Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products

Dennis Epple

Carnegie-Mellon University

In choosing the level of quality to purchase, the buyer of a differentiated product also chooses a point on the marginal price schedule for that product. Hence, in general, the demand functions for product characteristics cannot be consistently estimated by ordinary least squares. Market equilibrium results in a matching of characteristics of demanders and suppliers. This matching restricts the use of buyer and seller characteristics as instruments when estimating demand and supply functions for product characteristics. The paper develops these issues. A stochastic structure for hedonic equilibrium models is then proposed, identification results are presented, and estimation procedures are outlined.

I. Introduction

Hedonic models focus on markets in which a generic commodity can embody varying amounts of each of a vector of attributes. In empirical investigation of hedonic models, one issue of interest is determining how the price of a unit of the commodity varies with the set of attributes it possesses. The other subject of interest is estimating the demand and supply functions for attributes of the product. The purposes of this paper are to identify problems that must be confronted

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in identification and estimation of the parameters of such models and to discuss potential resolutions of these problems.

Early contributions to the theory of equilibrium in markets for differentiated products were provided by Tinbergen (1959). Rosen (1974) presented an integrated treatment of hedonic theory and the demand for and supply of differentiated products. He also outlined an econometric procedure for estimation of the demand and supply functions that determine the hedonic price function.

Section II of this paper contains a brief review of hedonic theory followed by a statement of the modeling strategy developed by Rosen. Applications of Rosen's modeling strategy have often used inappropriate estimation procedures that give rise to inconsistent estimates of the parameters of demand and supply functions. Conditions for identification and estimation procedures for models with linear demand and supply functions are proposed in Section III. It is argued that the key to resolution of identification and estimation problems in hedonic models is a precise statement of the orthogonality conditions between the measured variables and random components of such models. Moreover, when both demand and supply are endogenous, some seemingly natural specifications are shown to be incompatible with the conditions equilibrating demand and supply. Extension to models with nonlinear demand and supply functions is discussed in Section IV. Selected applications of hedonic theory are reviewed in Section V to illustrate how these issues arise in practice. A summary and concluding comments are presented in Section VI.

II. Hedonic Theory and Econometric Specification

Consider a generic commodity with the vector of attributes $\mathbf{z}$. The hedonic price function $p(\mathbf{z})$ specifies how the market price of the commodity varies as the characteristics vary. Rosen (1974) provides a theoretical framework in which $p(\mathbf{z})$ emerges from the interaction between suppliers and demanders of the commodity. The distribution of the quantity, as a function of $\mathbf{z}$, that is supplied and consumed is also endogenously determined. A brief summary of the theory follows to set the stage for a discussion of econometric issues.

A. *Outline of the Theoretical Framework*

Consumers who are assumed to purchase one unit of a product with characteristics $\mathbf{z}$ have the utility function $U(\mathbf{z}, x; \boldsymbol{\alpha})$, where $\mathbf{z}$ is a vector of m characteristics of the commodity of interest, x is the numeraire commodity, and $\boldsymbol{\alpha}$ is a vector of parameters characterizing the individual consumer. Consumers face the budget constraint $y = p(\mathbf{z}) + x$, where y is income. The joint probability distribution of y and $\boldsymbol{\alpha}$ in the population of consumers is $F(y, \boldsymbol{\alpha})$. Consumers are price takers: they take the functional form of $p(\mathbf{z})$ as given.

Maximization of utility subject to the budget constraint gives rise to a vector of m demand functions for characteristics:[1]

$$p_{\mathbf{z}} = \frac{U_{\mathbf{z}}(\mathbf{z}, y - p(\mathbf{z}); \boldsymbol{\alpha})}{U_x(\mathbf{z}, y - p(\mathbf{z}); \boldsymbol{\alpha})} = h(\mathbf{z}, y - p(\mathbf{z}); \boldsymbol{\alpha}). \tag{1}$$

In (1), $p_{\mathbf{z}}$ denotes the vector of first derivatives of the hedonic price function with respect to its arguments. The distribution function characterizing demand for the product can be derived using F and the m equations in (1).

Producers are assumed to have the cost function $C(M, \mathbf{z}; \boldsymbol{\beta})$, where M denotes the number of units of the product with characteristics $\mathbf{z}$ that the firm produces, and $\boldsymbol{\beta}$ is a vector of parameters characterizing the individual producer. The distribution function for $\boldsymbol{\beta}$ is $G(\boldsymbol{\beta})$. Producers are assumed to behave as price takers in maximizing profit:

$$p(\mathbf{z})M - C(M, \mathbf{z}; \boldsymbol{\beta}).$$

Thus the producers take the function $p(\mathbf{z})$ as given in making their production decisions.

