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Hedging against the Government: A Solution to the Home Asset Bias Puzzle

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Online Appendix

1 Appendix

1.1 Extensions of the two-period model

Here we describe the log-linearized version of the model where all the shocks are present. We log-linearize around the symmetric non-stochastic values of the variables in period 1.[1] We present the equations for the domestic agent only and the foreign agent's equations are analogous.

The domestic agent's optimality conditions for the two goods are:

$$\hat{C}_1^{Hf} - \hat{C}_1^{Hh} = \eta(\hat{P}_1^{Hh} - \hat{P}_1^{Hf}) \qquad \hat{C}_1^{Fh} - \hat{C}_1^{Ff} = \eta(\hat{P}_1^{Ff} - \hat{P}_1^{Fh}). \tag{1}$$

The definition of the consumption indices log-linearized gives

$$\hat{C}_1^{H} = a\hat{C}_1^{Hh} + (1-a)\hat{C}_1^{Hf} \qquad \hat{C}_1^{F} = a\hat{C}_1^{Ff} + (1-a)\hat{C}_1^{Fh} \tag{2}$$

and the corresponding welfare based price indices are

$$\hat{P}_1^{H} = a\hat{P}_1^{Hh} + (1-a)\hat{P}_1^{Hf} \qquad \hat{P}_1^{F} = a\hat{P}_1^{Ff} + (1-a)\hat{P}_1^{Fh}. \tag{3}$$

Next, the law of one price log-linearized gives

$$\hat{P}_1^{Hh} = \hat{S}_1 + \hat{P}_1^{Fh} \quad \hat{P}_1^{Hf} = \hat{S}_1 + \hat{P}_1^{Ff}. \tag{4}$$

Because we allow for consumption home bias and shocks lead to movements in relative prices, shocks will also affect the real exchange rate in our model. The real exchange rate follows

$$\hat{Q}_1 = (2a-1)(\hat{P}_1^{Hf} - \hat{P}_1^{Hh}). \tag{5}$$

The goods market clearing relations are given by

$$a\hat{C}_1^{Hh} + (1-a)\hat{C}_1^{Fh} + G_1^{Hh} + G_1^{Fh} = \hat{Y}_1^{H} \qquad a\hat{C}_1^{Ff} + (1-a)\hat{C}_1^{Hf} + G_1^{Hf} + G_1^{Ff} = \hat{Y}_1^{F}. \tag{6}$$

Finally, in the case where markets are complete, we have

$$\sigma(\hat{C}_1^{H} - \hat{C}_1^{F}) = \hat{Q}_1. \tag{7}$$

Let's derive the log-linear budget constraint. By the symmetry in the two countries and market clearing, we know that $E_0^{Hh} = E_0^{Ff} = \theta$ and $E_0^{Hf} = E_0^{Fh} = 1-\theta$. We denote this holding by θ to simplify notation and to make clear that it does not depend on the realization of the shocks in

[1] $\hat{\text{k}}$ for log- deviations of k and $\bar{\text{k}}$ for the value of the k in absence of shocks.

$t = 1$. Analogously, using the bond holdings' market clearing and assuming symmetry between the two governments' debt, i.e. $B_0^H = B_0^F$, we have that $B_0^{Hh} = B_0^{Ff} = z$ and $B_0^{Hf} = B_0^{Fh} = B - z$, where z and $B = B_0^H = B_0^F$ are simplified notation. These relations simplify the consumer budget constraint as

$$\hat{C}_1^H + \bar{\tau}\hat{\tau}_1^H = (\theta)(\hat{P}_1^{Hh} - \hat{P}_1^H + \hat{Y}_1^H) - (\theta - 1)(\hat{P}_1^{Hf} - \hat{P}_1^H + \hat{Y}_1^F) - R_0^H z \hat{P}_1^H - R_0^F (B - z)(\hat{P}_1^F - \hat{Q}_1). \quad (8)$$

The government budget constraint is, up to a first order approximation

$$\bar{\tau}\hat{\tau}_1^H = G_1^{Hh} + G_1^{Hf} - R_0^H B \hat{P}_1^H. \quad (9)$$

1.1.1 Complete markets

Proposition 2: Upto a first-order, (i) in the presence of nominal and government expenditure shocks, if (ii) the representative agent has CRRA utility, no preference bias in consumption, and CES preferences over domestic and foreign goods, (iii) the government taxes only domestic agents, and (iv) the government expenditure ratio between domestic goods and foreign goods is x , then (a) the agent holds only the bonds of her government (b) $x > 1$ implies home equity bias.

Proof: Eqn.(8) becomes:

$$\hat{C}_1^H + \bar{\tau}\hat{\tau}_1^H = (\theta)(\hat{P}_1^{Hh} - \hat{P}_1^H + \hat{Y}_1^H) - (\theta - 1)(\hat{P}_1^{Hf} - \hat{P}_1^H + \hat{Y}_1^F) - R_0^H z \hat{P}_1^H - R_0^F (B - z)(\hat{P}_1^F - \hat{Q}_1). \quad (10)$$

Here, we derive the portfolio allocation with a general a and σ and in the end we relate to the specific case of the proposition where $a = 0.5$. Using eqns.(9), (2), (6), and (7), we can write eqn.(10) as

$$G_1^{Hh} + G_1^{Hf} - G_1^{Fh} - G_1^{Ff} = \left(2(2 - \theta - a) + \frac{2a - 1}{\sigma}\right)\left(\hat{P}_1^{Hh} - \hat{P}_1^{Hf}\right) \quad (11)$$

$$+ R_0(B - z)\left(\hat{P}_1^H - \hat{P}_1^F\right) + R_0(B - z)\hat{Q}_1. \quad (12)$$

