UC Berkeley
UC Berkeley Previously Published Works

Title

Hedges: A Study In Meaning Criteria And The Logic Of Fuzzy Concepts

Permalink

https://escholarship.org/uc/item/0x0010nv

Author

Lakoff, George

Publication Date

1973

Peer reviewed

GEORGE LAKOFF

HEDGES: A STUDY IN MEANING CRITERIA AND THE LOGIC OF FUZZY CONCEPTS*

1. DEGREES OF TRUTH

Logicians have, by and large, engaged in the convenient fiction that sentences of natural languages (at least declarative sentences) are either true or false or, at worst, lack a truth value, or have a third value often interpreted as 'nonsense'. And most contemporary linguists who have thought seriously about semantics, especially formal semantics, have largely shared this fiction, primarily for lack of a sensible alternative. Yet students of language, especially psychologists and linguistic philosophers, have long been attuned to the fact that natural language concepts have vague boundaries and fuzzy edges and that, consequently, natural language sentences will very often be neither true, nor false, nor nonsensical, but rather true to a certain extent and false to a certain extent, true in certain respects and false in other respects.

It is common for logicians to give truth conditions for predicates in terms of classical set theory. 'John is tall' (or 'TALL(j)') is defined to be true just in case the individual denoted by 'John' (or 'j') is in the set of tall men. Putting aside the problem that tallness is really a relative concept (tallness for a pygmy and tallness for a basketball player are obviously different)[1], suppose we fix a population relative to which we want to define tallness. In contemporary America, how tall do you have to be to be tall? 5′8″? 5′9″? 5′10″? 5′11″? 6′? 6′2″? Obviously there is no single fixed answer. How old do you have to be to be middle-aged? 35? 37? 39? 40? 42? 45? 50? Again the concept is fuzzy. Clearly any attempt to limit truth conditions for natural language sentences to true, false and 'nonsense' will distort the natural language concepts by portraying them as having sharply defined rather than fuzzily defined boundaries.

Work dealing with such questions has been done in psychology. To take a recent example, Eleanor Rosch Heider (1971) took up the question of whether people perceive category membership as a clearcut issue or a matter of degree. For example, do people think of members of a given

species as being simply birds or nonbirds, or do people consider them birds to a certain degree? Heider's results consistently showed the latter. She asked subjects to rank birds as to the degree of their birdiness, that is, the degree to which they matched the ideal of a bird. If category membership were simply a yes-or-no matter, one would have expected the subjects either to balk at the task or to produce random results. Instead, a fairly well-defined hierarchy of 'birdiness' emerged.

(1) Birdiness hierarchy

>   robins
>   eagles
>   chickens, ducks, geese
>   penguins, pelicans
>   bats

Robins are typical of birds. Eagles, being predators, are less typical. Chickens, ducks, and geese somewhat less so. Penguins and pelicans less still. Bats hardly at all. And cows not at all.

A study of vegetableness yielded a similar hierarchical result:

(2) Vegetableness hierarchy

>   carrots, asparagus
>   celery
>   onion
>   parsley
>   pickle

Further experiments by Heider showed a distinction between central members of a category and peripheral members. She surmised that if subjects had to respond 'true' or 'false' to sentences of the form 'A (member) is a (category)' – for example, 'A chicken is bird' – the response time would be faster if the member was a central member (a good example of the category) than if it was a peripheral member (a not very good example of the category). On the assumption that central members are learned earlier than peripheral members, she surmised that children would make more errors on the peripheral members than would adults. (3) lists some of the examples of central and peripheral category members that emerged from

the study:

(3)	Category	Central Members	Peripheral Members
	Toy	ball, doll	swing, skates
	bird	robin, sparrow	chicken, duck
	fruit	pear, banana	strawberry, prune
	sickness	cancer, measles	rheumatism, rickets
	metal	copper, aluminum	magnesium, platinum
	crime	rape, robbery	treason, fraud
	sport	baseball, basketball	fishing, diving
	vehicle	car, bus	tank, carriage
	body part	arm, leg	lips, skin

I think Heider's work shows clearly that category membership is not simply a yes-or-no matter, but rather a matter of degree. Different people may have different category rankings depending on their experience or their knowledge or their beliefs, but the fact of hierarchical ranking seems to me to be indisputable. Robins simply are more typical of birds than chickens and chickens are more typical of birds than penguins, though all are birds to some extent. Suppose now that instead of asking about category membership we ask instead about the truth of sentences that assert category membership. If an X is a member of a category Y only to a certain degree, then the sentence 'An X is a Y' should be true only to that degree, rather than being clearly true or false. My feeling is that this is correct, as (4) indicates.

(4) Degree of truth (corresponding to degree of category membership)

        a. A robin is a bird.   (true)
        b. A chicken is a bird. (less true than a)
        c. A penguin is a bird. (less true than b)
        d. A bat is a bird.   (false, or at least very far from true)
        e. A cow is a bird.   (absolutely false)

Most speakers I have checked with bear out this judgement, though some seem to collapse the cases in (4a–c), and don't distinguish among them. My guess is that they in general judge the truth of sentences like those in (4) according to the truth of corresponding sentences like those in (5).

(5)   a. A robin is more of a bird than anything else.   (True)
        b. A chicken is more of a bird than anything else. (True)

    c. A penguin is more of a bird than anything else. (True)
    d. A bat is more of a bird than anything else.   (False)
    e. A cow is more of a bird than anything else.   (False)

That is, some speakers seem to turn relative judgments of category membership into absolute judgments by assigning the member in question to the category in which it has the highest degree of membership. As we shall see below, speakers who judge the sentences in (4) to have a pattern like those in (5) do make the distinctions shown in (4), but then collapse them to the pattern in (5).

2. Fuzzy logic

Although the phenomena discussed above are beyond the bounds of classical set theory and the logics based on it, there is a well-developed set theory capable of dealing with degrees of set membership, namely, fuzzy set theory as developed by Zadeh (1965). The central idea is basically simple: Instead of just being in the set or not, an individual is in the set to a certain degree, say some real number between zero and one.

(1) Zadeh's Fuzzy Sets

        In a universe of discourse $X = \{x\}$, a fuzzy set A is a set of ordered pairs $\{(x, \mu_A(x))\}$, where $\mu_A(x)$ is understood as the degree of membership of x in A. $\mu_A(x)$ is usually taken to have values in the real interval $[0, 1]$, though the values can also be taken to be members of any distributive complemented lattice.

        Union: $\mu_{A \cup B} = \max(\mu_A, \mu_B)$.
        Complement: $\mu_{A'} = 1 - \mu_A$
        Intersection: $\mu_{A \cap B} = \min(\mu_A, \mu_B)$
        Subset: $A \subseteq B$ iff $\mu_A(x) \leqslant \mu_B(x)$,   for all x in X.
        A fuzzy relation R^n is a fuzzy subset of X^n.

In most of the cases of fuzzy sets that we will be interested in, the membership function is not primitive. That is, in most cases membership functions will assign values between zero and one to individuals on the basis of some property or properties of those individuals. Take tallness for example. How *tall* one is considered to be depends upon what one's *height* is (plus various contextual factors) – and height is given in terms of

actual physical measurements. To see how the membership function for tallness might be given (in a fixed context) in terms of height, see Figures 1 and 2. As a subjective approximation we might say

HEIGHT	5'3"	5'5"	5'7"	5'9"	5'11"	6'1"	6'3"
DEGREE OF TALLNESS	0	0.1	0.3	0.55	0.8	0.95	1

Fig. 1. Subjective assignment of degrees of tallness of men relative to the population of contemporary America.
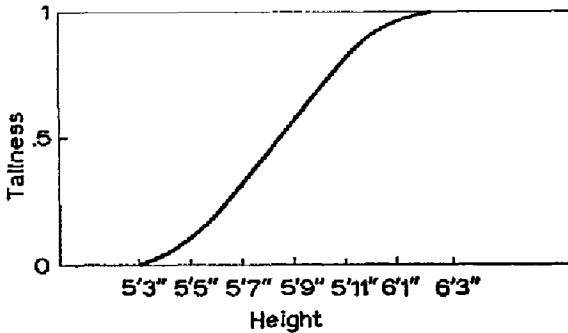

5'3" 5'5" 5'7" 5'9" 5'11" 6'1"   6'3"
Height
Fig. 2.

that if someone is smaller than 5'3", then he is not tall to any degree. If he is 5'7", we might say that he is tall to, say, degree 0.3. If he is 5'11", we might say that he is tall to, say, degree 0.8. And if he is over 6'3", then he is tall, period. The curve plotted in Figure 2 is not to be taken with great seriousness as to its exactitude. Undoubtedly the function which maps height into tallness is itself fuzzy. However, I do think that the curve in Figure 2 is not a bad approximation to my own intuitions about degrees of tallness. The curve has about the right shape. It rises continuously, as it should. It would be wrong to have a curve that falls or has several dips. It goes up from zero at about the right place and seems to hit one at about the right place. In short, there is far more right than wrong about it, which is what makes it an interesting approximation.

We should also ask how seriously we should take the fact that the function for tallness given in Figure 2 is continuous, assigning an infinite

number of values, in fact filling the uncountable infinity of values in the real interval between zero and one. After all, human beings cannot perceive that many distinctions. Perhaps it would be psychologically more real not to have an infinity of degrees of set membership, but rather some relatively small number of degrees, say the usual 7 ± 2. On the other hand, one might consider the interesting possibility that the finiteness of human perceptual distinctions is what might be called a surface phenomenon. It might be the case that the perception of degrees of tallness is based on an underlying continuous assignment of values like that given by the curve in Figure 2. The finite number of perceived distinctions would then result from 'low level' perceptual factors, though perhaps the number of perceived distinctions and their distribution would depend on the shape of the underlying curve (as indicated in Figure 3) and various contextual

Finite number of perceived intermediate values
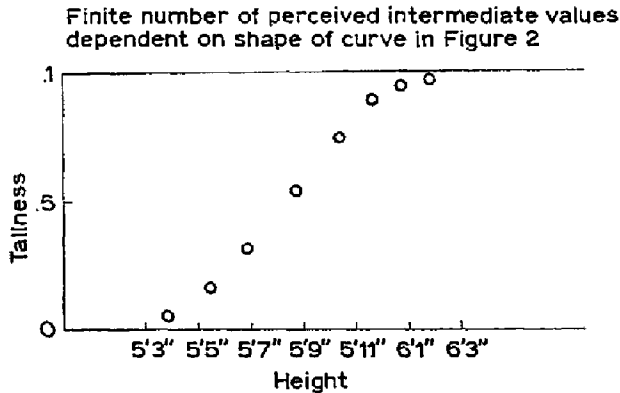dependent on shape of curve in Figure 2

Fig. 3.

factors. I think that the latter proposal has a high degree of plausibility, and I think that some of the facts discussed below will make it even more plausible. For this reason, I will stick to continuous assignments of values.

Given fuzzy sets we can, in a straightforward way extend classical propositional and predicate logics to the corresponding fuzzy logics. Take the syntax of a typical propositional logic with '¬', '∧', '∨', '→' as connectives and 'P', 'Q', ... as propositional variables. Let 'P', 'Q', ... have values in the closed interval [0, 1], and let $P, Q, \ldots$ stand for the values

of the propositional variables. Valuations for the connectives are defined
as follows:[2]

(2) $|\neg P| = 1 - |P|$
           $|P \wedge Q| = \min(|P|, |Q|)$
           $|P \vee Q| = \max(|P|, |Q|)$
           $|P \to Q| = 1$ iff $|P| \leqslant |Q|$.

Semantic entailment is defined as follows:

(3) $P \Vdash Q$ iff $|P| \leqslant |Q|$ in all models

As should be obvious:

           $P \Vdash Q$ iff $\Vdash P \to Q$.

To see what this means, let $P =$ 'John is tall' and $Q =$ 'Bill is rich'. Let
$P = 0.7$ and $Q = 0.4$, that is, suppose John is tall to degree 0.7 and Bill is
rich to degree 0.4. 'John is not tall' will be true to degree 0.3, while 'Bill
is not rich' will be true to degree 0.6. 'John is tall and Bill is rich' will be
true to degree 0.4, which is the minimum of 0.7 and 0.4. 'Either John is tall
or Bill is rich' will be true to degree 0.7, the maximum of 0.7 and 0.4.

