LIGHT MESON SPECTROSCOPY AND REGGE TRAJECTORIES IN THE RELATIVISTIC QUARK MODEL

Rudolf Faustov

Dorodnicyn Computing Centre RAS, Moscow

(in collaboration with Dietmar Ebert and Vladimir Galkin)

Ebert, Faustov, Galkin — Eur. Phys. J.C 60, 273-278 (2009)
Ebert, Faustov, Galkin — Phys. Rev. D 79, 114029 (2009)

OUTLINE

1. Introduction
2. Relativistic quark model
3. Mass spectra of light quark-antiquark mesons
4. Regge trajectories of light mesons
5. Masses of light tetraquarks in the diquark-antidiquark picture
6. Masses of light scalar mesons

INTRODUCTION

• Vast amount of experimental data on light meson with masses up to 2500 MeV is available. $\Longrightarrow$ The classification of these new data requires a better theoretical understanding of light meson mass spectra.

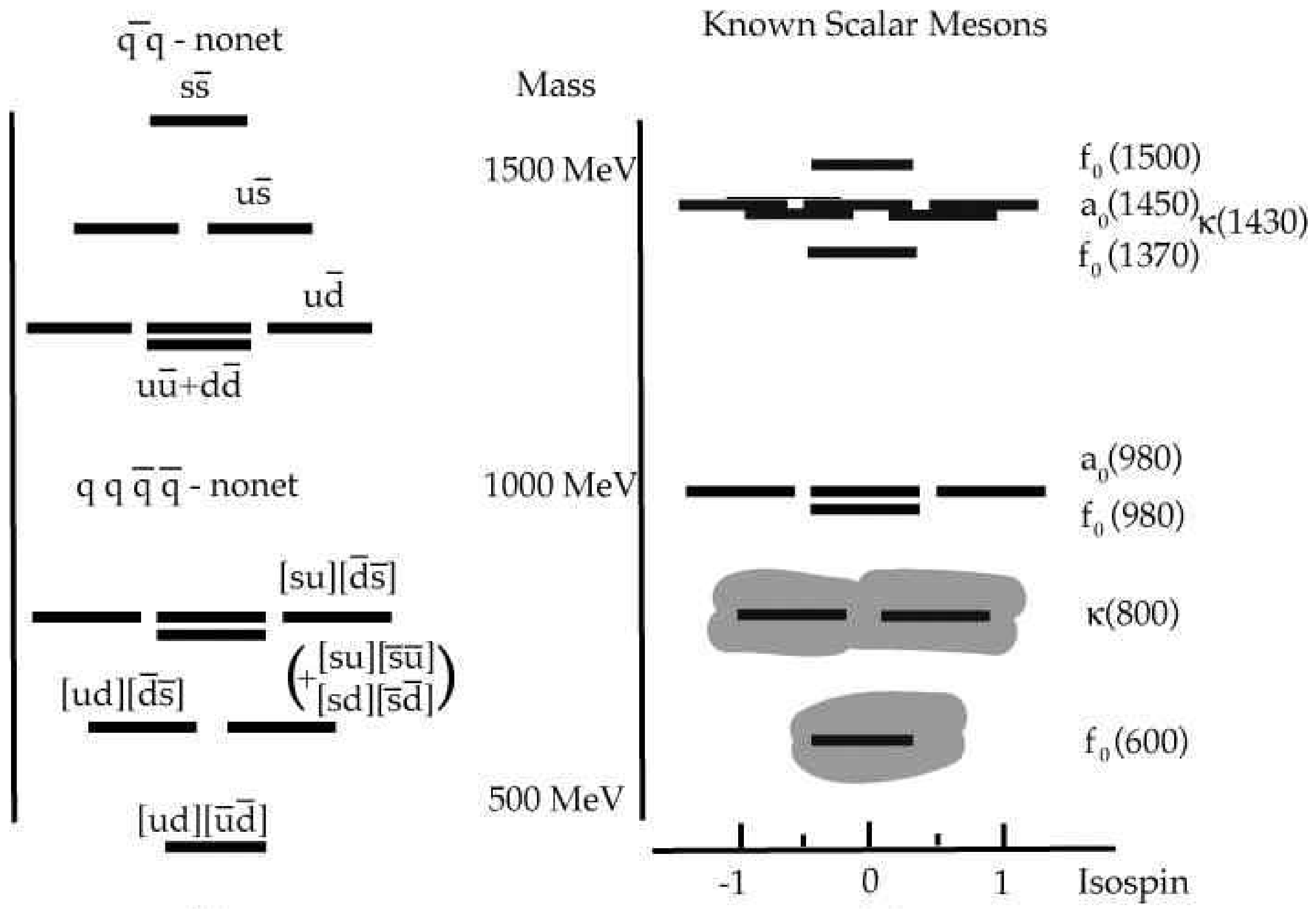
• Light exotic states (such as tetraquarks, glueballs, hybrids) predicted by quantum chromodynamics (QCD) are expected to have masses in this range.

• It is argued by Glozman et al. that the states of the same spin with different isospins and opposite parities are approximately degenerate in the interval 1700-2400 MeV. An intensive debate is going on now in the literature about whether the chiral symmetry is restored for highly excited states.

• Renewed interest to the Regge trajectories both in (M^2,J) and $(M^2,\ n_r)$ planes (M is the mass, J is the spin and n_r is the radial quantum number of the meson state): their linearity, parallelism and equidistance. $\Longrightarrow$ Assignment of experimentally observed mesons to particular Regge trajectories.

• Problem of scalar mesons:

- Abundance and peculiar properties of light scalars
- Experimental and theoretical evidence for the existence of $f_0(600)(\sigma)$, $K_0^*(800)(\kappa)$, $f_0(980)$ and $a_0(980)$ indicates that lightest scalars form a full $SU(3)$ flavour nonet.
- Inversion of the mass ordering of light scalars, which cannot be naturally understood in the $q\bar{q}$ picture. $\Longrightarrow$ Various alternative interpretations:
  ⋆ four-quark states (tetraquarks) and in particular diquark-antidiquark bound states
  ⋆ proximity of f_0/a_0 to the $K\bar{K}$ threshold led to the $K\bar{K}$ molecular picture.



Comparison of a traditional ideally mixed $q\bar{q}$ nonet of light mesons (like vector mesons) with the scalar diquark-antidiquark nonet and experimentally known light scalar mesons. Diquarks are considered in the colour antitriplet state.