Then, using eqns.(1), (2), (6), and (7), we get

$$G_1^{Hh} - G_1^{Hf} + G_1^{Fh} - G_1^{Ff} = (2a - 1)\left(\frac{2a - 1}{\sigma} + \frac{(1 - a)\eta}{a(2a - 1)} + \eta\left(\frac{1}{a} - 2a + 1\right)\right)\left(\hat{P}_1^{Hh} - \hat{P}_1^{Hf}\right). \quad (13)$$

Eqns.(13) and (11) determine the stochastic process for $\hat{P}_1^{Hh} - \hat{P}_1^{Hf}$. Once we realize that $\left(\hat{P}_1^H - \hat{P}_1^F\right)$ is an exogenous stochastic process independent of all the government expenditure processes, the only way the two equations are consistent is when $z = B$. For the foreign agent, the proof is the same. This shows that each agent holds only her own government's bond.

From eqn.(13) and using $z = B$, we have

$$G_1^{Hh} - G_1^{Fh} - G_1^{Hf} - G_1^{Ff} = \left[2(2 + \theta - a)(\frac{2a - 1}{\sigma})\right](\hat{P}_1^{Hh} - \hat{P}_1^{Hf}).$$

Using that $G_1^{Hh} = xG_1^{Hf}$ and $G_1^{Ff} = xG_1^{Fh}$, we can re-write eqn.(13) as

$$(x-1)(G_1^{Hf}-G_1^{Fh}) = (2a-1)\left[\frac{2a-1}{\sigma}+\frac{(1-a)\eta}{a(2a-1)}+\eta(\frac{1}{a}-2a+1)\right](\hat{P}_1^{Hh}-\hat{P}_1^{Hf}) \qquad (14)$$

and eqn.(11) as

$$(1+x)(G_1^{Hf}-G_1^{Fh}) = \left[2(2+\theta-a)+(\frac{2a-1}{\sigma})\right](\hat{P}_1^{Hh}-\hat{P}_1^{Hf}). \qquad (15)$$

Since eqns.(14) and (15) must hold for all realization of shocks and θ cannot be contingent on the shocks, we find that

$$\theta = \frac{1}{2}+\frac{1}{2}(2a-1)\left[\frac{1+x}{x-1}\left(\frac{2a-1}{\sigma}-\frac{4a\eta(a-1)}{2a-1}\right)+1-\frac{1}{\sigma}\right]. \qquad (16)$$

With $a=0.5$, $\eta=1$ and $\sigma=1$, the equivalent of eqn.(16) is $\theta=\frac{1}{x-1}+1$. With $a=0.5$ and general η and σ, eqn.(16) is $\theta=\frac{1}{2}\left(\frac{\eta(1+x)}{x-1}+1\right)$, and it follows trivially that $x>1$ implies $\theta>0.5$, that is, home bias in equity. This gives us the proof.

1.1.2 Incomplete Markets

Because markets are incomplete, we follow Devereux and Sutherland (2006) in order to compute equilibrium portfolios. This consists in satisfying a second order accurate approximation of the household Euler equation, which in our case has the form:

$$E_0\left[\left(\hat{C}_1^H-\hat{C}_1^F-\frac{\hat{Q}_1}{\sigma}\right)r_{x,1}^i\right]=0 \qquad (17)$$

where $r_{x,1}^i$, $i=1,2$ and 3, is the excess returns of all assets with respect to a reference asset, which in our case is the returns on foreign equity. Formally, the definitions are

$$\hat{r}_x^1 = \hat{Y}_1^H+\hat{P}_1^{Hh}-\hat{Y}_1^F-\hat{P}_1^{Hf}$$

$$\hat{r}_x^2 = -R_0^H\hat{P}_1^H-\hat{Y}_1^F-\hat{P}_1^{Hf}-\hat{P}_1^H$$

$$\hat{r}_x^3 = -R_0^F\hat{P}_1^F+\hat{Q}_1-\hat{Y}_1^F-\hat{P}_1^{Hf}-\hat{P}_1^H.$$

In other words, using the equilibrium conditions, we can write $\hat{C}_1^H-\hat{C}_1^F-\frac{\hat{Q}_1}{\sigma}=\Xi_a(\theta,z)*\zeta_1'$ and $\hat{r}_{x,1}=\Xi_b(\theta,z)*\zeta_1'$, where $\zeta_1=\left[\begin{array}{cccccc}\hat{Y}_1^H & \hat{Y}_1^F & G_1^{Hf} & G_1^{Fh} & \hat{P}_1^H & \hat{P}_1^F\end{array}\right]$ and $\hat{r}_{x,1}=\left[\begin{array}{ccc}\hat{r}_x^1 & \hat{r}_x^2 & \hat{r}_x^3\end{array}\right]$. Then the solution for θ and z is given by the following system of equations

$$\Xi_a(\theta,z)\Sigma\Xi_b(\theta,z)'=0 \qquad (18)$$

where $\Sigma=E_0\left[\zeta_1'\zeta_1\right]$. Again here we consider $\Sigma=I$.