HEIGHT	5'3	5'5"	5'7"	5'9"	5'11"	6'1"	6'3"	6'5"
DEGREE OF 'VERY TALL'	0	0	0.1	0.2	0.5	0.8	0.9	1.0

Fig. 4.   Subjective assignment of degrees to 'very tall' relative to the population of
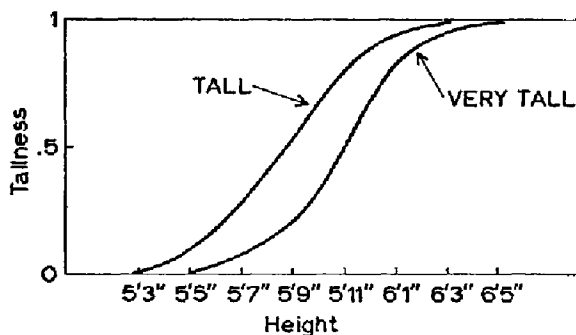contemporary American men.

Fig. 5.

For an example of how semantic entailment works, let $P=$'John is very tall' and $Q=$'John is tall'. Clearly P semantically entails Q. Consider an assignment of values to 'very tall', as in Figure 4 and Figure 5.

If you compare the tables in Figures 1 and 4, you will find that in each case, given a height, the value for 'VERY TALL' is *LESS THAN OR EQUAL TO* the value of 'TALL'. For example, at the height of 5'11", the value for 'VERY TALL' is 0.3, while the value for 'TALL' is 0.8. This can be seen clearly in Figure 5, where both curves are given. Thus, no matter what John's height is, the value of 'John is very tall' will be less than or equal to the value of 'John is tall', and by the above definition, the former will semantically entail the latter.

'$\rightarrow$' is the fuzzy logic correlate to material implication. '$P \rightarrow Q$' will hold in all cases where there is a real logical implication relation between P and Q, that is, where it is necessarily the case that $|P| \leqslant |Q|$. And like classical material implication, it will also hold in all cases where $|P|$ happens to be less than or equal to $|Q|$. If we restrict ourselves to propositional variables with only the values 0 and 1, '$P \rightarrow Q$' is indistinguishable from '$\neg P \vee Q$'. But when one considers the range of intermediate values between 0 and 1, '$P \rightarrow Q$' becomes very different from '$\neg P \vee Q$'. Given the above semantics, '$P \rightarrow P$' is a tautology, but '$\neg P \vee P$' is not.

Incidentally, I consider it a virtue of this system that '$\neg P \vee P$' is not a tautology. Suppose 'P' is 'This wall is red'. Suppose the wall is pretty red, say, to degree 0.6. Then 'This wall is red or not red' will be true to degree 0.6, according to the given semantics. This seems to me within the range of plausibility. Certainly one would not want to say that the sentence was true in such a situation. Similarly, '$P \wedge \neg P$' is not a contradiction in the above system. And similarly, the sentence 'This wall is red and not red' in the situation given where the wall is red to some extent seems to me not to be false, but rather to have a degree of truth.

Another fact about the fuzzy propositional logic given above is worth noting. Modus ponens is not only a valid form of inference, but it can be generalized so that it preserves degree of truth.

$$(4) \qquad \vdash_a P$$
$$\vdash P \rightarrow Q$$
$$\therefore \vdash_a Q$$

If we know, given our assumptions, that P is true at least to degree α, and

we know that $P \to Q$ is true, then we can be sure that Q is true at least to degree α.

We can get a better idea of what fuzzy propositional logic is like if we look at the classical tautologies that are valid and not valid in FPL.

(5)

NOT VALID IN FPL	*VALID IN FPL*
$P \lor \neg P$	$P \to P$
$P \to (Q \to P)$	$(P \to (Q \to R)) \to$
	$\quad ((P \to Q) \to (P \to R))$
$\neg P \to (P \to Q)$	$(\neg P \to \neg Q) \to (Q \to P)$
$((P \land Q) \to R) \leftrightarrow$	$\neg \neg P \leftrightarrow P$
$\quad (P \to (Q \to R))$	
$(P \to (Q \land \neg Q)) \to \neg P$	$(P \land \neg P) \to P$
$(P \land \neg P) \to Q$	$((P \to Q) \land \neg Q) \to \neg P$
$Q \to (P \lor \neg P)$	$(P \to Q) \to$
	$\quad ((Q \to R) \to (P \to R))$
The above are true in FPL in all models in which P, Q, and R are either 0 or 1.	De Morgan's Laws
	Associative Laws
	Distributive Law
	Commutative Laws

FPL reduces to ordinary propositional logic when the propositional variables are limited to the values 0 and 1.

Fuzzy propositional logic can be extended to fuzzy predicate logic in a straightforward way by defining valuations for predicates and for quantifiers. Let F be an n-place predicate. We define the value of $Fx_1, \ldots, x_n$ as the degree of membership of the ordered n-tuple $(x_1, \ldots, x_n)$ in the fuzzy set F, where $\bar{x}_i$ is the denotation of x_i on a given valuation. In other words,

$$|Fx_1, \ldots, x_n| = \mu_F(\bar{x}_1, \ldots, \bar{x}_n).$$

For example, suppose we have a one-place predicate TALL. The value of 'TALL(j)' is the degree to which the individual denoted by 'j' is a member of the fuzzy set TALL.

$$|\text{TALL}(j)| = \mu_{\text{TALL}}(j)$$

Valuations for quantifiers are straightforwardly defined. The value of '$\forall x F x$' is defined as minimum of the values of Fx for all assignments of x

to elements of the universe of discourse. The value of '$\exists x F x$' is the corresponding maximum.[3] In other words,

(6) $|\forall x F x| = \min\{|Fx|\}$ for all assignments to x of elements of
                                              the universe of discourse

       $|\exists x F x| = \max\{|Fx|\}$ for all assignments to x of elements of
                                              the universe of discourse.

We can get a fuzzy modal logic by adding operators '$\square$' and '$\Diamond$' and giving the following valuations.

(7) $|\square P|_w = \min\{|P|_{w'}\}$ for all w' such that Rww'

       $|\Diamond P|_w = \max\{|P|_{w'}\}$ for all w' such that Rww',
                                          where R is the alternativeness relation.

Note that the value of '$\square P$' will be equal to some α, $0 < \alpha < 1$, just in case the value of P never falls below α in any alternative world. If '$\square P$' is interpreted as meaning that P is a necessary truth, then in fuzzy modal logic, we will have degrees of necessary truth, since $|\square P|$ may fall in between zero and one. This raises the question of whether there are such things as statements which are necessarily true to a degree. The one type of possible example that comes to mind is an arithmetic statement that contains an approximation. For example, consider (8).

(8)   Approximately half of the prime numbers are of the form
                                                          $4N + 1$.

Is (8) true? Well, it depends on just what you mean by 'approximately'. But if you leave 'approximately' vague, as it is in normal English, then (8) is certainly not false, though one would, I think, hesitate to say that it is absolutely true. One would, however, have to ascribe to (8) a high degree of truth. And since (8) is a statement about arithmetic, one would be ascribing to (8) a high degree of *necessary* truth. Many other similar examples can be concocted by the reader. To take just one more example:

(9)   For almost all nonnegative integers N, 2^N is much greater
       than zero.

Here there are two vague concepts, 'almost all' and 'much greater than'. But allowing their usual fuzzy meanings, I think one would want to say

that (9) had a high degree of truth – necessary truth – though again I doubt
than one would want to say that (9) is absolutely true.

I have no particular philosophical ax to grind here. I am merely sug-
gesting that fuzzy modal logics might have some application in explicating
the status of arithmetical statements that contain vague words. To my
knowledge, the status of such statements has not been hitherto explicated.

We have been employing a many-valued logic in an attempt to provide
an initial explication of fuzziness in natural language. Many-valued logics
have also been used in an attempt to explicate the natural language no-
tion of a presupposition. It seems natural to ask what happens when fuzzy
logic mixes with presuppositional logic. One would not expect particularly
drastic results. But something rather drastic does happen – and the results
are I think surprising and interesting. Suppose that one were to try to
extend *FPL* to a presuppositional logic. Recall that *FPL* is already a
many-valued logic and that none of its values correspond to presuposi-
tion failure. To account for presupposition failure, we might expect to add
still another value or range of values. But it is not clear just what we will
be forced to.

Suppose initially that we try to extend *FPL* to a presuppositional logic,
keeping the valuation: $|\neg P| = 1 - |P|$. Let us assume the usual definition
of presupposition.[4]

(10)   *P* presupposes *Q*   iff   *P* ⊩ *Q* and ¬ *P* ⊩ *Q*.

Taking the above definition of entailment, we find that:

(11)   *P* presupposes *Q*   iff $|P| \leqslant |Q|$ and $1 - |P| \leqslant |Q|$ in all
          models.

But from (11), it follows that *P* presupposes *Q* only if $|Q| \geqslant 0.5$!! This is
a truly crazy result. Consider some examples.

(12)   a. The present king of France is bald.
          b. There is presently a king of France.
(13)   a. Dick Cavett regrets that he is tall.
          b. Dick Cavett is tall.

In each case, we would like to say that the (a) sentence presupposes the
(b) sentence – regardless of the truth value of the (b) sentence. But if we
accept (10) and we accept $|\neg P| = 1 - |P|$, then it will turn out that the

(a) sentences will presuppose the corresponding (b) sentences only if the values of the latter never fall below 0.5 in any model. Clearly this is undesirable. (12a) should presuppose (12b) even if there is no present king of France. (13a) should presuppose (13b), even though Dick Cavett happens to be tall, say, to degree 0.3.

One way to avoid this problem is by looking at standard valuations in a somewhat different way. Instead of assigning a truth value to a proposition, we can view standard valuations as assigning an ordered pair, (t, f), consisting of a truth value and a falsity value, whose sum is 1. For presuppositional logic, we propose (following an idea of Zadeh's) to extend the ordered pair to an ordered triple, (t, f, n), whose sum is 1, and whose third place would be interpreted as a nonsense value. A statement that was total nonsense would have a nonsense value of 1 and truth and falsity values of zero. The following valuation for $\neg P$ would be needed:

(14) $|\neg P| = (\alpha, \beta, \gamma)$ iff $|P| = (\beta, \alpha, \gamma)$, where $\alpha + \beta + \gamma = 1$

In other words, the truth value of $\neg P$ would be the falsity value of P and vice versa, and both P and $\neg P$ would have the same degree of nonsense.

In any presuppositional logic meeting these conditions, the above problem with the definition of presupposition does not arise. Since the value of $|\neg P|$ is not necessarily equal to $1 - |P|$, the value of $|Q|$ in (10) does not have to be 0.5 or greater. In fact, no conditions are placed on its value, as should be the case. The intuitive reason why this works is that the third place, the nonsense-value slot, provides the additional range of values to cover presupposition failure. Incidentally, given (14), the falsity value becomes redundant, since it can be computed from the truth- and nonsense-values. Consequently, we only need to assign ordered pairs, (α, γ), of truth- and nonsense-values since $\beta = 1 - (\alpha + \gamma)$.

I don't mean to suggest that the above solution is the only possible one, though it may well be the only one that permits both truth-functional negation and degrees of nonsense. Whether these are good things to have is another matter, which I will not take up here. (See Appendix II for a discussion of degrees of nonsense.)

For those interested in investigating such systems further, the connections with other presuppositional logics is worth considering with respect to the matter of how to define valuations for connectives. This will be taken up in Appendix II.

There are, of course, other issues to be considered in the study of fuzzy logic. For example, suppose we allow for fuzzy denotations. That is, suppose we generalize assignments of variables to individuals in the domain of discourse so that a variable x will denote an individual a to a degree α, $0 \leqslant \alpha \leqslant 1$. It might be useful for someone to investigate such systems, since there seem to be cases of fuzzy denotations in natural language. Consider (15).

(15)   a. The real numbers approximately equal to 5 are less than 1000.
         b. The real numbers approximately equal to 5 are less than 5.1.

These sentences contain a definite description which denotes only fuzzily. No nonfuzzy set of real numbers is picked out by 'the real numbers approximately equal to 5'. Yet we can make true statements about arithmetic using such descriptions. (15a) is one such statement. (15b) is another matter. (15b) sounds to me like a case of presupposition failure; at least it seems neither true nor false to any degree, but rather inappropriate. Such cases seem to me to be worthy of study.

Another interesting question concerns degree of entailment. We have defined '→' to have values zero and one. Suppose it were to take on intermediate values. Then one would like to generalize the notion of entailment to the notion of entailment to a degree α (written $\Vdash \alpha$), with (16) holding.

(16) $P \Vdash_\alpha Q$ iff $\Vdash_\alpha P \rightarrow Q$.

This will be discussed in Appendix I. Of course, in any such system we will want to talk about such concepts as 'degree of validity' and 'degree of theoremhood', which are natural concomitants of the notion 'degree of necessary truth'. If one wants a natural example of degree of entailment, consider (17) and (18).

(17) x is a bird.
(18) x flies.