RELATIVISTIC QUARK MODEL

Relativistic quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

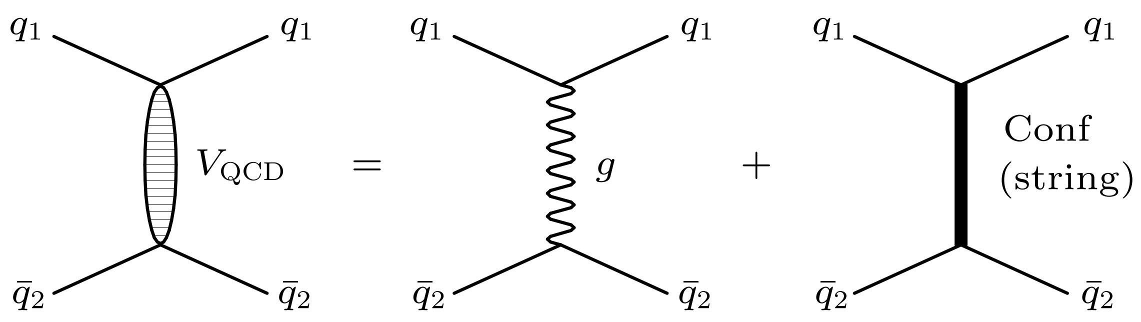
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from heavy meson sector

- **$q\bar{q}$ quasipotential**



$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{pmatrix} \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V^V_{\rm conf} + V^S_{\rm conf}$

In nonrelativistic limit

$$\left.\begin{array}{rcl} V^V_{\rm conf} & = & (1-\varepsilon)(Ar+B) \\ V^S_{\rm conf} & = & \varepsilon(Ar+B) \end{array}\right\} \quad \text{Sum :} \quad (Ar+B)$$

ε - mixing parameter

$$V_{\rm Coul}(r) = -\,\frac{4}{3}\,\frac{\alpha_s}{r}$$

$$V_{\rm Cornell}(r) = -\,\frac{4}{3}\,\frac{\alpha_s}{r} + Ar + B$$

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ${}^3P_J$ states and HQET

$(1 + \kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction (flux tube model)

Freezing of α_s for light quarks

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f, \quad \mu = \frac{2 m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

- **Light tetraquarks in diquark-antidiquark picture**

(qq')-interaction: $$V_{qq'} = \frac{1}{2}V_{q\bar{q}'}$$

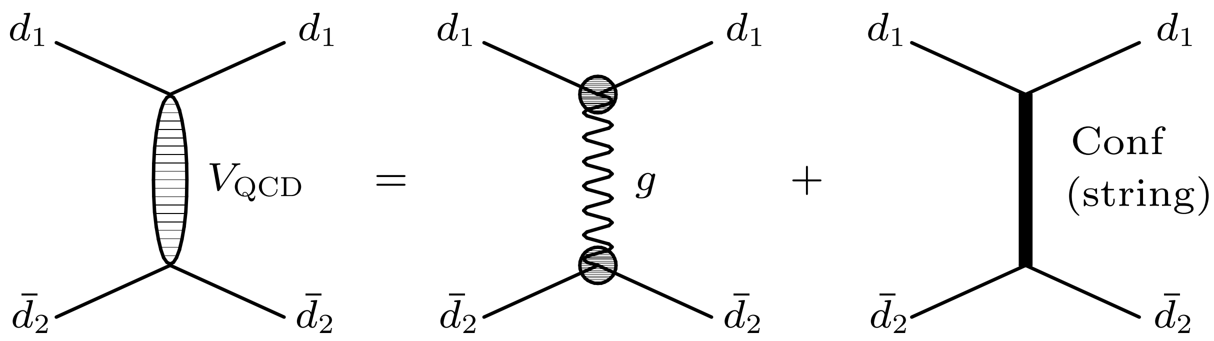
$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{2}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + \frac{1}{2}V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + \frac{1}{2}V^S_{\rm conf}(\mathbf{k})$$

$(d_1\bar{d}_2)$-interaction: $$d = (qq')$$

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d_1(P)|J_\mu|d_1(Q)\rangle}{2\sqrt{E_{d_1}E_{d_1}}}\frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k})\frac{\langle d_2(P')|J_\nu|d_2(Q')\rangle}{2\sqrt{E_{d_2}E_{d_2}}}$$
$$+\psi^*_{d_1}(P)\psi^*_{d_2}(P')\left[J_{d_1;\mu}J^{\mu}_{d_2}V^V_{\rm conf}(\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\right]\psi_{d_1}(Q)\psi_{d_2}(Q'),$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ -\dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark } (\mu_d = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})^\nu_\mu = -i(g_{\mu\rho}\delta^\nu_\sigma - g_{\mu\sigma}\delta^\nu_\rho)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction

MASSES OF LIGHT QUARK-ANTIQUARK MESONS

The quasipotential of $q\bar{q}$ interaction is extremely nonlocal in configuration space for arbitrary quark masses. To make it local

- ⋆ heavy quarks: nonrelativistic v/c or heavy quark $1/m_Q$ expansion
- ⋆ light quarks: highly relativistic, substitution

$$\epsilon_q(p) \equiv \sqrt{m_q^2 + \mathbf{p}^2} \to E_q = \frac{M^2 - m_{q'}^2 + m_q^2}{2M}$$

$q\bar{q}$ potential

$$V_{q\bar{q}}(r) = V_{\rm SI}(r) + V_{\rm SD}(r)$$

spin-dependent potential

$$V_{\rm SD}(r) = a_1\,\mathbf{LS}_1 + a_2\,\mathbf{LS}_2 + b\left[-\mathbf{S}_1\mathbf{S}_2 + \frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})\right] + c\,\mathbf{S}_1\mathbf{S}_2 + d\,(\mathbf{LS}_1)(\mathbf{LS}_2)$$

where e.g.

