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HEATING OF THE SOLAR CORONA BY THE RESONANT ABSORPTION OF ALFVEN WAVES

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INTRODUCTION

It has been suggested that resonance absorption of low frequency magnetohydrodynamic waves could be responsible for heating the solar corona (Ionson, 1982; Hollweg, 1984). In this paper an improved method for calculating the resonance absorption heating rate is discussed and the results are compared with observations in the solar corona. To accomplish this, the wave equation for a mildly dissipative, compressible plasma is derived from the linearized magnetohydrodynamic equations for a plasma with transverse Alfven speed gradients. For parameters representative of the solar corona, it is found that a two scale description of the wave motion is appropriate. The large scale motion, which can be approximated as nearly ideal, has a scale which is on the order of the width of the loop. The small scale wave, however, has a transverse scale much smaller than the width of the loop, 0.3-250 km, and is highly dissipative. These two wave motions are coupled in a narrow resonance region in the loop where the global wave frequency equals the local Alfven wave frequency. Formally this coupling comes about from using the method of matched asymptotic expansions to match the inner and outer (small and large scale) solutions. The resultant heating rate can be calculated from _either_ of these solutions. A formula derived using the outer (ideal) solution is presented, and shown to be consistent with observations of heating and line broadening in the solar corona.

DERIVATION OF THE BASIC EQUATIONS

The linearized momentum and induction equations can be written as

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{\nabla p}{\rho_0} + \frac{1}{4\pi\rho_0}\left[(\mathbf{B}_0\cdot\nabla)\,\mathbf{B} + (\mathbf{B}\cdot\nabla)\,\mathbf{B}_0 \right] + \beta^2\nabla^2\mathbf{v} + \gamma^2\nabla(\nabla\cdot\mathbf{v}) \qquad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = (\mathbf{B}_0\cdot\nabla)\mathbf{v} - \mathbf{B}_0(\nabla\cdot\mathbf{v}) - (\mathbf{v}\cdot\nabla)\,\mathbf{B}_0 + \alpha^2\nabla^2\mathbf{B} \qquad (2)$$

where $\alpha^2 = c^2/4\pi\sigma$, $\beta^2 = \eta\perp/\rho_0$ and $\gamma^2 = \eta_{\|}/\rho_0$. The functional forms for the viscosity coefficients, η, and the electrical conductivity, σ, are given in Braginskii (1965). By neglecting derivatives of the dissipation coefficients themselves, it is implicitly assumed in the equations above that dissipation processes are important only in a narrow layer, and that over this layer terms which are proportional to derivatives of the dissipation coefficients can be neglected. This is easily verified *a posteriori*. It is further assumed that all variations in $\mathbf{B}_0$ and ρ_0 are transverse to the field so that $v_A = v_A(x)$

only. The plasma is assumed to have $\beta \ll 1$ so that the transverse gradients of $\boldsymbol{B}_0$ can be neglected and the total pressure p can be approximated as $p = B_0 B_z/4\pi$. Take the time derivative of the momentum equation and use the induction equation to eliminate all derivatives of $\boldsymbol{B}$. Since dissipation is assumed to be small, one can neglect terms which are proportional to products of the dissipation coefficients. Assume that the loop is driven at z = 0 and that at z = L there is a perfectly reflecting boundary (see Hollweg, 1984 for more discussion on this point). Fourier transform the equation using

$$\mathbf{v}_\perp(x,y,z,t) = \frac{2}{L}\sum_{n=0}^{\infty} Cos\,(k_{zn}\,z)\int_{-\infty}^{\infty}\frac{d\,\omega}{2\pi}\,e^{-i\omega t}\,\mathbf{v}_{\perp n}(x,y,\omega) \qquad (3)$$

This form automatically satisfies the boundary condition at z = L as long as $k_{zn} = (2n + 1)\pi\ /\ (2L)$ with n = 0, 1, 2,... . Define the following dimensionless parameters $V_A{}^2(x) = v_A{}^2(x)/v_{A0}$, $R = v_{A0}/\omega\alpha^2$, $Pr_\parallel = \beta^2/\alpha^2$, and $Pr_\perp = \gamma^2/\alpha^2$, where v_{A0} is some typical value of the Alfven speed within the loop, R is the magnetic Reynolds number and $Pr_\perp$ and $Pr_\parallel$ the magnetic Prandlt numbers associated with shear and compressional viscosity. With these definitions, all lengths measured in units of d the transverse scale of the loop, and $\kappa^2(x) = (\omega^2 d^2/v_A{}^2(x) - k_z{}^2)$, the equation for the velocity can be written in the dimensionless form

$$\nabla_\perp(\nabla\cdot\mathbf{v}_\perp) + \kappa^2\mathbf{v}_\perp = \frac{i}{R\,V_A{}^2(x)}\left\{\frac{1}{\rho_0}\nabla^2\rho_0\mathbf{v}_{\perp n} + Pr_\perp\nabla^2\mathbf{v}_{\perp n} + Pr_\parallel\nabla_\perp(\nabla\cdot\mathbf{v})\right\} \quad (4)$$

