HEAT TRANSFER TO THE HIGHLY ACCELERATED TURBULENT BOUNDARY LAYER WITH AND WITHOUT MASS ADDITION

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Report HMT-6

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Abstract

Experimental heat transfer data are presented for a series of asymptotic accelerated turbulent boundary layers for the case of an impermeable wall, for several cases of blowing, and suction. The data are presented as Stanton number versus enthalpy thickness Reynolds number.

As noted by previous investigators, acceleration causes a depression in Stanton number when the wall is impermeable. Suction increases this effect, while blowing suppresses it. The combination of mild acceleration and strong blowing results in Stanton numbers which lie above the correlation for the same blowing but no acceleration.

Velocity and temperature profiles are presented, from which it is possible to deduce explanations for the observed behavior of the Stanton number. A prediction scheme is proposed which is demonstrated to quite adequately reproduce the Stanton number results, using correlations derived from the profiles.

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Nomenclature

English Letter Symbols

A^+	-	constant in the Van Driest damping factor
C_f	-	friction coefficient ($= \tau_o/(\rho u_\infty^2/2)$)
c_p	-	specific heat
D_v	-	Van Driest mixing-length damping factor
F	-	blowing fraction ($= v_w/u_\infty$)
H	-	boundary layer shape factor ($= \delta_1/\delta_2$)
h	-	convective heat transfer coefficient
K	-	acceleration parameter ($= (\nu/u_\infty^2)(du_\infty/dx)$)
k	-	mixing-length constant
ℓ	-	mixing-length
$\dot{q}$	-	heat transfer rate
t	-	temperature
t_w	-	wall, or surface, temperature
t_∞	-	free-stream temperature
u	-	velocity in x-direction
u_∞	-	free-stream velocity
u_τ	-	friction velocity ($= u/(u_\infty\sqrt{C_f/2})$)
v_w	-	velocity in y-direction at the wall (transpiration velocity), positive v_w for blowing, negative for suction
x	-	distance along surface
y	-	distance normal to surface

Greek Letter Symbols

γ	-	thermal conductivity
γ_t	-	turbulent thermal conductivity, or eddy conductivity
γ_{eff}	-	$(\gamma + \gamma_t)$
δ	-	99 percent thickness of momentum boundary layer
δ_1	-	displacement thickness $= \int_0^\infty (1 - \frac{u}{u_\infty}) dy$
δ_2	-	momentum thickness $= \int_0^\infty (1 - \frac{u}{u_\infty})(\frac{u}{u_\infty}) dy$
Δ_2	-	enthalpy thickness $= \int_0^\infty (\frac{u}{u_\infty})(\frac{t-t_\infty}{t_w-t_\infty}) dy$
λ	-	a turbulence length scale
μ	-	viscosity coefficient
μ_t	-	turbulent viscosity, or eddy viscosity
μ_{eff}	-	$(\mu + \mu_t)$
ν	-	kinematic viscosity (μ/ρ)
ρ	-	density
τ	-	shear stress
τ_w	-	shear stress at wall

Non-dimensional Groups

P^+	-	a pressure gradient parameter ($= K/(C_f/2)^{3/2}$)
Pr	-	Prandtl number $(\mu c_p/\gamma)$
Pr_t	-	turbulent Prandtl number $(\mu_t c_p/\gamma_t)$

Re_M - momentum thickness Reynolds number $(= u_\infty \delta_2/\nu)$

Re_H - enthalpy thickness Reynolds number $(= u_\infty \Delta_2/\nu)$

R_x - an integrated x-distance Reynolds number

$$\left(= \int_0^x \frac{u_\infty}{\nu}\, dx \right)$$

St - Stanton number $(= h/(u_\infty \rho c_p))$

t^+ - non-dimensional temperature $= \dfrac{(t-t_w)}{(t_\infty - t_w)} \dfrac{1}{u_\infty^+ St}$

u^+ - non-dimensional velocity $(= u/u_\tau)$

$v_w^{\ +}$ - non-dimensional blowing parameter $(= v_w/(u_\infty \sqrt{C_f/2}))$

y^+ - non-dimensional distance from wall $[= y u_\tau/\nu]$

Introduction

In 1965, Moretti and Kays (1) presented the results of an experimental investigation of heat transfer to a highly accelerated turbulent boundary layer. Of particular interest was the fact that for very strong accelerations Stanton number was observed to decrease abruptly and to approach what one would predict for a purely laminar boundary layer. These results have been frequently cited as evidence that a strong favorable pressure gradient tends to cause a retransition of a turbulent boundary layer to a laminar boundary layer. The phenomenon of retransition has been the subject of numerous recent studies, Launder (2), Launder and Stinchcombe (3), and Patel and Head (4), among others. The term "laminarization", suggested by Launder, has been frequently used instead of retransition.

It seems now to be generally agreed that a turbulent boundary layer will "laminarize", or undergo a retransition to a laminar boundary layer, in the presence of a sufficiently strong favorable pressure gradient. However, there is a very important region of technical applications in the range of moderately strong favorable pressure gradients where the boundary layer is definitely not laminar but where laminar-like behavior is observed and, in particular, Stanton number is observed to fall substantially below what would be predicted by earlier theories. The present paper is concerned with the heat transfer behavior in this region, including the effects of transpiration (blowing and suction).

Both Moretti and Kays, and Launder, proposed that a significant acceleration parameter, K, can be defined as follows:

$$K = \frac{\nu}{u_\infty^2} \frac{du_\infty}{dx} \tag{1}$$

Various combinations of K and the friction coefficient have also been proposed. For example, in the analysis of a Couette flow, the effect of a pressure gradient occurs in the form of a non-dimensional P^+ , which is related to K as follows:

$$P^+ = K/(C_f/2)^{3/2} \tag{2}$$

The parameter K , however, has the virtue of being entirely dependent upon externally imposed conditions and is, therefore, a convenient descriptor of the boundary conditions imposed upon the flow. Launder suggested that laminarization will occur when K is greater than 2×10^{-6} ; Moretti and Kays suggested 3.5×10^{-6} . The present paper is concerned with values of K in the range 0.0 to 2.5×10^{-6} , and thus is concerned with a region of what is believed to be stable turbulent boundary layers, although admittedly at $K = 2.5 \times 10^{-6}$ this last statement may be debatable.

Further insight into the significance of the parameter K can be gained by examination of the momentum integral equation of the boundary layer, and the energy integral equation of the boundary layer. For constant property flow along a flat plate, it is possible to express the momentum integral equation of the boundary layer in the following form:

$$\frac{dRe_M}{dR_x} = \frac{C_f}{2} - K(1 + H)Re_M + F \tag{3}$$

where $dR_x = u_\infty \rho dx/\mu$

$F = v_w/u_\infty$

For constant free-stream temperature and constant surface temperature, the corresponding form of the energy integral equation of the boundary layer is:

$$\frac{dRe_H}{dR_x} = St + F \tag{4}$$

Note that K appears explicitly only in the momentum equation, and of particular interest is the fact that a sufficiently large positive value of K can cause a decrease in Re_M. In fact, it appears that if K is maintained as a positive constant over a sufficient length of surface, and if F is zero or a positive constant, then the rate of change of Re_M will tend towards zero. This yields a boundary layer of constant momentum thickness Reynolds number, Re_M, which will be termed an "asymptotic" accelerating boundary layer. Exact solutions for asymptotic laminar boundary layers have been obtained (5), and Launder and Stinchcombe have demonstrated that such asymptotic boundary layers can be obtained for turbulent flows. Under such conditions, not only is Re_M constant, but also $C_f/2$ and the shape factor H. It also follows that under asymptotic conditions the velocity profiles at various stations along the surface will possess both inner and outer region similarity.

On the other hand, examination of equation (4) reveals that so long as F is zero or a positive constant, the energy thickness Reynolds number, Re_H, will always continue to grow. (The same conclusions apply for small negative values of F. However, strong suction leads to an asymptotic suction layer, regardless of K, with no growth in either Re_M or Re_H). One can conclude that for prolonged accelerations at any constant value of K and positive F, Re_M will approach a constant value, whereas Re_H will increase indefinitely.

This behavior suggests one reason why, even at moderate values of K, Stanton number will tend to decrease in an accelerated flow. Re_H can only increase indefinitely if the thermal boundary layer grows outside of the momentum boundary layer into a region of zero eddy conductivity and higher heat transfer resistance. This phenomena is discussed by Launder and Lockwood (13). It will be seen later that this is not the only reason for decreasing Stanton numbers in accelerated flows, but it is certainly a contributing factor.