$$c = \frac{2}{3E_1E_2}\Bigg[\Delta\bar{V}_{\rm Coul}(r) + \left(\frac{E_1 - m_1}{2m_1} - (1+\kappa)\frac{E_1 + m_1}{2m_1}\right) \times\left(\frac{E_2 - m_2}{2m_2} - (1+\kappa)\frac{E_2 + m_2}{2m_2}\right)\Delta V^V_{\rm conf}(r)\Bigg]$$

spin-independent potential

$$
\begin{aligned}
V_{\rm SI}(r) = {} & V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)^2}{4(E_1+m_1)(E_2+m_2)} \left\{ \frac{1}{E_1E_2} V_{\rm Coul}(r) \right. \\
& + \frac{1}{m_1m_2} \left(1 + (1+\kappa) \left[(1+\kappa) \frac{(E_1+m_1)(E_2+m_2)}{E_1E_2} \right. \right. \\
& \left. \left. \left. - \left(\frac{E_1+m_1}{E_1} + \frac{E_1+m_2}{E_2} \right) \right] \right) V_{\rm conf}^V(r) + \frac{1}{m_1m_2} V_{\rm conf}^S(r) \right\} \\
& + \frac{1}{4} \left(\frac{1}{E_1(E_1+m_1)} \Delta \tilde{V}_{\rm Coul}^{(1)}(r) + \frac{1}{E_2(E_2+m_2)} \Delta \tilde{V}_{\rm Coul}^{(2)}(r) \right) \\
& - \frac{1}{4} \left[\frac{1}{m_1(E_1+m_1)} + \frac{1}{m_2(E_2+m_2)} - (1+\kappa) \left(\frac{1}{E_1m_1} + \frac{1}{E_2m_2} \right) \right] \Delta V_{\rm conf}^V(r) \\
& + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)}{8m_1m_2(E_1+m_1)(E_2+m_2)} \Delta V_{\rm conf}^S(r) + \frac{1}{E_1E_2} \frac{\mathbf{L}^2}{2r} \bar{V}_{\rm Coul}'(r),
\end{aligned}
$$

Table 1: Masses of excited light ($q = u, d$) unflavored mesons (in MeV).

$n^{2S+1}L_J$	J^{PC}	Theory	Experiment				Theory	Experiment	
		$q\bar{q}$	$I=1$	mass	$I=0$	mass	$s\bar{s}$	$I=0$	mass
1^1S_0	0^{-+}	154	π	139.57			743		
1^3S_1	1^{--}	776	ρ	775.49(34)	ω	782.65(12)	1038	φ	1019.455(20)
1^3P_0	0^{++}	1176	a_0	1474(19)	f_0	1200-1500	1420	f_0	1505(6)
1^3P_1	1^{++}	1254	a_1	1230(40)	f_1	1281.8(6)	1464	f_1	1426.4(9)
1^3P_2	2^{++}	1317	a_2	1318.3(6)	f_2	1275.1(12)	1529	f_2'	1525(5)
1^1P_1	1^{+-}	1258	b_1	1229.5(32)	h_1	1170(20)	1485	h_1	1386(19)
2^1S_0	0^{-+}	1292	π	1300(100)	η	1294(4)	1536	η	1476(4)
2^3S_1	1^{--}	1486	ρ	1465(25)	ω	1400-1450	1698	φ	1680(20)
1^3D_1	1^{--}	1557	ρ	1570(70)	ω	1670(30)	1845		
1^3D_2	2^{--}	1661					1908		
1^3D_3	3^{--}	1714	ρ_3	1688.8(21)	ω_3	1667(4)	1950	φ_3	1854(7)
1^1D_2	2^{-+}	1643	π_2	1672.4(32)	η_2	1617(5)	1909	η_2	1842(8)
2^3P_0	0^{++}	1679			f_0	1724(7)	1969		
2^3P_1	1^{++}	1742	a_1	1647(22)			2016	f_1	1971(15)
2^3P_2	2^{++}	1779	a_2	1732(16)	f_2	1755(10)	2030	f_2	2010(70)
2^1P_1	1^{+-}	1721					2024		
3^1S_0	0^{-+}	1788	π	1816(14)	η	1756(9)	2085	η	2103(50)
3^3S_1	1^{--}	1921	ρ	1909(31)	ω	1960(25)	2119	φ	2175(15)
1^3F_2	2^{++}	1797			f_2	1815(12)	2143	f_2	2156(11)
1^3F_3	3^{++}	1910	a_3	1874(105)			2215	f_3	2334(25)
1^3F_4	4^{++}	2018	a_4	2001(10)	f_4	2018(11)	2286		

Table 1: (continued)

$n^{2S+1}L_J$	J^{PC}	Theory $q\bar{q}$	Experiment $I=1$	mass	$I=0$	mass	Theory $s\bar{s}$	Experiment $I=0$	mass
1^1F_3	3^{+-}	1884					2209	h_3	2275(25)
2^3D_1	1^{--}	1895	ρ	1909(31)			2258	ω	2290(20)
2^3D_2	2^{--}	1983	ρ_2	1940(40)	ω_2	1975(20)	2323		
2^3D_3	3^{--}	2066					2338		
2^1D_2	2^{-+}	1960	π_2	1974(84)	η_2	2030(20)	2321		
3^3P_0	0^{++}	1993	a_0	2025(30)	f_0	1992(16)	2364	f_0	2314(25)
3^3P_1	1^{++}	2039	a_1	2096(123)			2403		
3^3P_2	2^{++}	2048	a_2	2050(42)	f_2	2001(10)	2412	f_2	2339(60)
3^1P_1	1^{+-}	2007	b_1	1960(35)	h_1	1965(45)	2398		
4^1S_0	0^{-+}	2073	π	2070(35)	η	2010(50)	2439		
4^3S_1	1^{--}	2195	ρ	2265(40)	ω	2205(30)	2472		
1^3G_3	3^{--}	2002	ρ_3	1982(14)	ω_3	1945(20)	2403		
1^3G_4	4^{--}	2122	ρ_4	2230(25)	ω_4	2250(30)	2481		
1^3G_5	5^{--}	2264	ρ_5	2300(45)	ω_5	2250(70)	2559		
1^1G_4	4^{-+}	2092					2469		
3^3D_1	1^{--}	2168	ρ	2149(17)			2607		
3^3D_2	2^{--}	2241	ρ_2	2225(35)	ω_2	2195(30)	2667		
3^3D_3	3^{--}	2309	ρ_3	2300(60)	ω_3	2278(28)	2727		
3^1D_2	2^{-+}	2216	π_2	2245(60)	η_2	2248(20)	2662		

Table 1: (continued)

