

## Heavy Quark Free Energies and the Renormalized Polyakov Loop in Full QCD

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We study the renormalized free energy of a heavy quark anti-quark pair in the different colour channels in full QCD at finite temperature. Similarities and differences to the quenched case are discussed and the temperature dependence as well as their short distance behavior are analyzed. The asymptotic large distance behavior of the free energy is used to define the non-perturbatively renormalized Polyakov loop which is well behaved in the continuum limit.

### §1. Introduction

The study of the fundamental forces between quarks is a key to the understanding of QCD and the occurrence of different phases at high temperatures ( $T$ ) and densities ( $\mu$ ). The free energy of a static quark anti-quark pair, separated by a distance  $r$ , is a good tool to analyze the  $r$  and  $T$  dependence of the forces, potentials and entropy contributions of static quarks in the medium at non-zero  $T$  and  $\mu$ . The short and intermediate distance regime of those observables,  $rT \lesssim 1$ , is relevant for the discussion of in-medium modifications of heavy quark bound states which are sensitive to thermal modifications of the heavy quark potential.

Here we will discuss first results on the heavy quark free energies in different color channels in QCD with dynamical quarks. Results for the quenched theory and a detailed description about the renormalization procedure can be found in Refs. 1) and 2). While in earlier studies of the heavy quark free energy in full QCD<sup>3)</sup> only the color averaged operators were analyzed, we have a quite detailed description of the different color channels in the quenched theory. Below the deconfinement phase transition, the free energies show the same linearly rising behavior at large distances governed by the string tension. Above  $T_c$ , the free energies are exponentially screened at large distances ( $rT \gg 1$ ) due to the generation of a chromoelectric (Debye) mass. At very small separations,  $rT \ll 1$ , one gets into the perturbative regime, where the relevant scale is set by the distance  $r$  and no temperature effects are seen, even at high temperatures in the deconfined phase. In this region, the singlet free energy is well described by the zero temperature potential and the running coupling depends on the dominant scale, i.e.  $g = g(r)$ .

In contrast to a linear rising potential in the quenched theory, in QCD with dynamical fermions the free energies below  $T_c$  show a different behavior at large separations. Due to the possibility of pair creation the string between the two *test*

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<sup>\*)</sup> Presented by O. Kaczmarek.

quarks can *break*, leading to a constant free energy at large separations. At very small distances, again the dominant scale is given by the distance  $r$  and the free energies are expected to be well described by the zero temperature potential, i.e. zero temperature perturbation theory at sufficiently small  $r$ .

Polyakov loop correlation functions are generally used to analyze the temperature dependence of confinement forces and the screening in the high temperature phase of QCD. They are directly related to the *change in free energy* arising from the presence of a static quark anti-quark pair in a thermal medium,  $\langle \text{Tr}W(\vec{x})\text{Tr}W^\dagger(0) \rangle \sim \exp(-F_{\bar{q}q}(r = |\vec{x}|, T)/T)$ .<sup>1),4)</sup>

In §2 we will describe the behavior of the free energy in different color channels and in the rest of the paper we will then concentrate on results from calculations of the singlet free energy,  $F_1(r = |\vec{x}|, T)/T = -\ln\langle \text{Tr}W(\vec{x})W^\dagger(0) \rangle$ . The operators used to calculate  $F_1$  as well as the octet free energy,  $F_8$ , are not gauge invariant. Calculations thus have been performed in Coulomb gauge. It has been shown that this approach is equivalent to using a suitably defined gauge invariant (non-local) operator for the singlet free energy.<sup>5)</sup>

## §2. The renormalized free energy

We will analyze in the following the properties of heavy quark anti-quark pairs in a thermal heat bath of gluons and dynamical quarks. To analyze the heavy quark free energies we have calculated Polyakov loop correlation functions on configurations generated with the p4-action in the fermionic sector and a Symanzik improved gauge action in the gauge sector. On  $16^3 \times 4$  lattices we have used 2-flavors of dynamical quarks with a quark mass of  $m/T = 0.4$ . The zero temperature potential and the temperature scale were obtained from the data of Ref. 6).