We know that not all birds fly, but we might well want to say that once a bird has a certain degree of birdiness, say 0.7, then it flies. We might then want to say that (17) entails (18) to degree 0.7.

The purpose of this discussion of fuzzy logic has been to show that one need not throw up one's hands in despair when faced by the problems of vagueness and fuzziness. Fuzziness can be studied seriously within formal semantics, and when such a serious approach is taken, all sorts of interesting questions arise. For me, some of the most interesting questions are raised by the study of words whose meaning implicitly involves fuzziness – words whose job is to make things fuzzier or less fuzzy. I will refer to such words as 'hedges'. A small list (which is far from complete) appears on the following page.

3. HEDGES

Let us begin with a hedge that looks superficially to be simple: *sort of*. Just as *very* is an intenisfier in that it shifts values to the right and steepens the curve (see Figure 5), so *sort of* is, in part at least, a deintensifier in that it shifts the curve to the left and makes it less steep. However it also drops off sharply to zero on the right. Consider the notion 'sort of tall' in Figure 9 below. The values for 'sort of tall' are greatest when you are of intermediate height. If you are of less than intermediate height, then the values for 'sort of tall' are greater than those for 'tall'. But above intermediate height the values for 'sort of tall' drop off sharply. If you're really tall, you're not sort of tall.

The same thing is true in the case of birdiness.

(1) a. A robin is sort of a bird.   (False – it is a bird, no question, about it)
     b. A chicken is sort of a bird. (True, or very close to true)
     c. A penguin is sort of a bird. (True, or close to true)
     d. A bat is sort of a bird.   (Still pretty close to false)
     e. A cow is sort of a bird.   (False)

Sort of is a predicate modifier, but one of a type that has not been previously studied in formal semantics in that its effect can only be described in terms of membership functions for fuzzy sets. It takes values that are true or close to true and makes them false while uniformly raising values in the low to mid range of the scale, leaving the very low range of the scale constant. The effect of *sort of* cannot even be described in a two-valued system, where sentences are either true or false and individuals are either set members or not. Consider again example (4) of section I, where

we saw that there were speakers who did not distinguish between the (a), (b), and (c), sentences, but rather lumped them together as all being true, as in the corresponding sentences of example (5) of section I. However, even such speakers distinguish the (a) sentence in example (1) of this section from the (b) and (c) sentences. In order for them to do this, they must have been able to make an underlying distinction in degree of birdiness between robins on the one hand and chickens and penguins on the other. The effect of the predicate modifier *sort of* depends upon just such a dis-

---

SOME HEDGES AND RELATED PHENOMENA

sort of	in a real sense
kind of	in an important sense
loosely speaking	in a way
more or less	mutatis mutandis
on the _____ side (tall, fat, etc.)	in a manner of speaking
roughly	details aside
pretty (much)	so to say
relatively	a veritable
somewhat	a true
rather	a real
mostly	a regular
technically	virtually
strictly speaking	all but technically
essentially	practically
in essence	all but a
basically	anything but a
principally	a self-styled
particularly	nominally
par excellence	he calls himself a ...
largely	in name only
for the most part	actually
very	really
especially	(he as much as ...
exceptionally	-like
quintessential(ly)	-ish
literally	can be looked upon as
often	can be viewed as
more of a _____ than anything else	pseudo-
almost	crypto-
typically/typical	(he's) another (Caruso/Lincoln/ Babe Ruth/...)
as it were	_____ is the _____ of _____
in a sense	(e.g., America is the Roman Empire of
in one sense	the modern world. Chomsky is the DeGaulle of Linguistics. etc.)

tinction. There are other types of predicate modifiers that reveal such distinctions.

(2)	a. A robin is a bird par excellence.	(true)
	b. A chicken is a bird par excellence.	(false)
	c. A penguin is a bird par excellence.	(false)
(3)	a. A chicken is a typical bird.	(false)
	b. In essence, a chicken is a bird.	(true)
(4)	a. In a manner of speaking, a bat is a bird.	(true or close to true)
	b. In a manner of speaking, a cow is a bird.	(false)
	c. In a manner of speaking, a chicken is a bird.	(nonsense – (c) presupposes that chickens are not really birds, which is false).

As (2) reveals, *par excellence* requires the highest degree of category membership. Robins fit, chickens and penguins don't. *Typical*, in (3), also requires a high degree of membership, which is why chickens don't fit. But a high degree of membership isn't sufficient for *typical*, as (5) shows.

(5)	a. A robin is a typical bird.	(true)
	b. An eagle is a typical bird.	(false, or at least far from true)

Even though eagles seem to rank high in birdiness, the fact that they are predators makes them atypical of birds. What examples (2) – (5) seem to show is that people do make the full range of distinctions in the birdiness hierarchy. Though these distinctions may be subtle, they can be thrown into clear relief by hedges.

But hedges do not merely reveal distinctions of degree of category membership. They can also reveal a great deal more about meaning. Consider (6).

(6)	a. Esther Williams is a fish.
	b. Esther Williams is a regular fish.

(6a) is false, since Esther Williams is a human being, not a fish. (6b), on the other hand, would seem to be true, since it says that Esther Williams

swims well and is at home in water. Note that (6b) does not assert that
Esther Williams has gills, scales, fins, a tail, etc. In fact, (6b) presupposes
that Esther Williams is not literally a fish and asserts that she has certain
other characteristic properties of a fish. Bolinger (1972) has suggested
that *regular* picks out certain 'metaphorical' properties. We can see what
this means in an example like (7).

(7)   a. John is bachelor.
         b. John is a regular bachelor.

(7b) would not be said of a bachelor. It might be said of a married man
who acts like a bachelor – dates a lot, feels unbound by marital respon-
sibilities, etc. In short, *regular* seems to assert the connotations of 'bach-
elor', while presupposing the negation of the literal meaning. (7) reveals
the same fact, though perhaps more clearly.

(8)   a. Sarah is a spinster.
         b. Sarah is a regular spinster.

(8b) asserts that Sarah has certain characteristic properties of spinsters –
presumably that she is prissy and disdains sexual activity. (8b) would not
be said of someone who was literally a spinster, but might be said either
of a married woman or a girl who was not yet past marriageable age who
acted like a spinster. What (8b) asserts is the connotation of 'spinster' –
prissiness and lack of sexual activity, while presupposing the negation of
the literal meaning.

   If this account of the meaning of *regular* is essentially correct, a rather
important conclusion follows. It is usually assumed that the connotations
of words are part of pragmatics – the wastebasket of the study of meaning.
Certainly most philosophers seem to take it for granted that connotations
and other pragmatic aspects of meaning are irrelevant to the assignment
of truth values (leaving aside sentences containing indexical expressions).
Truth is usually taken to involve literal or denotative meaning alone. Yet
in sentences with *regular*, such as (6b), (7b) and (8b), the truth value of the
sentences as a whole depends not upon the literal meaning of the predicates
involved, but strictly upon their connotations! What this indicates, I think,
is that semantics cannot be taken to be independent of pragmatics, but
but that the two are inextricably tied together.

   In the above discussion I used the terms 'literal meaning' and 'connota-

tion' as though they were adequate to describe at least informally the types of meaning components affected by hedges and related words. But as might be expected the situation is more complex. We can see this if we try to find some hedges that are opposites of *regular*, ones which pick out literal meaning alone. Two promising candidates are *strictly speaking* and *technically*.

(9)   a. A whale is technically a mammal.
        b. Strictly speaking a whale is a mammal.

Technically and *strictly speaking* seem to have the same effect in (9a) and (b). However, in other sentences they produce radically different results.

(10)   a. Richard Nixon is technically a Quaker.   (true)
        b. Strictly speaking, Richard Nixon is a Quaker.   (false)
(11)   a. Ronald Reagan is technically a cattle rancher.   (true)
        b. Strictly speaking, Ronald Reagan is a cattle rancher.
                                                         (false)
(12)   a. Strictly speaking, George Wallace is a racist.
        b. Technically, George Wallace is a racist.

As (10) and (11) show, *technically* picks out some definitional criterion, while *strictly speaking* requires both the definitional criterion and other important criteria as well. Richard Nixon may be a Quaker in some definitional sense, but he does not have the religious and ethical views characteristic of Quakers. He meets the definitional criterion, but not other important criteria. Ronald Reagan meets the definitional criterion for being a cattle rancher since he seems to have bought cattle stocks as a tax dodge (which is reported how he avoided 1970 income taxes). However, he does not meet all of the primary criteria for being a cattle rancher. Note that, as (12) shows, *technically* seems to mean *only technically*, that is it asserts that the definitional criteria are met but that some important criterion for category membership is not met. Hence the strangeness of (12b).

Strictly speaking contrasts interestingly with *loosely speaking*.

(13)   a. Strictly speaking, a whale is a mammal.
        b. Loosely speaking, a whale is a fish.

(13) shows the need for distinguishing between important or primary properties on the one hand and secondary properties on the other hand.

(13a) says that whales classify as mammals if we take into account im-
portant criteria for distinguishing mammals from fish. For example, they
give live birth and breathe air. (13b) seems to say that we can classify
whales as fish if we ignore the primary properties and take into account
certain secondary properties, for example, their general appearance and
the fact that they live in water. Thus, we need to distinguish between
primary and secondary criteria for category membership.

However, *loosely speaking* still differs sharply from *regular*, as the
following examples show:

(14)   a. Harry is a regular fish.
        b. Loosely speaking, Harry is a fish.
(15)   a. Loosely speaking, a whale is a fish.
        b. A whale is a regular fish.

What is strange about (14b) is that it asserts that Harry is a member of
the category fish to some degree by virtue of having some secondary
property of fish. (14a) simply says that he swims well and at is home in
water, while it presupposes that he is not a member of the category fish to
any degree whatever. The distinction between (14a) and (14b) indicates
that we must distinguish between those properties capable of conferring
some degree of category membership and those properties which happen
to be characteristic of category members, but do not confer category
membership to any degree at all. No matter how well you swim, that
won't make you a fish to any degree at all. But if you are a living being,
live in the water, are shaped like a fish, and your only limbs are flippers
and a tail, it would seem that, like the whale, you are loosely speaking,
that is by virtue of secondary criteria, a member of the category fish to
some extent. Note that (15b) is odd in that it presupposes that the whale
is not a member of the category fish to any extent.

An adequate account of the functioning of characteristic-though-in-
cidental properties should provide an understanding of at least one type
of metaphor. Suppose I say 'John is a fish'. I am using a metaphor to in-
dicate either that he swims well or that he is slimy (in the nonliteral sense).
The mechanism for this is, I think, something like the following. Since it
is presupposed that the subject, John, is not literally a member of the
category fish, one cannot be asserting membership in that category if the
sentence is to make sense. Instead, the sentence is understood in essen-

tially the same way as 'John is a regular fish', that is, the contextually most important incidental-though-characteristic properties are asserted. (For a discussion of metaphor and fuzzy logic, see Reddy, 1972.)

By looking at just four hedges – *technically, strictly speaking, loosely speaking* and *regular* – we have seen that we must distinguish at least four types of criteria for category membership:

(16)   TYPES OF CRITERIA
        1. Definitional ⎤ – capable of conferring category member-
        2. Primary   ⎬ ship to a certain degree depending on
        3. Secondary ⎦ various factors
        4. Characteristic though incidental – not capable of confer-
                                     ring category member-
                                     ship to any degree, but
                                     contributes to degree of
                                     category membership if
                                     some degree of member-
                                     ship is otherwise estab-
                                     lished.

These distinctions are necessary for even a primitive account of how such hedges function. Such a primitive account is given in (17).

(17)   An Informal and Inadequate Approximation to an Under-
        standing of Some Hedges
        TECHNICALLY – Truth value depends upon values of defi-
                              nitional criteria alone. Implies that at
                              least one primary criterion is below the
                              threshold value for simple category
                              membership.
        STRICTLY SPEAKING – Truth value depends on value of
                              definitional and primary criteria.
                              Values for each criterion must be
                              above certain threshold values.
        LOOSELY SPEAKING – Truth value depends primarily on
                              secondary criteria. Implies that
                              threshold values for definitional
                              and primary criteria are insuf-

> ficient to confer category member-
> ship.
>
> REGULAR – Truth value depends upon characteristic-
> though-incidental criteria. It is presupposed
> that the values of other criteria are insufficient
> to establish any degree of category member-
> ship.

The facts in (17) cannot be handled within the framework of fuzzy logic as developed above, since they require a distinction between types of criteria for category membership. Nor can they, so far as I know, be handled by any logic developed to date. Let us consider what type of logic would be needed to handle such cases.

4. FUZZY LOGIC WITH HEDGES

Let each predicate F be assigned two values, a vector value $\|F\|$ and an absolute value $|F|$. The absolute value will be the membership function for a fuzzy set.