An acceleration at constant K is particularly easy to establish experimentally with an incompressible fluid, since it can be shown from continuity that flow between two convergent flat surfaces yields a nearly constant K when the blowing fraction, F, is uniform. Asymptotic constant K boundary layers are even more convenient for experimental study because once the asymptotic condition has been closely approached, it is relatively easy to accurately deduce the friction coefficient using equation (3). Furthermore, it is a simple matter to obtain a nearly asymptotic boundary layer by arranging a starting length with constant free-stream velocity (before acceleration) such that Re_M at the beginning of acceleration is close to the anticipated asymptotic value, thereby avoiding a lengthy transition region. For most of the experimental results presented by Moretti and Kays, Re_M was considerably greater than the asymptotic value at the beginning of acceleration even though K was nearly constant, and thus the reported heat transfer results were primarily in a region of rapidly decreasing Re_M. Such accelerated boundary layers will be referred to as "overshot"; obviously boundary layers can also be "undershot" if the value of Re_M before acceleration is less than the asymptotic value.

The Stanford Heat and Mass Transfer Apparatus was designed for accurate measurement of local heat transfer coefficients along a flat surface through which transpiration (either blowing or suction) can take place in any prescribed manner, and over which free-stream velocity can be varied in any arbitrary manner. Extensive experimental results obtained from this apparatus for the case of constant free-stream velocity and the entire spectrum of blowing and suction have been presented in Moffat and Kays (6), and Simpson, Moffat, and Kays (7). The apparatus is also ideally suited for a study of the behavior of asymptotic accelerated turbulent boundary layers with blowing or suction. This paper is a brief summary of a few of the results of such an investigation. As such, it is a continuation of the work of Moretti and Kays, but differing in two major respects:

(a) An attempt is made to obtain close to asymptotic boundary layers, and thus to carry out a more controlled experiment;

(b) The additional effects of blowing and suction on acceleration are studied, with emphasis on certain unexpected results of the coupling of blowing and acceleration.

More complete and extensive data resulting from this investigation will be presented in a later paper.

Objectives of This Paper

The specific objectives of this paper are to:

(a) Present the results of a systematic series of heat transfer experiments on asymptotic accelerated turbulent boundary layers for a series of values of the acceleration parameter K up to 2.5×10^{-6}, and blowing fraction, F, from -0.002 to $+0.006$.

(b) Present representative velocity and temperature profiles, and on the basis of these profiles to attempt to explain the physical phenomena observed.

(c) Present some results of an analytic prediction scheme, based on a finite difference solution of the boundary layer equations, to demonstrate a mathematical model of the phenomena observed.

Apparatus and Data Reduction

The Stanford Heat and Mass Transfer Apparatus contains a 24-segment porous plate eight feet long and 18 inches wide, which forms the bottom surface of a rectangular flow duct. The main stream flow and the transpiration flow are both air. Each of the 24 segments is provided with separately controllable transpiration flow and electric power. Fig. 1 shows a cross-section of one segment. The balsa wood insulation on the walls of the plenum, the pre-plate, and the honeycomb flow straighteners serve to ensure uniform air temperature entering the working plate. Five thermocouples are imbedded in the plate, in the center six inch span. The working plate is 0.25 inches thick, made of sintered bronze with an average particle diameter of 0.005 inches. Heater wires are imbedded in grooves in the bottom of the plate, close enough together so that the top surface of the plate is uniform in temperature to within 0.04^{o}F at maximum power and blowing. Pressure drop through the working plate is approximately 12 inches of water at maximum blowing, so that the maximum streamwise pressure gradient (approximately 0.5 inches of water per segment width) has only a small effect on the distribution of the transpiration flow.

Two different top covers, shown in Fig. 2, were used for the test duct. One with a single hinge line across it, and one with two hinge lines, permitted constant K flows to be established by setting the desired slope of the top surface. Static pressures were measured with side-wall taps spaced 2 inches apart in the flow direction. Static pressure traverses of the main stream and boundary layer showed no more than 0.002 inches variation across a plane in the accelerating region.

Temperature and velocity traverses were made with manually operated micrometer driven traverse gear. Flat mouthed total pressure probes were used with tips 0.012 inches high and 0.040 wide. Temperature traverses were made using iron-constantan thermocouples with junctions flattened to 0.009 inches.

Stanton numbers reported here are based on the heat transfer from the plate to the boundary layer as deduced by an energy balance on each plate.

$$\dot{q} = \text{Net Power} - \text{ECONV} - \Sigma\text{QRAD} - \Sigma\text{QCOND}$$

ECONV measures the energy transport associated with the transpiration flow. Radiation from the top and bottom of the plate is calculated, based on measured emissivities of the plate. Heat is also lost by conduction from the center span of the plate to the ends of the plate and to the casting. All corrections were evaluated as functions of plate temperature and transpiration rate and appropriately entered into the data program.

A somewhat more detailed description is presented by Moffat and Kays (6).

Qualification of the Experimental System

Validity of the data reduction program as a mathematical model of the apparatus was established by a series of energy balance tests conducted with no main stream flow. The energy balances closed within 2% for most blowing cases and 4% for most suction cases. The Stanton numbers reported here are believed to be reliable to within 0.0001 units, for the blown cases, and 0.0002 units for suction.

Free-stream turbulence intensities were found to be between 0.8 and 1.2% although velocity profiles taken in the uniform velocity section satisfy Coles' criterion for "normal" boundary layers (8).

Two side effects must be investigated before the observed change in Stanton number behavior can be attributed solely to the effects of acceleration. It must be shown that the data are not influenced by surface roughness and that data for various uniform velocities will display a universal relationship when plotted against enthalpy thickness Reynolds number.

Surface roughness and velocity effects were investigated by a series of tests at 40, 86 and 126 fps. Stanton number data shows the same relationship to enthalpy thickness Reynolds number for all three velocities, although the velocity profiles show a slight drop in u^+ for the data at 126 fps. Plate roughness elements, considered as half the particle diameter, are calculated to remain inside the viscous region of the boundary layer as best as this can be determined.

Two-dimensionality of a flow can only be established by elaborate probing of the boundary layers. This was not done. Secondary evidence, however, can be had by comparing enthalpy thickness derived from plate heat transfer measurements with values determined from temperature and velocity profiles. Such checks show agreement within 8% for all blowing runs. This is within the uncertainty calculated for the enthalpy thickness integrals using the method of Kline and McClintock (9).

Results

Stanton number data are shown in Figs. 3 through 6, plotted against enthalpy thickness Reynolds number, Re_H . Each figure shows the effect of varying K while holding F constant. Surface temperatures were held constant, for all tests, at approximately 100°F, while free-stream stagnation temperature was 60-70°F. The Stanton numbers were corrected to approximately constant property conditions by the factor $(T_w/T_\infty)^{0.4}$.

Fig. 3 shows the data for $F = 0.0$ using solid symbols to represent data in the accelerating region and hollow symbols for the constant velocity approach. Note that acceleration immediately depresses the Stanton number below the constant velocity results, with the magnitude of the depression increasing as K increases.

The data for $K = 2.5 \times 10^{-6}$ behaves almost as would be expected from a laminar boundary layer, based on the rate of change of Stanton number as enthalpy thickness increases. Shape factors, determined from the velocity profiles are approximately 1.4 to 1.5 for this acceleration, suggesting that the boundary layer is still turbulent (the shape factor for the asymptotic laminar layer is 2.0). No effort was made to measure turbulence intensities inside the layer.

It can be seen from Fig. 3 that the boundary layers were slightly "overshot", in that the momentum thickness Reynolds numbers <u>decrease</u> in the flow direction. The boundary layers are believed to be close to the asymptotic condition at the points where Re_M is marked in the accelerating region.

Fig. 4 presents Stanton number data for the same values of K as does Fig. 3, but with blowing: F = +0.002 . All of the data for the different values of K lie much closer to the baseline data, taken from Moffat and Kays (6). The spread in the data is reduced, and the entire pattern is shifted upward. Acceleration at $K = 0.75 \times 10^{-6}$ now results in a slight rise in Stanton number above the uniform velocity case, rather than a drop, and even the strongest acceleration ($K = 2.5 \times 10^{-6}$) produces only a relatively minor depression.

The upward shift indicated by Fig. 4 is seen much more clearly in the results at higher blowing (Fig. 5: F = +0.0062), and the opposite trend is observed for suction (Fig. 6: F = -0.002). In the presence of strong blowing, even a moderate acceleration ($K = 0.77 \times 10^{-6}$) causes a dramatic upward shift from the uniform velocity values for the same blowing. Moderate suction, F = -0.002, increases the spread between the data for various K values and causes a general downward shift relative to the uniform velocity results.

The combination of blowing and acceleration can thus result in either an increase or a decrease in Stanton number (at fixed Re_H) in spite of the fact that either condition, applied alone, results in a decrease. Stanton number is thus not simply related to Re_H , K , and F even for the restricted case studied here of asymptotic boundary layers. It is not unreasonable to suppose that highly "overshot" or "undershot" layers will display somewhat different characteristics, raising the number of variables from 3 to 5. Experimental studies of these effects are planned for the near future, as well as investigations into the behavior of the boundary layer under conditions of variable K, and in the recovery region downstream of an acceleration.