In the case where there is no consumption bias of the agent, that is $a=0.5$, it implies that $\hat{Q}_1=0$. Using demand and market-clearing conditions as well as the budget and resource constraints, we get

$$\hat{C}_1^H - \hat{C}_1^F = \begin{pmatrix} (-1+2\theta)(1-\frac{1}{\eta}) \\ -(-1+2\theta)(1-\frac{1}{\eta}) \\ -(1+2(\theta-\frac{1}{2})\frac{1}{\eta}\frac{1-x}{1+x}) \\ (1+2(\theta-\frac{1}{2})\frac{1}{\eta}\frac{1-x}{(1+x)}) \\ 2R_0(B-z) \\ -2R_0(B-z) \end{pmatrix}' \begin{pmatrix} \hat{Y}_1^H \\ \hat{Y}_1^F \\ G_1^{Hf} \\ G_1^{Fh} \\ \hat{P}_1^H \\ \hat{P}_1^F \end{pmatrix}$$

while $\hat{r}_x = \begin{pmatrix} \hat{r}_x^1 \\ \hat{r}_x^2 \end{pmatrix}$ is given by

$$\hat{r}_{x,1} = \begin{pmatrix} 1-\frac{1}{\eta} & \frac{1}{\eta}-1 & \frac{1}{\eta}\frac{x-1}{1+x} & -\frac{1}{\eta}\frac{x-1}{1+x} & 0 & 0 \\ -\frac{1}{2\eta} & \frac{1}{2\eta}-1 & \frac{1}{2\eta}\frac{x-1}{1+x} & -\frac{1}{2\eta}\frac{x-1}{1+x} & -R_0 & 0 \end{pmatrix} \begin{pmatrix} \hat{Y}_1^H \\ \hat{Y}_1^F \\ G_1^{Hf} \\ G_1^{Fh} \\ \hat{P}_1^H \\ \hat{P}_1^F \end{pmatrix}.$$

Then, using the methodology above (Eqn.(18)), we have a system of two equations and two variables that gives the following allocations:

$$\theta = \frac{1}{2} + \frac{1}{2}\frac{\eta\left(x^2-1\right)}{(\eta-1)^2(1+x)^2 + 2(1-x)^2}$$

$$z = B.$$

Proposition 3: Upto a first-order, (i) in the presence of nominal, endowment, and government expenditure shocks, if (ii) the representative agent has log-utility, preference bias in consumption for domestic goods, and CES preferences over domestic and foreign goods, (iii) the government taxes only domestic agents, and (iv) the government expenditure ratio between domestic goods and foreign goods is x , then (a) there is home nominal bond bias (b) $x > \frac{a}{1-a}$ implies home equity bias.

Proof: The set of equations to be solved is given below. We can re-write the system as

$$\begin{pmatrix} -1 & 1 & 1 & -1 \\ a+\frac{(\theta-a)}{\eta} & 1-a-\frac{(\theta-a)}{\eta} & 0 & 0 \\ a & 0 & 1-a & 0 \\ 0 & 1-a & 0 & a \end{pmatrix} \begin{pmatrix} \hat{C}_1^{Hh} \\ \hat{C}_1^{Hf} \\ \hat{C}_1^{Fh} \\ \hat{C}_1^{Ff} \end{pmatrix} =$$

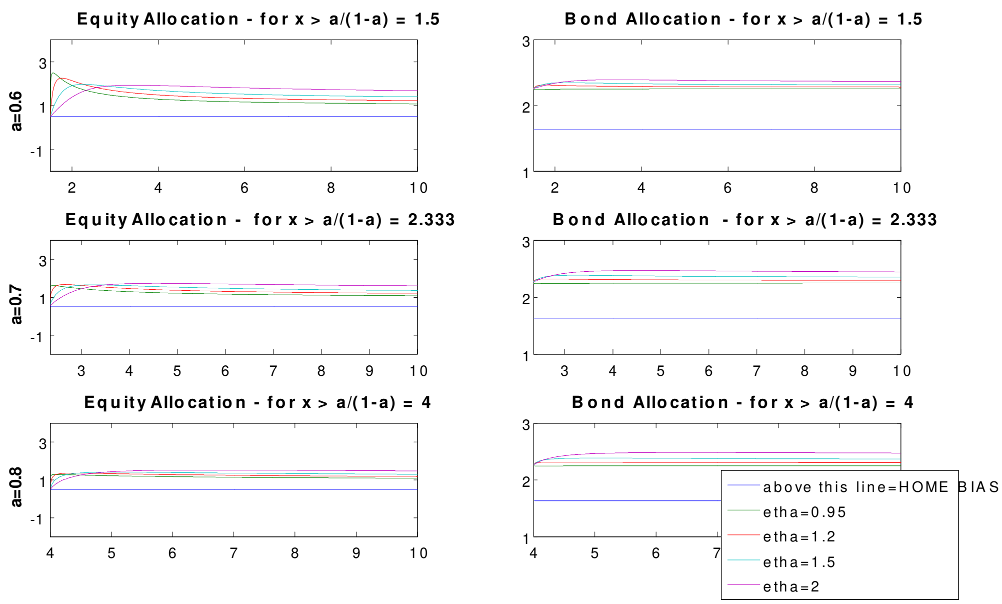
$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \theta & -(\theta-1) & -x-1 & 0 & R_0(B-z) & -R_0(B-z) \\ 1 & 0 & -x & -1 & 0 & 0 \\ 0 & 1 & -1 & x & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{Y}^H \\ \hat{Y}^F \\ G^{Hf} \\ G^{Fh} \\ \hat{P}^H \\ \hat{P}^F \end{pmatrix}$$

We can solve for the consumption levels in terms of the shocks. Using also that $\hat{Q}_t = (2a-1)(\hat{P}^{Hf} - \hat{P}^{Hh})$ we can re-write excess returns as

$$r_{x,t}=\begin{pmatrix}1 & -1 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & -R_0 & 0\end{pmatrix}\begin{pmatrix}\hat{Y}^H\\ \hat{Y}^F\\ G^{Hf}\\ G^{Fh}\\ \hat{P}^H\\ \hat{P}^F\end{pmatrix}+\begin{pmatrix}1\\ a\end{pmatrix}\frac{(\hat{C}^{Hh}-\hat{C}^{Hf})}{\eta}.$$

We can use eqn.(18) for θ and z, using $\sigma=1$. The resulting expressions for θ and z, while closed-form, are very cumbersome to put in text. We have shown using a symbolic package that $x>\frac{a}{1-a}$ is the sufficient condition to generate home bias in equity.

