viscous flow. Sharma and Agarwal[4] reconsidered this problem using an improved formulation suggested by Sharma[5]. Sharma and Singh[6] have studied the problem of heat transfer in the flow of a second-order fluid between two enclosed rotating discs.

In the present investigations we are concerned with the heat transfer in the steady incompressible flow of a second-order fluid between two enclosed discs rotating with different angular velocities in the following two cases:

(i) When the discs rotate in the same sense with equal angular velocity.
(ii) When both discs rotate with different angular velocities.

II. Formulation Of The Problem

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll[7] can be written as

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \qquad \ldots(1)$$

where

$$d_{ij} = \frac{1}{2}(u_{i,\,j} + u_{j,\,i}),$$

$$e_{ij} = \frac{1}{2}(a_{i,\,j} + a_{j,\,i}) + u^m{}_{,\,i}\,u_{m,\,j}$$

$$c_{ij} = d_i^m d_{mj} \qquad \ldots(2)$$

where p is the hydrostatic pressure, τ_{ij} is the stress tensor, u_i and a_i are velocity and acceleration vector.

The equation (1) together with the momentum equation for no extraneous force

$$\rho\left(\frac{\partial u_i}{\partial t} + u^m u_{i,\,m}\right) = \tau^m_{i,\,m} \qquad \ldots(3)$$

and the equation of continuity for incompressible fluid

$$u^i_{,i} = 0 \qquad \ldots(4)$$

where ρ is the density of the fluid and (,) represents covariant differentiation, forms the set of governing equations. The material constants $\mu_1,\ \mu_2$ and μ_3 are known as coefficients of Newtonian-viscosity, elastico-viscosity and cross-viscosity respectively, whose values are given by $\mu_1 = 18.5,\ \mu_2 = -0.2$ and $\mu_3 = 1.0$ (all expressed in CGS units for 5.46% solutions of poly-isobutylene in cetane at $30°C$ as suggested by Markowitz[8])

In a three dimensional cylindrical set of co-ordinate $(r,\ \theta,\ z)$, the system consists of two finite discs of radius r_s (coinciding with the plane $z = 0$ and $z = z_0 < r_s$) rotating with different velocities. The lower disc is rotating with angular velocity Ω whenever the upper disc is rotating with different angular velocity $N\Omega$ in an incompressible second-order fluid forming the part of a co-axial cylindrical casing. The symmetrical radial steady out flow has a small mass rate 'm' for radial outflow and '$-m$' for radial inflow. The inlet conditions is taken as a simple radial source flow along the z-axis starting from radius r_0. The lower and upper discs are maintained at constant temperature T_a and T_b respectively.

The energy equation describing the transport of thermal energy is

$$\rho c_v \frac{DT}{Dt} = k\nabla^2 T + \Phi \qquad \ldots(5)$$

where $c_v,\ k$ and ρ are the specific heat, thermal conductivity and density of fluid; T is the temperature and Φ is the viscous dissipations. In equation (5), the first term represents the contribution of convection, the second term is due to viscous dissipation which is given by

$$\Phi = \tilde{\tau}^i_j d^j_i, \qquad \ldots(6)$$

where $\tilde{\tau}^i_j$ is the deviatoric stress tensor and the dissipation function Φ indicates the energy which is dissipated into heat due to friction in the fluid. The boundary conditions on the velocity profile are:

$$z = 0 \ : \quad u = 0, \quad v = r\Omega, \quad w = 0, \quad T = T_a$$
$$z = z_0 \ : \quad u = 0, \quad v = Nr\Omega, \quad w = 0, \quad T = T_b \qquad \ldots(7)$$

where $(u,\ v,\ w)$ are the velocity components along the cylindrical system of axis $(r,\ \theta,\ z)$. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as[6].

$$U = -\xi H'(\zeta) + \xi^{-1}\left(\frac{R_m}{R_z}\right) M'(\zeta)$$
$$V = \xi G(\zeta) + \xi^{-1}\left(\frac{R_l}{R_z}\right) L(\zeta)$$
$$W = 2H(\zeta) \qquad \ldots(8)$$

where $U = \dfrac{u}{\Omega z_0}, V = \dfrac{v}{\Omega z_0}, W = \dfrac{w}{\Omega z_0},\ \ T^* = \left(\dfrac{c_v}{\nu_1 \Omega}\right) T,\ \Phi^* = \dfrac{\Phi}{\mu_1 \Omega^2}, \xi = \dfrac{r}{z_0},$ $\zeta = \dfrac{z}{z_0}$ are dimensionless quantities and $H(\zeta),\ G(\zeta),\ L(\zeta),\ M'(\zeta)$ are dimensionless functions of the dimensionless variable ζ. $R_m\left(= \dfrac{m}{(2\pi\rho z_0 \nu_1)}\right)$, $R_l\left(= \dfrac{l}{(2\pi\rho z_0 \nu_1)}\right)$ are dimensionless numbers to be called Reynolds number of net radial outflow and circulatory flow respectively (R_m is negative for a net radial inflow), $R_z\left(=\frac{\Omega z_0^2}{\nu_1}\right)$ be the flow Reynold's number. The small mass rate 'm' of the radial outflow is represented by

$$m = 2\pi\rho\int_0^{z_0} ru\,dz \qquad \ldots(9)$$

Using expression (8), the boundary conditions transform for G, L and H into the following form:

$$\zeta = 0 \quad : \quad G = 1, \quad L = 0, \quad H = 0, \quad H' = 0$$
$$\zeta = 1 \quad : \quad G = N, \quad L = 0, \quad H = 0, \quad H' = 0 \qquad \ldots(10)$$