(1) $|F| = \mu_L.$

Suppose the membership function is itself a function of a k-tuple of criteria, that is, of other membership functions:

(2)   Suppose $\mu_F = f(\mu_{G_1}, ..., \mu_{G_k}).$

For example, if μ_F is the membership function for the fuzzy set of birds, then μ_{G_1} might be the membership function for the fuzzy set of animals with wings, and μ_{G_2} might be the membership function for the fuzzy set of animals with feathers, etc. We define the k-tuple $(\mu_{G_1}, ..., \mu_{G_k})$ as the vector value of the predicate F and call each element of the k-tuple a 'meaning component'.

(3) $\|F\| = (\mu_{G_1}, ..., \mu_{G_k}).$

Corresponding to the four types of meaning components discussed above in Section 3 – definitional, primary, secondary, and characteristic-though-incidental – we define four functions: *def, prim, sec,* and *char*. These will, when applied to the vector value of a predicate F, pick out the appropriate

meaning components and form a new function, which itself will be a membership function for a fuzzy set. For the sake of discussion, let us assume that *def, prim, sec* and *char* each forms the intersection of the meaning components that they pick out of the vector value of F. This is obviously over-simplified; various complex combinations will be needed. We are now in a position to give a first approximation to valuations for the hedges *technically* (TECH), *strictly speaking* (STR), *loosely speaking* (LOOS), and *regular* (REG).

(3)   Let TECH, STR, LOOS, and REG be predicate modifiers.
$|TECH(F)| = def(\|F\|) \cap NEG(prim(\|F\|))$ where:
$$NEG(f) = 1 - f$$
$|STR(F)| = def(\|F\|) \cap prim(\|F\|)$
$|LOOS(F)| = sec(\|F\|) \cap NEG(def(\|F\|) \cap prim(\|F\|))$
$|REG(F)| = char(\|F\|) \cap NEG(def(\|F\|) \cap prim(\|F\|) \cap$
$$\cap sec(\|F\|)).$$

(3) is simply a formal way of saying what is said informally in (17) in Section 3. For example, the value of *technically* is the value of the definitional criteria of the predicate modified intersected with the value of the negative of the primary criteria.

As will be seen below, the analysis given in (3) is inadequate in various ways. But inadequacies aside, (3) could not actually be applied in a fuzzy logic unless vector values were assigned to all the predicates and unless the functions *def, prim, sec* and *char* were defined in terms of those vector values. But that means that we have to seriously study the meaning of predicates in a way that has not previously been done. One thing that the study of hedges in formal terms like (3) can do is to give us a technique for doing the empirical study of meaning components. By comparing the truth conditions for predicates both unhedged and then with various hedges, we may be able to sort out the meaning components.

So far, we have looked only at hedges whose truth conditions depend on vector values. One might ask a whether there are any hedges whose valuations depend only on absolute values. Zadeh has claimed that such hedges do exist. Though I think his analyses are inadequate in certain important respects, there is also something right about them. Zadeh's basic idea is that there is a small number of basic functions that, in combination, produce a wide range of modifiers specifically, absolute value

modifiers, for fuzzy predicates. Aside from the Boolean functions of inter-
section, union, and complementation (which we will write NEG), Zadeh
(1971a, 1972) has suggested the following:

(4)   Some Zadeh Functions
        Concentration: $\mu_{CON(F)} = \mu_F^2$
        Dilation: $\mu_{DIL(F)} = \mu_F^{1/2}$
        Contrast intensification: $\mu_{INT(F)}(x) = 2\mu_F(x)$,
                            for $0 \leqslant \mu_F(x) \leqslant 0.5$.
                    $\mu_{INT(F)}(x) = 1 - 2(1 - \mu_F(x))^2$,
                            for $0.5 \leqslant \mu_F(x) \leqslant 1$.
        Convex combination: $\mu_F = w_1\mu_{G_1} + w_1\mu_{G_2} + \cdots + w_k\mu_{G_k}$,
                    where w_i is in (0,1) and
                            $w_1 + \cdots + w_k = 1$.

Convex combination is simply a weighted sum. Though I suggested
above that functions like *prim* and *sec* were intersections of the primary
and secondary criteria, respectively, for a given predicate, it seems more
likely that such functions are actually weighted sums. The effects of CON,
DIL, and INT are given in the following diagrams.

CON lowers the values and makes the curve steeper. If the curve is
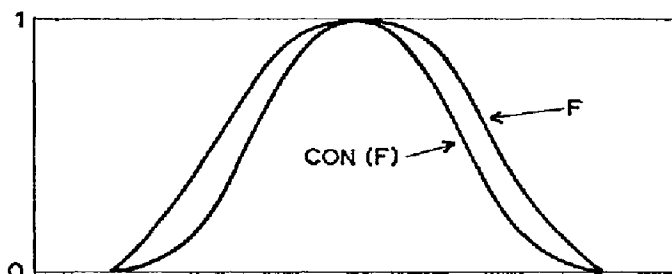
Fig. 6.

bell-shaped, CON pulls the values in toward the center as shown in Figure
6. DIL raises the values and makes the curve less steep. If the curve is
bell-shaped, DIL spreads the values out as shown in Figure 7. INT raises
higher values and lowers lower values, thus making for greater contrast,
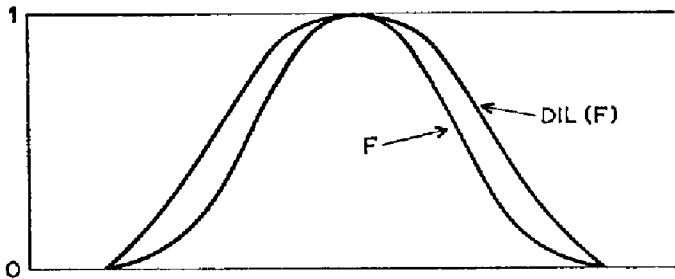as shown in Figure 8. Note that the following relations hold.
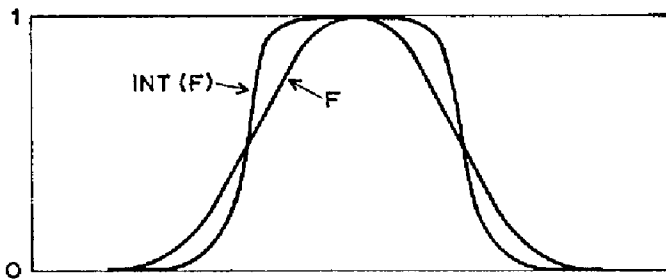
Fig. 7.

Fig. 8.

(5)   a. $\mu_{\text{DIL}}(f) = \mu_{\text{NEG(CON(NEG}(F)))}$

       b. $\mu_{\text{INT}(F)}(x) = \mu_{\text{MULT(CON}(F))}(x)$ for $0 \leqslant x \leqslant 0.5.$

       c. $\mu_{\text{INT}(F)}(x) = \mu_{\text{NEG(MULT(CON(NEG}(F))))}(x)$ for $0.5 \leqslant x \leqslant 1,$

$$\text{where } \mu_{\text{MULT}(F)} = 2\mu_F$$

In these definitions Zadeh happened to use squares, square roots, and factors of two. However, he does not intend those exact numbers to be taken seriously. What he does intend to be taken seriously is the kinds of effects these functions have on the curve. Whether 3 or 1.745 would be better numbers than 2 in such functions is irrelevant, so long as CON pulls the curve in, DIL spreads it out and INT heightens contrasts – and so long as the relations given in (5) continue to hold. In fact, all these functions may well be fuzzy themselves, so talk of an exact multiple or power in the equations may make no sense.

The point of Zadeh functions is to define valuations for modifiers using them. Zadeh has suggested the following as approximations:
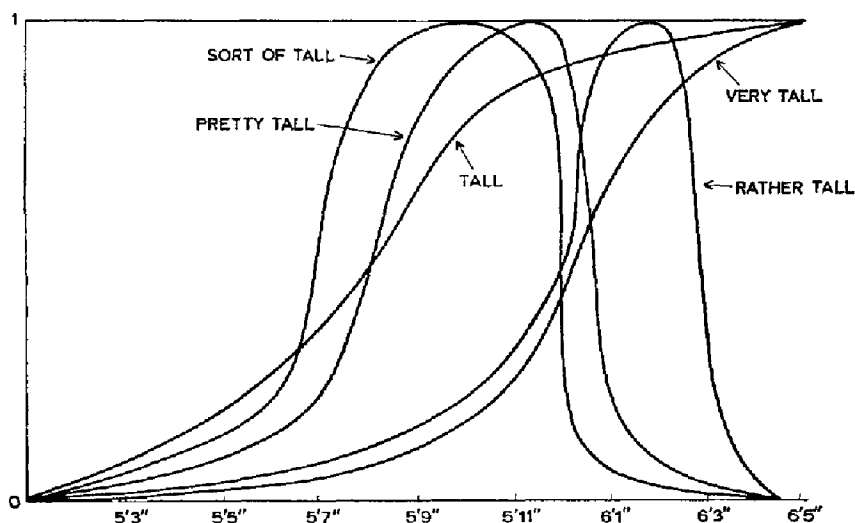
Fig. 9.

(6) Some examples of modifier valuations using Zadeh functions

$$|VERY(F)| = CON(|F|)$$
$$|SORT\ OF(F)| = INT(DIL(|F|)) \cap INT(DIL(NEG(|F|)))$$
$$|PRETTY(F)| = INT(|F|) \cap NEG(INT(CON(|F|)))$$
$$|RATHER(F)| = INT(CON(F))$$
$$\text{or } INT(CON(|F|)) \cap NEG(CON(|F|))$$
$$[\text{rather, but not very}].$$

Figure 9 gives some idea of what the modified curves for TALL would be like. Whatever the shortcomings of the valuations in (6), I think there is something basically right about Zadeh's idea, and if there is, then there is a rather remarkable consequence: algebraic functions play a role in natural language semantics! Certainly the basic idea seems to be right – given the curve for TALL, one should be able to define derived curves for VERY TALL, RATHER TALL, PRETTY TALL and SORT OF TALL. Algebraic functions of some sort or other would seem to be necessary (though perhaps not sufficient) for the characterization of such derived curves.

Let us now return to the fact that people seem not to perceive an in-finite gradation of, say, tallness, but seem rather to perceive some relatively

small finite number of discrete values for tallness. Two possible ways of describing this were suggested above. (1) Restrict fuzzy logic to some finite number of values. (2) Keep an infinity of values, but assume the existence of a 'low level' perceptual apparatus that determines the number and distribution of the perceived values depending on the shape of the curve and various contextual factors. I think (2) is correct on a number of grounds. First, the number of perceived values seems to be variable, to change somewhat from concept to concept and context to context. This means that there would be no single fixed number of values to which we could restrict fuzzy logic. Second, the number and location of the perceived values seem to depend on the shape of the curve. Since such concepts as PRETTY TALL will be a complex function of TALL (if Zadeh is anywhere near correct). There will be no way to guarantee, given assumption (1), that the perceived values for TALL will map directly, via that function, onto the perceived values for PRETTY TALL, not to mention RATHER TALL, SORT OF TALL, VERY VERY TALL, etc. A good example is a case where VERY is iterated a large number of times with a concept where a limit will be approached, as in VERY VERY VERY VERY CLOSE TO 1000. One could probably perceive something like the usual 7 ± 2 values for CLOSE TO 1000, but the number seems to drop considerably for VERY VERY VERY VERY CLOSE TO 1000, which is not surprising on Zadeh's account of VERY, since a repeated squaring of the values for a curve like CLOSE TO 1000 will produce a function that approaches a vertical line. I think considerations like this force us to reject alternative (1) in favor of (2), which is, I think, an interesting result.

5. SOME INADEQUACIES OF THE TREATMENT OF HEDGES IN SECTION 4

I don't want to give the impression that I take the proposals in section IV to be correct in all or even most details. Hedges have barely begun to be studied and I have discussed only a handful. I have no doubt that the apparatus needed to handle the rest of them will have to be far more sophisticated. In fact, it is easy to show that far more sophisticated apparatus will be needed to handle merely the hedges discussed so far. Moreover I think that four types of criteria is far to few, though I have not done further investigation.

5.1. *Dependence upon Context*

The valuations for hedges given in Section 4 were independent of context. However, it is fairly easy to show that any adequate treatment will have to take context into account. Consider (1).

(1)   Technically, this TV set is a piece of furniture.