We report in the figure below the numerical results on asset holdings as we vary x in this set-up for $\sigma=1.5$ and different values of a and η..



1.2 Calibrated dynamic production economy

1.2.1 First-order approximation

To solve the model, we use an approximation around a symmetric non-stochastic steady state where the net foreign assets of the countries, wealth of the consumer less government debt, is equal to zero. In addition, steady state aggregate consumption, aggregate price levels, and aggregate output are equal to one while government spending is equal to zero. Next, we present the system of log-linearized equations that are relevant for determining portfolio holdings.

Consumer The FOCs of the home agent log-linearized give

$$-\sigma\hat{C}_t^H = E_t\left[-\sigma\hat{C}_{t+1}^H + \hat{P}_t^H + \hat{R}_t^H - \hat{P}_{t+1}^H\right] \tag{19}$$

$$-\sigma\hat{C}_t^H = E_t\left[-\sigma\hat{C}_{t+1}^H + \hat{P}_t^F + \hat{R}_t^F - \hat{P}_{t+1}^F + \hat{Q}_{t+1} - \hat{Q}_t\right] \tag{20}$$

$$-\sigma\hat{C}_t^H = E_t\left[-\sigma\hat{C}_{t+1}^H + \beta\hat{q}_{t+1}^H - \hat{q}_t^H + (1-\beta)\Pi_{t+1}^H\right] \tag{21}$$

$$-\sigma\hat{C}_t^H = E_t\left[-\sigma\hat{C}_{t+1}^H + \beta\hat{q}_{t+1}^F - \hat{q}_t^F + (1-\beta)\Pi_{t+1}^F + \hat{Q}_{t+1} - \hat{Q}_t\right] \tag{22}$$

$$\nu\hat{L}_t^H = -\sigma\hat{C}_t^H + \hat{w}_t^H - \left(\frac{\bar{\tau}^L}{1-\bar{\tau}^L}\right)\hat{\tau}_t^L \tag{23}$$

while the FOCs of the foreign agent log-linearized and combined with FOCs of the domestic agent gives

$$-\sigma\left[\hat{C}_t^H - \hat{C}_t^F\right] + \hat{Q}_t = -\sigma E_t\left[\hat{C}_{t+1}^H - \hat{C}_{t+1}^F\right] + E_t\left[\hat{Q}_{t+1}\right]. \tag{24}$$

Similarly the definitions of the various aggregate indices and the demand curve can be written as

$$\hat{C}_t^H = a\hat{C}_t^{Hh} + (1-a)\hat{C}_t^{Hf} \qquad \hat{P}_t^H = a\hat{P}_t^{Hh} + (1-a)\hat{P}_t^{Hf} \tag{25}$$

$$\hat{C}_t^{Hh} - \hat{C}_t^{Hf} = \eta(\hat{P}_t^{Hf} - \hat{P}_t^{Hh}). \tag{26}$$

The real exchange rate can be expressed as

$$\hat{Q}_t = \hat{S}_t + \hat{P}_t^F - \hat{P}_t^H \tag{27}$$

and the law of one price in aggregate terms as

$$\hat{P}_t^{Hh} = \hat{S}_t + \hat{P}_t^{Fh} \qquad \hat{P}_t^{Hf} = \hat{S}_t + \hat{P}_t^{Ff}. \tag{28}$$

Finally, the consumer's budget constraint can be written as

$$\begin{aligned}\hat{C}_t^H + \bar{B}\hat{W}_t^H = {} & \frac{1}{\beta}\bar{B}(\hat{W}_{t-1}^H + \hat{r}_t^1) - \left(\bar{\tau}^L\bar{w}\right)\hat{\tau}_t^L + \\ & \left(1-\bar{\tau}^L\right)\bar{w}(\hat{w}_t^H + \hat{L}_t^H) + \frac{\bar{B}^{Hf}}{\beta}\hat{r}_{x,t}^1 + \frac{\bar{q}^H\bar{A}^{Hh}}{\beta}\hat{r}_{x,t}^2 + \frac{\bar{q}^F\bar{A}^{Hf}}{\beta}\hat{r}_{x,t}^3\end{aligned} \tag{29}$$

where

$$\hat{r}_t^1 = \hat{P}_{t-1}^H + \hat{R}_{t-1}^H - \hat{P}_t^H \tag{30}$$

is the real return on domestic nominal bonds and

$$\hat{r}_{x,t}^1 = \left(\hat{P}_{t-1}^F + \hat{R}_{t-1}^F - \hat{P}_t^F + \hat{Q}_t - \hat{Q}_{t-1}\right) - \hat{r}_t^1 \tag{31}$$

$$\hat{r}_{x,t}^2 = \left(\beta\hat{q}_t^H - \hat{q}_{t-1}^H + (1-\beta)\Pi_t^H\right) - \hat{r}_t^1 \tag{32}$$

$$\hat{r}^3_{x,t} = \left(\beta \hat{q}^F_t - \hat{q}^F_{t-1} \ + (1-\beta)\Pi^F_t + \hat{Q}_t - \hat{Q}_{t-1}\right) - \hat{r}^1_t \tag{33}$$

are the excess returns of foreign nominal bonds, domestic equity, and foreign equity over domestic nominal bonds.