The condition on M on the boundaries are obtainable from the equation (9) for m as follows

$$M(1) - M(0) = 1, \qquad \ldots(11)$$

which on choosing the discs as streamlines reduces to:

$$M(1) = 1 \qquad M(0) = 0 \qquad \ldots(12)$$

Using equation (1) and expression (8) in equation (3) and neglecting the squares and higher powers of $\frac{R_m}{R_z}$ (assumed small) the dimensionless equation of motion for transverse component of the velocity is-

$$0 = -2\Omega^2 z_0\xi(HG' - H'G) - \Omega^2 z_0\left(\frac{R_m}{R_z}\right)\left(\frac{2M'G}{\xi}\right) - \Omega^2 z_0\left(\frac{R_l}{R_z}\right)\left(\frac{2HL'}{\xi}\right)$$
$$+ \nu_1\frac{\Omega}{z_0}\left\{\xi G'' + \left(\frac{R_l}{R_z}\right)\left(\frac{L''}{\xi}\right)\right\} + \left(\frac{2\nu_2}{z_0}\right)\left[\left(\frac{R_l}{R_z}\right)\left(\frac{\Omega^2}{\xi}\right)(H''L' + H'''L\right.$$
$$+ HL''' + H'L'') + (\Omega^2\xi)(HG''' - H''G') + \left(\frac{R_m}{R_z}\right)\left(\frac{2\Omega^2}{\xi}\right)(M'G''$$
$$\left. + M''G')\right] + \left(\frac{2\nu_3\Omega^2}{z_0}\right)\left\{\xi(H'G'' - H''G') + \left(\frac{R_l}{R_z}\right)\left(\frac{1}{\xi}\right)(H''L' + H'''L\right.$$
$$\left. + H'L'') + \left(\frac{R_m}{R_z}\right)\left(\frac{1}{\xi}\right)(2M''G' + M'G'')\right\} \qquad \ldots(13)$$

After eliminating the pressure p from the equation of motion of radial and axial component we get–

$$-2\Omega^2 z_0\xi(HH''' + GG') + \left(\frac{R_m}{R_z}\right)\left(\frac{2\Omega^2 z_0}{\xi}\right)(H'M'' + HM''') - \left(\frac{R_l}{R_z}\right)\times$$
$$\left(\frac{2\Omega^2 z_0}{\xi}\right)(LG' + L'G) - \left(\frac{\nu_1\Omega}{z_0}\right)\left\{\left(\frac{R_m}{R_z}\right)\left(\frac{M^{iv}}{\xi}\right) - \xi H^{iv}\right\} - \left(\frac{2\nu_2}{z_0}\right)\times$$
$$\left[-\Omega^2\xi(2H''H''' + H'H^{iv} + HH^{v} + 4G'G'') + \left(\frac{R_m}{R_z}\right)\left(\frac{\Omega^2}{\xi}\right)(2H'''M''\right.$$
$$+ H^{iv}M' + 2H''M''' + 2H'M^{iv} + HM^{v}) - \left(\frac{R_l}{R_z}\right)\left(\frac{2\Omega^2}{\xi}\right)(2L'G''$$

$$+ L''G' + LG''')\Bigg] - \left(\frac{2\nu_3\Omega^2}{z_0}\right)\left\{\left(\frac{R_m}{R_z}\right)\left(\frac{1}{\xi}\right)(H^{iv}M' + 2H'''M''\right.$$

$$+ 2H''M''' + H'M^{iv}) - \left(\frac{R_l}{R_z}\right)\left(\frac{1}{\xi}\right)(3L'G'' + 2L''G' + LG''')$$

$$\left. -\xi(H'H^{iv} + 3G'G'' + 2H''H''')\right\} = 0 \qquad \ldots(14)$$

The energy equation (5) take the dimensionless form as:

$$U\frac{\partial T^*}{\partial \xi} + W\frac{\partial T^*}{\partial \zeta} = \frac{1}{PR_z}\left(\frac{\partial T^*}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial T^*}{\partial \xi} + \frac{\partial^2 T^*}{\partial \zeta^2}\right) + \Phi^* \qquad \ldots(15)$$

where $P\left(=\frac{\mu_1 c_v}{k}\right)$ is the prandtl number.

III. Solution Of The Problem

On equating the coefficients of ξ and $\frac{1}{\xi}$ from the equations (13) and (14) and assuming $R_m \sim R_z$ and expanding the function H, G, L, M in the ascending powers of R_z (assumed small such that R_z^3 and higher power of R_z are neglected) as:

$$G(\zeta) = G_0(\zeta) + R_z G_1(\zeta) + R_z^2 G_2(\zeta)$$
$$L(\zeta) = L_0(\zeta) + R_z L_1(\zeta) + R_z^2 L_2(\zeta)$$
$$H(\zeta) = H_0(\zeta) + R_z H_1(\zeta) + R_z^2 H_2(\zeta)$$
$$M(\zeta) = M_0(\zeta) + R_z M_1(\zeta) + R_z^2 M_2(\zeta) \qquad \ldots(16)$$

We get the set of differential equations in terms of $G_0, G_1, G_2; L_0, L_1, L_2; H_0, H_1, H_2; M_0, M_1, M_2$ and their derivatives which when integrated subject to the boundary conditions:

$$G_0(0) = 1,\ G_1(0) = 0,\ G_2(0) = 0 \qquad G_0(1) = N,\ G_1(1) = 0,\ G_2(1) = 0$$
$$H_0(0) = 0,\ H_1(0) = 0,\ H_2(0) = 0 \qquad H_0(1) = 0,\ H_1(1) = 0,\ H_2(1) = 0$$
$$H_0'(0) = 0,\ H_1'(0) = 0,\ H_2'(0) = 0 \qquad H_0'(1) = 0,\ H_1'(1) = 0,\ H_2'(1) = 0$$
$$L_0(0) = 0,\ L_1(0) = 0,\ L_2(0) = 0 \qquad L_0(1) = 0,\ L_1(1) = 0,\ L_2(1) = 0$$
$$M_0(0) = 0,\ M_1(0) = 0,\ M_2(0) = 0 \qquad M_0(1) = 0,\ M_1(1) = 0,\ M_2(1) = 0$$
$$M_0'(0) = 0,\ M_1'(0) = 0,\ M_2'(0) = 0 \qquad M_0'(1) = 0,\ M_1'(1) = 0,\ M_2'(1) = 0$$
$$\ldots(17)$$

Gives the values of $G_0, G_1, G_2; H_0, H_1, H_2; L_0, L_1, L_2; M_0, M_1, M_2$.