Velocity profiles with $K = 1.45 \times 10^{-6}$ and $F = 0$ are shown on Fig. 7 with solid symbols, while one profile in the non-accelerating region of this run is shown with hollow symbols. This figure shows some of the important characteristics of accelerated turbulent boundary layers, and asymptotic boundary layers in particular. Note that the three profiles in the accelerated region are close to similar in both the inner and outer regions, and the boundary layer is not significantly growing at successive stations along the surface. The usual rise in u^+ in the "wake" region has disappeared, and the viscous inner region has significantly grown so that u^+ lies above the non-accelerated curve in the middle region.

On the basis of examination of these velocity profiles, as well as other asymptotic profiles at different values of K, it is concluded that increasing K causes an increase in the thickness of the viscous region and a decrease in the values of u^+ and y^+ at the outer edge of the boundary layer. Such a trend with increasing K must ultimately lead to a disappearance of the turbulent region entirely, i.e., a laminar boundary layer.

A succession of temperature profiles taken under the same flow conditions are shown on Fig. 8. Two trends are apparent. In the inner region the curves in the accelerated region come together, but with a greater slope than in the non-accelerated region, evidently a direct result of the thickening of the viscous region near the wall. In the outer region the important observation is that the thermal boundary layer continues to grow at successive stations along the surface, unlike the momentum boundary layer, and is seen to penetrate into a region where the velocity gradient is small, or zero. Stanton number varies inversely as the maximum value of t^+, and thus the observed decrease in Stanton number in Fig. 3 is seen also in Fig. 8.

In summary, it appears that the depression in Stanton number observed in accelerating flows results from a combination of an increase in the viscous region thickness, and the growth of the thermal boundary layer beyond the momentum boundary layer.

Prediction Method

The heat transfer data presented here are only of limited value unless they can be used as the basis of some kind of prediction method that can be employed in design. However, any attempt at satisfactory overall empirical correlation of the data shown on Figs. 3 to 6 would appear to be a virtually hopeless task because of the great variety of possible conditions and resulting behavior. For constant free-stream velocity, the data of Moffat and Kays (6), and subsequent work on the same project not yet published, show that Stanton number can be expressed as a simple function of Re_H , and a blowing parameter, and is only weakly dependent upon any other parameters. Thus a reasonably satisfactory prediction scheme can be developed using the integral energy equation. The data for acceleration, but no blowing (i.e., Fig. 3), show a certain orderliness, but even a superficial examination of these results suggests that Stanton number is a function of at least Re_H , K , and the value of Re_H (or Re_M) where acceleration starts, and these data only represent the behavior under essentially constant K conditions. When blowing or suction are superimposed, the number of variables even for uniform F and K is obviously out of hand, and behavior such as seen in Fig. 5 discourages any attempt at simple correlation.

The obvious next step is to attempt to correlate the experimental data at a more fundamental level by devising empirical correlations which can be used in mathematical models of the momentum and energy exchange processes. Not only can perhaps the desired generality be obtained, but a better understanding of the physics as well.

The scheme to be described here is based on a finite-difference solution of the momentum and thermal energy differential equations of the boundary layer, using the Spalding/Patankar (10) program as the basic mathematical tool. Any desired physical model of the momentum and energy exchange processes can be inserted into the program, subject only to the restriction that the equations are in parabolic form, so that one must be willing to use the concept of eddy viscosity and eddy conductivity. However, one is free to evaluate these quantities in any way desired, and the possibilities range from direct empirical correlations, to deductions based on solution of the turbulence energy equation, which can be solved simultaneously by the same program, if desired.

For present purposes a direct empirical mixing-length correlation is used, based on a modification of the Van Driest mixing-length hypothesis. The equations used are as follows:

$$\mu_{eff} = \mu + \mu_t \qquad \text{effective viscosity}$$

$$\mu_t = \rho \ell^2 \left(\frac{du}{dy}\right) \qquad \text{turbulent viscosity}$$

$$\ell = kyD_v \quad \text{for} \quad y<(\lambda\delta/k)$$

mixing-length

$$\ell = \lambda\delta D_v \quad \text{for} \quad y>(\lambda\delta/k)$$

where δ is the 99% momentum boundary layer thickness

$k = 0.44$ mixing-length constant

$\lambda = 0.25\ Re_M^{-1/8}[1 - 67.5(v_w/u_\infty)]$ turbulence length scale

if $\lambda < 0.085$, $\lambda \doteq 0.085$

$$D_v = 1 - \exp(-y\rho\sqrt{\tau/\rho}\ /A^+\mu)$$

Van Driest damping factor

$$A^+ = 4.42/(v_w^+ + 0.17) - 1133P^+ + f(P^+, v_w^+)$$

where $f(P^+, v_w^+) = -1990(-P^+ \, v_w^{+\,0.25})^{1.10}$ for $v_w^+ > 0.0$

$f(P^+, v_w^+) = 6.78(-P^+)^{0.7}(-v_w^+)^{1.4}$ for $v_w^+ < 0.0$

empirical correlation of the effects of transpiration and acceleration

$\gamma_{eff} = \tilde{\gamma} + \gamma_t$ effective conductivity

$\gamma_t = \mu_t c_p / Pr_t$ turbulent conductivity

$$Pr_t = (1/Pr)\left[1 - 0.1(26/A^+)^{0.4}\sqrt{\mu_t/\mu}\right](1 + 20P^+)$$

if $Pr_t < 0.86$, $Pr_t = 0.86$

turbulent Prandtl number

Although the quantity of empirical input appears formidable, it should be pointed out that correlating λ with Re_M and v_w/u_∞ has only a minor influence, and that is at low Reynolds numbers only. A constant value, $\lambda = 0.085$, will also yield results close to those to be shown. Similarly, constant turbulent Prandtl number, $Pr_t = 0.90$, will yield Stanton numbers in close agreement with those to be shown. The rather complex expression used is based on direct measurements of Pr_t (11) which indicate a variation through the boundary layer starting high near the wall. In the prediction scheme it was found that such a variation in Pr_t is needed to obtain accurate temperature profiles. $Pr_t = 0.90$ is simply an effective average.

The core of the correlation scheme is in the expression for A^+. Note that for $v_w^+ = 0$ and $K = 0$, $A^+ = 26$, a frequently used value. This correlation is presented as only tentative and illustrative of what can be done. Essentially A^+ is related to the thickness of the viscous sublayer (in y^+ coordinates), and the correlation reflects the thickening of this region in a favorable pressure gradient as observed in Fig. 7. It also includes a decrease in thickness observed for blowing, as determined from the data of Simpson, et al (7), and a cross-coupling effect.

One further point should be noted. In attempting to apply the above model to an acceleration with $K = 2.5 \times 10^{-6}$, it was found that although a substantial decrease in St occurs, quantitative agreement with the data shown in Fig. 3 was not as good as for lower values of K, and there were also qualitative differences. The only reasonable modification in the model that yields results in good agreement with the data of Fig. 3 involves forcing the eddy viscosity, and thus eddy conductivity, to zero in the outer part of the boundary layer. Since the shear stress, τ, is very low in the outer half of a highly accelerated boundary layer the damping function, D_v, already has the desired characteristic, but the damping is evidently not great enough; the difference between low turbulence and zero turbulence in the outer half of the boundary layer has a negligible influence on the momentum equation, and the resulting velocity profile, but a decisive influence on the energy equation. For $K > 1.5 \times 10^{-6}$ this damping is accomplished in the program by artificially reducing τ to zero in the equation for D_v at an appropriate point. It is emphasized, however, that this artifice is used for only one of the nine runs to be shown, and further investigation of this region of high K, where complete laminarization is certainly near, is definitely needed.

Some sample results of predictions based on the above described model are shown in Figs. 9, 10, and 11. The imposed boundary conditions correspond closely in each case to the test results in Figs. 3 to 6.

With the exception of the region just following the start of acceleration, the agreement with the experiments is, in every case, quite good. The difficulty at the beginning of acceleration is an understandable and easily correctable one. The correlation for A^+ is based on velocity profiles for equilibrium boundary layers, i.e., the asymptotic cases such as shown in Fig. 7. The viscous sublayer of the real boundary layer does not instantaneously assume its new equilibrium configuration when a new pressure gradient is imposed; there is obviously a lag, and detailed examination of the experimental data shows this lag very clearly. Launder and Jones (12) propose a reasonable and simple scheme for introducing such a lag into the calculation, and the authors intend to investigate this scheme shortly.

The most spectacular success of the prediction method, setting aside the problem just discussed, is shown on Fig. 11. Here the unexpected increase in Stanton number with a mild acceleration, seen experimentally on Fig. 5, comes through very clearly.

The main conclusion which can be drawn from the results of the analysis is that the primary effects of acceleration, transpiration, and a combination of both, can be introduced into the analysis merely through the constant in the Van Driest damping term, A^+ . If A^+ is evaluated properly, everything else follows. The fact that Stanton number sometimes increases and sometimes decreases with acceleration is merely attributable to the response of the boundary layer equations to the imposed conditions.