As Eleanor Heider (personal communication) has observed, there is no generally recognized technical definition accepted throughout American culture (or any other) that will tell you whether a particular TV set is or is not a piece of furniture. The range of TV sets goes from small portable ones that can easily be carried (perhaps in one's pocket) to large consoles with fancy wooden cabinets. But whether a given TV set is technically a piece of furniture will vary with the situation. For example, insurance companies or movers may set different rates for furniture, appliances, and other personal property. In such situations, technical standards have to be set and it is doubtful that there will be much uniformity. Yet, the truth or falsity or even the appropriateness of (1) in a given context will depend on what those standards are, if there are any. Moreover, different cultures, subcultures, or even individuals may differ as to which criteria for a given predicate are primary and which are secondary. In fact, it would not be surprising to find that which criteria were considered primary and which secondary depended on context. Consider (2).

(2)   a. Strictly speaking, Christine Jorgenson is a woman.
        b. Strictly speaking, Christine Jorgenson is a man.

One can imagine contexts in which either (2a) or (2b) would be true or very close to true. Take contexts where current sex is what matters, for example, job applications, sexual encounters, examinations for venereal disease, choice of rest room, etc. In such situations, (2a) would be true and (2b) would not. Take, on the other hand, situations where former sex might matter, for example, psychological studies of early childhood, classification with respect to military benefits, etc. In such situations, one could imagine that (2b) might be true and (2a) not. That is, current sex might be primary for determining manhood vs. womanhood in some contexts and former sex primary in others.

5.2. *Modifiers that Affect the Number of Criteria Considered*

Under Zadeh's proposals for the definition of words like VERY and
SORT OF, such modifiers affect only the absolute values of the predicates
modified. However, consider cases like VERY SIMILAR and SORT OF
SIMILAR. Things are similar or dissimilar not just to degrees, but also in
various respects. In judging similarity one picks out a certain number of
contextually important criteria, and determines degree of similarity on
the basis of how closely the values match for the criteria chosen. In deter-
mining the values for VERY SIMILAR, there are two possibilities. First,
one can, for the fixed number of criteria considered in judging mere simi-
larity, require that the values assigned to the various criteria be closer.
Secondly, one can require that more criteria be taken into account. For
example, consider (1).

(1)   a. Richard Nixon and Warren G. Harding are similar.
         b. Richard Nixon and Warren G. Harding are very similar.

In judging (1a) to be true to a certain degree, one might take into account
merely their records as president. One might then want to go on to assert
(1b) by taking into account other criteria, for instance, the personal lives,
moral values, etc.

SORT OF has the opposite effects when applied to SIMILAR.

(2)   a. George Wallace and Adolf Hitler are similar.
         b. George Wallace and Adolf Hitler are sort of similar.

(2b) can be a hedge on (2a) in two different respects. First, on the given
criteria considered, one may require less closeness of values for (2b) than
for (2a). Secondly, in judging the degree of truth of (2b) versus (2a), one
may take fewer criteria into account.

These considerations show that an adequate account of the meanings
of VERY and SORT OF cannot be given simply in terms of how they
affect the absolute values of the predicates they modify; one must take
into account the way they change the consideration of vector values. In
the case of similarity that includes both the closeness of selected vector
values and the number of them.

5.3. *Some Hedges Must Be Assigned Vector Values*

In the treatment given in Section 4, all of the hedges were assigned only

absolute values. That this is inadequate can be seen by considering an expression like VERY STRICTLY SPEAKING, as in (1).

(1)   a. Strictly speaking, Sam is not the kind of person we want to hire.
        b. Very strictly speaking, Sam is just the kind of person we do not want to hire.

One can imagine a situation in which one might say (1a) and then follow it up with (1b). Suppose one were running a business and had certain criteria for filling a certain job – objective qualifications, honesty, personality traits, etc., with some criteria being more important than others. Given that Sam did not measure up according to the primary criteria, one might accurately say (1a), though perhaps nothing stronger. Suppose that, one then isolated the most important of the primary criteria and looked at how Sam ranked with respect to those. With respect to those, he might not merely be unqualified but might actually be injurious to the business. One might then be in a position to make the stronger statement (1b). One of the things that VERY does, when applied to STRICTLY SPEAKING, is further restrict the number of categories considered most important: this can be viewed as changing the weights assigned to various criteria at the upper end of the spectrum. This is, incidentally, the opposite of what it does when applied to SIMILAR – and and I have no idea why. Be that as it may, VERY seems to operate on the vector value of STRICTLY SPEAKING, not just on the absolute value. This means that we must find a way of assigning vector values to hedges like STRICTLY SPEAKING. The same is true of LOOSELY SPEAKING, as expressions like VERY LOOSELY SPEAKING show. In this case, however, one of the effects of VERY is to *increase* the number of criteria considered – or at least increase the weights assigned to the lower end of the spectrum – the opposite of what happened in the case of VERY STRICTLY SPEAKING. Any adequate description of the meaning of VERY will have to take such considerations into account. Another thing suggested by these facts is that there may not be a strict division between primary and secondary criteria; rather there may be a continuum of weighted criteria, with different hedges picking out different cut-off points in different situations.

5.4. *Perhaps Values Should Not Be Linearly Ordered, But only Partially Ordered*

So far we have discussed the concept 'true to a certain degree'; we have paid hardly any attention to the concept 'true in a certain respect'. Any serious study of hedges like IN SOME RESPECTS, IN A SENSE, IN A REAL SENSE, etc. requires it. What these hedges seem to do is say there are certain criteria which, if given great weight, would make the statement true. Consider (1).

(1)   a. In some respects, Nixon has helped the country.
         b. In a sense, J. Edgar Hoover was a great man.
         c. In a real sense, Nixon is a murderer.

But very often, sentences without such hedges are meant to be taken in the same way.

(2)   Nixon is a murderer and he's not a murderer.

The usual sense of (2) is not either a statement of a contradiction nor a statement that Nixon is a murderer to a degree. Rather it would usually be understood as saying that if you take into account certain criteria for being a murderer, Nixon qualifies, while if you give prominence to other criteria, he doesn't qualify. On a reading such as this, sentence (2) could be true. But one of the inadequacies of fuzzy logic as we have set it up is that we have no way of assigning values in such a way that (2) comes out to be true. Even though $P \wedge \neg P$ is not a contradiction in FPL, it is still constrained so that it cannot have a value greater than 0.5 – that is, it has to come out to be more false than true in FPL. The reason is, of course, that FPL does not take account of the notion 'truth in a certain respect'. Any attempt to incorporate such a notion into FPL would lead to having to give up a linear sequence of values in favor of a lattice of values. I have not investigated at all just what would have to be done to FPL to incorporate the notion of 'truth in a certain respect' – and I hope that the problem will be taken up by logicians. It is an important problem, since a great deal of ordinary discourse involves that notion. When a member of the New Left says:

(3)   Nixon is a murderer.

and the local Republican spokesman replies

(4)   Nixon is not a murderer.

the disagreement is not over the facts of the world. They may agree completely on just what Nixon has and hasn't done. The disagreement is one of values. What criteria should be considered important in conferring membership in the category of murderers? The issue is by no means trivial. Similar cases arise every day in most people's speech. Any serious account of human reasoning will require an understanding of such cases.

5.5. *More Problems With VERY*

We saw above in the discussions of VERY SIMILAR and VERY STRICTLY SPEAKING that the meaning of VERY cannot be adequately represented simply by taking a function of the absolute value of the predicate modified; vector values must be taken into consideration. There are other considerations that seem to me to indicate this. According to Zadeh's treatment of VERY, in which

$$|\text{VERY}(F)| = \mu_{\text{CON}(F)} = \mu_F^2,$$

the curve for VERY TALL hits the values 0 and 1 at exactly the same places as the curve for TALL. This seems to me to be counterintuitive. It seems to me that it can be absolutely true that someone is tall without it being absolutely true that he is very tall. The situation, of course, gets worse with VERY VERY TALL, VERY VERY VERY TALL, etc., since according to Zadeh's treatment, they all hit the value 1 at the same place as TALL. The difference between Zadeh's proposal and what seems to me to be more correct can be seen in Figures 10 and 11.
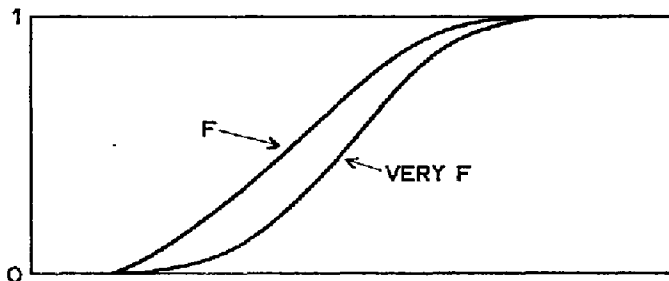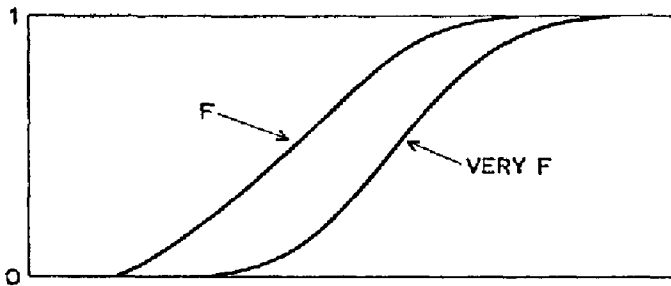

Fig. 10.

Fig. 11.

In Figure 10, the curve for TALL has been modified by the function CON to give the curve for VERY TALL. In Figure 11, the curve for VERY TALL has in addition been shifted over to the right, which seems to me to be more correct. However, there is no way to get the effect of such a shift simply by having VERY operate on the absolute value of TALL. Rather any function giving the values for VERY TALL in Figure 11 would have to range over *heights*, which would be included in the vector value for TALL. Assuming $\mu_{TALL} = f(h)$, for some function f ranging over heights, to get a shift to the right as shown in Figure 11 we would have to take some constant c and compute $f(h-c)$; then to get the curve for VERY TALL we would need apply the function CON as Zadeh suggests. Thus $\mu_{VERY\ TALL} = CON(f(h-c))$. How one arrives at the constants for each given predicate modified would be a serious problem. More likely, one would not be subtracting a constant but rather some function of heights, $g(h)$, which would grow smaller the more one iterated VERY. The reason for the latter suggestion is that as one iterates occurrences of VERY the curve is not shifted further and further to the right, but rather reaches a point of diminishing returns. One of the reasons for Zadeh's suggestion that VERY be represented by a function that raises the value to a power is that one gets such a result automatically. My feeling at present is that a complete understanding of VERY is very far from our grasp.

5.6. *Restrictions on the Occurrence of Modifiers*

Some modifiers can apply to other modifiers, but the combinations are quite limited. We get VERY STRICTLY SPEAKING, but not VERY RATHER. Moreover, there are firm restrictions on what modifiers can

modify what predicates. We get NEARLY EQUAL TO 5, but not VERY
EQUAL TO 5, though we get VERY CLOSE TO 5. A few of these
restrictions follow automatically from certain of the above proposals.
For example, Zadeh's suggestion for VERY accounts for the odd redund-
ancy we find in VERY EQUAL TO 5, since squaring the graph for
EQUAL TO 5 will not change any of the values, which are either 0 or 1.
But most other restrictions on the occurrence of hedges seem not to follow
automatically from what has been said above. Such restrictions should
follow automatically from any adequate account.

Hedges raise some interesting questions:

A.   How Do Hedges Interact With Performatives?
           Take a sentence like (1).

(1)   Technically, I *said* that Harry was a bastard.

What (1) would generally be taken to mean is that I said it but I didn't
mean it. That is, TECHNICALLY in (1) seems to be cancelling the im-
plicature that if you say something, you mean it. Or suppose a sergeant
says (2).

(2)   You might want to close that window, Private Snurg.

I think it would be appropriate to describe such a situation by the sen-
tences in (3).

(3)   a. Strictly speaking, the sergeant didn't order the private to
              close the window.
           b. Essentially, the sergeant did order the private to close the
              window.

Obviously hedges interact with felicity conditions for utterances and with
rules of conversation. An investigation of the subject should be revealing.
    In addition, Robin Lakoff (personal communication) has observed
that certain verbs and syntactic constructions convey hedged perfor-
matives.

(4)   a. I suppose (guess/think) that Harry is coming.
           b. Won't you open the door?

(4a) is a hedged assertion. (4b) is a 'softened' request. An investigation of these would also be revealing.

B.   Are There Hedges in Lexical Items?

Robin Lakoff has suggested that one might want to describe a word like 'pink' as a hedge between red and white. This is also suggested by the metaphorical term 'pinko', which is a hedge on 'red'.