$n^{2S+1}L_J$	J^{PC}	Theory $q\bar{q}$	Experiment $I=1$	mass	$I=0$	mass	Theory $s\bar{s}$	Experiment $I=0$	mass
2^3F_2	2^{++}	2091	a_2	2100(20)	f_2	2141(12)	2514		
2^3F_3	3^{++}	2191	a_3	2070(20)			2585		
2^3F_4	4^{++}	2284			f_4	2320(60)	2657		
2^1F_3	3^{+-}	2164	b_3	2245(50)			2577		
4^3P_0	0^{++}	2250			f_0	2189(13)	2699		
4^3P_1	1^{++}	2286	a_1	2270(50)	f_1	2310(60)	2729		
4^3P_2	2^{++}	2297	a_2	2280(30)	f_2	2297(28)	2734		
4^1P_1	1^{+-}	2264	b_1	2240(35)	h_1	2215(40)	2717		
2^3G_3	3^{--}	2267	ρ_3	2260(20)	ω_3	2255(15)	2743		
2^3G_4	4^{--}	2375					2819		
2^3G_5	5^{--}	2472					2894		
2^1G_4	4^{-+}	2344	π_4	2250(15)	η_4	2328(30)	2806		
5^1S_0	0^{-+}	2385	π	2360(25)	η	2320(15)	2749		
5^3S_1	1^{--}	2491					2782		
1^3H_4	4^{++}	2234	a_4	2237(5)	f_J	2231.1(35)	2634		
1^3H_5	5^{++}	2359					2720		
1^3H_6	6^{++}	2475	a_6	2450(130)	f_6	2465(50)	2809		
1^1H_5	5^{+-}	2328					2706		

- **Important conclusion:** Light scalars below 1 GeV cannot be described as $q\bar{q}$ mesons in our model

- **State mixing**

Using the $\eta - \eta'$ mixing scheme, which accounts for the axial vector anomaly (Feldmann, Kroll, Stech) with mixing angle $\phi = 38^\circ$ and the decay constant ratio $y \equiv f_q/f_s = 0.81$ for our values of $M_{\eta_{s\bar{s}}} = 743$ MeV and the pion mass $M_\pi = 154$ MeV we get

$M_\eta = 573$ MeV $\qquad M_{\eta'} = 989$ MeV

$M_\eta^{\rm exp} = 547.853 \pm 0.0024$ MeV $\qquad M_{\eta'}^{\rm exp} = 957.66 \pm 0.24$ MeV

Strange meson states (L_L) with $J = L$ are the mixtures of spin-triplet (^3L_L) and spin-singlet (^1L_L) states:

$$\begin{aligned} K_J &= K(^1L_L)\cos\varphi + K(^3L_L)\sin\varphi, \\ K'_J &= -K(^1L_L)\sin\varphi + K(^3L_L)\cos\varphi, \qquad J = L = 1, 2, 3\ldots \end{aligned}$$

Mixing angle $\varphi \approx 44^\circ$ for all considered states in our model

Table 2: Masses of excited strange mesons (in MeV).

$n^{2S+1}L_J$	J^P	Theory	Experiment		$n^{2S+1}L_j$	J^P	Theory	Experiment	
		$q\bar{s}$	$I=1/2$	mass			$q\bar{s}$	$I=1/2$	mass
1^1S_0	0^-	482	K	493.677(16)	3^1S_0	0^-	2065		
1^3S_1	1^-	897	K^*	891.66(26)	3^3S_1	1^-	2156		
1^3P_0	0^+	1362	K_0	1425(50)	2^3D_1	1^-	2063		
1^3P_2	2^+	1424	K_2^*	1425.6(15)	2^3D_3	3^-	2182		
$1P_1$	1^+	1412	K_1	1403(7)	$2D_2$	2^-	2163	K_2	2247(17)
$1P_1$	1^+	1294	K_1	1272(7)	$2D_2$	2^-	2066		
2^1S_0	0^-	1538			3^3P_0	0^+	2160		
2^3S_1	1^-	1675	K^*		3^3P_2	2^+	2206		
1^3D_1	1^-	1699	K^*	1717(27)	$3P_1$	1^+	2200		
1^3D_3	3^-	1789	K_3^*	1776(7)	$3P_1$	1^+	2164		
$1D_2$	2^-	1824	K_2	1816(13)	1^3G_3	3^-	2207		
$1D_2$	2^-	1709	K_2	1773(8)	1^3G_5	5^-	2356	K_5^*	2382(24)
2^3P_0	0^+	1791			$1G_4$	4^-	2285		
2^3P_2	2^+	1896			$1G_4$	4^-	2255		
$2P_1$	1^+	1893			2^3F_4	4^+	2436		
$2P_1$	1^+	1757	K_1	1650(50)	$2F_3$	3^+	2348	K_3	2324(24)
1^3F_2	2^+	1964	K_2^*	1973(26)	2^3G_5	5^-	2656		
1^3F_4	4^+	2096	K_4^*	2045(9)	$2G_4$	4^-	2575	K_4	2490(20)
$1F_3$	3^+	2080							
$1F_3$	3^+	2009							

REGGE TRAJECTORIES

a) The (J, M^2) Regge trajectory:

$$J = \alpha M^2 + \alpha_0$$

b) The (n_r, M^2) Regge trajectory:

$$n_r \equiv n - 1 = \beta M^2 + \beta_0,$$

where α, β are the slopes and α_0, β_0 are intercepts.

QCD string with two light quarks at the ends gives the slopes:

$$\alpha = \frac{1}{2\pi\sigma}, \qquad \beta = \frac{1}{4\pi\sigma} \qquad \Longrightarrow \quad \alpha/\beta = 2.$$

where σ is the string tension which is equal to the slope of the linear confining potential A

The quasiclassical picture for the massless Salpeter equation with a linear confining potential:

$$(2p + Ar)\psi = M\psi,$$

gives for the Regge slopes

$$\alpha = \frac{1}{8A}, \qquad \beta = \frac{1}{4\pi A} \qquad \Longrightarrow \quad \alpha/\beta = \pi/2.$$