Summary and Conclusions

In this paper experimental data on heat transfer to close to asymptotic accelerated turbulent boundary layers, with and without transpiration, have been presented. It is shown that acceleration causes a depression in Stanton number for the case of no transpiration, and for suction. For an accelerated boundary layer with blowing it is shown that acceleration can cause an increase in Stanton number under certain conditions.

Examination of velocity and temperature profiles suggest that acceleration causes an increase in the thickness of the viscous sublayer. It has been shown earlier that blowing causes a decrease in sublayer thickness, while suction increases thickness. Acceleration can cause the momentum thickness Reynolds number to decrease, and an acceleration at a constant value of the parameter K will lead to a constant value of momentum thickness Reynolds number. The enthalpy thickness Reynolds will always increase, however, (except for strong suction), with the result that prolonged acceleration will lead to penetration of the thermal boundary layer beyond the momentum boundary layer. The decrease in Stanton number observed for accelerated boundary layers is believed to result from a combination of the effects of a thicker sublayer and a thermal boundary layer penetrating beyond the momentum boundary layer.

Finally, a mathematical model based on the Van Driest mixing-length hypothesis, and incorporating the observed effects of acceleration and transpiration on the sublayer thickness, is shown to be capable of quite satisfactorily reproducing the experimental data for accelerations up to $K = 2.5 \times 10^{-6}$, and a wide range of blowing or suction. Means for improving the model are discussed.

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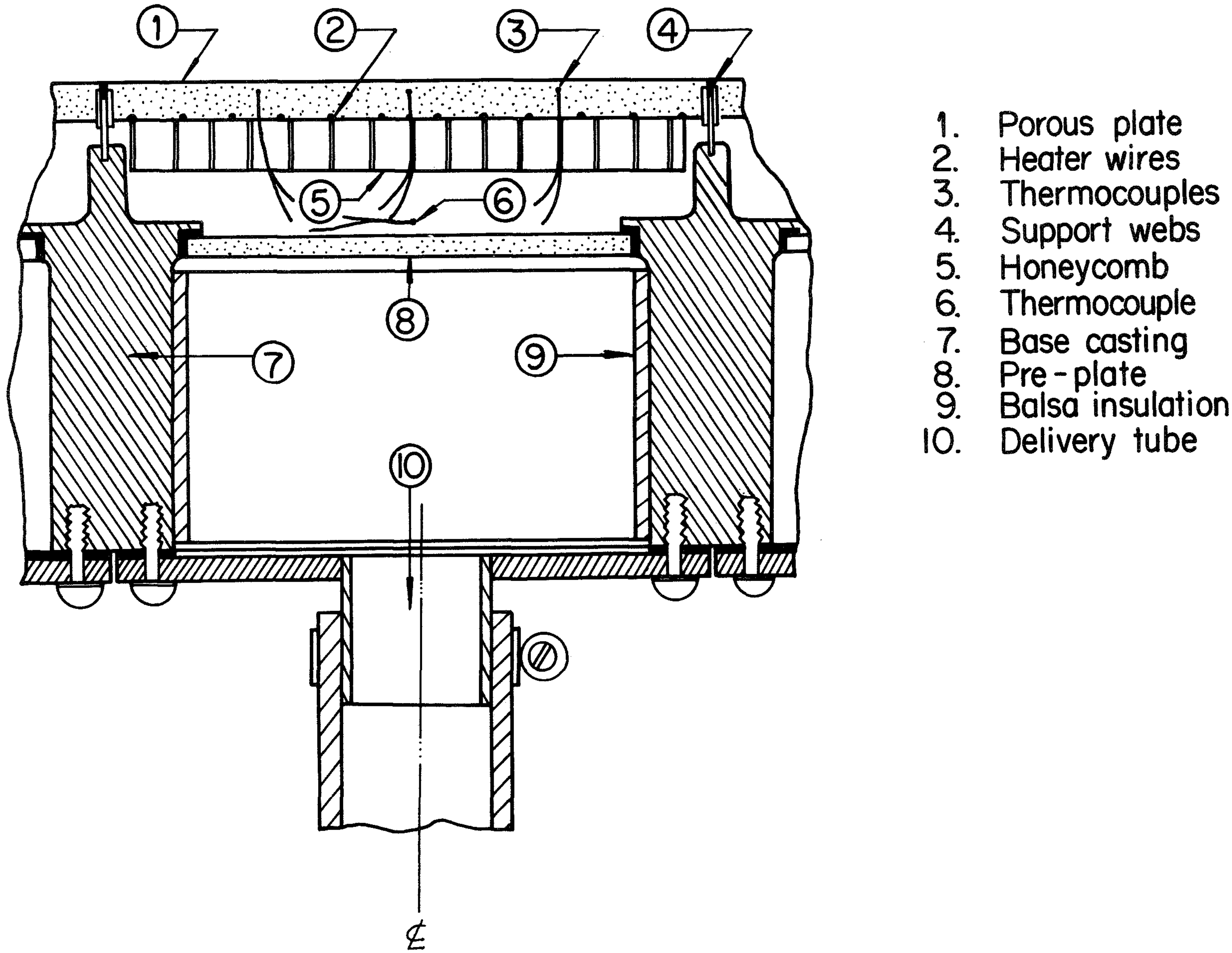

Fig. 1 - A segment of the porous test plate showing transpiration system and plate heating system.

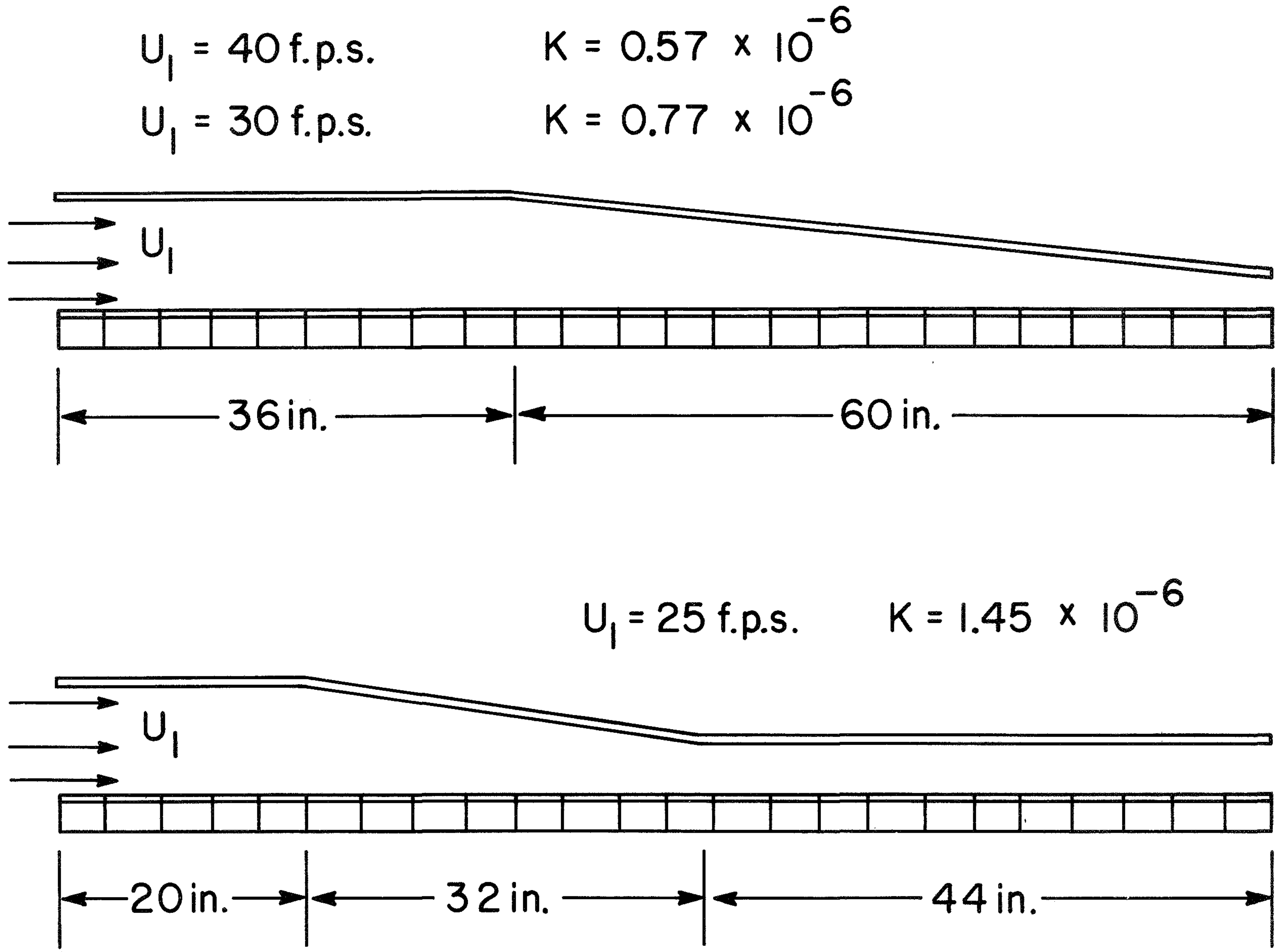

Fig. 2 - Arrangement of top covers of test duct to obtain constant K acceleration.