Characterization of product supply is different in the short run than in the long run. Rosen considers three cases: a short run in which only M is variable by the producer, a short run in which the producer can vary both M and $\mathbf{z}$, and a long run in which plants can be added or retired. The second of these cases is discussed here for purposes of illustration. First-order profit-maximizing conditions for this case are

$$p_{\mathbf{z}} = \frac{C_{\mathbf{z}}(M, \mathbf{z}; \boldsymbol{\beta})}{M} \tag{2a}$$

$$p(\mathbf{z}) = C_M(M, \mathbf{z}; \boldsymbol{\beta}). \tag{2b}$$

The right-hand side of (2a) contains the m-dimensional vector of first partials of C with respect to $\mathbf{z}$, with each element divided by M. The distribution of quantity supplied as a function of $\mathbf{z}$ is derived using G and equations (2) in the same manner as the distribution of quantities demanded is derived from F and equation (1).

Equilibrium requires that the quantity demanded of the product

[1] In this paper, the usual convention of referring to these equations as demand functions is adopted, but, as McConnell and Phipps (1985) point out, these equations are actually marginal rate of substitution functions. These functions reduce to demand functions only when the marginal utility of money is constant.

with attributes $\mathbf{z}$ equal the quantity supplied for all $\mathbf{z}$. Sattinger (1980) provides a detailed exposition of the solution of hedonic equilibrium models.

Tinbergen's (1959) contribution to the theory of equilibrium in markets for differentiated products is formulation of an equilibrium model in the labor market when different individuals have different values of a vector of skills. As the following example illustrates, his model can be reformulated as a model of equilibrium in a market for an arbitrary differentiated commodity. The model is an instructive example of a closed-form solution for hedonic equilibrium, and it is also a special case of the model considered in Section III.

Example: Let the utility function of a typical consumer be

$$U = -(\mathbf{z} - \boldsymbol{\alpha})'\frac{\boldsymbol{\theta}}{2}(\mathbf{z} - \boldsymbol{\alpha}) + x,$$

where $\boldsymbol{\theta}$ is a positive definite diagonal matrix common to all consumers. The vector of taste parameters $\boldsymbol{\alpha}$ differs across consumers. In the population of consumers, $\boldsymbol{\alpha}$ is normally distributed with mean $\boldsymbol{\tau}$ and diagonal covariance matrix $\boldsymbol{\Sigma}$: $\boldsymbol{\alpha} \sim N(\boldsymbol{\tau}, \boldsymbol{\Sigma})$. Let the distribution of products available be exogenous and normally distributed with mean $\boldsymbol{\xi}$ and diagonal covariance matrix $\boldsymbol{\Omega}$: $\mathbf{z} \sim N(\boldsymbol{\xi}, \boldsymbol{\Omega})$. Then the hedonic price function that equilibrates this market is[2]

$$p(\mathbf{z}) = \boldsymbol{\psi}'\mathbf{z} + \mathbf{z}'\frac{\boldsymbol{\Pi}}{2}\mathbf{z},$$

where $\boldsymbol{\psi} = \boldsymbol{\theta}(\boldsymbol{\tau} - \boldsymbol{\Sigma}^{1/2}\boldsymbol{\Omega}^{1/2}\boldsymbol{\xi})$ and $\boldsymbol{\Pi} = -\boldsymbol{\theta}(I - \boldsymbol{\Sigma}^{1/2}\boldsymbol{\Omega}^{1/2})$.

This example illustrates that the parameters $\boldsymbol{\psi}$ and $\boldsymbol{\Pi}$ of the equilibrium price function depend on the parameters characterizing preferences, $\boldsymbol{\theta}$, on those characterizing the distribution of preferences, $(\boldsymbol{\tau}, \boldsymbol{\Sigma})$, and on those characterizing the distribution of supply, $(\boldsymbol{\xi}, \boldsymbol{\Omega})$. It is also possible (Epple 1984) to derive a closed-form expression with both demand and supply endogenous by extending the example above with quadratic production costs and normally distributed supplier characteristics. This also yields a quadratic expression for the equilibrium price function in which the parameters of suppliers' cost functions also appear. Closed-form solutions with other functional forms in two-factor models of production are provided by Sattinger (1980).