Firms The aggregate production technology log-linearized yields

$$\hat{Y}^H_t = \hat{A}^H_t + \hat{L}^H_t \tag{34}$$

while profits and the pricing rule are given by

$$\hat{\Pi}^H_t = \theta \left[-\bar{w} \left(\hat{w}^H_t + \hat{Y}^H_t - \hat{A}^H_t \right) + \left(\hat{P}^{Hh}_t - \hat{P}^H_t + \hat{Y}^H_t \right) \right] - \left(\frac{\bar{\tau}^\Pi}{1-\bar{\tau}^\Pi} \right) \hat{\tau}^\pi_t$$

$$\hat{P}^{Hh}_t - \hat{P}^H_t = \hat{w}^H_t - \hat{A}^H_t . \tag{35}$$

Government The government budget constraint can be expressed as

$$\bar{B}\left(\hat{B}_t - \hat{P}^H_t\right) = \beta^{-1}\bar{B}\left(\hat{R}^H_{t-1} + \hat{B}_{t-1} - \hat{P}^H_t\right) - \left(1 + \frac{1}{y}\right)\bar{\tau}^L \bar{w}\left(\hat{\tau}^L_t + \hat{w}^H_t + \hat{L}^H_t\right) + G^{Hh}_t + G^{Hf}_t . \tag{36}$$

Moreover, we can write the monetary policy rule as

$$\hat{R}^H_t = \gamma\left(\hat{P}^H_t - \hat{P}^H_{t-1}\right) + \epsilon_{r,t}$$

and the fiscal policy rule as

$$\hat{\tau}^L_t + \hat{w}^H_t + \hat{L}^H_t = \phi\left(\hat{B}_t - \hat{P}^H_t\right).$$

The relationship between government spending on domestic and foreign goods, and the tax revenues through labor and profits can be written as

$$G^{Hh}_t = x G^{Hf}_t \tag{37}$$

$$\hat{\tau}^L_t + \hat{w}^H_t + \hat{L}^H_t = \hat{\tau}^\pi_t + \hat{Y}^H_t - \theta\left(\hat{P}^H_t - \hat{P}^{Hh}_t\right) - (\theta-1)\left(\hat{w}^H_t - \hat{A}^H_t\right). \tag{38}$$

Market clearing The market clearing condition for goods at the aggregate level can be expressed as

$$a\hat{C}^{Hh}_t + (1-a)\hat{C}^{Fh}_t + G^{Hh}_t + G^{Fh}_t = \hat{Y}^H_t \qquad \hat{C}^{Ff}_t + \hat{C}^{Hf}_t + G^{Hf}_t + G^{Ff}_t = \hat{Y}^F_t \tag{39}$$

while the asset market clearing condition in terms of steady state values are given by

$$\bar{E}^{Hh} + \bar{E}^{Fh} = 1 \qquad \bar{E}^{Hf} + \bar{E}^{Ff} = 1 \tag{40}$$

$$\bar{B}^{Hh} + \bar{B}^{Fh} = \bar{B} \qquad \bar{B}^{Hf} + \bar{B}^{Ff} = \bar{B}^F . \tag{41}$$

1.2.2 Solution for steady-state portfolio

The technique used to determine steady state portfolio holdings under incomplete markets is the same as described earlier for the two-period model. That is, we take second order approximations of the domestic and foreign consumer's euler equations and combine them to get

$$E_t \left[\left(\hat{C}^H_{t+1} - \hat{C}^F_{t+1} - \frac{\hat{Q}_{t+1}}{\sigma} \right) \hat{r}^i_{x,t+1} \right] = 0; \quad i = 1:3. \tag{42}$$

Then, we solve the system of equations up to first order approximation for given values of $\bar{B}^{Hf}$, $\bar{A}^{Hh}$, and $\bar{A}^{Hf}$. Then, we check if the resulting dynamics of variables satisfy the euler equations upto second order accuracy. We iterate until a fixed-point for the asset holdings is found.

1.2.3 Moments of the calibrated model

We report below some standard open economy business cycle moments produced from a simulation of our calibrated model. The data counterparts are taken from Backus and Kehoe (1992) and Corsetti e tal (2008).

Moments of calibrated Model		
Volatilities		
	Data	Model
Consumption	1.47%	1.86%
Trade Balance	0.97%	1.77%
St. Dev. Relative to GDP		
	Data	Model
Real Exchange Rate	3.9	0.5
Terms of Trade	1.7	1.0
Consumption	0.9	1.0
Employment	1.2	0.3
Correlations		
	Data	Model
Consumption and GDP	0.82	0.37
Trade Balance and GDP	-0.2	0.32
ToT and Relative GDPs	-0.33	-0.73
ToT and Relative Consumption	-0.74	-0.92

1.3 Extensions of the dynamic production economy

Here we provide details on the extended model that features a term structure of government debt and capital as an input in production.

A representative agent at home maximizes the expected present discounted value of utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^H\right)^{1-\sigma}}{1-\sigma} - \lambda \frac{\left(L_t^H\right)^{1+\nu}}{1+\nu} \right] \quad 0 < \beta < 1,\ \sigma > 0,\ \nu > 0,\ \lambda > 0$$

where C_t^H is the composite domestic consumption good and L_t^H is domestic labor supply. The agent is subject to the period budget constraint

$$C_t^H + Z_t^{H,m} \frac{B_t^{Hh,m}}{P_t^H} + Z_t^{F,m} \frac{B_t^{Hf,m}}{P_t^F} Q_t + q_t^H\ E_t^{Hh} + q_t^F\ E_t^{Hf} Q_t = (1-\tau_t^L) w_t^H L_t^H + r_t^H K^H$$

$$+ \frac{\left(1+\rho Z_t^{H,m}\right) B_{t-1}^{Hh,m}}{P_t^H} + \frac{\left(1+\rho Z_t^{F,m}\right) B_{t-1}^{Hf,m}}{P_t^F} Q_t + \left(q_t^H\ + \Pi_t^H\right) E_{t-1}^{Hh} + \left(q_t^F\ + \Pi_t^F\right) Q_t E_{t-1}^{Hf}.$$

where $B_{t-1}^{Hh,m}$, $B_{t-1}^{Hf,m}$, E_{t-1}^{Hh}, and E_{t-1}^{Hf} are holdings of domestic nominal bonds, foreign nominal bonds, claims to aggregate after-tax profits of domestic firms, and claims to aggregate after-tax profits of foreign firms purchased in period $t-1$ to be brought into period t. The domestic agents thus simply owns the capital stock K^H and rents it out to firms at the rate r_t^H in a competitive market. Note that here the bonds are a general portfolio of infinitely many bonds and ρ determines the average maturity of debt. We describe this set-up in detail later below.