Phenomenological analysis and some ADS/QCD models favour:

$$\alpha = \beta = 1/(2\pi A)$$

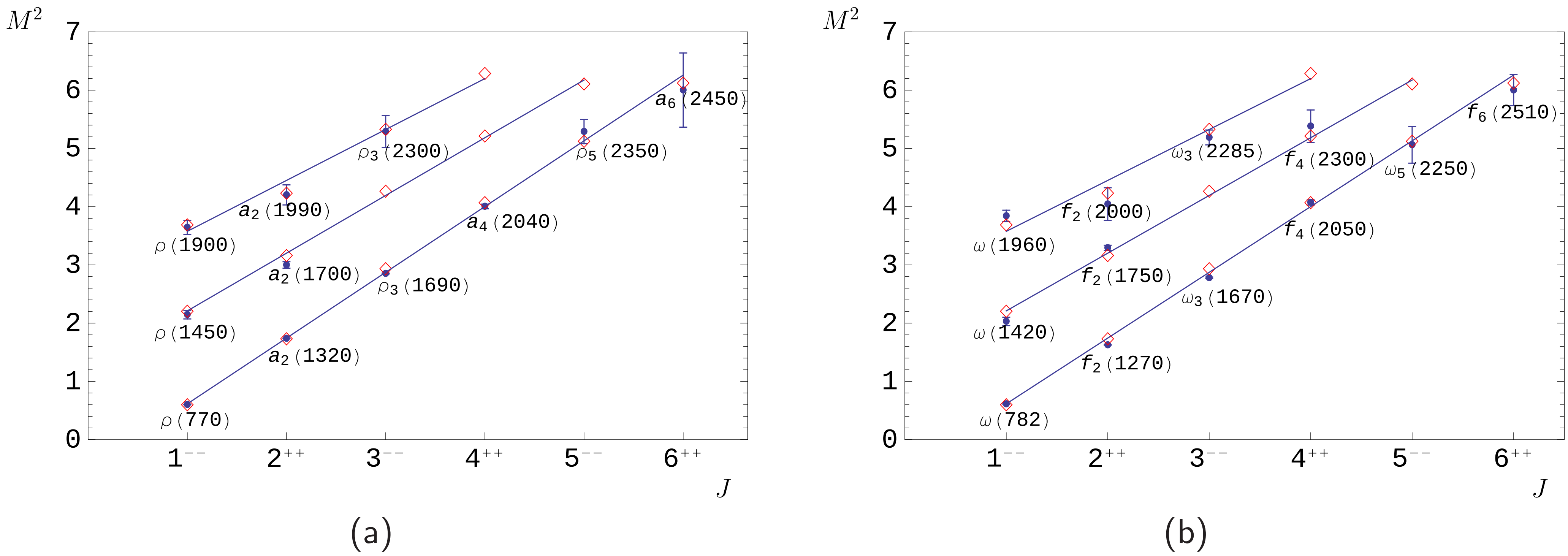


Figure 1: Parent and daughter (J, M^2) Regge trajectories for isovector (a) and isoscalar (b) light mesons with natural parity. Diamonds are predicted masses. Available experimental data are given by dots with error bars and particle names. M^2 is in GeV^2.

$P = (-1)^J$ – natural parity
$P = (-1)^{J-1}$ – unnatural parity

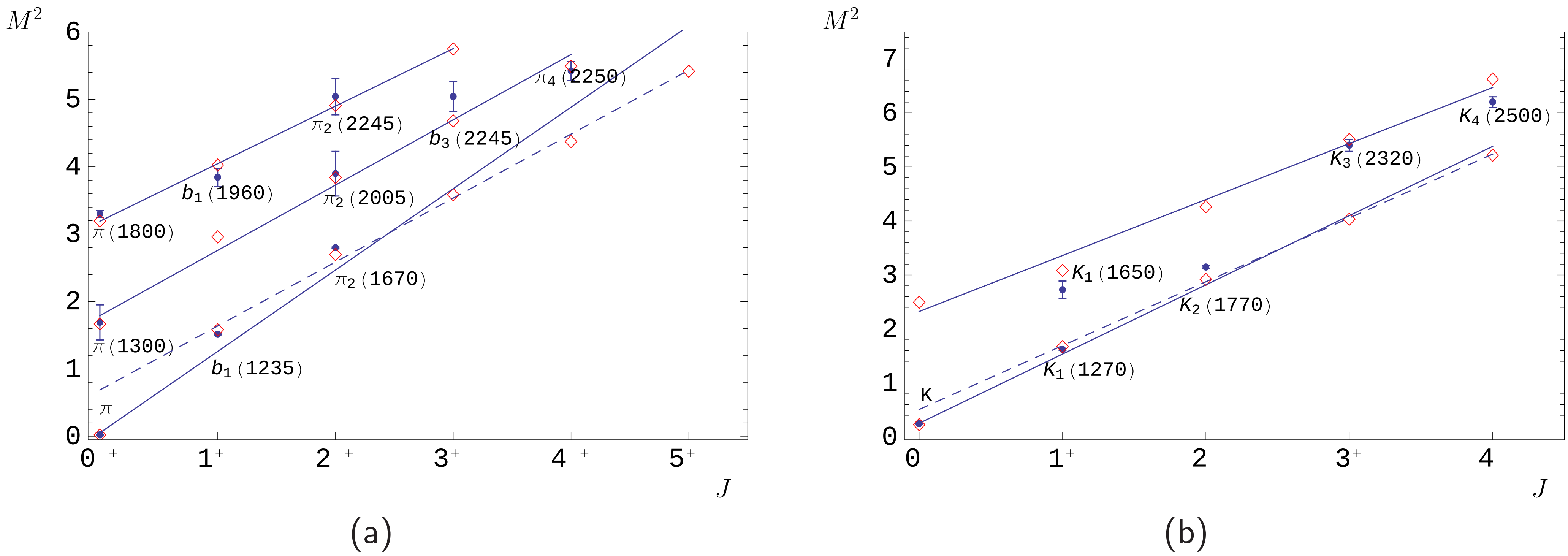


Figure 2: Parent and daughter (J, M^2) Regge trajectories for isovector (a) and isodoublet (b) light mesons with unnatural parity. Dashed line corresponds to the Regge trajectory, fitted without π and K.