The static quark sources are described by the Polyakov loop,

$$W(\vec{x}) = \prod_{\tau=1}^{N_\tau} U_0(\vec{x}, \tau) \quad (2.1)$$

with  $U_0(\vec{x}, \tau) \in SU(3)$  being defined on the link in time direction. The free energies in the singlet and octet channels are then defined as<sup>4),7)</sup>

$$e^{-F_1(r)/T+C} = \frac{1}{3} \text{Tr} \langle W(\vec{x})W^\dagger(0) \rangle, \quad (2.2)$$

$$e^{-F_8(r)/T+C} = \frac{1}{8} \langle \text{Tr}W(\vec{x})\text{Tr}W^\dagger(0) \rangle - \frac{1}{24} \text{Tr} \langle W(\vec{x})W^\dagger(0) \rangle, \quad (2.3)$$

where  $r = |\vec{x}|$  and  $C$  is a suitably chosen renormalization constant. Furthermore, one can consider the color averaged free energy defined through the correlation function

$$e^{-F_{\bar{q}q}(r)/T+C} = \frac{1}{9} \langle \text{Tr}W(\vec{x})\text{Tr}W^\dagger(0) \rangle. \quad (2.4)$$

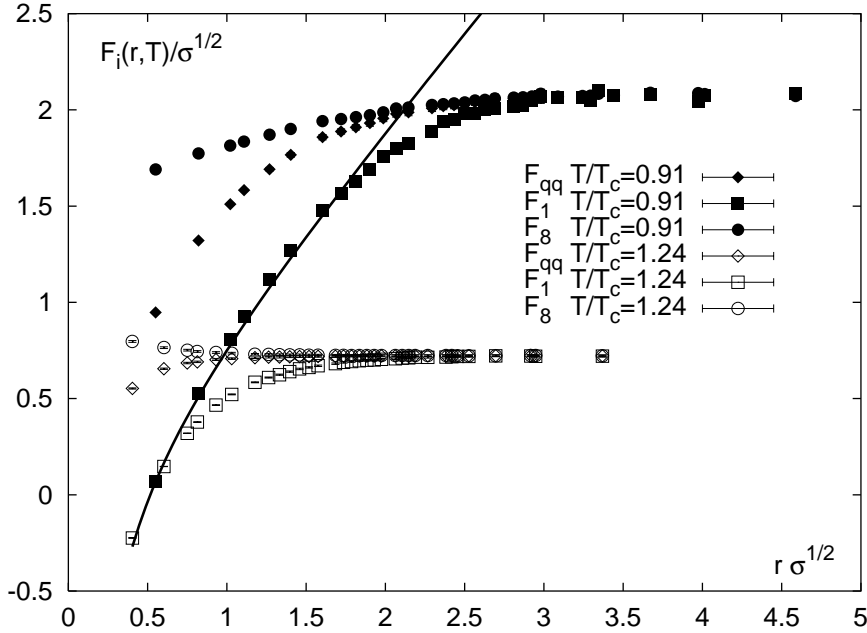


Fig. 1. Heavy quark free energies for 2-flavors of dynamical quarks at a quark mass of  $m/T = 0.40$  on  $16^3 \times 4$  lattices. Shown are the different color channels, the singlet ( $F_1$ ), octet ( $F_8$ ) and color averaged ( $F_{\bar{q}q}$ ) renormalized to the zero- $T$  potential obtained from<sup>6)</sup>(solid line).

This can be written as thermal average over free energies in singlet and octet channels

$$e^{-F_{\bar{q}q}(r)/T} = \frac{1}{9}e^{-F_1(r)/T} + \frac{8}{9}e^{-F_8(r)/T}. \tag{2.5}$$

At distances much shorter than the inverse temperature ( $rT \ll 1$ ) the dominant scale is set by  $r$  and the running coupling will be controlled by this scale and become small for ( $r \ll 1/\Lambda_{\text{QCD}}$ ). In this limit the singlet and octet free energies are dominated by one-gluon exchange and become calculable within ordinary zero temperature perturbation theory, i.e. are given by the singlet and octet heavy quark potential. We have used this to fix the constant  $C$  in (2.2), (2.3) and (2.5) by matching the singlet free energy to the zero temperature heavy quark potential at short distances.