Hence we get the values of the functions H, G, L, M calculated under the assumption of smallness of $\left(\frac{R_m}{R_z}\right)$ and equivalence of the order of R_m and R_l correct to the square of flow Reynolds number R_z, and first power of $\left(\frac{R_m}{R_z}\right)$ are:

$$H = \left(\frac{R_z}{60}\right)\Big[\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2 + N^2(\zeta^5 - 3\zeta^3 + 2\zeta^2)$$

$$-N(2\zeta^5 - 5\zeta^4 + 4\zeta^3 - \zeta^2)\Big]$$

$$G = 1 + (N-1)\zeta + \left(\frac{R_z^2}{6300}\right)\Big[20\zeta^7 - 140\zeta^6 + 357\zeta^5 - 420\zeta^4 + 210\zeta^3 - 27\zeta$$

$$-N^3(20\zeta^7 - 63\zeta^5 + 35\zeta^4 + 8\zeta) - N(60\zeta^7 - 280\zeta^6 + 441\zeta^5 - 280\zeta^4$$

$$+ 70\zeta^3 - 11\zeta) + N^2(60\zeta^7 - 140\zeta^6 + 21\zeta^5 + 175\zeta^4 - 140\zeta^3 + 24\zeta)\Big]$$

$$+\left(\frac{R_z^2}{30}\right)(\tau_1 + \tau_2)\Big[N^3(\zeta^5 - 3\zeta^3 + 2\zeta^2) - (\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2)$$

$$+N(3\zeta^5 - 10\zeta^4 + 11\zeta^3 - 4\zeta^2) - N^2(3\zeta^5 - 5\zeta^4 + \zeta^3 + \zeta^2)\Big]$$

$$L = \left(\frac{R_m}{R_l}\right)R_z\left[\frac{1}{5}\Big\{(3\zeta^5 - 10\zeta^4 + 10\zeta^3 - 3\zeta) - N(3\zeta^5 - 5\zeta^4 + 2\zeta)\right.$$

$$\left. + 20(\tau_1 + \tau_2)(N-1)(2\zeta^3 - 3\zeta^2 + \zeta)\Big\}\right]$$

$$M' = 6(\zeta - \zeta^2) + \left(\frac{R_z^2}{4200}\right)\Big[60\zeta^8 - 300\zeta^7 + 588\zeta^6 - 294\zeta^5 - 630\zeta^4 + 840\zeta^3$$

$$-263\zeta^2 - \zeta + N^2(60\zeta^8 - 180\zeta^7 + 168\zeta^6 - 294\zeta^5 + 420\zeta^4 - 263\zeta^2$$

$$+89\zeta) - 2N(60\zeta^8 - 240\zeta^7 + 238\zeta^6 + 126\zeta^5 - 105\zeta^4 - 280\zeta^3 + 247\zeta^2$$

$$-46\zeta)\Big] - \left(\frac{R_z^2}{525}\right)(\tau_1 + \tau_2)\Big[105\zeta^6 - 595\zeta^4 + 1190\zeta^3 - 939\zeta^2 + 239\zeta$$

$$+ N^2(105\zeta^6 - 630\zeta^5 + 980\zeta^4 - 910\zeta^3 + 636\zeta^2 - 181\zeta) - N(210\zeta^6$$

$$-630\zeta^5 + 385\zeta^4 + 280\zeta^3 - 303\zeta^2 + 58\zeta)\Big]$$

$$+\frac{8}{5}R_z^2(\tau_1 + \tau_2)^2(N-1)^2(5\zeta^4 - 10\zeta^3 + 6\zeta^2 - \zeta) \qquad \ldots(18)$$

where $\nu_1 = \frac{\mu_1}{\rho}$, $\nu_2 = \frac{\mu_2}{\rho}$ and $\nu_3 = \frac{\mu_3}{\rho}$ are the kinematic-viscosity, elastico- viscosity and cross-viscosity respectively, $\tau_1\left(=\frac{\nu_2}{z_0^2}\right)$ and $\tau_2\left(=\frac{\nu_3}{z_0^2}\right)$ are the elastico-viscous and cross-viscous dimensionless parameters.

Using (8) in equation (15) and neglecting terms of order $\left(\frac{R_m}{R_z}\right)^2$ and higher, we get

$$\left[\left(-\xi H' + \frac{1}{\xi}\left(\frac{R_m}{R_z}\right)M'\right)\frac{\partial T^*}{\partial \xi} + 2H\frac{\partial T^*}{\partial \zeta}\right]$$
$$= \frac{1}{PR_z}\left[\frac{\partial^2 T^*}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial T^*}{\partial \xi} + \frac{\partial^2 T^*}{\partial \zeta^2}\right] + \frac{1}{2}\left[24H'^2 + 4\frac{R_m}{R_z}\left(\frac{R_l}{R_m}G'L' - H''M''\right)\right.$$
$$+ 48\tau_1 R_z(H'^3 + HH'H'') + 2R_m\tau_1\left\{\frac{R_l}{R_m}(6H'L'G' + 6LG'H'' + 2HG'L''\right.$$
$$\left. + 2HL'G'') + (-6H'H''M'' - 2HH''M'' - 2M'H''^2 - 2HM''H''' + 4M'G'^2)\right\}$$
$$+ 48\tau_2 R_z H'^3 + 2R_m\tau_2\left\{(-3M'H''^2 - 6H'H''M'' + 3M'G'^2) + \frac{R_l}{R_m}(6H'L'G'\right.$$
$$\left.\left. + 6LG'H'')\right\}\right] + \frac{1}{2}\xi^2\left[2(H''^2 + G'^2) + \tau_1 R_z(4H'H''^2 + 4H'G'^2\right.$$
$$\left. + 4HH''H''' + 4HG'G'') + 6\tau_2 R_z(H'H''^2 + H'G'^2)\right] \qquad \ldots(19)$$