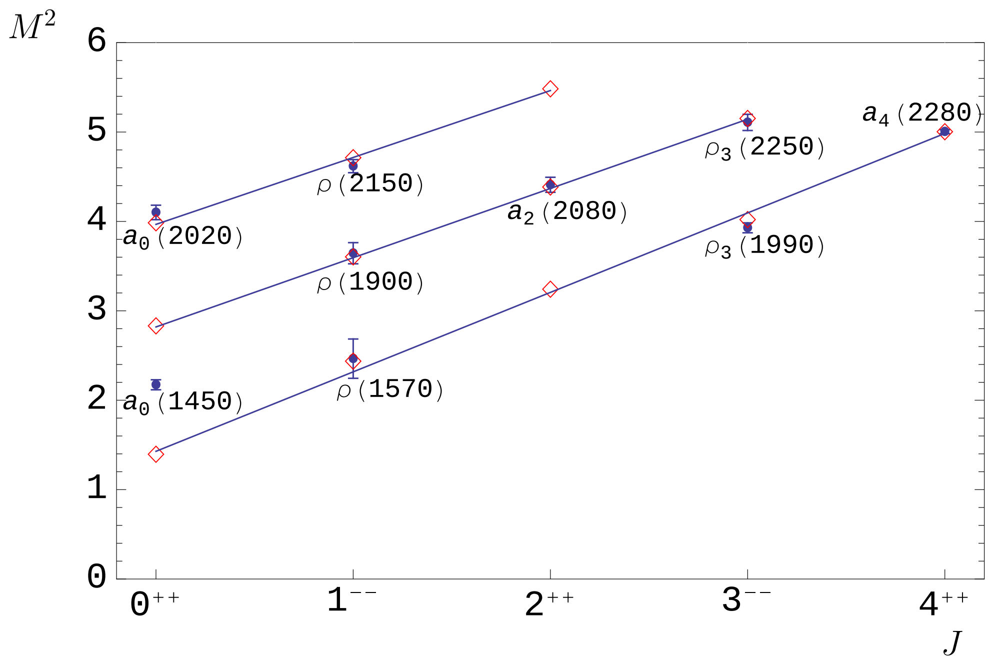


Figure 3: Parent and daughter (J, M^2) Regge trajectories for isovector light $q\bar{q}$ mesons with natural parity (a_0).

Note: $a_0(1450)$ and $\rho(1700)$ do not lie on the corresponding Regge trajectories $\Rightarrow$ possible exotic nature

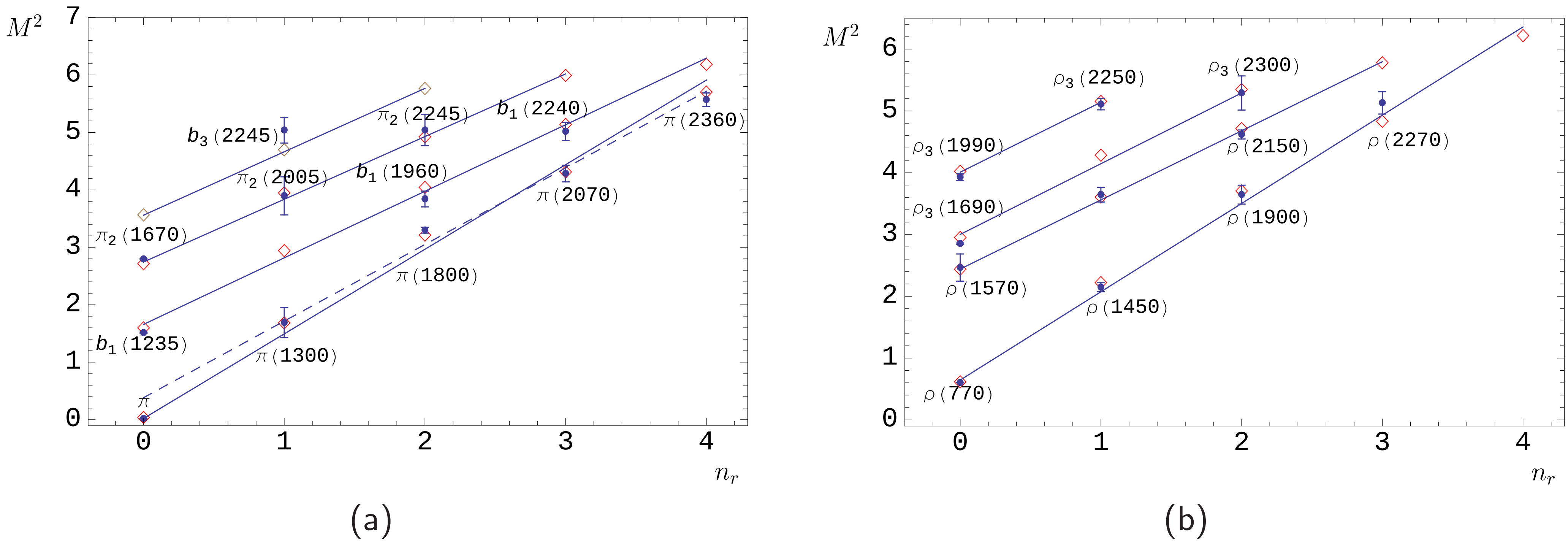


Figure 4: The (n_r, M^2) Regge trajectories for spin-singlet isovector mesons π, b_1, π_2 and b_3 (a) and $\rho(^3S_1)$, $\rho(^3D_1)$, $\rho_3(^3D_3)$ and $\rho_3(^3G_3)$ (b) (from bottom to top). The dashed line corresponds to the Regge trajectory, fitted without π.

The quality of fitting the π meson Regge trajectories both in (J, M^2) and (n_r, M^2) planes is significantly improved if the ground state π is excluded from the fit (the χ^2 is reduced by more than an order of magnitude and becomes compatible with the values for other trajectories).

In the kaon case omitting the ground state also improves the fit but not so dramatically as for the pion.

$\Longrightarrow$ the special role of the pion originating from the chiral symmetry breaking.

Table 3: Fitted parameters of the (J, M^2) parent and daughter Regge trajectories for light mesons with natural and unnatural parity ($q = u, d$).

Trajectory	natural parity		unnatural parity	
	α (GeV^{-2})	α_0	α (GeV^{-2})	α_0
$q\bar{q}$	ρ		π	
parent	0.887 ± 0.008	0.456 ± 0.018	$0.828 \pm 0.057^*$	$-0.025 \pm 0.034^*$
daughter 1	1.009 ± 0.019	-1.232 ± 0.074	1.031 ± 0.063	-1.846 ± 0.217
daughter 2	1.144 ± 0.113	-3.092 ± 0.540	1.171 ± 0.009	-3.737 ± 0.042
$q\bar{q}$	a_0		a_1	
parent	1.125 ± 0.035	-1.607 ± 0.104	1.014 ± 0.036	-0.658 ± 0.120
daughter 1	1.291 ± 0.003	-3.640 ± 0.011	1.148 ± 0.012	-2.497 ± 0.050
daughter 2	1.336 ± 0.022	-5.300 ± 0.102	1.154 ± 0.014	-3.798 ± 0.007
$q\bar{s}$	K^*		K	
parent	0.839 ± 0.004	0.318 ± 0.012	$0.780 \pm 0.022^\dagger$	$-0.197 \pm 0.036^\dagger$
daughter	0.942 ± 0.046	-1.532 ± 0.209	0.964 ± 0.072	-2.240 ± 0.296
$s\bar{s}$	φ		$\eta_{s\bar{s}}$	
parent	0.728 ± 0.011	0.234 ± 0.034	0.715 ± 0.023	-0.444 ± 0.068
daughter 1	0.721 ± 0.089	-1.072 ± 0.047	0.718 ± 0.032	-1.786 ± 0.157
daughter 2	0.684 ± 0.039	-2.047 ± 0.226	0.729 ± 0.010	-3.174 ± 0.057