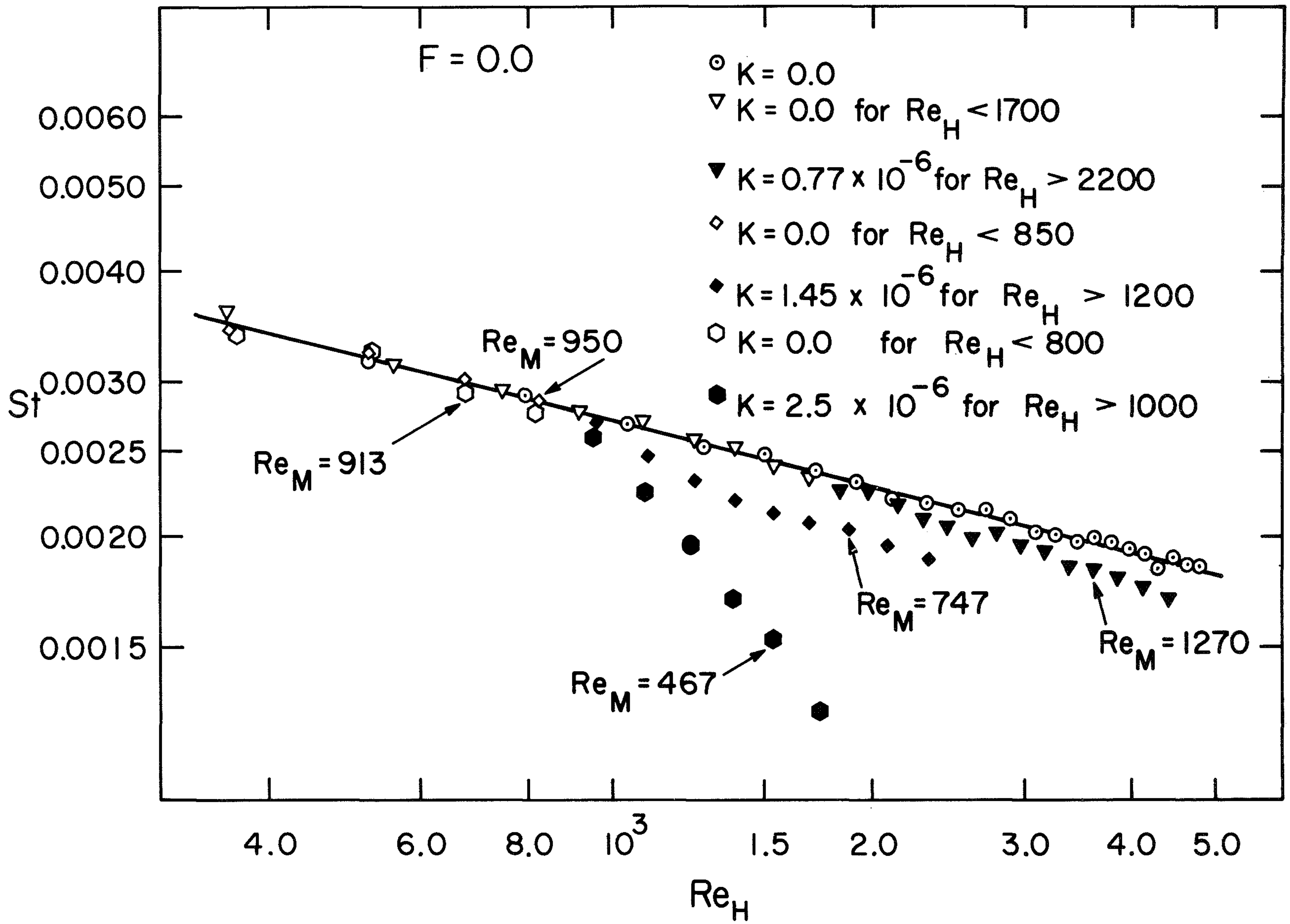

Fig. 3 - Heat transfer results for four values of the acceleration parameter, K , for no transpiration, F = 0.0 .

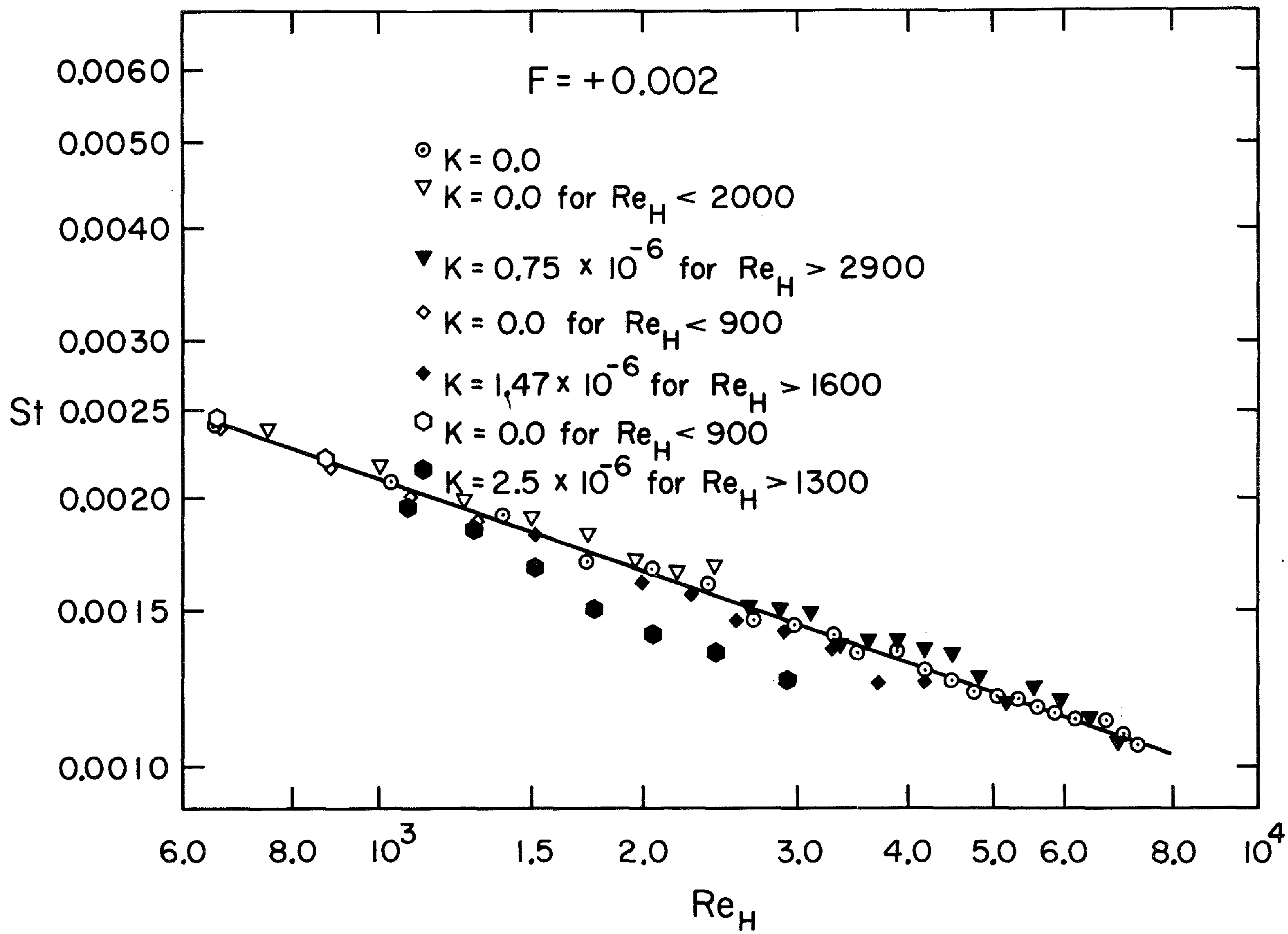

Fig. 4 - Heat transfer results for four values of the acceleration parameter, K, for moderately strong blowing, $F = +0.002$.