The appropriate form for T^* as suggested from equation (19) is

$$T^* = T_a^* + \phi(\zeta) + \xi^2\psi(\zeta) \qquad \ldots(20)$$

where T_a^* denotes the dimensionless temperature on the lower disc and ϕ, ψ are the dimensionless functions of ζ. Substituting equation (20) into (19) and equating the coefficients of ξ^2 and terms independent of ξ on both sides of the equation thus obtained, we get

$$\frac{1}{P}(4\psi + \phi'') = 2R_m\left[M'\psi - \left\{\left(\frac{R_l}{R_m}\right)G'L' - H''M''\right\}\right] + 2R_z\left[H\phi' - 6H'^2\right.$$
$$- \frac{1}{2}\tau_1 R_m\left\{\left(\frac{R_l}{R_m}\right)(6H'L'G' + 6LG'H'' + 2HG'L'' + 2HL'G'')\right.$$
$$\left. + (-6H'H''M'' - 2HH''M'' - 2M'H''^2 - 2HM''H''' + 4M'G'^2)\right\}$$
$$- \frac{1}{2}\tau_2 R_m\left\{(-3M'H''^2 - 6H'H''M'' + 3M'G'^2) + \left(\frac{R_l}{R_m}\right)(6H'L'G'\right.$$

$$+6LG'H'')\Big\}\Big] - 24R_z^2[\tau_1(H'^3 + HH'H'') + \tau_2 H'^3] \quad \ldots(21)$$

$$\frac{1}{P}\psi'' = R_z[-2H'\psi + 2H\psi' - H''^2 - G'^2] - R_z^2[2\tau_1(H'H''^2 + H'G'^2$$
$$+HH''H''' + HG'G'') + 3\tau_2(H'H''^2 + H'G'^2)] \quad ..(22)$$

The expressions for H, G, L and M' have been considered upto $(R_z)^2$ and hence the appropriate forms for ϕ and ψ should be as follows:

$$\left.\begin{aligned} \phi &= \phi_0 + R_z\phi_1 + R_z^2\phi_2 \\ \psi &= \psi_0 + R_z\psi_1 + R_z^2\psi_2 \end{aligned}\right\} \quad \ldots(23)$$

where ϕ_0, ψ_0 etc. are functions of ζ.

The boundary conditions on the temperature in terms of ϕ_0, ψ_0 etc. can be written as:

$$\zeta = 0 \;:\; \phi_0 = 0,\; \phi_1 = 0,\; \phi_2 = 0; \qquad \psi_0 = 0,\; \psi_1 = 0,\; \psi_2 = 0$$
$$\zeta = 1 \;:\; \phi_0 = S,\; \phi_1 = 0,\; \phi_2 = 0; \qquad \psi_0 = 0,\; \psi_1 = 0,\; \psi_2 = 0$$
$$\ldots(24)$$

where $S = \dfrac{(T_b - T_a)c_v}{(\nu_1\Omega)}$

Substituting (23) into (21) and (22) and equating the terms independent of R_z, coefficient of R_z and R_z^2 on both sides of the resulting equations, we get a set of equations containing ϕ_0, ψ_0 etc. and their derivatives. The solution of this set of equations satisfying the boundary condition (24) is given by:

$$\psi_0 = 0, \qquad \psi_1 = \frac{1}{2}P(N-1)^2\zeta(1-\zeta), \qquad \psi_2 = 0$$

$$\phi_0 = S\zeta$$

$$\phi_1 = \frac{1}{6}P(N-1)^2(\zeta^4 - 2\zeta^3 + \zeta) + PR_m\Big[\frac{1}{10}P(N-1)^2(2\zeta^6 - 6\zeta^5 + 5\zeta^4 - \zeta)$$
$$-\frac{1}{15}\Big\{\zeta^6 - 9\zeta^5 + 21\zeta^4 - 27\zeta^3 + 18\zeta^2 - 4\zeta + N^2(\zeta^6 + 3\zeta^5 - 9\zeta^4 + 13\zeta^3$$
$$-12\zeta^2 + 4\zeta) - 2N(\zeta^6 - 3\zeta^5 + 6\zeta^4 - 7\zeta^3 + 3\zeta^2)\Big\} - \Big\{2\tau_1(\zeta^4 - 2\zeta^3$$
$$+2\zeta^2 - \zeta) + \frac{1}{2}\tau_2(5\zeta^4 - 10\zeta^3 + 8\zeta^2 - 3\zeta)\Big\}(N-1)^2\Big]$$

$$\phi_2 = \frac{PS}{(12600)}\Big[(10\zeta^7 - 70\zeta^6 + 147\zeta^5 - 105\zeta^4 + 18\zeta) + N^2(10\zeta^7 - 63\zeta^5$$
$$+70\zeta^4 - 17\zeta) - N(20\zeta^7 - 70\zeta^6 + 84\zeta^5 - 35\zeta^4 + \zeta)\Big] \quad \ldots(25)$$

Using equations (20), (23) and (25) we get the solution of equation (19) the dimensionless form of the temperature as:

$$\frac{T - T_a}{T_b - T_a} = \frac{T^* - T_a^*}{T_b^* - T_a^*} = \zeta + \left(\frac{PR_z^2}{12600}\right)\Big\{10\zeta^7 - 70\zeta^6 + 147\zeta^5 - 105\zeta^4 + 18\zeta$$
$$+ N^2(10\zeta^7 - 63\zeta^5 + 70\zeta^4 - 17\zeta) - N(20\zeta^7 - 70\zeta^6 + 84\zeta^5 - 35\zeta^4$$
$$+\zeta)\Big\} + EP\Bigg[\frac{1}{6}(N-1)^2(\zeta^4 - 2\zeta^3 + \zeta) + \frac{R_m}{30}\Big\{3P(N-1)^2(2\zeta^6$$
$$- 6\zeta^5 + 5\zeta^4 - \zeta) - 2\Big\{\zeta^6 - 9\zeta^5 + 21\zeta^4 - 27\zeta^3 + 18\zeta^2 - 4\zeta$$
$$+ N^2(\zeta^6 + 3\zeta^5 - 9\zeta^4 + 13\zeta^3 - 12\zeta^2 + 4\zeta) + 2N(-\zeta^6 + 3\zeta^5 - 6\zeta^4$$
$$+ 7\zeta^3 - 3\zeta^2)\Big\} - \Big\{60\tau_1(\zeta^4 - 2\zeta^3 + 2\zeta^2 - \zeta) + 15\tau_2(5\zeta^4 - 10\zeta^3$$
$$+ 8\zeta^2 - 3\zeta)\Big\}(N-1)^2\Big\} + \frac{1}{2}\xi^2(N-1)^2\zeta(1-\zeta)\Bigg] \qquad \ldots(26)$$

where $E\left(= \frac{\Omega z_0^2}{c_v(T_b - T_a)}\right)$ is the Eckert number.