Moreover, P_t^H is the aggregate domestic price level, P_t^F is the aggregate foreign price level, Q_t is the real exchange rate, q_t^H is the (real) price of one unit of claim to domestic profits, q_t^F is the (real) price of one unit of claim to foreign endowment, Π_t^H is after-tax aggregate real profits of domestic firms, Π_t^F is after-tax aggregate real profits of foreign firms, $Z_t^{H,m}$ is the price of domestic bonds, $Z_t^{F,m}$ is the price of foreign bonds, τ_t^L is the rate of labor income tax, and w_t^H is the real wage at home.

The composite consumption good C_t^H is a CES aggregate of domestic C_t^{Hh} and foreign C_t^{Hf} final goods. The home consumption good C_t^{Hh} is produced in differentiated brands c_t^{Hh} by a continuum of monopolistically competitive home firms indexed j and of measure 1, and is defined as

$$C_t^{Hh} = \left[\int_0^1 c_t^{Hh}(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \theta > 1$$

where the elasticity of substitution among the brands is given by θ. Similarly, the foreign consumption good C_t^{Hf} is produced in differentiated brands c_t^{Hf} by a continuum of monopolistically competitive foreign firms indexed f and of measure 1, and is defined as

$$C_t^{Hf} = \left[\int_0^1 c_t^{Hf}(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$$

where the elasticity of substitution among the brands is given by θ.

As is well known, expenditure minimization by the agent will imply a utility-based aggregate price index at home, P_t^H. Expenditure minimization will also imply the following domestic price level of the home consumption good $P_t^{Hh} = \left[\int_0^1 p_t^{Hh}(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$, where $p_t^{Hh}(j)$ is the domestic price level of brand j of the domestic good, and the following domestic price level of the foreign consumption good $P_t^{Hf} = \left[\int_0^1 p_t^{Hf}(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$, where $p_t^{Hf}(f)$ is the domestic price level of brand f of the foreign good.

Similarly, given the definition of the consumption goods and the price levels, manipulation of the demand curves at the brand level gives

$$\frac{c_t^{Hh}(j)}{C_t^{Hh}} = \left(\frac{p_t^{Hh}(j)}{P_t^{Hh}}\right)^{-\theta} \qquad \frac{c_t^{Hf}(j)}{C_t^{Hf}} = \left(\frac{p_t^{Hf}(j)}{P_t^{Hf}}\right)^{-\theta}.$$

The law of one price holds among the tradable brands and hence we have

$$p_t^{Hh}(j) = S_t \; p_t^{Fh}(j) \quad p_t^{Hf}(f) = S_t \; p_t^{Ff}(f)$$

where $p_t^{Fh}(j)$ and $p_t^{Ff}(f)$ are the foreign price level of price of the brand j of the domestic good and brand f of the foreign good.

Given the definition of the consumption indices and the price indices resulting from expenditure minimization, the optimization problem of the consumer results in

$$\begin{aligned}
\frac{1}{\left(C_t^H\right)^{\sigma}} &= E_t\left[\frac{\beta \; P_t^H\left(1+\rho Z_{t+1}^{H,m}\right)}{\left(C_{t+1}^H\right)^{\sigma} P_{t+1}^H Z_t^{H,m}}\right] = E_t\left[\frac{\beta \; P_t^F\left(1+\rho Z_{t+1}^{F,m}\right) Q_{t+1}}{\left(C_{t+1}^H\right)^{\sigma} P_{t+1}^F Z_t^{F,m} Q_t}\right] \\
\frac{1}{\left(C_t^H\right)^{\sigma}} &= E_t\left[\frac{\beta \; \left(q_{t+1}^H + \Pi_{t+1}^H\right)}{\left(C_{t+1}^H\right)^{\sigma} q_t^H}\right] = E_t\left[\frac{\beta \left(q_{t+1}^F + \Pi_{t+1}^F\right) Q_{t+1}}{\left(C_{t+1}^H\right)^{\sigma} q_t^F Q_t}\right]
\end{aligned}$$

$$\lambda \left(L_t^H\right)^{\nu} = \left(C_t^H\right)^{-\sigma} (1-\tau_t^L) w_t^H.$$

Each brand j of the domestic good is produced by a single home firm j using the following production function

$$y_t^H(j) = A_t^H \left(l_t^H(j)\right)^{\alpha} \left(k_t^H(j)\right)^{1-\alpha}$$

where $y_t^H(j)$ is the domestic output of brand j, A_t^H is the country-specific productivity shock that follows an exogenous process, $l_t^H(j)$ is the labor demand by firm j, and $k_t^H(j)$ is capital used in production by firm j and α is the share of labor. Firms hire labor and capital in a competitive market taking the wage and rental rate of capital as given. The labor and capital used is homogenous across all firms j. The firms are identical except for the fact that they produce differentiated brands for the same good. The process for productivity is given by $\log A_t^H = \rho_A \; \log A_{t-1}^H + \epsilon_{a,t}$. The supply of capital is fixed in the aggregate.