In Fig. 1 the renormalized free energies in the different color channels for two temperatures are plotted. At small distances  $F_1$  coincides with the  $T=0$ -potential. For the temperature of  $0.91 T_c$  we see no thermal effect up to a distance of  $r\sqrt{\sigma} \approx 1.5$  where string breaking sets in and leads to a constant value at larger separations. For  $T=1.24 T_c$  the thermal effect sets in at  $r\sqrt{\sigma} \approx 0.7$ . The singlet free energy,  $F_1$ , shows the usual screened Coulomb like behavior approaching a temperature dependent constant value at large distances. In all color channels the free energies reach the same constant (*cluster*) value at large separations above as well as below  $T_c$ .

While the singlet potential is attractive, the octet potential is repulsive at short distances. From Eq. (2.5) it follows that in this limit the color averaged free energy will be dominated by the singlet contribution. We may then deduce from (2.5) also

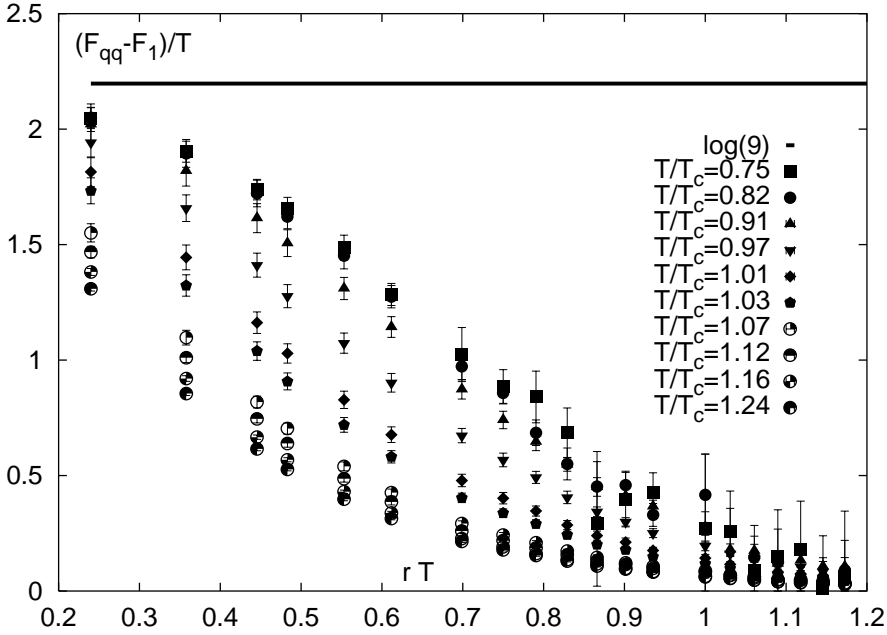


Fig. 2. The difference of the color averaged and singlet free energies for 2-flavors of dynamical quarks at a quark mass of  $m/T = 0.40$  on  $16^3 \times 4$  lattices. The black line indicates the expected asymptotic short distance limit (solid line).

the asymptotic short distance behavior of  $F_{\bar{q}q}$  and  $F_1$ ,

$$\lim_{r \rightarrow 0} (F_{\bar{q}q}(r, T) - F_1(r, T)) = T \ln 9 \quad \text{for all } T. \tag{2.6}$$

In Fig. 2 this asymptotic behavior at small distances is reached, although for the higher temperatures the distances analyzed here are not small enough to get into the regime where (2.6) is fulfilled. At large distances the difference between the singlet and color averaged free energies vanishes and therefore  $F_1$ ,  $F_{\bar{q}q}$  and consequently also  $F_8$  reach the same *cluster* value at large separations.

### §3. Short vs long distances

In the following we will concentrate on the singlet free energies and analyze their short and long distance behavior. In Fig. 3 we show  $F_1(r, T)$  in the temperature range of  $0.75 < T/T_c < 1.25$ . We see again the  $T$ -independence at sufficiently small distances. The thermal effects set in at relatively large distances of  $r\sqrt{\sigma} \approx 2$  for the lowest temperatures and move towards smaller distances with increasing  $T$ .

At large distances  $F_1$  is *screened* and reaches constant values for all temperatures, which can be explained by string breaking below  $T_c$  and screening due to the generation of screening masses above the transition temperature.