The amount of heat transfer from the lower and upper discs are

$$Q_a = \frac{1}{\pi(\xi^2 - \xi_0^2)}\int_{\xi_0}^{\xi} 2\pi\xi q_a \, d\xi \qquad \ldots(27)$$

and

$$Q_b = \frac{1}{\pi(\xi^2 - \xi_0^2)}\int_{\xi_0}^{\xi} 2\pi\xi q_b \, d\xi \qquad \ldots(28)$$

respectively.

$q_a\left(= -\left(\frac{k}{z_0}\right)\frac{\partial T}{\partial \zeta}\Big|_{\zeta=0}\right)$ and $q_b\left(= -\left(\frac{k}{z_0}\right)\frac{\partial T}{\partial \zeta}\Big|_{\zeta=1}\right)$ are heat fluxes on the lower and upper discs. Therefore the average Nusselt numbers on the lower and upper discs are

$$Nu_a = \frac{Q_a z_0}{(T_b - T_a)k}$$
$$= -\Bigg[1 + \frac{PR_z^2}{12600}\left(18 - 17N^2 - N\right) + EP\Big\{\frac{1}{6}(N-1)^2$$
$$+ \frac{R_m}{30}\left\{-3P(N-1)^2 + 8 - 8N^2 + 60\tau_1(N-1)^2 + 45\tau_2(N-1)^2\right\}$$
$$+ \frac{1}{4}(\xi^2 + \xi_0^2)(N-1)^2\Big\}\Bigg] \qquad \ldots(29)$$

$$Nu_b = \frac{Q_b z_0}{(T_b - T_a)k} = -\Bigg[1 + \frac{PR_z^2}{12600}\left(-17 + 18N^2 - N\right) + EP\Big\{-\frac{1}{6}(N-1)^2$$

$$+\frac{R_m}{30}\left\{3P(N-1)^2+8-8N^2-60\tau(N-1)^2-45\tau_2(N-1)^2\right\}$$
$$-\frac{1}{4}(\xi^2+\xi_0^2)(N-1)^2\Bigg\}\Bigg] \quad \ldots(30)$$

IV. Results And Discussion

The dimensionless form of the radii at which there is no recirculation for the cases of net radial outflow $(R_m > 0)$ and net radial inflow $(R_m < 0)$ respectively, satisfying the following conditions

$$\text{I} \qquad R_m > 0 \qquad \left(\frac{\partial U}{\partial \zeta}\right)_{\zeta=0} \geq 0, \qquad \left(\frac{\partial U}{\partial \zeta}\right)_{\zeta=1} \leq 0 \quad \ldots(31)$$

$$\text{II} \qquad R_m\left(= -R_n < 0\right) \qquad \left(\frac{\partial U}{\partial \zeta}\right)_{\zeta=0} \leq 0, \qquad \left(\frac{\partial U}{\partial \zeta}\right)_{\zeta=1} \geq 0 \quad \ldots(32)$$

Case I: When both the discs rotate in the same sense with equal angular velocity $(N = 1)$:

It is evident from equation (26) that there is no effect of the second-order parameter τ_1, τ_2 and ξ, E, R_m on the temperature profile. It is also observed that the temperature variation with ζ is linear. The average Nusselt numbers in this case are constants for the constant value of R_z, P.

Case II: When both the discs rotate with different angular velocities:

The maximum value of the dimensionless radii ξ for no-recirculation viz. ξ_1 (for $m > 0$) and ξ_2 (for $m < 0$) are obtained from the condition (31) and (32) respectively as:

$$\frac{\xi_1^2}{R_m} = \frac{25200 - R_z^2[6720(N-1)^2\tau^2 + 8\tau(239 - 181N^2 - 58N) + 1 - 89N^2 - 92N]}{70R_z^2(4N^2 + 2N - 6)}$$

$$\frac{\xi_2^2}{R_n} = \frac{25200 - R_z^2[6720(N-1)^2\tau^2 - 8\tau(181 - 239N^2 + 58N) + N^2 - 92N - 89]}{70R_z^2(-4 + 6N^2 - 2N)}$$

where $\tau = \tau_1 + \tau_2$ and $R_m = -R_n (R_n > 0)$ for $m < 0$.

The behaviour of the dimensionless temperature $\frac{(T - T_a)}{(T_b - T_a)}$ with ζ at $P = 20,$ $E = 0.02, R_z = 0.5, N = 2$ for different values of $\tau_2 = 0, 1, 2$ in cases of $R_m(= -0.02) < 0; R_m(= 0.02) > 0$ when $\xi = 20$ and $\xi = 1$ is represented through figure-1 and figure-2 respectively.

Table-1 Variation of Temperature with ζ for different values of τ_2 at $N = 2$ in Recirculation region $(\xi = 20)$ in case of $R_m < 0$ and $R_m > 0$

	$R_m = -0.02$			$R_m = 0.02$		
ζ	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$
0	0	0	0	0	0	0
0.1	7.30652	7.305394	7.304267	7.30245	7.303577	7.304704
0.2	13.012049	13.01018	13.008311	13.004689	13.006557	13.008427
0.3	17.116175	17.113848	17.111523	17.106415	17.108742	17.111069
0.4	19.618626	19.616053	19.613482	19.607544	19.610117	19.61269
0.5	20.519262	20.516613	20.513964	20.50808	20.510729	20.51338
0.6	19.818079	19.815506	19.812933	19.808004	19.810577	19.81315

0.7	17.515198	17.512873	17.510546	17.50725	17.509577	17.511904
0.8	13.610898	13.609029	13.607161	13.6057	13.60757	13.609438
0.9	8.105617	8.104489	8.103363	8.103265	8.104391	8.105518
1	1	1	1	1	1	1

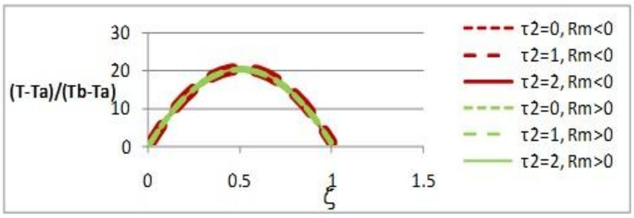

Figure-1 Variation of Temperature with ζ for different values of τ_2 at $N = 2$ in Recirculation region $(\xi = 20)$ in case of $R_m < 0$ and $R_m > 0$

It is clear from the numerical values of the temperature that the temperature decreases with an increase in τ_2 in case of net radial inflow $(R_m < 0)$ whenever increases in case of net radial outflow $(R_m > 0)$ through out the gap length in both the figures-1 $(\xi = 20)$ and figure-2 $(\xi = 1)$. Due to small increments in temperature with respect to τ_2 the branches of the graphs are overlapping in both the figures. In figure-1 the temperature is maximum at the middle of the gap length and minimum (equal to zero) at the lower disc. Temperature is increasing near the lower disc and after attaining its maximum value in the middle it starts decreasing and attains the value 1 on the upper disc whenever in figure-2 it is zero at the lower disc and attains its maximum value 1 on the upper disc.