The optimal choice of inputs by firms is given by

$$\frac{w_t^H l_t^H(j)}{r_t^H k_t^H(j)} = \frac{\alpha}{1-\alpha}$$

where r_t is the rental rate of capital and the nominal marginal cost is given by

$$MC_t = \frac{1}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \frac{P_t^H \left(w_t^H\right)^{\alpha} \left(r_t^H\right)^{1-\alpha}}{A_t^H}.$$

Firm j maximizes real profits, that is revenue less labor costs, given by

$$\frac{p_t^{Hh}(j)\ y_t^H(j)}{P_t^H} - w_t^H l_t^H(j) - r_t^H k_t^H(j)$$

leading to the familiar pricing equation

$$p_t^{Hh}(j) \ = \frac{\theta}{\theta-1}\frac{P_t^H}{A_t^H}\left(\frac{\left(w_t^H\right)^\alpha\left(r_t^H\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}\ \alpha^\alpha}\right)$$

where monopolistically competitive firms charge a price that is a mark-up times the nominal marginal cost MC_t.

The aggregate after tax real profits of the firms in the domestic economy can be written as

$$\Pi_t^H = (1-\tau_t^\pi)(P_t^{Hh} - \frac{\left(w_t^H\right)^\alpha\left(r_t^H\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}\ \alpha^\alpha A_t^H}P_t^H)\frac{Y_t^H}{P_t^H}$$

and the optimization decision of the individual domestic firms gives

$$\frac{P_t^{Hh}}{P_t^H} = \frac{\theta}{\theta-1}\frac{\left(w_t^H\right)^\alpha\left(r_t^H\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}\ \alpha^\alpha A_t^H}.$$

The optimization problem of the foreign firms is entirely analogous and is not presented here to conserve space.

For government debt, rather than considering only one-period debt, we follow Woodford (2001) and allow for the existence of a general portfolio of government debt $B_t^{H,m}$ that has price $Z_t^{H,m}$.[2] Households engage on trade of this general government debt instrument. In particular, this instrument's payment structure is $\rho^{T-(t+1)}$ for $T > t$ and $0 \leq \rho \leq 1$. Thus, the value of this portfolio of bond issued in period t in $t+j$ is given by $Z_{t+j}^{H,m-j} = \rho^j Z_{t+j}^{H,m}$. We interpret this general instrument as a portfolio of infinitely many bonds, whose weights are given by $\rho^{T-(t+1)}$. ρ thus determines the average maturity of government debt: when $\rho = 0,$ all debt is of one-period maturity.[3] The period home government budget constraint in this set-up is then given by

$$\begin{aligned} Z_t^{H,m}\frac{B_t^{H,m}}{P_t^H} \quad &= \quad \frac{B_{t-1}^{H,m}}{P_t^H}\left(1+\rho Z_t^{H,m}\right) - \tau_t^L w_t^H L_t^H - \tau_t^\pi\left(P_t^{Hh} - \frac{\left(w_t^H\right)^\alpha\left(r_t^H\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}\ \alpha^\alpha A_t^H}P_t^H\right)\frac{Y_t^H}{P_t^H} \\ &\quad +G_t^{Hh}\frac{P_t^{Hh}}{P_t^H} + G_t^{Hf}\frac{P_t^{Hf}}{P_t^H} \end{aligned}$$

where $B_t^{H,m}$ is total nominal debt issued by the home government in period t and G_t^{Hh} and G_t^{Hf} respectively are the home government's spending on domestic and foreign good. The ratio of labor tax revenue vs. profit tax revenue is for simplicity, constant

[2]Here we use the index m to denote the fact that this bond has a certain duration in period t.
[3]The average maturity of the portfolio is given by $(1-\beta\rho)^{-1}$.

$$\tau_t^L w_t^H L_t^H = y\left[\tau_t^{\pi}\left(P_t^{Hh} - \frac{\left(w_t^H\right)^{\alpha}\left(r_t^H\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A_t^H}P_t^H\right)\frac{Y_t^H}{P_t^H}\right]$$

where y is a parameter of our model.

We assume here that government spending over the differentiated brands of the domestic and foreign goods is defined in the same way as for the consumer with the same elasticity of substitution over the brands. That is,

$$G_t^{Hh} = \left[\int_0^1 g_t^{Hh}(j)^{\frac{\theta-1}{\theta}}dj\right]^{\frac{\theta}{\theta-1}} \qquad G_t^{Hf} = \left[\int_0^1 g_t^{Hf}(f)^{\frac{\theta-1}{\theta}}df\right]^{\frac{\theta}{\theta-1}}.$$

The ratio of government spending over domestic vs. foreign good is for simplicity, constant

$$G_t^{Hh} = xG_t^{Hf}$$

where x is a parameter of our model. Government spending follows an exogenous process

$$G_t^H = G_t^{Fh}\left(\frac{P_t^{Fh}}{P_t^F Q_t}\right) + G_t^{Ff}\left(\frac{P_t^{Ff}}{P_t^F}\right) = \rho_G\ G_{t-1}^{Hh} + \epsilon_{g,t}.$$

We do not consider explicit optimal government policy and use simple rules as descriptions of government policy. The government conducts monetary policy using a version of the interest rate rule given by

$$\left(\frac{1+\rho Z_{t+1}^{H,m}}{Z_t^{H,m}}\right) = \gamma_0\left(P_t^H/P_{t-1}^H\right)^{\gamma}\exp(\epsilon_{r,t}^H)$$

where the interest rate shock follows the exogenous process $\log\epsilon_{r,t}^H = \rho_R\log\epsilon_{r,t-1}^H + e_{r,t}$ and fiscal policy using a rule for total tax revenue responding to real value of debt

$$\tau_t^L w_t^H L_t^H + \tau_t^{\pi}\left(P_t^{Hh} - \frac{\left(w_t^H\right)^{\alpha}\left(r_t^H\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A_t^H}P_t^H\right)\frac{Y_t^H}{P_t^H} = \phi_0\left(\frac{B_t^H}{P_t^H}\right)^{\phi}.$$

Again, the foreign government's description is completely analogous and symmetric.