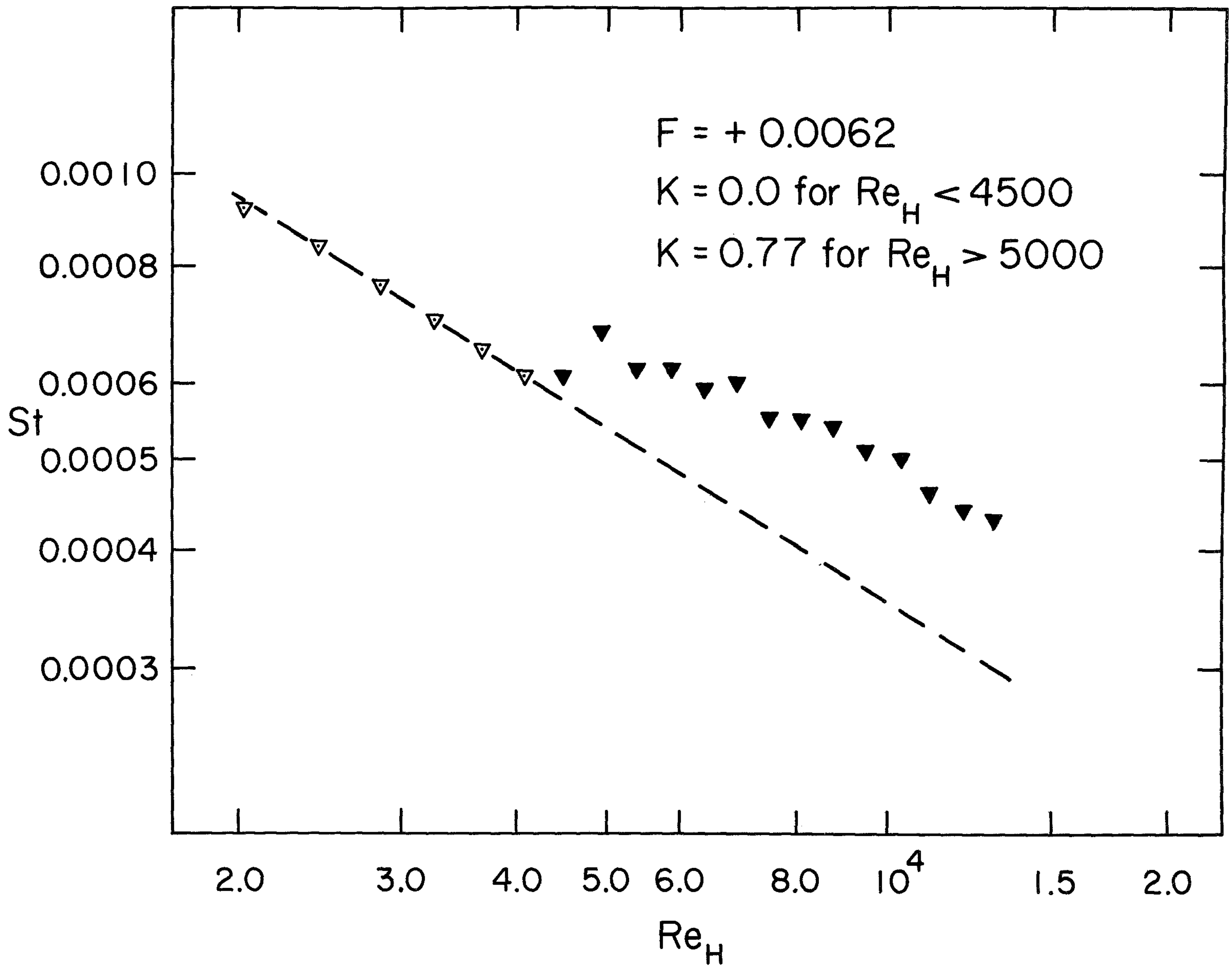

Fig. 5 - Heat transfer results for a case of strong blowing, and moderate acceleration.

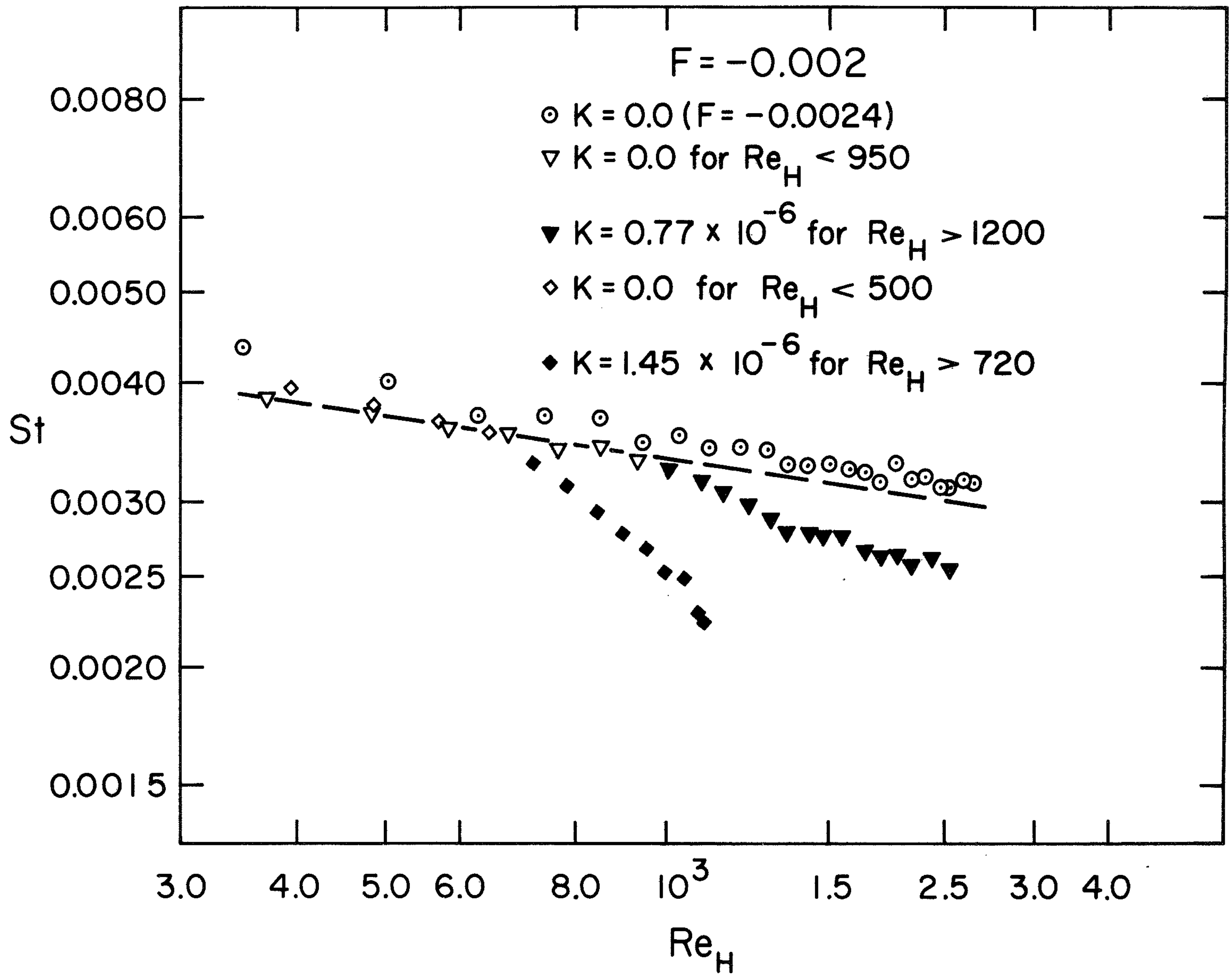

Fig. 6 - Heat transfer results for three values of the acceleration parameter, K, for moderately strong suction, $F = -0.002$.