Table-2 Variation of Temperature with ζ for different values of τ_2 at $N = 2$ in No-Recirculation region $(\xi = 1)$ in case of $R_m < 0$ and $R_m > 0$

	$R_m = -0.02$			$R_m = 0.02$		
ζ	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$
0	0	0	0	0	0	0
0.1	0.12452	0.123864	0.123209	0.12045	0.121106	0.121762
0.2	0.244049	0.243071	0.242093	0.236689	0.237667	0.238645
0.3	0.358175	0.357068	0.355961	0.348414	0.349522	0.350629
0.4	0.466627	0.465482	0.464338	0.455545	0.45669	0.457834
0.5	0.569264	0.568114	0.566964	0.55808	0.55923	0.56038
0.6	0.666079	0.664935	0.66379	0.656005	0.65715	0.658294
0.7	0.757201	0.756093	0.754986	0.749251	0.750358	0.751465
0.8	0.842902	0.841924	0.840946	0.837704	0.838682	0.83966
0.9	0.923622	0.922967	0.922311	0.921271	0.921927	0.922583
1	1	1	1	1	1	1

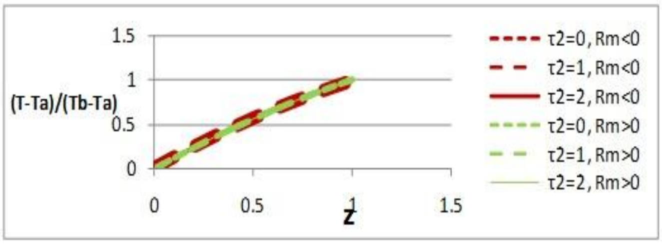

Figure-2 Variation of Temperature with ζ for different values of τ_2 at $N = 2$ in No- Recirculation region $(\xi = 1)$ in case of $R_m < 0$ and $R_m > 0$

In figure-3 $(\xi = 20)$ and figure-4 $(\xi = 1)$, the value of the temperature at $N = 1$ is equal to ζ for $R_m < 0$ and $R_m > 0$. At $P = 20,\ E = 0.02,\ R_z = 0.5,\ \tau_2 = 2$ for different values of $N = 1, 2, 3$ the temperature is increasing with an increase in the value of the ratio of angular velocities in case of net radial inflow and outflow in both the figures 3 and 4. The graph of the temperature is linear at $N = 1$ with minimum at lower disc and maximum at the upper disc. At $N = 2$ and $N = 3$, the temperature is maximum in the middle of the gap length in figure-3 and maximum on the upper disc in figure-4.

Table-3 Variation of Temperature with ζ for different values of N at $\tau_2 = 2$ in Recirculation region $(\xi = 20)$ in case of $R_m < 0$ and $R_m > 0$

	$R_m = -0.02$			$R_m = 0.02$		
ζ	$N = 1$	$N = 2$	$N = 3$	$N = 1$	$N = 2$	$N = 3$
0	0	0	0	0	0	0
0.1	0.1	7.304267	28.923027	0.1	7.304704	28.918385
0.2	0.2	13.008311	51.445034	0.2	13.008427	51.432682
0.3	0.3	17.111523	67.562813	0.3	17.111069	67.542694
0.4	0.4	19.613482	77.274048	0.4	19.61269	77.248703
0.5	0.5	20.513964	80.5774	0.5	20.51338	80.551071
0.6	0.6	19.812933	77.472527	0.6	19.81315	77.449875
0.7	0.7	17.510546	67.960114	0.7	17.511904	67.944824
0.8	0.8	13.607161	52.041874	0.8	13.609438	52.035286
0.9	0.9	8.103363	29.720556	0.9	8.105518	29.720497
1	1	1	1	1	1	1

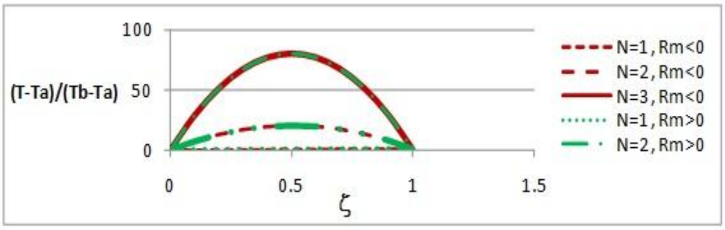

Figure-3 Variation of Temperature with ζ for different values of N at $\tau_2 = 2$ in Recirculation region $(\xi = 20)$ in case of $R_m < 0$ and $R_m > 0$

Table-4 Variation of Temperature with ζ for different values of N at $\tau_2 = 2$ in No- Recirculation region $(\xi = 1)$ in case of $R_m > 0$ and $R_m < 0$

	$R_m = -0.02$			$R_m = 0.02$		
ζ	$N = 1$	$N = 2$	$N = 3$	$N = 1$	$N = 2$	$N = 3$
0	0	0	0	0	0	0
0.1	0.1	0.123209	0.195027	0.1	0.121762	0.190386
0.2	0.2	0.242093	0.373036	0.2	0.238645	0.360683
0.3	0.3	0.355961	0.530814	0.3	0.350629	0.510691
0.4	0.4	0.464338	0.66605	0.4	0.457834	0.640706
0.5	0.5	0.566964	0.777403	0.5	0.56038	0.751069
0.6	0.6	0.66379	0.864532	0.6	0.658294	0.841874
0.7	0.7	0.754986	0.928122	0.7	0.751465	0.912831
0.8	0.8	0.840946	0.969887	0.8	0.83966	0.963302
0.9	0.9	0.922311	0.992581	0.9	0.922583	0.992523
1	1	1	1	1	1	1