* fit without π: $\alpha = (1.053 \pm 0.059)$ GeV^{-2} , $\alpha_0 = -0.725 \pm 0.170$

† fit without K: $\alpha = (0.846 \pm 0.013)$ GeV^{-2} , $\alpha_0 = -0.431 \pm 0.042$

Table 4: Fitted parameters of the (n_r, M^2) Regge trajectories for light mesons.

Meson	β (GeV^{-2})	β_0	Meson	β (GeV^{-2})	β_0
$q\bar{q}$			$s\bar{s}$		
π	$0.679 \pm 0.023^*$	$-0.018 \pm 0.014^*$	$\eta_{s\bar{s}}$	0.559 ± 0.009	-0.315 ± 0.026
$\rho(^3S_1)$	0.700 ± 0.023	-0.451 ± 0.060	φ	0.597 ± 0.009	-0.662 ± 0.031
a_0	0.830 ± 0.032	-1.214 ± 0.109	f_0	0.566 ± 0.009	-1.156 ± 0.039
a_1	0.840 ± 0.037	-1.401 ± 0.134	f_1	0.561 ± 0.013	-1.224 ± 0.058
b_1	0.863 ± 0.030	-1.431 ± 0.106	h_1	0.575 ± 0.015	-1.292 ± 0.066
$a_2(^3P_2)$	0.867 ± 0.036	-1.585 ± 0.134	f_2	0.581 ± 0.007	-1.370 ± 0.031
$\rho(^3D_1)$	0.894 ± 0.013	-2.182 ± 0.050			
π_2	0.916 ± 0.032	-2.514 ± 0.134			
$\rho_3(^3D_3)$	0.874 ± 0.041	-2.623 ± 0.189			
$a_2(^3F_2)$	0.891 ± 0.010	-2.881 ± 0.043			
a_3	0.890 ± 0.014	-3.254 ± 0.066			
b_3	0.906 ± 0.015	-3.225 ± 0.071			
a_4	0.899 ± 0.016	-3.672 ± 0.084			

* fit without π: $\beta = (0.750 \pm 0.032)$ GeV^{-2}, $\beta_0 = -0.287 \pm 0.109$

In our model

$$\langle\alpha\rangle/\langle\beta\rangle \approx 1.3$$

and for light mesons without s-quark

$$\langle\beta\rangle \approx 0.85\ \mathrm{GeV}^{-2} \approx 1/(2\pi A) = 0.88\ \mathrm{GeV}^{-2}$$

MASSES OF LIGHT TETRAQURKS

Light tetraquarks are considered in the diquark-antidiquark picture:

1. Masses and form factors of light diquarks are calculated

2. Tetraquark is considered as a bound diquark-antidiquark state

We take the diquark masses and form factors form our previous studies of heavy baryons in the heavy quark-light diquark picture

Table 5: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[\ldots]$ and symmetric $\{\ldots\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		our	NJL	BSE	BSE	Lattice
$[u,d]$	S	710	705	737	820	694(22)
$\{u,d\}$	A	909	875	949	1020	806(50)
$[u,s]$	S	948	895	882	1100	
$\{u,s\}$	A	1069	1050	1050	1300	
$\{s,s\}$	A	1203	1215	1130	1440	

In the diquark-antidiquark picture of tetraquarks both scalar S (antisymmetric in flavour $[\ldots]$) and axial vector A (symmetric in flavour $\{\ldots\}$) diquarks are considered $\Longrightarrow$
Structure of the light tetraquark ground ($1S$) states (C is defined only for neutral self-conjugated states):

- Two states with $J^{PC} = 0^{++}$:

$$X(0^{++}) = S\bar{S}$$
$$X(0^{++\prime}) = A\bar{A}$$

- Three states with $J^{PC} = 1^{+\pm}$:

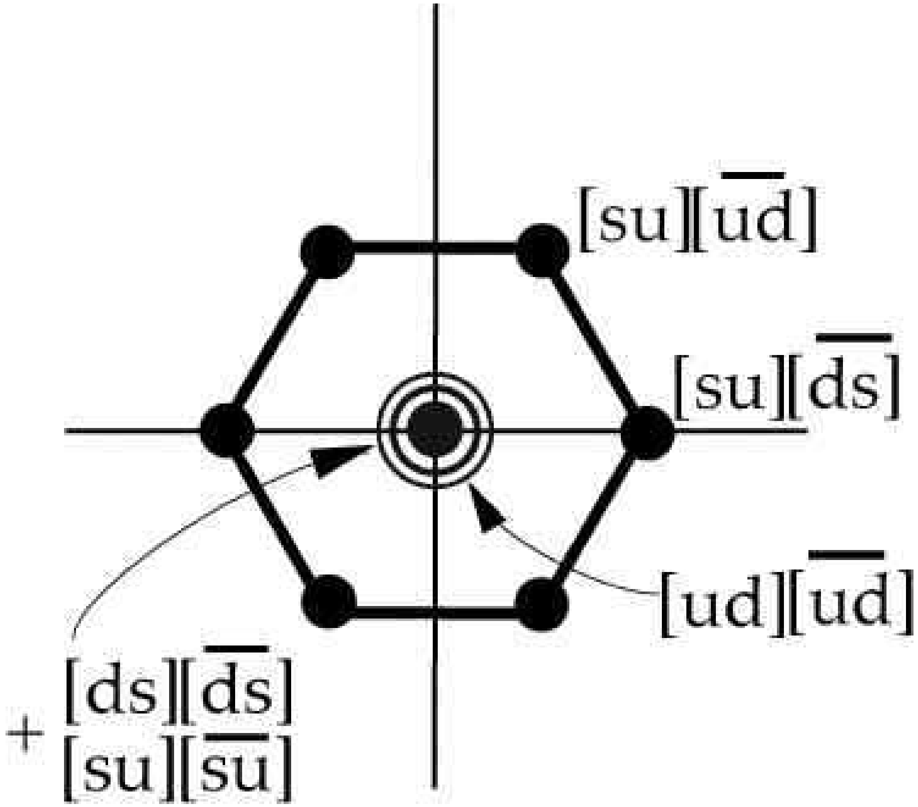
$$X(1^{++}) = \frac{1}{\sqrt{2}}(S\bar{A} + \bar{S}A)$$
$$X(1^{+-}) = \frac{1}{\sqrt{2}}(S\bar{A} - \bar{S}A)$$
$$X(1^{+-\prime}) = A\bar{A}$$