To analyze at what distances these screening effects set in, we introduce the

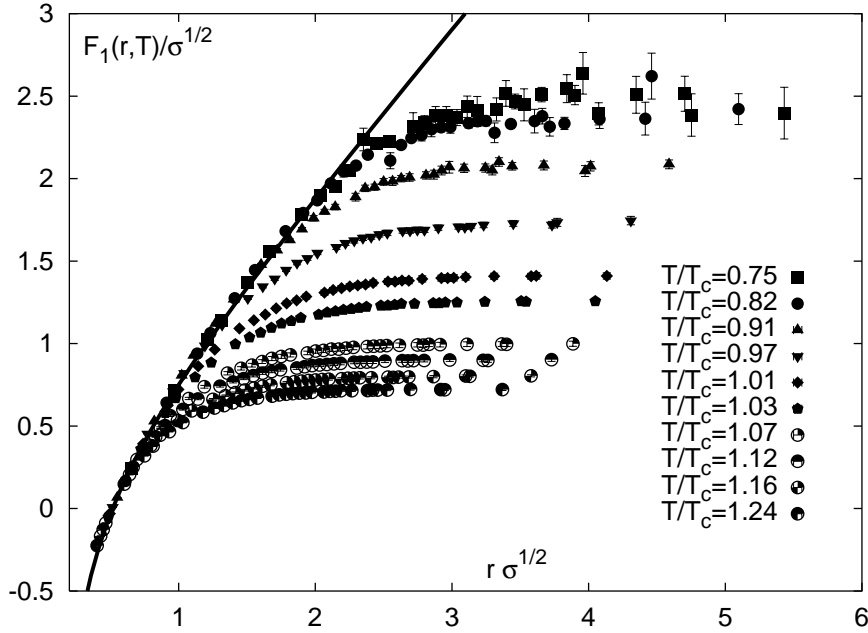


Fig. 3. Heavy quark free energies in the singlet channel for 2-flavors of dynamical quarks at a quark mass of  $m/T = 0.40$  on  $16^3 \times 4$  lattices renormalized to the zero- $T$  potential obtained from<sup>6)</sup> (solid line).

color singlet screening function as

$$S_1(r, T) = -\frac{3}{4}r(F_1(r, T) - F_1(\infty, T)). \tag{3.1}$$

At short distances this quantity should approach the running coupling constant,  $\alpha = g^2/4\pi$ , and thus is expected to drop logarithmically, while at large distances it carries information about screening and is expected to drop exponentially. Consequently we expect that  $S_1(r, T)$  will exhibit a maximum at some intermediate distance which we can identify as the point separating the short distance physics from the large distance regime.

Figure 4 shows the data on  $S_1(r, T)$  on both linear (left) and logarithmic (right) scales. At small temperatures a clear maximum is visible at distances of  $rT \simeq 0.45$ . At higher temperatures indications for a tendency to develop a maximum are visible, but the distances analyzed here are not small enough to verify this. We find that the maximum, i.e. the point separating the short from the large distance regime, occurs at  $rT \simeq 0.45$  at  $0.75T_c$  and slowly shifts to smaller values at high temperatures, e.g. at  $T_c$  it is at  $rT \simeq 0.4$ . Beyond this length scale  $S_1(r, T)$  drops rapidly and thus exhibits screening. The distances analyzed here are too small to really see the simple exponential form, as distances  $rT \geq 1$  are needed to separate the long distance regime from the short distance one.

The strong thermal effects seen in Fig. 4 even at small distances are to a large extent caused by our normalization in (3.1), which forces  $S_1(r, T)$  to approach zero