The terms on the left hand side of this equation are the terms obtained from ideal MHD. Solutions of the ideal equation for various forms of the Alfven speed profile have been discussed by several authors in the context of solar coronal heating (Ionson, 1982; Rae and Roberts 1982) and in the context of solar wind acceleration in coronal holes by Davila (1985). These investigations have demonstrated that this equation describes the propagation of MHD surface waves and guided wave modes in an inhomogeneous plasma.

The terms on the right are all due to dissipation and are therefore non-ideal MHD terms. In the solar corona these terms are "small" in most locations since they are all multiplied by the inverse of the magnetic Reynolds number R which is on the order of 10^{12} for typical solar parameters. The exception to this ordering is at the location $x = x_A$ where $\kappa^2(x_A) = 0$. At this location the first term on the left can only be balanced by the dissipative terms on the right even though they are small. Sufficiently far from x_A, in the outer region, the ideal MHD solution is a reasonable approximation to the actual solution. However, near x_A, in the inner region, the character of the solution changes (the equation changes from second to fourth order) and the assumption of ideal MHD is not valid. This is a classic example of a singular perturbation (Nayfeh, 1981). In the following paragraphs singular perturbation theory and the method of matched asymptotic expansions will be used to obtain a solution which is valid both outside and inside the resonance layer. To do this one must first reduce the coupled equations for the vector components of the velocity to a single equation for one velocity component, say v_x.

To illustrate the basic idea of the resonance absorption layer and to demonstrate the matched asymptotic solution method, let us assume that the dominant dissipation mechanism is shear viscosity. This assumption cannot be completely justified at this point so it must be regarded simply as an *ansatz*. An investigation which incorporates ohmic, compressive viscous and shear viscous dissipation self consistently in a numerical solution is currently underway. The results of this investigation will be published when they become available. Under these assumptions the wave equation can be written

$$\nabla_{\perp}(\nabla\cdot\mathbf{v}_{\perp_n}) + \kappa^2\mathbf{v}_{\perp_n} = i\epsilon\ \nabla^2\mathbf{v}_{\perp_n} \tag{5}$$

where $\epsilon = (1 + \mathrm{Pr}\perp)/\ (R\ V_A{}^2)$. An equation for v_{xn} correct to first order in ϵ can be obtained for the apppropriate ordering $1 \gg k_y{}^2 \gg \kappa_z{}^2 \gg \epsilon k_y{}^2$

$$\frac{d}{dx}\frac{\kappa^2}{k_y{}^2}\frac{dv_{xn}}{dx} - \kappa^2 v_{xn} = \frac{i\epsilon}{k_y{}^2}\frac{d^4 v_{xn}}{dx^4} \tag{6}$$

The outer solution is obtained by expanding the velocity, v_{xn}, as a power series in the small parameter ϵ. The lowest order term must then satisfy

$$\frac{d}{dx}\frac{\kappa^2}{k_y^2}\frac{dv_{xn}^{(0)}}{dx} + \kappa^2 v_{xn}^{(0)} = 0 \tag{7}$$

Detailed solutions of this equation have been obtained before (Rae and Roberts, 1982; and Lee and Roberts, 1986). For our purposes it is only necessary to obtain the solution near the resonance layer, i.e. where $x \to x_A$ with x_A defined by the resonance condition $\kappa^2(x_A) = 0$. In this region, the first term dominates and the solution can be approximated as

$$v_{xn}^{(0)} \approx A\ ln\ (x - x_A) \tag{8}$$

where A is the wave amplitude determined by matching boundary conditions at the driver.