- One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = A\bar{A}.$$

Lightest scalar tetraquarks

The lightest $S\bar{S}$ scalar (0^{++}) tetraquark states form the SU(3) flavour nonet:



- one tetraquark ($[ud][\bar{u}\bar{d}]$) with neither open or hidden strangeness ($Q = 0$ and $I = 0$);
- four tetraquarks ($[sq][\bar{u}\bar{d}]$, $[\bar{s}\bar{q}][ud]$, $q = u, d$) with open strangeness ($Q = 0, \pm 1$, $I = \frac{1}{2}$);
- four tetraquarks ($[sq][\bar{s}\bar{q}']$) with hidden strangeness ($Q = 0, \pm 1$, $I = 0, 1$).

Table 6: Masses of light unflavored diquark-antidiquark ground state ($\langle \mathbf{L}^2 \rangle$=0) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Theory mass	Experiment $I=0$	mass	$I=1$	mass
$(qq)(\bar{q}\bar{q})$						
0^{++}	$S\bar{S}$	596	$f_0(600)$ (σ)	400-1200		-
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	672				
0^{++}	$A\bar{A}$	1179	$f_0(1370)$	1200-1500		
1^{+-}	$A\bar{A}$	1773				
2^{++}	$A\bar{A}$	1915	$f_2(1910)$	1903(9)		
			$f_2(1950)$	1944(12)		
$(qs)(\bar{q}\bar{s})$						
0^{++}	$S\bar{S}$	992	$f_0(980)$	980(10)	$a_0(980)$	984.7(12)
1^{++}	$(S\bar{A} + \bar{S}A)/\sqrt{2}$	1201	$f_1(1285)$	1281.8(6)	$a_1(1260)$	1230(40)
1^{+-}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	1201	$h_1(1170)$	1170(20)	$b_1(1235)$	1229.5(32)
0^{++}	$A\bar{A}$	1480	$f_0(1500)$	1505(6)	$a_0(1450)$	1474(19)
1^{+-}	$A\bar{A}$	1942	$h_1(1965)$	1965(45)	$b_1(1960)$	1960(35)
2^{++}	$A\bar{A}$	2097	$f_2(2010)$	2011(70)	$a_2(1990)$	2050(45)
			$f_2(2140)$	2141(12)	$a_2(2080)$	2100(20)
$(ss)(\bar{s}\bar{s})$						
0^{++}	$A\bar{A}$	2203	$f_0(2200)$	2189(13)		-
1^{+-}	$A\bar{A}$	2267	$h_1(2215)$	2215(40)		-
2^{++}	$A\bar{A}$	2357	$f_2(2340)$	2339(60)		-

Table 7: Masses of strange diquark-antidiquark ground state ($\langle \mathbf{L}^2 \rangle$=0) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State	Diquark	Theory	Experiment	
J^P	content	mass	$I = \frac{1}{2}$	mass
$(qq)(\bar{s}\bar{q})$ or $(sq)(\bar{q}\bar{q})$				
0^+	$S\bar{S}$	730	$K_0^*(800)$ (κ)	672(40)
1^+	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1057		
0^+	$A\bar{A}$	1332	$K_0^*(1430)$	1425(50)
1^+	$A\bar{A}$	1855		
2^+	$A\bar{A}$	2001	$K_2^*(1980)$	1973(26)

- Lightest scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be interpreted in our model as light tetraquarks composed from a scalar diquark and antidiquark $(S\bar{S})$. Therefore, the $f_0(980)$ and $a_0(980)$ tetraquarks contain, in comparison to the $q\bar{q}$ picture, an additional pair of strange quarks which gives a natural explanation why their masses are heavier than the strange $K_0^*(800)$ (κ).

- $a_0(1450)$ should be predominantly a tetraquark state composed from axial vector diquark and antidiquark $(A\bar{A})$. The exotic scalar state $X(1420)$ from the "Further States" Section of PDG could be its isotensor partner. On the other hand $s\bar{q}(1^3P_0)$ interpretation is favored for $K_0^*(1430)$. This picture naturally explains the experimentally observed proximity of masses of the unflavoured $a_0(1450)$ and $f_0(1500)$ with the strange $K_0^*(1430)$.

- Rather low mass values of the 1^+ tetraquark states are predicted: $(\{ud\}[\bar{u}\bar{d}] \pm \{\bar{u}\bar{d}\}[ud])/\sqrt{2}$, 672 MeV, and of their strange partner $([qs]\{\bar{u}\bar{d}\} \pm [\bar{q}\bar{s}]\{ud\})$, 1057 MeV. Such axial vector states are not observed experimentally.

CONCLUSIONS

- Completely relativistic treatment of the light quark dynamics allowed us to get masses of the π and K mesons in agreement with experimental data in the considered model, where the chiral symmetry is explicitly broken by the constituent quark masses.
- The lightest scalar $q\bar{q}$ (1^3P_0) states have masses above 1 GeV in our model.
- The calculated masses of light mesons reproduce the linear Regge trajectories both in the (J, M^2) and (n_r, M^2) planes. The slope of the orbital excitations α was found to be in average 1.3 times larger than the slope of the trajectories of radial excitations β.
- Possible experimental candidates for the states populating the Regge trajectories were identified. Predictions for the masses of the missing states were presented. Our results in some cases differ from the previous phenomenological prescriptions. Future experimental data can help in discriminating between the theoretical predictions
- The chiral symmetry is not restored for highly excited states in our model. This should be expected since the Lorentz-scalar part of the confining potential explicitly breaks the chiral symmetry.
- Masses of the ground state light tetraquarks were calculated in the diquark-antidiquark picture and the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equations. The diquark structure was taken into account by using diquark-gluon form factors in terms of diquark wave functions.
- It was found that the lightest scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be naturally described in our model as diquark-antidiquark bound systems.