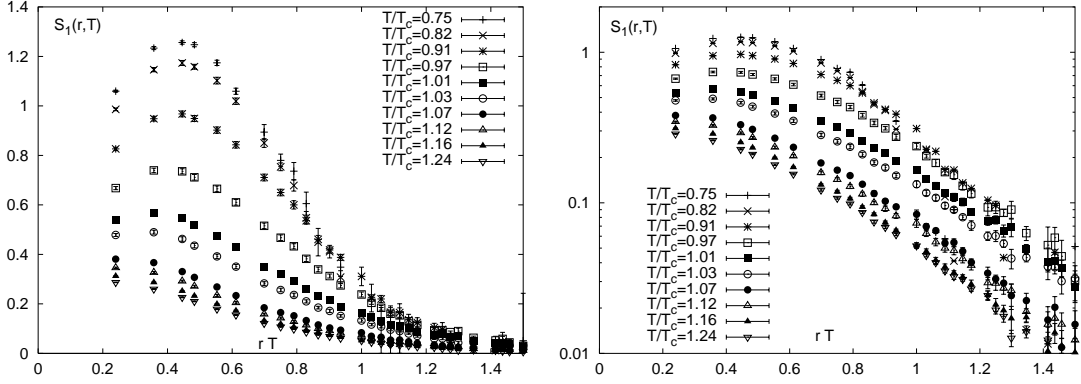


Fig. 4. The color singlet screening function on linear (left) and logarithmic (right) scales.

at large separations, but introduces an artificial temperature dependence at short distances.

Looking at the temperature dependence of the singlet free energies in Fig. 3,  $F_1(r, T)$  decreases with increasing temperature at fixed  $r$ , indicating that there is a positive entropy,  $S = -\frac{\partial F_1}{\partial T}$ , contribution at large distances, while it is close to zero, due to the (asymptotic)  $T$ -independence of  $F_1(r, T)$ , at small  $r$ .

#### §4. The renormalized Polyakov loop

The Polyakov loop, calculated on the lattice, is ultra-violet divergent and needs to be renormalized to become a meaningful observable in the continuum limit. We will do so by renormalizing the free energies at short distances. Assuming that no additional divergences arise from thermal effects and that at short distances the heavy quark free energies will not be sensitive to medium effects, renormalization is achieved through a matching of free energies to the zero temperature heavy quark potential. Using the large distance behavior of the renormalized free energies we can then define the renormalized Polyakov loop which is well behaved also in the continuum limit.

Using the renormalized free energies from Fig. 3, i.e. the asymptotic values in Fig. 5, we can define the renormalized Polyakov loop,<sup>1)</sup>

$$L_{\text{ren}} = \exp\left(-\frac{F_1(r = \infty, T)}{2T}\right). \tag{4.1}$$

In Fig. 6 we show the results for  $L_{\text{ren}}$  in full QCD compared to the quenched results obtained from Ref. 1). In quenched QCD it is zero below  $T_c$  by construction, as the free energy goes to infinity in the limit of infinite distance. From the results of different values of  $N_\tau$ , it is apparent that  $L_{\text{ren}}$  does not depend on  $N_\tau$  and therefore is well behaved in the continuum limit.

The renormalized Polyakov loop in full QCD is no longer zero below  $T_c$ . Due to string breaking the free energies reach a constant value at large separations

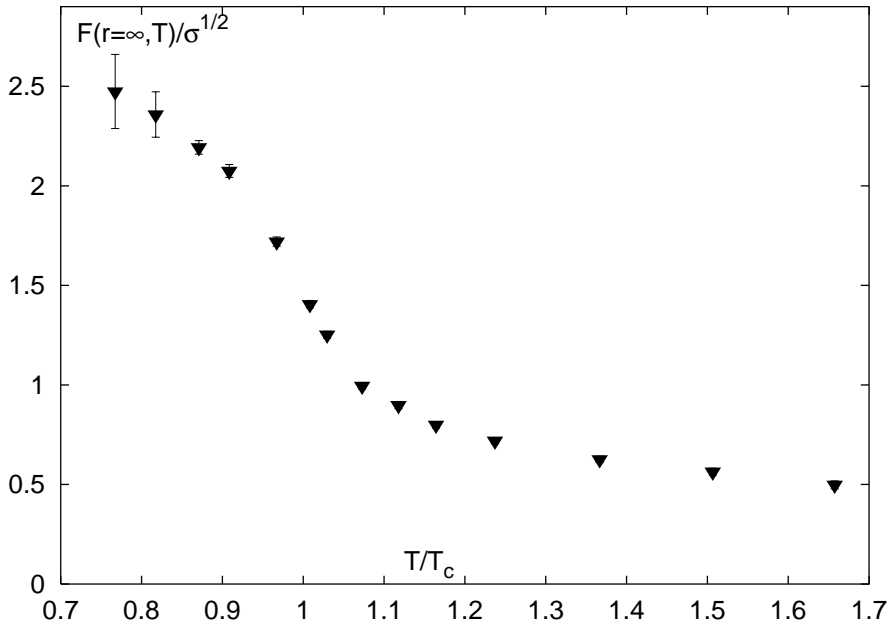


Fig. 5. The asymptotic large distance values of the free energies.