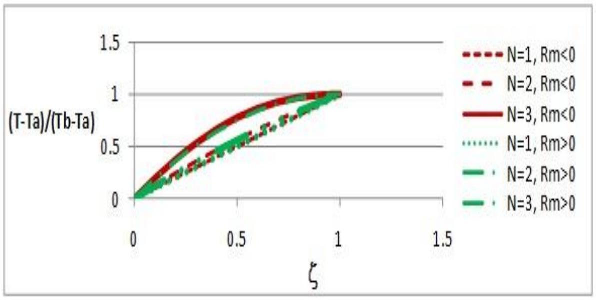

Figure-4 Variation of Temperature with ζ for different values of N at $\tau_2 = 2$ in No- Recirculation region $(\xi = 1)$ in case of $R_m < 0$ and $R_m > 0$

Figure-5 represents the variation of Nu_a (Nusselt number at the lower disc) with ξ at $P = 20,\ E = 0.02,\ R_z = 0.5,\ \xi_0 = 5$ for different values of $\tau_2 = 0, 1, 2$. The average Nusselt number (Nu_a) on the lower disc is negative throughout the entire radial region and increases with an increase in τ_2 in case of $R_m < 0$ whenever decreases in case of $R_m > 0$. It is also clear that Nu_a is decreasing with an increase in ξ in both the cases $(R_m > 0$ and $R_m < 0)$. The behaviour of Nu_b (average Nusselt number on the upper disc) in figure-6 is just reverse to that of Nu_a (see figure-5). Nu_b is positive throughout the entire radial region and attains its minimum value at the lower disc and maximum at the upper disc.

Table-5 Variation of Nu_a with ξ for different values of τ_2 at $N = 2,\ \xi_0 = 5$ in case of $R_m < 0$ and $R_m > 0$

	$R_m = -0.02$			$R_m = 0.02$		
ξ	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$
0	-3.56843	-3.55963	-3.55083	-3.52363	-3.53243	-3.54123
1	-3.66843	-3.65963	-3.65083	-3.62363	-3.63243	-3.64123
2	-3.96843	-3.95963	-3.95083	-3.92363	-3.93243	-3.94123
3	-4.46843	-4.45963	-4.45083	-4.42363	-4.43243	-4.44123
4	-5.16843	-5.15963	-5.15083	-5.12363	-5.13243	-5.14123
5	-6.06843	-6.05963	-6.05083	-6.02363	-6.03243	-6.04123
6	-7.16843	-7.15963	-7.15083	-7.12363	-7.13243	-7.14123
7	-8.46843	-8.45963	-8.45083	-8.42363	-8.43243	-8.44123
8	-9.96843	-9.95963	-9.95083	-9.92363	-9.93243	-9.94123
9	-11.6684	-11.6596	-11.6508	-11.6236	-11.6324	-11.6412
10	-13.5684	-13.5596	-13.5508	-13.5236	-13.5324	-13.5412

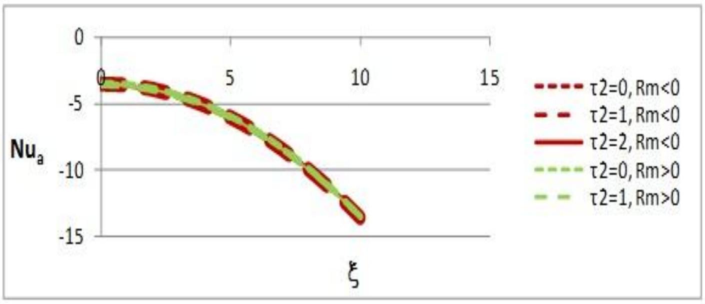

Figure-5 Variation of Nu_a with ξ for different values of τ_2 at $N = 2,\ \xi_0 = 5$ in case of $R_m < 0$ and $R_m > 0$

Table-6 Variation of Nu_b with ξ for different values of τ_2 at $N = 2, \xi_0 = 5$ in case of $R_m > 0$ and $R_m < 0$

	$R_m = -0.02$			$R_m = 0.02$		
ξ	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$	$\tau_2 = 0$	$\tau_2 = 1$	$\tau_2 = 2$
0	1.555235	1.546435	1.537635	1.536035	1.544835	1.553635
1	1.655235	1.646435	1.637635	1.636035	1.644835	1.653635
2	1.955235	1.946435	1.937635	1.936035	1.944835	1.953635
3	2.455235	2.446435	2.437635	2.436035	2.444835	2.453635
4	3.155235	3.146435	3.137635	3.136035	3.144835	3.153635
5	4.055235	4.046435	4.037635	4.036035	4.044835	4.053635
6	5.155235	5.146435	5.137635	5.136035	5.144835	5.153635
7	6.455235	6.446435	6.437635	6.436035	6.444835	6.453635
8	7.955235	7.946434	7.937635	7.936035	7.944835	7.953635
9	9.655234	9.646435	9.637634	9.636035	9.644835	9.653635
10	11.55524	11.54643	11.53764	11.53604	11.54484	11.55364

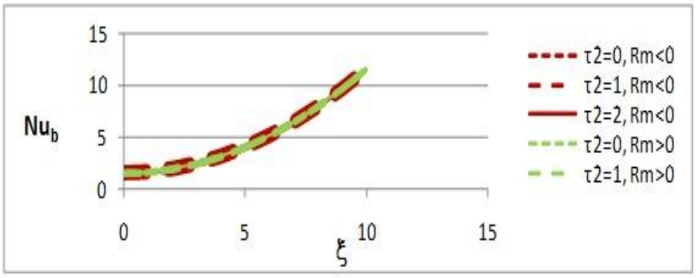

Figure-6 Variation of Nu_b with ξ for different values of τ_2 at $N = 2, \xi_0 = 5$ in case of $R_m < 0$ and $R_m > 0$

The variation Nu_a and Nu_b with ξ at $P = 20, E = 0.02, R_z = 0.5, \xi_0 = 5$ for different values of $N = 1, 2, 3$ through figure-7 and 8 respectively. The value of Nu_a is negative throughout the radial region for all values of N. In case of $N = 1$ both Nu_a and Nu_b for net radial inflow and outflow have same values -1 for all values of ξ. In case of $N = 2, 3$ Nu_a have negative values whenever Nu_b have positive values. Nu_a is decreasing with an increase in velocity ratio N in both cases $R_m < 0$ and $R_m > 0$ (see figure-7) whenever the behaviour of Nu_b (see figure-8) is just reverse to that of Nu_a.