C.   What are the Primitive Fuzzy Concepts in Natural Language?

We will say that a fuzzy set is primitive if its membership function cannot be decomposed, that is, if there is no function f such that $\mu_A = = f(\mu_{B_1}, ..., \mu_{B_k})$. The question as to what such primitives are in natural language is a fundamental question about the nature of the human mind. The question has, of course, been raised innumerable times before, but to my knowledge the possibility that the primitives themselves might be fuzzy has not been discussed.

D.   What are the Possible Types of Membership Functions?

The membership function for each nonprimitive concept F is representable as a function of some finite number of other membership functions: $\mu_F = f(\mu_{G_1}, ..., \mu_{G_k})$. What are the possible f's? What constraints are there on them?

6. CONCLUSIONS

6.1. *The Logic of Fuzzy Concepts Can Be Studied Seriously*

Fuzzy concepts have had a bad press among logicians, especially in this century when the formal analysis of axiomatic and semantic systems reached a high degree of sophistication. It has been generally assumed that such concepts were not amenable to serious formal study. I believe that the development of fuzzy set theory by Zadeh and the placement of it by Scott (see Appendix I) within the general context of recent work in modal and many-valued logics makes such serious study possible.

6.2. *In Natural Language, Truth is a Matter of Degree, Not an Absolute*

Heider (1971) has shown that category membership is a matter of degree. Sentences asserting category membership of an individual or object correspondingly display a degree of truth. This is made clear by the study of

modifiers like SORT OF, PAR EXCELLENCE, TYPICAL, IN ES-
SENCE, and IN A MANNER OF SPEAKING (see Section 3 above),
whose effect on truth conditions can only be made sense of if the corre-
sponding sentences without those modifiers admit of degrees of truth.

6.3. *Fuzzy Concepts Have Internal Structure*

The study of hedges like TECHNICALLY, STRICTLY SPEAKING,
LOOSELY SPEAKING, and REGULAR requires the assignment of
vector values to the predicates they modify. Each component is a mean-
ing criterion, itself a membership function for a fuzzy set. There are at
least four types of meaning criteria, three of which are capable of con-
ferring category membership to some degree, one of which is not.

6.4. *Semantics is Not Independent of Pragmatics*

The study of the hedge REGULAR by Bolinger 1972 reveals that
sentences with REGULAR assert connotations, not any aspect of literal
meaning. Connotations are considered to be part of pragmatics and, as
such, to have nothing to do with truth conditions, since semantics has been
assumed to be independent of pragmatics. However, since the truth condi-
tions of sentences with REGULAR depend only on connotations, it
follows that if connotations are part of pragmatics, then semantics is not
independent of pragmatics. Since connotations are closely tied to the
real-world situation, it seems reasonable to maintain the traditional view
that connotations are part of pragmatic information.

6.5. *Algebraic Functions Play a Role in the Semantics of Certain Hedges*

Hedges like SORT OF, RATHER, PRETTY, and VERY change distri-
bution curves in a regular way. Zadeh has proposed that such changes can
be described by simple combinations of a small number of algebraic func-
tions. Whether or not Zadeh's proposals are correct in all detail, it seems
like something of the sort is necessary. (See Figure 9.)

6.6. *Perceptual Finiteness Depends on an Underlying Continuum of Values*

Since people can perceive, for each category, only a finite number of gra-
dations in any given context, one might be tempted to suggest that fuzzy
logic be limited to a relatively small finite number of values. But the study

of hedges like SORT OF, VERY, PRETTY, and RATHER, whose effect seems to be characterizable at least in part by algebraic functions, indicates that the number and distribution of perceived values is a surface matter, determined by the shape of underlying continuous functions. For this reason, it seems best not to restrict fuzzy logic to any fixed finite number of values. Instead, it seems preferable to attempt to account for the perceptual phenomena by trying to figure out how, in a perceptual model, the shape of underlying continuous functions determines the number and distribution of perceived values.

6.7. *The Logic of Hedges Requires Serious Semantic Analysis for All Predicates*

In a fuzzy predicate logic *with hedges,* (*FPrLH*) the notion of a valuation is fundamentally more complex than the corresponding notion in other logics developed to date. The reason is that each predicate must be assigned a vector value as well as an absolute value and the models for each *FPrLH* must contain functions mapping vector values into absolute values, as well as the functions *prim, sec, def,* and *char.* What this amounts to is that the assignment of truth values in an *FPrLH* requires a much deeper analysis of meaning than in a classical predicate logic. In fact, by comparison, the assignment of values to predicates in a classical predicate logic is a triviality. For each *n*-place predicate we set up in a model a corresponding (classical) set of *n*-tuples of individuals. Thus, an expression like 'BIRD(x)' is true on an assignment of individuals to variables just in case the individual denoted by x is in the set of birds. Nothing is said about whether it has to have wings or a beak, whether it typically flies, what its body structure is, how it reproduces, whether it has feathers, etc. Nor is anything said in classical predicate logic about what type of criteria these are and how they contribute to degree of category membership. In a fuzzy predicate logic with hedges, *all* these matters must be taken into account in every valuation for the predicate BIRD. The reason is that all of these matters enter into the assignment of truth values when BIRD is modified by one or another of the set of hedges. Simply saying that an individual is or is not in the set of birds will tell you next to nothing about how to evaluate sentences where BIRD is modified by a hedge. In short, fuzzy predicate logic with hedges requires serious semantic analysis for all predicates.

6.8. *Claim: Hedges Show That Formal Semantics is the Right Approach to the Logic of Natural Language and That Axiomatic Theories Will Be Inadequate*

Considering the cleverness of logicians in devising axiomatizations, this claim should be hedged considerably. However, I think it will turn out to be correct. Suppose Zadeh is right in suggesting that hedges like SORT OF, PRETTY, VERY, etc. require algebraic functions such as those discussed above to account for their meaning, at least in part. It seems to me unlikely that one is going to be able to get complete axiomatizations for fuzzy predicate logics containing such hedges. At least, the question should be raised as a challenge to logicians. If my guess is correct, then we will have learned something very deep and important about natural languages and how they differ from artificial languages.

6.9. *In Addition to Degrees of Truth, Degrees of Nonsense are Needed to Account for Certain Hedges*

Suppose P presupposes Q and Q has some intermediate degree of truth. Does P make sense? Is it complete nonsense? Or does it have an intermediate degree of nonsense? A study of the hedge TO THE EXTENT THAT IT MAKES SENSE TO SAY THAT ... indicates that intermediate degrees of nonsense are necessary. Moreover, Fuzzy Presuppositional Logics with intermediate degrees of nonsense cannot be handled by Van Fraassen's supervaluations. (For discussion, see Appendix II.)
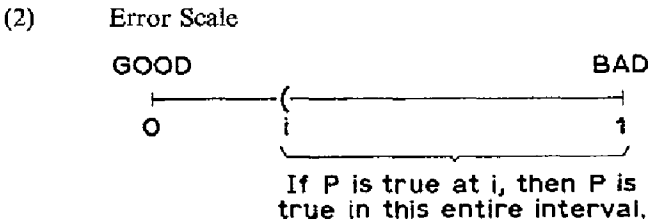
APPENDIX I: A SUGGESTION OF SCOTT'S

Dana Scott (personal communication) has suggested a method for setting up fuzzy propositional logic in a way that shows its relation to modal and many-valued logics in general. The Kripke semantics for modal logics is based on the notion of a 'possible world', that is, a complete and consistent assignment of truth values to every proposition, in other words, a classical (two-valued) valuation. A model for a classical modal logic contains a set of possible worlds (that is, a set of classical valuations) and a two-place alternativeness relation between worlds (that is, a relation between classical valuations). Scott has suggested a semantics for propositional fuzzy

logic that looks like a modal semantics, though as we will see below, it differs from classical modal semantics.

Take a set of (two-valued) valuations – one for each number in the real interval [0, 1]. Let the alternativeness relation be '$\leq$'. Let '$V_i(P)$' stand for 'P is true in valuation i'. Constrain the set of valuations as in (1).

(1)   If $V_i(P)$, then for all j, $i \leq j$, $V_j(P)$.

We can represent this diagrammatically as in (2).

(2)   Error Scale

GOOD   BAD

If P is true at i, then P is
true in this entire interval.

We will say that P deviates from absolute truth to degree i iff i is the greatest lower bound of the valuations in which P is true.

Scott has set this up in the reverse of the way we set up the semantics for *FPL*. We spoke of degree of truth. In Scott's treatment, we have what might be called degree of error or degree of deviation from absolute truth. To get Zadeh values, take $1 -$ Scott's values.

Scott defines valuations for $\neg$, $\wedge$, $\vee$, and $\to$ as follows.

(3) $V_i(\neg P)$ iff not $V_{1-i}(P)$
       $V_i(P \wedge Q)$ iff $V_i(P)$ and $V_i(Q)$
       $V_i(P \vee Q)$ iff $V_i(P)$ or $V_i(Q)$
       $V_i(P \to Q)$ iff $(\forall j)$ $V_j(P)$ implies $V_j(Q)$

The words 'not', 'and', 'or', and 'implies' in (3) stand for the corresponding connectives in classical propositional calculus, as used in the metalanguage. The point is to show how the connectives of fuzzy propositional logic can be defined in terms of the connectives of classical propositional logic and sequences of two-valued valuations.

Truth (absolute truth) turns out to be truth in all valuations, like necessity in an S5 modal system. If one wants, one can define an operator '$\square$' so that '$\square P$' is interpreted as 'P is true' then,

(4) $V_i(\Box P)$ iff $V_0(P)$, that is, iff $(\forall j)\, V_j(P)$.

In *FPL* we defined '$\rightarrow$' as taking only the values 0 and 1. However, it might be interesting to investigate fuzzy propositional logics where '$\rightarrow$' takes on intermediate values as well. I have not thought about any systems that might be motivated by empirical considerations taken from the study of natural language. However, the literature on many-valued logics contains extensions of our '$\rightarrow$' to intermediate values, which are motivated on purely formal grounds. One such case is a many-valued system of Gödel's (see Rescher, 1969, p. 44), which contains the same definitions of '$\neg$', '$\wedge$' and '$\vee$' as *FPL* and the following definition of '$\rightarrow$'.

(5) Gödel's '$\rightarrow$'

$$|P \rightarrow Q| = \begin{cases} 1 & \text{iff} \quad |P| \leqslant |Q| \\ |Q| & \text{iff} \quad |P| > |Q|. \end{cases}$$

Translated into Scott's treatment of fuzzy logic, we get:

(6) Scott's Version of Gödel's '$\rightarrow$'

$$V_i(P \rightarrow Q) \quad \text{iff} \quad \forall j,\, i \leqslant j,\, [V_j(P) \text{ implies } V_j(Q)].$$

As Scott observed (personal communication), (6) is an intuitionistic-style implication. As McKinsey and Tarski (1948) showed, intuitionistic logic has an S4 semantics; i.e., a modal semantics in which the alternativeness relation is reflexive and transitive, but not symmetric. In (6), '$\leqslant$' serves as an alternativeness relation, relating valuations i and j. '$\leqslant$' is, of course, reflexive and transitive, but not symmetric. The *FPL* '$\rightarrow$' is the S5 counterpart of Gödel's S4 '$\rightarrow$'.

Another interesting extension of the *FPL* '$\rightarrow$' to intermediate values is that found in the many-valued generalizations of the 3-valued system of Łukasiewicz (see Rescher, 1969, p. 36).

(7) Łukasiewicz' '$\rightarrow$'

$$|P \rightarrow Q| = \begin{cases} 1 & \text{iff} \quad |P| \leqslant |Q| \\ 1 - |P| + |Q|, & \text{iff} \quad |P| > |Q|. \end{cases}$$

Translated into Scott's treatment, we get:

(8) Scott's Version of Łukasiewicz' '$\rightarrow$'

$$V_i(P \rightarrow Q) \quad \text{iff} \quad \forall j,\, k \text{ such that}$$
$$i + j \leqslant k\, [V_j(P) \text{ implies } V_k(Q)].$$

Intuitively, Łukasiewicz' implication in a fuzzy logic can be thought of

as putting a constraint on the amount of error (or deviation from absolute truth) accumulated by an application of modus ponens. Suppose we assume P and $P \to Q$, and we deduce Q. If P has degree of deviation from truth j, and $P \to Q$ has degree of deviation from truth i, then the degree of deviation from truth of Q, namely k, must be less than or equal to the sum of i and j. Note that if $i = 0$, we get the *FPL* '$\to$'.

To me, the most interesting thing about (8) is that it is reminiscent of definitions of relevant entailment. '$i+j \leqslant k$' can be viewed as a 3-place alternativeness relation. If one replaces '$i+j \leqslant k$' with '$R(i, j, k)$' in (8) we get the general form of the definition or relevant entailment (see Meyer and Routley). This suggests a method for studying the relation between fuzzy logics and relevant entailment systems.