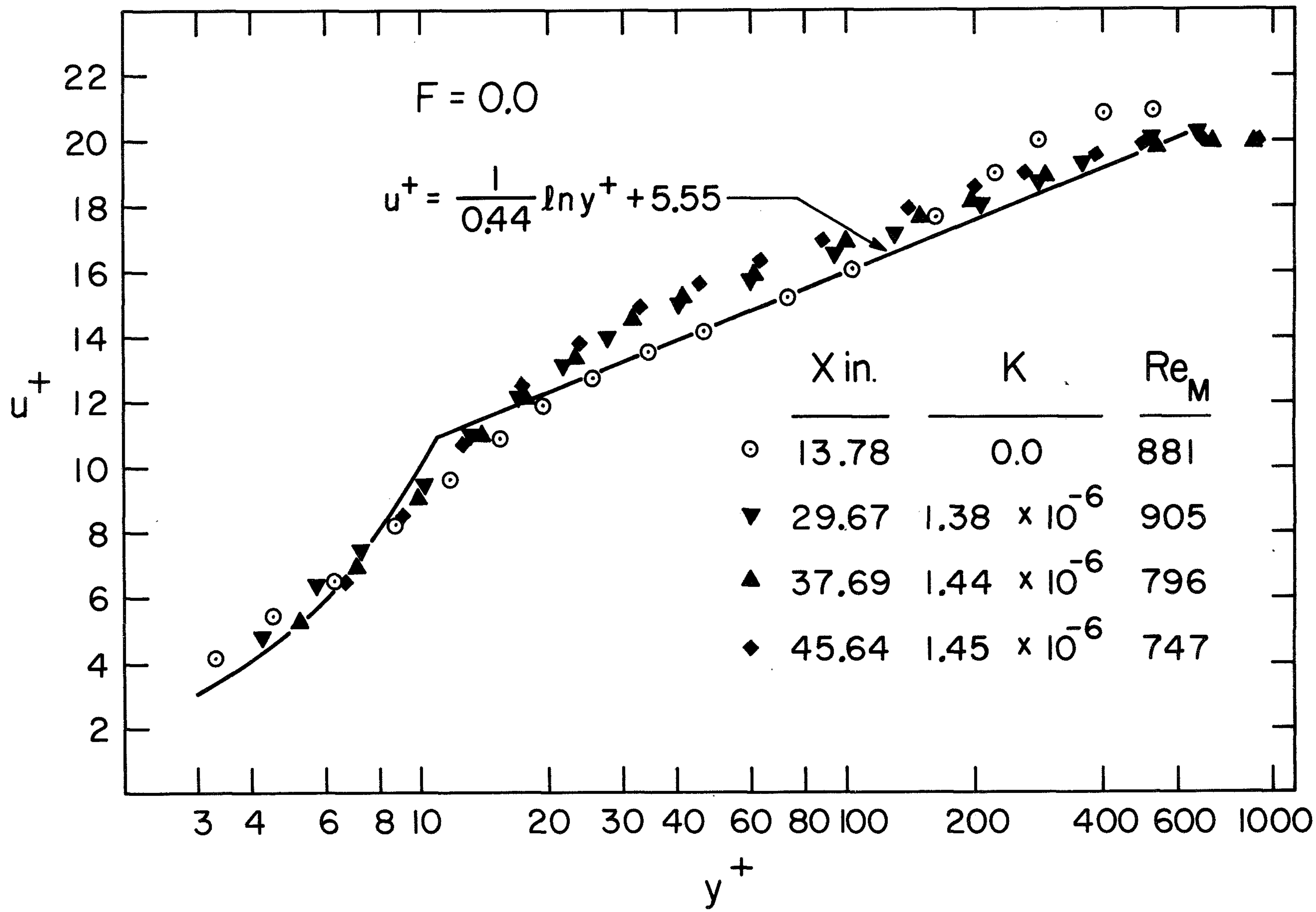

Fig. 7 - Velocity profiles for a nearly asymptotic boundary layer at approximately $K = 1.45 \times 10^{-6}$, $F = 0.0$.

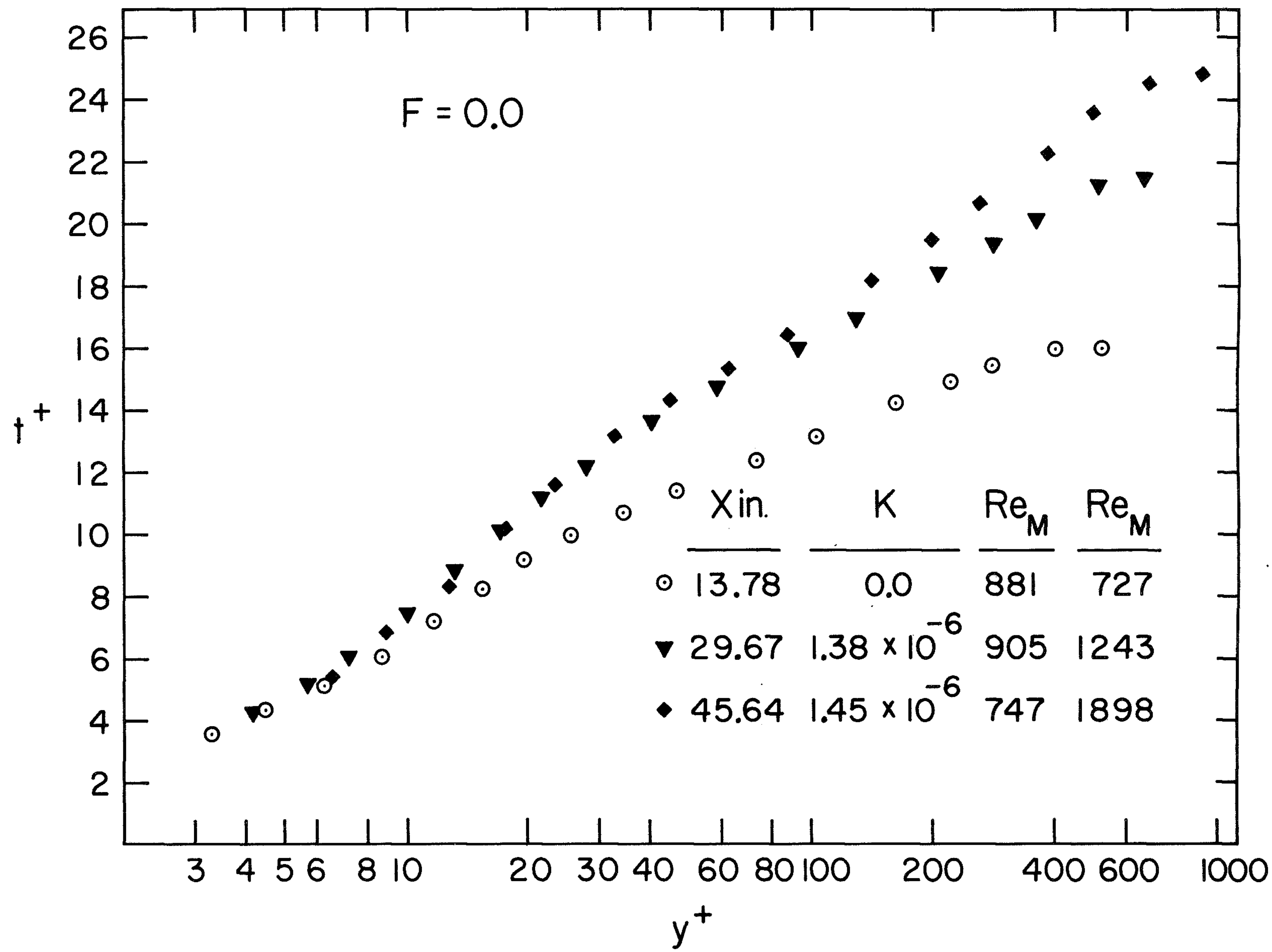

Fig. 8 - Temperature profiles for the same case shown in Fig. 7 .

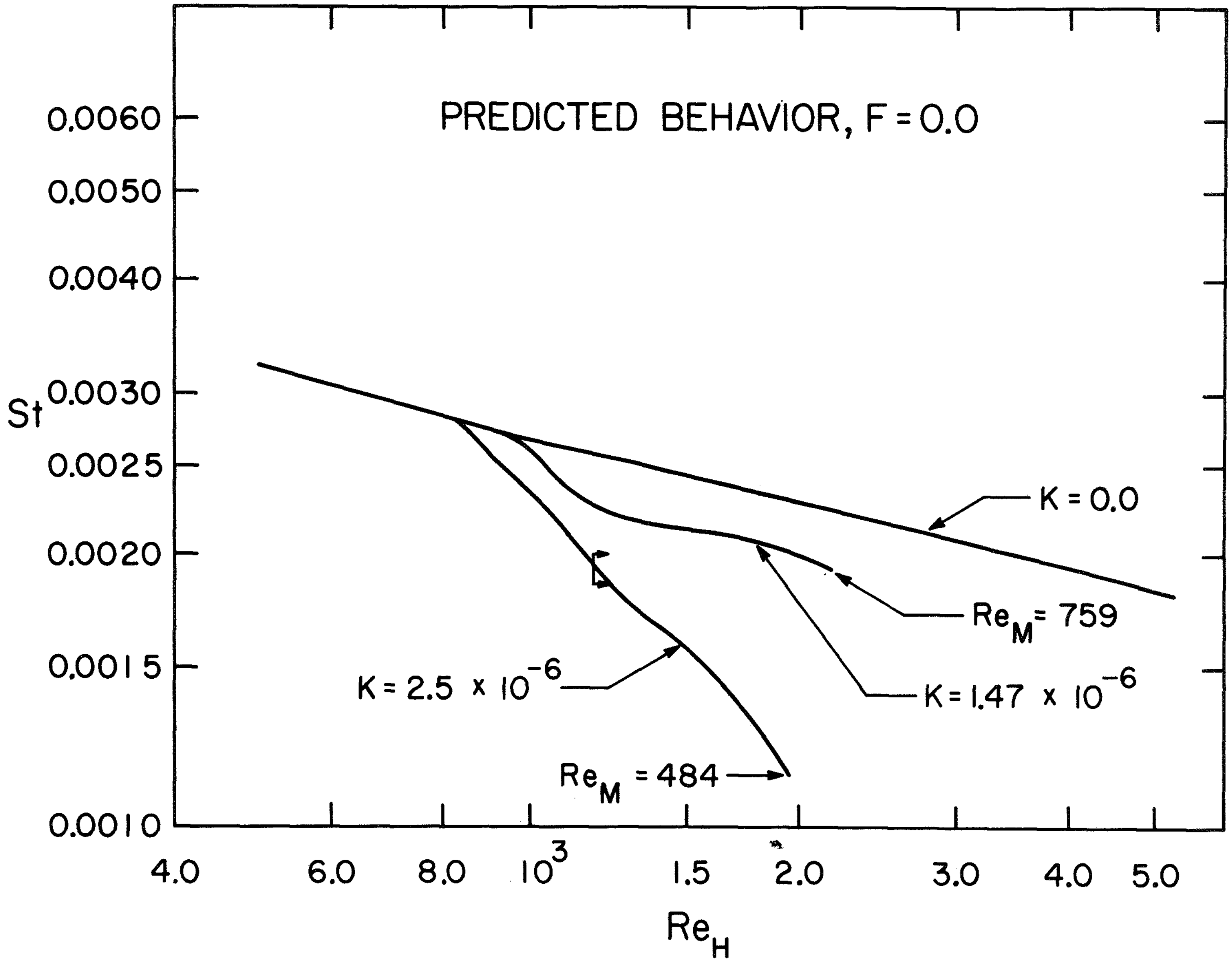

Fig. 9 - Predicted heat transfer performance for three values of K, for no transpiration, $F = 0.0$.