Table-7 Variation of Nu_a with ξ at $\tau_2 = 2, \xi_0 = 5$ for different values of N in case of $R_m < 0$ and $R_m > 0$

	$R_m = -0.02$			$R_m = 0.02$		
ξ	$N = 1$	$N = 2$	$N = 3$	$N = 1$	$N = 2$	$N = 3$
0	-1	-3.55083	-11.2226	-1	-3.54123	-11.2012
1	-1	-3.65083	-11.6226	-1	-3.64123	-11.6012
2	-1	-3.95083	-12.8226	-1	-3.94123	-12.8012

3	-1	-4.45083	-14.8226	-1	-4.44123	-14.8012
4	-1	-5.15083	-17.6226	-1	-5.14123	-17.6012
5	-1	-6.05083	-21.2226	-1	-6.04123	-21.2012
6	-1	-7.15083	-25.6226	-1	-7.14123	-25.6012
7	-1	-8.45083	-30.8226	-1	-8.44123	-30.8012
8	-1	-9.95083	-36.8226	-1	-9.94123	-36.8012
9	-1	-11.6508	-43.6226	-1	-11.6412	-43.6012
10	-1	-13.5508	-51.2226	-1	-13.5412	-51.2012

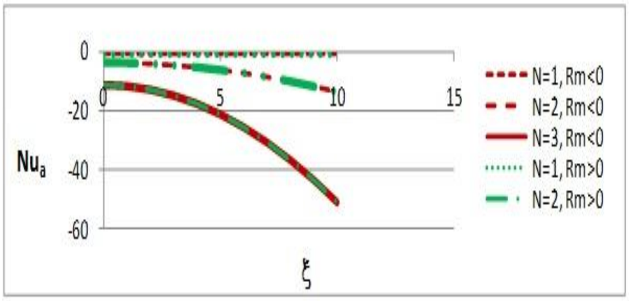

Figure-7 Variation of Nu_a with ξ at $\tau_2 = 2, \xi_0 = 5$ for different values of N in case of $R_m < 0$ and $R_m > 0$

Table-8 Variation of Nu_b with ξ at $\tau_2 = 2, \xi_0 = 5$ for different values of N in case of $R_m < 0$ and $R_m > 0$

	$R_m = -0.02$			$R_m = 0.02$		
ξ	$N = 1$	$N = 2$	$N = 3$	$N = 1$	$N = 2$	$N = 3$
0	-1	1.537635	9.186851	-1	1.553635	9.233784
1	-1	1.637635	9.58685	-1	1.653635	9.633784
2	-1	1.937635	10.78685	-1	1.953635	10.83378
3	-1	2.437635	12.78685	-1	2.453635	12.83378
4	-1	3.137635	15.58685	-1	3.153635	15.63378
5	-1	4.037635	19.18685	-1	4.053635	19.23378
6	-1	5.137635	23.58685	-1	5.153635	23.63378
7	-1	6.437635	28.78685	-1	6.453635	28.83378
8	-1	7.937635	34.78685	-1	7.953635	34.83378
9	-1	9.637634	41.58685	-1	9.653635	41.63378
10	-1	11.53764	49.18685	-1	11.55364	49.23378

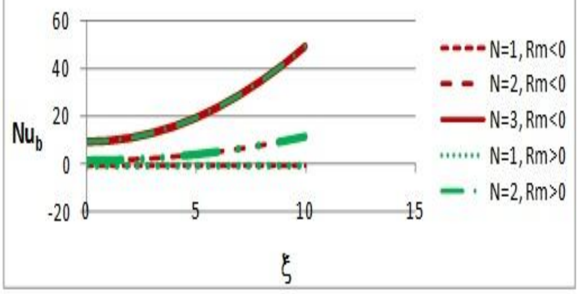

Figure-8 Variation of Nu_b with ξ at $\tau_2 = 2, \xi_0 = 5$ for different values of N in case of $R_m < 0$ and $R_m > 0$

V. Conclusions

It is evident from the expressions of the functions H, G, L, M' and the graphs of this problem that at $N = 1$ the results are identical to those obtained by Sharma and Singh[6].

Since Nu_a is negative throughout the entire radial region it follows that the heat is being transferred from the fluid to the lower disc and the amount of heat transfer decreases with an increase in τ_2 in case of $R_m > 0$ whenever increases in case of $R_m < 0$. It is also concluded that due to $Nu_b > 0$ for all values of ξ heat is being transferred from the fluid towards the upper disc and the amount of heat transfer is being increased with an increase in the angular velocity of the upper disc.

References

[1] Smith, R.D., Experimental cooling of radial flow turbines, A.S.M.E., paper No. 54-A-245(1945).

[2] Emmons, H.W. and S.L.Soo, Development and preliminary test of a rotating viscometer. Harvard University research report, Cambridge, Massachusetts, May 1952.

[3] Soo, S.L., R.W. Besant and Z.N. Sarafa, the nature of heat transfer from an enclosed rotating disc, Z. Angew, Math. Phys., 13 (1962) p 297.

[4] Sharma, S.K. and R.S. Agarwal, Heat transfer from an enclosed rotating disc, Appl. Set. Res. 16 (1966) p 204.

[5] Sharma, S.K., On the flow over an enclosed rotating disc, Proc. VIII Cong. On Theor and Appl., India, 8 (1963) p 77.

[6] Sharma, H.G and K.R. Singh, Heat transfer in the flow of a second-order fluid between two enclosed rotating disc, Int. J. Heat Mass Transfer, 28 (1985) p 304.

[7] Coleman, B.D. and W. Noll, Recent result in the continuum theory of visco-elastic fluids, Annals., N. Y. Acad. Sci. 89 (1961) p 672.