As the example above shows, by following Tinbergen's lead, one can derive closed-form solutions for equilibrium in models that may

[2] That this is the solution can be verified by direct calculation. The first- and second-order conditions for a maximum for each consumer are satisfied, and supply equals demand for products of all qualities.

prove general enough for many empirical applications. However, closed-form solutions will not be available for arbitrary choices of functional forms for the demand, cost, and distribution functions. Recognizing this, Rosen proposed an estimation procedure that does not rely on the availability of a closed-form solution.

B. Issues in Econometric Estimation

Let the empirical counterpart of the demand functions in (1) be denoted[3]

$$p_i(\mathbf{z}) = F_i(\mathbf{z}, \mathbf{x}_1, y - p(\mathbf{z})), \quad i = 1, \ldots, m, \tag{3}$$

where $\mathbf{x}_1$ is a vector of empirical counterparts of $\boldsymbol{\alpha}$, and $p_i(\mathbf{z})$ denotes the partial derivative of $p(\mathbf{z})$ with respect to characteristic i. If the utility function has the property that the marginal utility of money is constant, then $y - p(\mathbf{z})$ will not appear in the demand functions (1) and hence not in (3). If the marginal utility of money is not constant, $y - p(\mathbf{z})$ will appear; y alone will not appear.

Let the supply functions for characteristics corresponding to (2) be denoted

$$p_i(\mathbf{z}) = G_i(\mathbf{z}, \mathbf{x}_2, p(\mathbf{z})), \quad i = 1, \ldots, m, \tag{4}$$

where the vector $\mathbf{x}_2$ is the empirical counterpart of $\boldsymbol{\beta}$. When M is fixed exogenously for technological or other reasons, (4) is the empirical counterpart of (2a), and $p(\mathbf{z})$ will not appear. When M is endogenous, one may eliminate M by solving (2b) for M and substituting the result into (2a). Product price, $p(\mathbf{z})$, will generally appear in the resulting equations. Therefore, when M is endogenous, $p(\mathbf{z})$ will generally appear in the supply functions (4).

Rosen outlines a two-step procedure in which the first step is to estimate $p(\mathbf{z})$ by choice of the functional form that best fits the data. The next step is to compute the partial derivatives of this function, evaluate the derivatives at points corresponding to the sample values of $\mathbf{z}$, and use the resulting variables as the dependent variables to estimate (3) and (4).

If the two steps described above are applied using ordinary least squares for each step,[4] the coefficient estimates of the second stage

[3] It is assumed throughout that an equilibrium exists, that the equilibrium price function is continuous and has continuous derivatives, and that the demand and supply relationships can be solved for the characteristics, $\mathbf{z}$.

[4] Rosen does not propose that a particular estimation method such as ordinary least squares be used. Rather, he outlines an econometric modeling strategy, presumably intending that choice of an estimation procedure be determined as appropriate for particular applications. In applications, however, ordinary least squares and other methods have often been used inappropriately.