Given an extension of *FPL* '$\to$' to intermediate values, we automatically get a notion of degree of entailment via the definition:

(9) $P \Vdash_\alpha Q$ iff $\Vdash_\alpha P \to Q$
        P entails Q to degree α iff the value of '$P \to Q$' never falls below α in any valuation.

Naturally, each different extension of *FPL* '$\to$' gives us a different notion of degree of entailment.

If propositional fuzzy logic has a modal semantics, as we have just seen, then what kind of semantics does fuzzy modal logic have? The answer is straightforward. Instead of having a possible world w being a single valuation, V_w, think of a possible world as a sequence of valuations: $V_{0w}, \ldots, V_{iw}, \ldots, V_{1w}$. The value of '$\square P$' is then characterized by:

(10) $V_{iw}(\square P)$ iff $\forall_{w'}$ such that $Rww' [V_{iw'}(P)]$.

The beauty of Scott's suggestion is that it would allow us to treat modal and many-valued logics in a single framework, one in which they can be understood in terms of two-valued valuations and classical connectives in the metalanguage. I have, of course, considered only a special case, one where the set of valuations is linearly ordered by the relation $\leqslant$ and corresponds $1-1$ with the real interval $[0, 1]$. In the case of a 3-valued logic, there would be only 3 linearly ordered valuations. In the case of a Boolean-valued logic, the valuations would form a Boolean algebra. In a modal logic, the valuations would be structured by an alternativeness relation.

Contra Zadeh (1971, p. 33), fuzzy sets cannot be represented as sequences of classical sets, since complementation would not work correctly. This remarkable fact follows directly from the fact that negation cannot be classical in Scott's system, as we are about to see.

In an earlier version of this paper, published in the *Proceedings of the Eighth Regional Meeting of the Chicago Linguistics Society*, there was an important misprint. The definition of negation was mistakenly given as (A) instead of (B), which was Scott's suggested definition.

(A) $V_i(\neg P)$ iff not $V_i(P)$
(B) $V_i(\neg P)$ iff not $V_{1-i}(P)$.

We can see why Scott wanted (B) rather than (A) by looking at his definition of conjunction. Let us begin with a simple case.

(11) Suppose the value of P is 0.3 and the value of Q is 0.6:
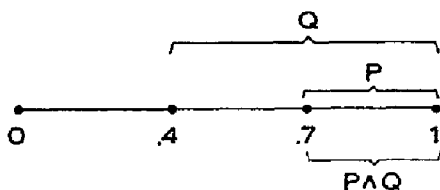
In this situation, '$P \wedge Q$' will be true in just those valuations in which 'P' is true and so it will have the value 0.3, just as in *FPL*. But now what would the value of '$P \wedge \neg Q$' be in such a situation? The answer will depend on which definition of negation we choose. Let us consider both possibilities.
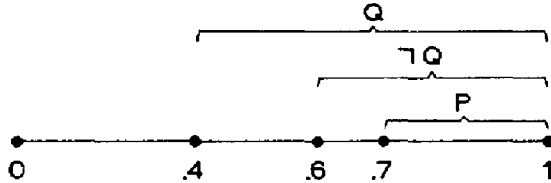
(12) The value of 'P' is 0.3, the value of 'Q' is 0.6, and the value of '$\neg Q$' is 0.4.

(a) Definition (A) yields:
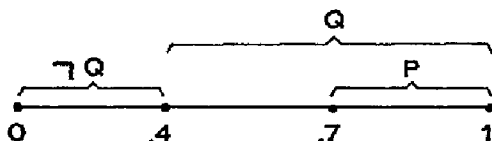
There is no valuation in which 'P' is true and '$\neg Q$' is true.
Therefore, the value of '$P \wedge \neg Q$' is zero, which does not accord with *FPL*.

(b) Definition (B) yields:

'$P \wedge \neg Q$' is true in every valuation in which 'P' is true.

Therefore the value of '$P \wedge \neg Q$' is 0.3, which accords with *FPL*.

The effect of the '$1 - i$' in definition (B) is to make sure that the evaluations take place at the same end of the scale, so that the definitions of conjunction and disjunction will work correctly. Definition (B) gives the right answer, while (A) gives the wrong answer. But not without cost. Note that in (2b), 'Q' and '$\neg Q$' are both true in every valuation between 0.6 and 1, while in the valuations between 0 and 0.4, neither 'Q' nor '$\neg Q$' is true. In short, the two-valued valuations represented in (12b) are anything but classical. One may or may not want to consider this a flaw. Perhaps the appropriate way to look at many-valued logic is as a modal logic in which the individual two-valued valuations are nonclassical in this way. Note that they are not completely nonclassical however. The classical definitions of conjunction and disjunction will still hold; but negation becomes a modal operator under definition (B).[5] For many logicians, much of the appeal of Scott's system may be lost if the valuations are not classical. I think this would be a pity. Such systems are beautiful and interesting regardless of whether the valuations are classical.

APPENDIX II: FUZZY PRESUPPOSITIONAL LOGIC

In Section 2, we observed that there was an inconsistency between defining negation as $|\neg P| = 1 - |P|$ and defining presuppositions so that P presupposes Q iff $P \Vdash Q$ and $\neg P \Vdash Q$. We observed that we could get around the inconsistency if we let the values assigned be triples (α, β, γ) consisting of a truth-value, a falsity-value, and a nonsense-value, such that $\alpha + \beta + \gamma = 1$. Negation would then be defined by:

(1) $|\neg P| = (\alpha, \beta, \gamma)$ iff $|P| = (\beta, \alpha, \gamma)$.

As in all presuppositional logics, the negation in (1) interchanges the truth and falsity values and preserves the nonsense value.

Given (1), there are two possible variations one can consider. One can permit degrees of nonsense, letting γ range over the real interval $[0, 1]$. Or one can consider systems where every proposition either makes sense or it doesn't, restricting γ to the values 0 and 1. If one makes the latter decision, presuppositional fuzzy logic can be handled by a system of supervaluations of the sort developed by Van Fraassen. Suppose P contains a presupposition failure. Then in a system where negation is defined as in (1), the value of P would be $(0, 0, 1)$, and the value of $\neg P$ would also be $(0, 0, 1)$. In a supervaluation treatment, we would assign as values not triples but simply ordinary truth-values, except that in the case of presupposition failure, a value would not be assigned. We could then define negation so that $|\neg P|$ would be $1 - |P|$ just in case P was assigned a value. If P was not assigned a value, then $\neg P$ would not be assigned a value.

In a technical sense, the supervaluation approach could get around the problem without having triples assigned as values, provided it was assumed that every sentence either made complete sense or was complete nonsense. However, if we allow intermediate degrees of nonsense, no supervaluation approach will work. But even if we restrict every sentence either to making complete sense or being complete nonsense, the supervaluation approach has a serious defect. As we said, supervaluations would not assign a value *in the case of presupposition failure*. In nonfuzzy logics, it is clear what constitutes a presupposition failure, namely, if one or more of the presupposed sentences is false. But in fuzzy logic it is not clear what constitutes a presupposition failure, since the presupposed sentences can have values intermediate between 0 and 1. Thus, in the supervaluation approach, we would have to make an arbitrary decision as to what constitutes a presupposition failure, so that we would know when not to assign a truth value. We could, for example, decide that if a presupposed sentence had any value less than 1, that is, if it deviated from absolute truth at all, that would constitute a presupposition failure. Or we could decide that there was a presupposition failure only when the value of some presupposed sentence was 0, that is, when it was absolutely false. Or we could pick 0.5 as a designated value and say that we had a presupposition failure only when the value of P fell below 0.5, that is, when P was more false than true. Or we could arbitrarily designate any

other value. The point is that in the supervaluation approach the choice is arbitrary. This, I think, is a irremediable weakness in that approach.

What provides troubles for the supervaluation approach is that a presupposed sentence may be fuzzy, that is, that it may take on a truth value intermediate between 0 and 1. My feeling about this is that when P presupposes Q and Q has the intermediate truth value α, then P should have the intermediate nonsense value $1 - \alpha$. In other words, P lacks sense to the degree that its presuppositions lack truth. The truer its presuppositions get, the more P makes sense. I think that there are examples that justify this intuition.

(2)   a. Dick Cavett regrets that he is tall.
       b. Dick Cavett is tall.

(3)   a. Sam was surprised that he had approximately $10000 in his savings account.
       b. Sam had approximately $10000 in his savings account.
       c. Sam had $9992 in his savings account.
       d. Sam had $9950 in his savings account.
       e. Sam had $9500 in his savings account.
       f. Sam had $9200 in his savings account.
          etc.

(2a) presupposes (2b). As it happens, Dick Cavett happens to be 5'7" tall, which is tall to about degree 0.3. Consequently (2) doesn't make much sense. But suppose Cavett were 5'9". Then (2a) would start making more sense. Or suppose he were 5'11". (2a) would make even more sense. And if Cavett were 6'4", then (2a), whether true or false, would make perfect sense in most situations.

(3a) is a similar example. (3a) presupposes (3b). But (3b) is fuzzy – it depends on what counts as an approximation to having $10000 in ones savings account. Suppose (3c) were the case. Then I think (3b) would be true no matter what, and (3a) would make perfect sense. If (3d) were the case, I think most people in most situations would still want to say that (3b) was true and that (3a) made sense. If (3e) were true, the truth of (3b) would become questionable. In many situations (3b) would have a high degree of truth given the truth of (3e), and (3a) would pretty much make sense. When we get down to (3f), however, the degree of truth of (3b) gets lower, and it makes less sense to say (a). And so on.

Interestingly enough, there is a hedge in English which depends upon there being intermediate degrees of nonsense. In fact, its function is to remove intermediate degrees of nonsense, while not changing complete nonsense. The expression is: TO THE EXTENT THAT IT MAKES SENSE TO SAY THAT .... Consider the following example.

(4)   a. J. L. Austin was a *good* linguist. [main stress on *good*]
        b. J. L. Austin was a linguist.
        c. To the extent that it makes sense to say that J. L. Austin was a linguist, he was a *good* linguist.

J. L. Austin was primarily a philosopher, not a linguist. Yet his analyses of various natural language phenomena could certainly be considered linguistics, in fact, excellent linguistics. Thus, though (4b) is not strictly true, I don't think it would be correct to say that it was strictly false. I would say that it had some intermediate value. (4a) presupposes (4b). Thus, (4a) has an intermediate nonsense value. To some extent, (4a) seems to make sense and to some extent it seems not to. However, (4c) makes perfect sense. The effect of the *to*-phrase in (4c) has been to remove the intermediate nonsense value of (4a). Note incidentally that the *to*-phrase of (4c) presupposes (4d).

(4d)   To some extent it makes sense to say that J. L. Austin was a linguist.

Given that (4c) makes perfect sense, (4d) must be true, which seems intuitively correct. Note that (4d) asserts (truth-fully!) that there is some intermediate degree to which a proposition of (4b) makes sense.
Compare (4) with (5).

(5)   a. J. L. Austin was a *good* king of France.
        b. J. L. Austin was the king of France.
        c. To the extent to which it makes sense to say that J. L. Austin was the king of France, he was a *good* king of France.[6]
        d. To some extent it makes sense to say that J. L. Austin was the king of France.

(5b) is utterly false – it is not true to any degree. (5a), which presupposes (5b), is complete nonsense (given the facts of this world). (5d) is false.

And (5c) is complete nonsense, just like (5a). I think that the disparity between the sentences in (4) and those in (5) shows that sentences of English can take on intermediate nonsense values when their presuppositions take on intermediate truth values. Thus, I think there is real motivation for investigating fuzzy presuppositional logics that can take on intermediate nonsense values.

Let us return then to the investigation of systems where the values assigned are ordered triples whose sum is 1 and where negation is defined as in (1) above. How do we go about defining conjunction and disjunction? To get some idea of how this can be done, let us look at the corresponding definitions in the classic 3-valued systems of Bochvar and Łukasiewicz (See Rescher, 1969).

(6) Bochvar's Conjunction and Disjunction

$\wedge$	N	F	T		$\vee$	N	T	F
N	N	N	N		N	N	N	N
F	N	F	F		T	N	T	T
T	N	F	T		F	N	T	F

(7) Łukasiewicz' Conjunction and Disjunction

$\wedge$	F	N	T		$\vee$	T	N	F
F	F	F	F		T	T	T	T
N	F	N	N		N	T	N	N
T	F	N	T		F	T	N	F

There are certain general principles governing the determination of values in these systems. They are as follows:

  I. The value of the conjunction (disjunction) is the same as the value of one of the component sentences (both, if they have the same value).
  II. In each case there is a hierarchy of values that determines the component sentence whose value will be assigned to the entire sentence. The hierarchies are given in (8).