The inner solution can be obtained by considering the scale stretching transformation given by $\zeta = (x - x_A)/a$, where a is the small scale parameter to be determined in the problem. For the case presented here $a^3 = \epsilon/\lambda$ where $\lambda = -k_z{}^2\ d(\ln\ \rho)/dx$. Using the shear viscosity and resistivity coefficients given in Braginskii one can estimate $a \approx 0.3$ km. This is below the resolution of current instruments. Using the largest viscosity coefficient in Braginskii one can estimate $a_{max} \approx 250$ km. If this second estimate is more nearly correct, these sheets could be observed with instruments with resolution on the order of 0.1 arc second. Using the transformation described above, the inner equation can be written

$$\frac{d}{d\zeta}\left[\frac{d^2}{d\zeta^2} - i\zeta\right]\frac{dv_{xn}}{d\zeta} = -ik_y^{\,2}a^2\zeta v_{xn} \qquad (9)$$

Assume a power law expansion of v_{xn} with a^2 as the expansion parameter, then the solution of the zeroth order equation is

$$v_{xn}^{(0)}(\zeta) = C_1 - C_2\int_0^\infty dp\ \exp\left(-\frac{p^3}{3}\right)\left[\frac{1 - \exp(-ip\zeta)}{ip}\right] \qquad (10)$$

In the limit as $\zeta \to \infty$ it can be shown that

$$v_{xn}(\zeta) \approx -iC_2\ \ln(x - x_A) \qquad (11)$$

This shows that by proper choice of the constants, namely $C_2 = iA$, the inner and outer solutions match as is physically required.

Although in the steady state the details of the velocity profile within the resonance layer do depend on the dissipation mechanism (eqn 10), the heating rate does not. The result presented below is obtained by considering the outer solution and integrating the Poynting flux over the entire surface of the resonace layer (Chen and Hasagawa, 1974). An equivalent result could be obtained from the inner solution by integrating the volumetric heating rate of Braginskii (1965) over the volume of the resonance layer. The result is

$$H = A_s\frac{B_0^{\,2}}{8\pi\omega d}\left[-\frac{1}{k_y^{\,2}}\frac{d(\kappa^2)}{dx}\right]|A|^2\ Im\left\{\ \ln^*(x - x_A + i\phi)\right\}\Bigg|_{x_1}^{x_2} \qquad (12)$$

where x_1 and x_2 denote the positions of the two surfaces of the resonance layer and A_s is the surface area of the resonance layer. If we consider the limit of this equation as $(x_1 - x_2) \to 0$ and use causality to choose the proper analytic continuation (McPherson and Pridmore-Brown, 1966; Kapraff and Tataronis, 1977; Mok and Einaudi, 1985; Einaudi and Mok, 1985; Bertin et al, 1986) the resulting heating rate is

$$H = A_s\frac{B_0^{\,2}}{8\ \pi\omega d}\frac{k_z^{\,2}}{k_y^{\,2}}\ \pi\ |A|^2 \qquad (13)$$

To determine whether this heating rate is consistent with observational constraints, let us simply equate it to the observed radiation rate in soft x-rays to obtain an expression for the RMS velocity amplitude required to explain the observed emission. Using typical values of the parameters, B_0=100 G, d=5×10^8 cm, $\Lambda(T)=10^{-22}$ (Rosner et al. 1978), P=300 sec, $k_y=2k_z$ and $\int n_e^{\,2}dl$ = 10^{28-29} (Webb et al. 1986) one obtains an estimate of V_{rms} = 2-6 km/sec using the following expression.

$$V_{rms} = \left(\frac{\Lambda(T) \frac{2\pi d}{P} \int n_e^2 \, dl}{\pi \, (B_0^2/8\pi) \, (k_z^2/k_y^2)} \right)^{1/2} \tag{14}$$

This is comparable to the observed value of 10-20 km/sec regularly seen from observations of non-thermal line broadening in the corona.

CONCLUSIONS

The primary conclusion to be drawn from these calculations is that to the level of the approximation adopted here, the observations of the heating rate and non-thermal line broadening in the solar corona are consistent with heating by the resonance absorption mechanism. This basic agreement is gratifying but several problems remain. The plane symmetry assumed here is highly idealized. It has been shown in the plasma physics literature that for the tokamak problem introducing cylindrical symmetry has presented no new physics. Nevertheless, when considering the heating rate to an accuracy of say factors of 2-5 the geometry factors must be properly accounted for. Second, observations of the turbulent power spectrum at the base of the corona are badly needed as input for the theory. These observations should be carried out in ions which are present at or above the transition region temperatures. It seems that EUV observations would be the most appropriate. Finally, other sources of dissipation must be considered. For although the heating rate is independent of the dissipation mechanism for any reasonable value of the coefficients, the amplitude of the velocity inside the resonance layer, and the width of the layer, both depend on the magnitude of the dissipation coefficient. High resolution instruments such as POF may be able to observe velocities within the narrow resonace regions in the reasonably near future. Therefore it is worthwhile to consider theoretically the observational consequences of various dissipation mechanisms now. In the work presented here, shear viscosity was assumed to be the dominant dissipation mechanism. This is not necessarily the case for solar conditions.

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