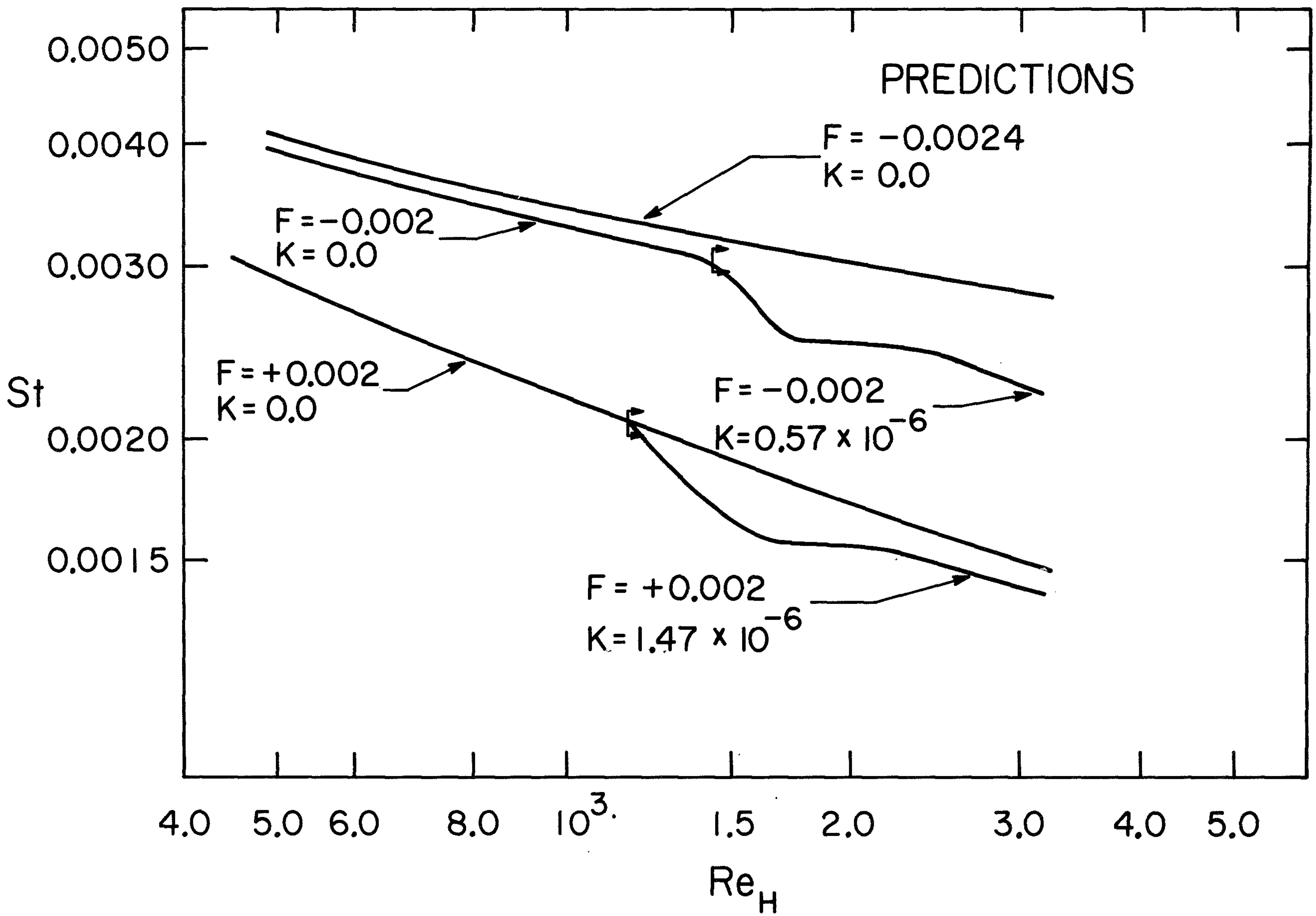

Fig. 10 - Predicted heat transfer performance showing some effects of blowing, suction, and combinations of transpiration and acceleration.

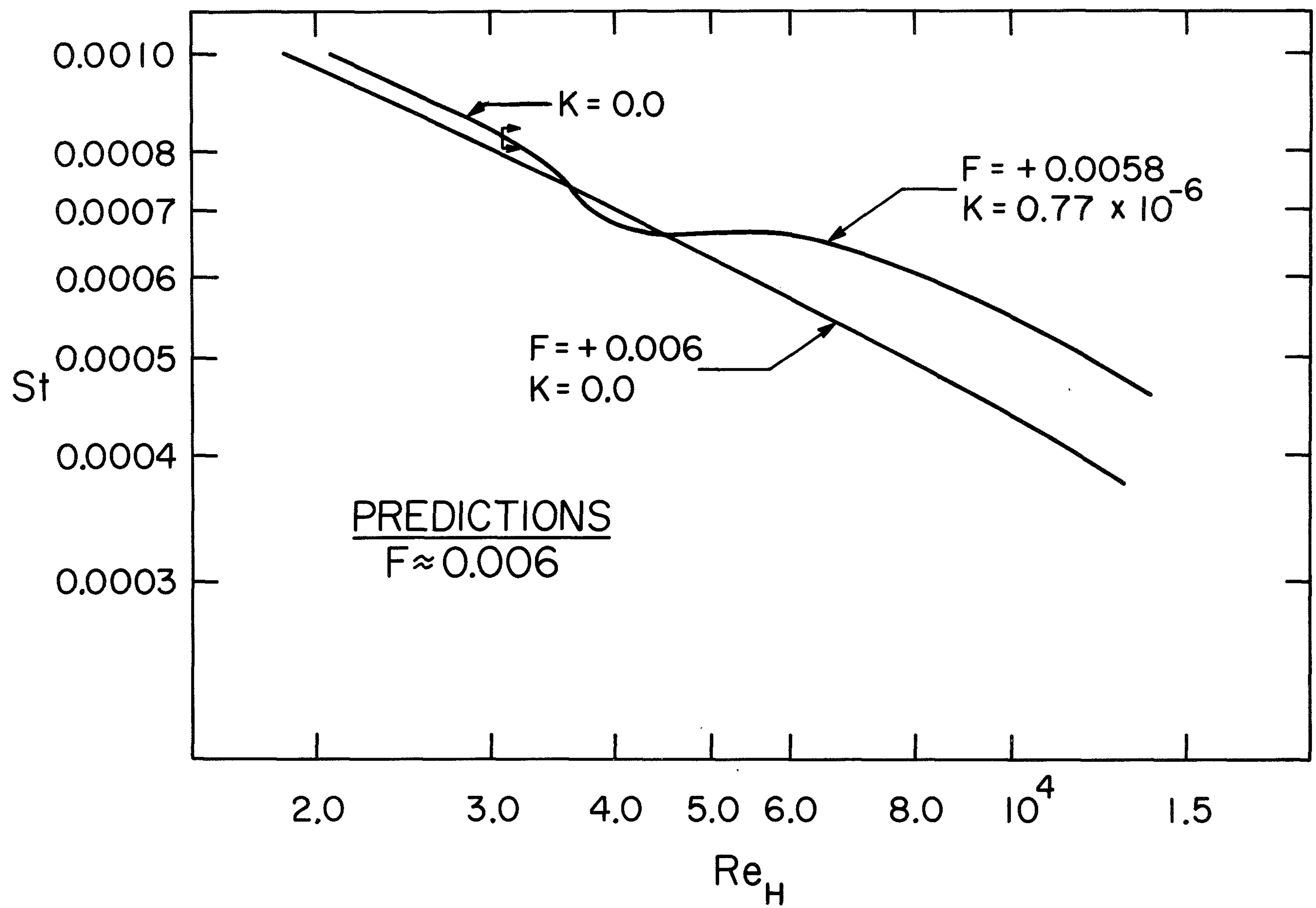

Fig. 11 - Predicted heat transfer performance for a case of strong blowing, and moderate acceleration, corresponding approximately to the conditions of Fig. 5 .