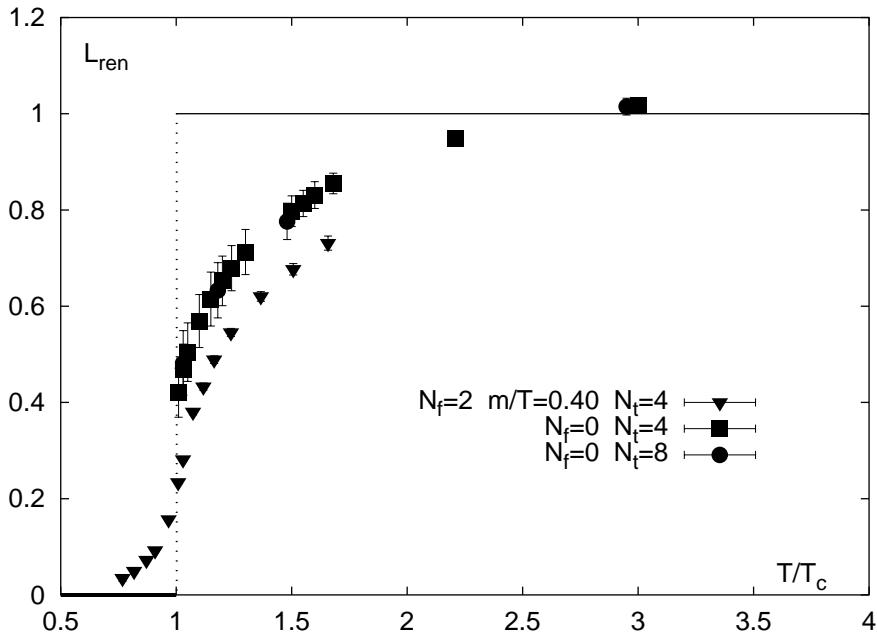


Fig. 6. The renormalized Polyakov loop in full QCD compared to the quenched results.<sup>1)</sup>

leading to a non-zero value of  $L_{ren}$ . The renormalized Polyakov loop is no longer an order parameter for finite quarks mass, but still indicates a clear signal for a phase

change at  $T_c$ . It is small below  $T_c$  and shows a strong increase close to the critical temperature. In the temperature range we have analyzed,  $L_{\text{ren}}$  is smaller in full QCD compared to the quenched case. In Ref. 6) no major quark mass effects were visible in the color averaged free energies below a quark mass of  $m/T = 0.4$ . To verify this for the singlet and octet channel, a more detailed analysis of the mass and also flavor dependence is needed.

## §5. Conclusions

We have discussed the renormalized free energies in the different color channels for QCD with dynamical quarks. The results in 2-flavor QCD show that the concepts, developed in quenched QCD, can be extended to full QCD. A more precise analysis of the temperature and mass (and flavor) dependence, as well as a closer look at the short distance regime, is needed. For a detailed analysis of screening phenomena at high temperatures, larger lattices are needed, as separations of  $rT \geq 1$  are required to obtain screening masses from the exponential fall off. We have shown that there is a quite different behavior for short and large distances and that those regimes are separated around  $rT \simeq 0.4$  near  $T_c$ . From the analysis of the screening function we find indications of the running of the coupling at small distances, but smaller lattice cut-offs are needed to analyze this behavior in more detail.

The large distance behavior of the free energy was used to calculate the renormalized Polyakov loop. We observe visible differences to the quenched results.  $L_{\text{ren}}$  is no longer zero below  $T_c$ , but still indicates different behavior in both phases and a strong increase close to  $T_c$ . It is smaller compared to the quenched case in the temperatures region we have studied.

The extension of the analysis described here to non-zero density using a Taylor expansion<sup>8)</sup> in  $\mu$  is straightforward and in progress.

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