Market clearing for goods implies

$$\begin{aligned} c_t^{Hh}(j) + c_t^{Fh}(j) + g_t^{Hh}(j) + g_t^{Fh}(j) &= A_t^H\left(l_t^H(j)\right)^{\alpha}\left(k_t^H(j)\right)^{1-\alpha} \\ c_t^{Ff}(f) + c_t^{Hf}(f) + g_t^{Hf}(f) + g_t^{Ff}(f) &= A_t^F\left(l_t^F(j)\right)^{\alpha}\left(k_t^F(j)\right)^{1-\alpha} \end{aligned}$$

while market clearing for assets implies

$$E_t^{Hh} + E_t^{Fh} = 1 \qquad E_t^{Hf} + E_t^{Ff} = 1$$
$$B_t^{Hh,m} + B_t^{Fh,m} = B_t^{H,m} \qquad B_t^{Hf,m} + B_t^{Ff,m} = B_t^{F,m}$$

and total labor demand by firms equaling labor supply and capital being fixed in the aggregate implies

$$\int_0^1 l_t^H(j)dj = L_t^H$$

$$\int_0^1 k_t^H(j)dj = K_t^H = K^H.$$

We next provide results for asset holdings when we shut down government spending shocks in the model

Domestic Holdings - Model Extension	
No Govnt Expenditure Shock	
Bonds	-525%
Equity	-163%

1.4 VAR methodology

We have

$$\hat{b}_{t-1} = -E_{t-1}\left[\sum_{s=0}^{\infty}(\bar{\rho})^s\,\tilde{r}_{t+s}\right] - \bar{\rho}\left[E_{t-1}\sum_{s=0}^{\infty}(\bar{\rho})^s\left[\hat{d}_{t+s}\right]\right]$$

which we rewrite as

$$\hat{b}_t = -E_t\left[\sum_{s=1}^{\infty}(\bar{\rho})^{s-1}\,\tilde{r}_{t+s}\right] - \bar{\rho}\left[E_t\sum_{s=1}^{\infty}(\bar{\rho})^{s-1}\left[\hat{d}_{t+s}\right]\right].$$

Now lets derive the VAR representation that we need to compute the RHS of this expression. We will estimate the following VAR(p) where $y_t = \left(\tilde{r}_t,\ d_t,\ \hat{b}_t\right)'$

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + .. + \ \phi_p y_{t-p} + \epsilon_t.$$

Next, it is convenient to write the VAR (p) in a VAR(1) form, by stacking appropriately

$$\bar{y}_t = \phi\bar{y}_{t-1} + \bar{\epsilon}_t$$

where $\bar{y}_t = \left(y_t',\ y_{t-1}', \ldots, y_{t-p+1}'\right)'$, $\bar{\epsilon}_t = \ (\bar{\epsilon}_t', 0)$, and ϕ is the companion matrix

$$\phi = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & .. & \phi_{p-1} & \phi_p \\ I & 0 & 0 & ... & 0 & 0 \\ 0 & I & 0 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & .. \\ 0 & 0 & 0 & .. & I & 0 \end{bmatrix}.$$

Here I is a $3*3$ identity matrix and 0 is a $3*3$ matrix of zeros.

Then, given this VAR (1) notation, we have

$$E_t \bar{y}_{t+s} = \phi^s \bar{y}_t.$$

Also, define the following vectors that pick out the relevant elements from $\bar{y}_t$

$$e^r = [1\ 0_{p-1}]\,;\ e^r \bar{y}_t = \tilde{r}_t$$

$$e^d = [0\ 1\ 0_{p-2}]\,;\ e^d \bar{y}_t = d_t$$

$$e^b = [0\ 0\ 1\ 0_{p-3}]\,;\ e^b \bar{y}_t = \hat{b}_t.$$

Next, we can then derive the two components of the RHS of the intertermporal budget constraint

$$b_t^r = -\sum_{s=1}^{\infty} (\bar{\rho})^{s-1} E_t \tilde{r}_{t+s}$$

$$= -e^r \sum_{s=1}^{\infty} (\bar{\rho})^{s-1} \phi^s \bar{y}_t$$

$$= -e^r \phi \left(I - \bar{\rho}\phi\right)^{-1} \bar{y}_t.$$

Similarly, we have

$$b_t^d = -\bar{\rho} \sum_{s=1}^{\infty} (\bar{\rho})^{s-1} E_t \hat{d}_{t+s}$$

$$= -e^d\ \bar{\rho} \sum_{s=1}^{\infty} (\bar{\rho})^{s-1} \phi^s \bar{y}_t$$

$$= -e^d \left(\bar{\rho}\right) \phi \left(I - \bar{\rho}\phi\right)^{-1} \bar{y}_t.$$

Given the estimates of ϕ and our resulting estimates for b_t^d, we then compute the correlations between $\tilde{r}_{t+1}$ and b_t^d and between π_{t+1} and -b_t^d.

References

[1] Backus, D. K., P. J. Kehoe, 1992, "International Evidence of the Historical Properties of Business Cycles," American Economic Review, 82, 864-88.

[2] Corsetti, G., L. Dedola, S. Leduc, 2008, "International Risk Sharing and the Transmission of Productivity Shocks," Review of Economic Studies, 75, 444-73.