(8) The Value Hierarchies

> Bochvar Conjunction: N, F, T
> Bochvar Disjunction: N, T, F
> Łukasiewicz Conjunction: F, N, T
> Łukasiewicz Disjunction: T, N, F

III. General Principles Determining the Hierarchies

    a. In conjunctions, T occupies the lowest place in the hierarchy.
      In disjunctions, F occupies the lowest place in the hierarchy.
    b. In the Bochvar system, N occupies the highest place in the
      hierarchies.
      In the Lukasiewicz system, N occupies the intermediate
      place in the hierarchies.

Given I, II, and III and the hierarchies in (8), we can give general direc-
tions for computing the tables in (6) and (7).
  (9) How to Compute (6) and (7)

    a. If one (or both) of the component sentences has the highest
      value in the hierarchy, the conjunction (disjunction) has
      that value.
    b. If not, then if one (or both of the component sentences) has
      the next highest value, then the entire conjunction (disjunc-
      tion) has that value.
    c. Otherwise the conjunction (disjunction) has the lowest
      value in the hierarchy.

Given this characterization of the Bochvar and Lukasiewicz connec-
tives for 3-valued logic, we can get Bochvar-style and Lukasiewicz-style
connectives for fuzzy presuppositional logics. We keep principles I, II,
and III and the hierarchies in (8). However, in fuzzy presuppositional
logics, the values assigned to propositions are not merely T, F, and N.
Instead we have ordered triples of numerical values (α, β, γ). We will refer
to T, F, and N as 'value-types' and α, β, and γ will be the numerical values
of those types. The hierarchies in (8) now are hierarchies of value-types
rather than values. We can now give directions for computing the values
for Bochvar-style and Lukasiewicz-style conjunctions and disjunctions in
fuzzy presuppositional logics.
  (10) How to Compute Bochvar-style and Lukasiewicz-style Connec-
tives
    a. If one of the component sentences has the highest numer-
      ical value for the highest value-type in the hierarchy, then
      the conjunction (disjunction) has the same triple of numer-
      ical values as that component sentence.

b. If the numerical values for the highest value-type are the same, then if one of the component sentences has the highest numerical value for the next-highest value-type in the hierarchy, then the conjunction (disjunction) has the same triple of numerical values as that component sentence.

c. Otherwise, both component sentences have the same triple of values and the conjunction (disjunction) has that triple.

An example of how these systems work is given in the chart in (10). On the right hand side we have listed whether the entire conjunction or disjunction gets the value for component sentence P or component sentence Q.

(10)

	P	Q	$P \wedge_B Q$	$P \wedge_B Q$	$P \wedge_L Q$	$P \wedge_L Q$
a.	(0.2,0.1,0.7)	(0.3,0.5,0.2)	P	P	Q	Q
b.	(0.2,0.5,0.3)	(0.4,0.3,0.3)	P	Q	P	Q
c.	(0.4,0.2,0.4)	(0.3,0.2,0.5)	Q	Q	Q	P
d.	(0.5,0.3,0.2)	(0.5,0.2,0.3)	Q	Q	P	Q

In (10a): For both Bochvar connectives, N is highest in the hierarchy. P has the highest N-value, namely, 0.7. So both connectives are assigned the triple for P. F is highest in the Łukasiewicz conjunction hierarchy. Q has the highest F-value, namely, 0.5. T is the highest in the Łukasiewicz disjunction hierarchy. Q has the highest T-value, namely, 0.3. So both Łukasiewicz connectives are assigned the triple for Q.

In (10b): For Bochvar conjunction, N is highest in the hierarchy. But P and Q have the same N-value, namely, 0.3. F is next highest in the Bochvar conjunction hierarchy. P has the highest F-value, 0.5; so the Bochvar conjunction is assigned the triple for P. For Bochvar disjunction, N is again highest in the hierarchy and again the N-values are the same for P and Q. T is next highest in the Bochvar disjunction hierarchy. Q has the highest T-value, so the Bochvar disjunction is assigned the triple for Q. The rest of the examples are obvious.

So far as implication is concerned, my feeling is that the degree of nonsense for '$P \rightarrow Q$' should always be 0. I feel that it always makes sense to ask whether or to what degree P implies Q. Moreover, I feel that implication is based solely on truth values, not on nonsense values. Therefore

one can incorporate into fuzzy presuppositional logics the same definitions of implication used in fuzzy propositional logics.

I should make clear that the above definitions of Bochvar-style and Łukasiewicz-style connectives are purely a technical exercise. I certainly do not believe that any of them accurately represents the meaning of natural language *and* or *or*. In fact, these connectives are incredibly simple-minded compared to the complexities of natural language conjunction (see Robin Lakoff, 1971). My purpose here is simply to get the study of fuzzy presuppositional logics off the ground in the hope that others will carry it further. One interesting question to consider is whether the Bochvar-style and Łukasiewicz-style systems described above can be translated into the format suggested by Scott (see Appendix I). So far, I have not been able to do it.

APPENDIX III: FUZZY THEOREMS

The following is a well-known theorem of Euclidean geometry.

(1)   The three lines each of which bisect one side of a given triangle and go through the opposite angle, meet in a point.

Zadeh (personal communication) has observed that this theorem can be 'fuzzified' to a sentence which is also true in the Euclidean plane.

(2)   Three lines, each of which *approximately* bisects one side of a given triangle and goes through the opposite angle, form a triangle inside the given triangle and *much* smaller than it.

Though (2) is true in the Euclidean plane, it cannot be deduced from Euclid's postulates, since they provide no way of dealing with hedges like *approximately* and *much* (as in *much larger than*). (2) is an example of a sentence which is true in a model of Euclid's postulates but which is not deduceable from the postulates. Though Gödel showed that such cases must exist for arithmetic, his examples were of a very much different sort. (2) is an interesting curiosity, an intuitively obvious truth of Euclidean geometry which cannot be deduced from Euclid's postulates.

University of California

NOTES

The original impetus for the study of hedges came from the work of Heider (1971), Alston (1964), Ross (1970), and Bolinger (1972). The formal parts are based on the development of fuzzy set theory by Lofti Zadeh, Dept. of Electrical Engineering, U. of California, Berkeley. Professor Zadeh has been kind enough to discuss this paper with me often and at great length and many of the ideas in it have come from those discussions. In addition I would like to thank the following people whose discussed these ideas with me and who contributed to what little understanding I have of the subject: Ann Borkin, Herb Clark, Alan Dershowitz, Hubert Dryfus, Charles Fillmore, Jim Fox, Dov Gabbay, Richard Grandy, Charles Guignon, Eleanor Heider, Peter Kenen, Robin Lakoff, John Lawler, Robert LeVine, David Lewis, Ruth Barcan Marcus, James Matisoff, Jim McCawley, Robert Nozick, Michael Reddy, Haj Ross, Dana Scott, and Bas van Fraassen.

[1] Tallness is also relative to point of view. In a given population, someone who is 5′5″ may consider someone who is 5′10″ to be relatively tall, while someone who is 6′4″ might not. We will ignore such factors at present, noting that they will have to be taken into account.

[2] Another system of fuzzy propositional logic can be found in Goguen (1971). Goguen, however, chose a different semantics for conjunction and disjunction. The system given here is identical to the system $S_{\aleph}$ discussed in Rescher, 1969, p. 47.

[3] Actually we should have least upper bound for max and greatest lower bound for min, since we are dealing with the real numbers and so may have sequences which approach a limit but do not reach it.

[4] Lauri Karttunen (1971) has shown that (10) is inadequate and must be revised to include modalities as in (10′).

(10′) P presupposes Q iff $\Diamond P \Vdash Q$ and $\Diamond \neg P \Vdash Q$.

However, he has not yet figured out which $\Diamond$ will work. (See the discussion in Herzberger (1971).) But whichever $\Diamond$ turns out to be adequate, (10)) will at least be a necessary if not sufficient condition for logical presuppositions. Therefore, everything we have to say will hold for Karttunen's notion of presupposition.

[5] From here it is a small step to a demonstration that fuzzy sets cannot be represented as sequences of classical sets. Suppose we try to represent a fuzzy set P with degrees of membership in the interval [0, 1] as a sequence of classical sets $P_0, ..., P_i, ..., P_1, ...,$ with one set corresponding to each point in the interval. For each P_i, we assume that $x \in P_i$ or $x \notin P_i$, and if $x \in P_i$, then for all j, $j > i$, $x \in P_j$. We then say that $\mu_P(x) = k$ iff P_k is the greatest lower bound of $\{P_i / x \in P_i\}$. Classical intersection and union can easily be defined as follows: Intersection: $x \in P_i \cap Q_i$ iff $x \in P_i$ and $x \in Q_i$. Union: $x \in P_i \cup Q_i$ iff $x \in P_i$ or $x \in Q_i$. But complementation is a problem. Consider definitions (A) and (B). (A) Classical complementation: $x \in \bar{Q}_i$ iff $x \notin Q_i$. (B) 'Modal' complementation: $x \in \bar{Q}_i$ iff $x \notin Q_{1-i}$.

Now suppose that $\mu_P(x) = 0.3$ and $\mu_Q(x) = 0.6$. According to Zadeh's fuzzy set theory, $\mu_{\bar{Q}}(x) = 0.4$ and $\mu_{P \cap \bar{Q}}(x)$ should equal 0.3. (A) gives us a situation like that

pictured in figure (12a) in the text; in other words, we will get the wrong answer that $\mu_{P \cap \bar{Q}}(x) = 0$. With definition (B), we will get a situation like that pictured in figure (12b), and will get the right answer for fuzzy set theory. Unfortunately, (B) is not the definition of complementation in classical set theory. (B) gives rise to a nonstandard set theory, what might be called a 'modal' set theory.

⁶ Note that it is also nonsensical to say (i), though not as bad as (5).

(i)   To the extent to which it makes sense to say that J. L. Austin was a philosopher, he was a *good* philosopher.

(i)   is nonsense to some degree unless there is some reason to doubt that Austin was a philosopher.

BIBLIOGRAPHY

Alston, William: 1964, *Philosophy of Language*. Prentice-Hall, Chapter 5.
Bolinger, Dwight: 1972, *Degree Words*, Mouton.
Goguen, J. A.: 1971, 'The Logic of Inexact Concepts', *Synthese*.
Heider, Eleanor Rosch: 1971, 'On the Internal Structure of Perceptual and Semantic Categories', unpublished paper, Pyschology Dept., U. of California, Berkeley.
Herzberger, Hans: 1971, 'Remarks on Karttunen's Notion of Presupposition', unpublished paper, Philosophy Dept., U. of Toronto.
Karttunen, Lauri: 1971, 'Some Observations on Factivity', *Papers in Linguistics* 4, 55–70.
Keenan, Edward: 1969, 'A Logical Base for English', U. of Pennsylvania Dissertation. Distributed by Transformation and Discourse Analysis Project, Linguistics Dept., U. of Pennsylvania, Philadelphia, Pa.
Lakoff, Robin: 1971, 'If's, And's, and But's About Conjunction', in *Studies in Linguistic Semantics* (ed. by Fillmore and Langendoen), Holt.
McKinsey, J. C. C. and Tarki, A.: 1948, 'Some Theorems About the Sentential Calculi of Lewis and Heyting', *Journal of Symbolic Logic*.
Meyer, Robert and Routley, Richard: 1971, 'Semantics of Entailment I', in *Truth, Syntax, Modality* (ed. by H. Leblanc), North-Holland Publ. Co., Amsterdam, forthcoming.
Reddy, Michael: 1972, 'Reference and Metaphor in Human Language', Unpublished U. of Chicago Dissertation, English Dept.
Rescher, Nicholas: 1969, *Many-Valued Logics*, McGraw-Hill.
Ross, John Robert: 1970, 'A Note on Implicit Comparatives', *Linguistic Inquiry* 1. 36–63.
Slote, Michael: 1966, 'The Theory of Important Criteria', *Journal of Philosophy*.
van Fraassen, Bas: 1971, *Formal Semantics and Logic*, Macmillan.
Zadeh, Lofti: 1965, 'Fuzzy Sets', *Information and Control* 8, 338–53.
Zadeh, Lofti: 1971a, 'Quantitative Fuzzy Semantics', *Information Sciences* 3, 159–76.
Zadeh, Lofti: 1971b, 'Fuzzy Languages and Their Relation to Human Intelligence', Memorandum No. ERL-M302, Electronic Research Laboratory, of California, Berkeley.
Zadeh, Lofti: 1972, 'Fuzzy Set-Theoretic Interpretation of Hedges', Unpublished paper, Dept. of Electrical Engineering, U. of California, Berkeley.