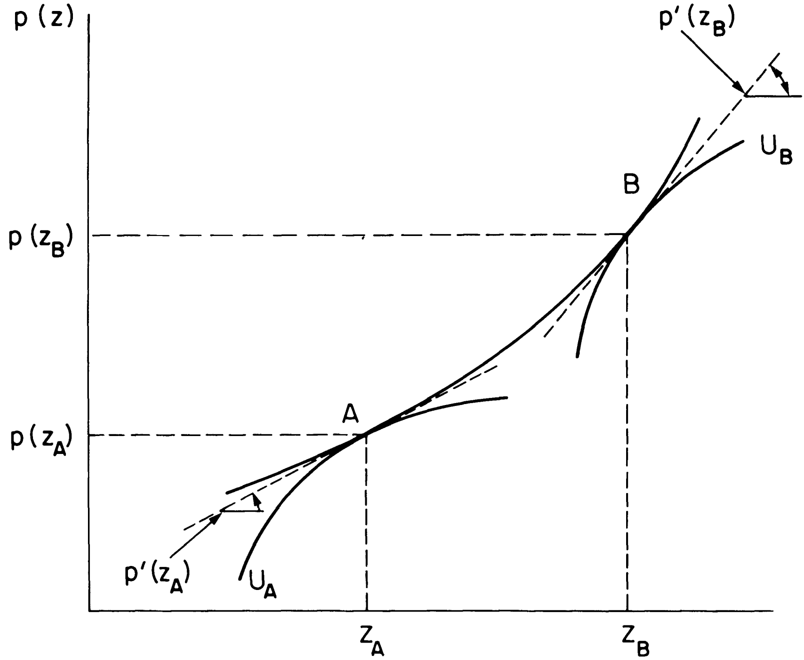

FIG. 1

will not be estimated consistently regardless of the sources of error in the equations. Formal demonstration of this claim in the context of a specific model is presented in Section III. An intuitive explanation is offered here. By construction, the dependent variables in (3) and (4) are functions of **z**. As a result, when error terms are appended to (3) and (4), the **z** will generally be correlated with those error terms. Ordinary least squares will, in general, give inconsistent estimates when the dependent variable appears on both sides of the equation being estimated.

This problem arises even when the distribution of supply is exogenous and only the demand and hedonic price function parameters are being estimated. This is illustrated in figure 1, which shows the hedonic price function taken as exogenous by individuals. It also shows indifference curves of two individuals that are tangent to the hedonic function. Individual A chooses point *A* while B chooses point *B*. Note that in choosing a location on the hedonic price function, individuals simultaneously choose quality and the slope of the hedonic function. Thus, individual A simultaneously chooses z_A and $p'(z_A)$. Similarly, B chooses z_B and $p'(z_B)$. This is the simultaneity reflected in the demand function, and it is present whether the distribution of characteristics supplied in the market is exogenous or endogenous.

It might appear that the problem can be easily remedied by using $\mathbf{x}_1$ and/or $\mathbf{x}_2$ as instruments in estimating equations (3) and (4). This procedure may sometimes be appropriate. However, it will be shown that when both supply and demand are endogenous, one cannot assume that *all* elements of *both* $\mathbf{x}_1$ and $\mathbf{x}_2$ are uncorrelated with the error terms in *both* (3) and (4).

III. Models with Linear Demand and Supply Functions

While the model considered in this section is a particular form of the general model, it proves to be an attractive specification in practice.[5] It is also a form that permits straightforward illustration of issues that arise with other functional forms.

The strategy of the presentation is as follows. First, a hedonic model is presented in which the sources of error are unspecified. Next, it is shown that the equilibrium conditions of the model impose some surprising restrictions on the error terms—restrictions that rule out some seemingly natural estimation strategies. On the basis of these findings, it is argued that a systematic treatment of sources of error is required, and such a treatment is then offered.

Suppose that observations are available for K geographically or temporally distinct markets indexed by k.[6] Let the hedonic price equation for an observation from market k be[7]

$$p_k = \gamma_k + \boldsymbol{\psi}_k'\mathbf{z}_k + \frac{\mathbf{z}_k'\boldsymbol{\Pi}_k\mathbf{z}_k}{2} + \zeta_k, \tag{5}$$

where γ_k is a scalar, $\boldsymbol{\psi}_k$ is a vector, and $\boldsymbol{\Pi}_k$ is a symmetric matrix. The subscripts indicate that the parameters of the hedonic price equation differ by market. Let the demand and supply functions be

$$\frac{\partial p_k}{\partial \mathbf{z}_k} = \mathbf{A}_1\mathbf{z}_k + \mathbf{H}_1\mathbf{x}_{1k} + \mathbf{v}_{1k}, \tag{6d}$$

$$\frac{\partial p_k}{\partial \mathbf{z}_k} = \mathbf{A}_2\mathbf{z}_k + \mathbf{H}_2\mathbf{x}_{2k} + \mathbf{v}_{2k}. \tag{6s}$$

[5] This model is introduced by Witte, Sumka, and Erekson (1979). Their application will be discussed in Sec. V. While the specification is an attractive one from the standpoint of estimation, its suitability for a particular application must also be judged by how well it fits the data. Halvorsen and Pollakowski (1981) have proposed use of the Box-Cox transformation to choose a functional form for the hedonic price equation.

[6] See Diamond and Smith (1985) for a discussion of various strategies for developing multimarket data bases.

[7] These equations and all equations that follow hold for each observation of a sample of $j = 1, \ldots, J$ observations. For notational convenience no j subscript is employed, but it should be understood that the equations refer to an arbitrarily chosen observation.

Here $(\partial p_k/\partial \mathbf{z}_k)$ is an $m \times 1$ vector of partial derivatives of p_k with respect to the characteristics, and $\mathbf{v}_{1k}$ and $\mathbf{v}_{2k}$ are vectors of error terms. In (6) the constant vector is included as an element of both $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ to capture the intercepts of the demand and supply equations, $y - p(\mathbf{z})$ is assumed to be one of the elements of $\mathbf{x}_{1k}$ in the demand functions (6d), and $p(\mathbf{z})$ is assumed to be one of the elements of $\mathbf{x}_{2k}$ in the supply functions (6s).

Note that the parameter matrices $\mathbf{A}_1$, $\mathbf{H}_1$, $\mathbf{A}_2$, and $\mathbf{H}_2$ in these equations have no k subscript. This indicates that the form of the demand and supply functions is the same in all markets. The form of the demand and supply functions depends on the tastes of consumers with a given set of characteristics or on the technology of producers with a given set of characteristics. Therefore, the parameters of these demand and supply functions should not vary across markets. In contrast, the shape of the hedonic price function depends on the distributions of characteristics of demanders and suppliers. If these distributions vary from market to market, the parameters of the price function will differ across markets as well.

In the discussion that follows, I assume that the objective is to obtain estimates of the parameters of demand and supply functions ($\mathbf{A}_1$, $\mathbf{H}_1$, $\mathbf{A}_2$, $\mathbf{H}_2$) and the parameters of the hedonic price function in each market (γ_k, $\boldsymbol{\psi}_k$, $\boldsymbol{\Pi}_k$; $k = 1, \ldots, K$). For part of the discussion, it will be assumed that supply is exogenous. For that case, equation (6s) does not apply. Instead, the distribution of product characteristics $\mathbf{z}_k$ in each market is given exogenously. The relationships (5) and (6d) then characterize the way in which the exogenously given distribution of product characteristics is allocated among consumers.

The following points will be shown: (1) Whether supply is exogenous or endogenous, ordinary least squares applied to the demand equations (6d) will not yield consistent parameter estimates. (2) In hedonic applications, the price equation (5) is typically estimated by ordinary least squares when supply is exogenous. It is also customary to assume that the $\mathbf{x}_{1k}$ are exogenous (i.e., uncorrelated with ζ_k and the $\mathbf{v}_{1k}$). Under these conditions, ordinary least squares estimation of the price equation (5) will be consistent only if the error term ζ_k in the price equation is uncorrelated with the error vector $\mathbf{v}_{1k}$ in the demand equations. (3) When demand and supply are endogenous, the seemingly natural strategy of treating all elements of $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ as exogenous (i.e., uncorrelated with ζ_k, $\mathbf{v}_{1k}$, and $\mathbf{v}_{2k}$) is *not* consistent with the structure of the model.

To establish the first point, solve (5) for $(\partial p_k/\partial \mathbf{z}_k)$, substitute the result into (6d), and solve for $\mathbf{z}_k$ to obtain

$$\mathbf{z}_k = (\boldsymbol{\Pi}_k - \mathbf{A}_1)^{-1}(\mathbf{H}_1\mathbf{x}_{1k} + \mathbf{v}_{1k} - \boldsymbol{\psi}_k). \tag{7}$$

Equation (7) implies that vectors $\mathbf{z}_k$ and $\mathbf{x}_{1k}$ cannot both be uncorrelated with $\mathbf{v}_{1k}$. Thus ordinary least squares estimation of (6d) will not yield consistent parameter estimates. When supply is endogenous, the same argument applies to establish that the supply equations cannot be estimated by ordinary least squares. The first point is thus established.

The second claim is also an immediate consequence of (7). For ordinary least squares estimation of the price function to be consistent, the $\mathbf{z}_k$ on the right-hand side of (5) must be uncorrelated with the error term ζ_k. Postmultiply (7) by ζ_k and invoke the assumptions that ζ_k has zero mean and is uncorrelated with $\mathbf{x}_{1k}$. This will establish that ζ_k can be uncorrelated with $\mathbf{z}_k$ when $\mathbf{x}_{1k}$ is exogenous only if ζ_k is also uncorrelated with $\mathbf{v}_{1k}$.

Perhaps the most surprising claim is point 3. It is established by using the following equilibrium condition. Solve (5) for $(\partial p_k/\partial \mathbf{z}_k)$, substitute the result into (6), solve each of the equations in (6) for $\mathbf{z}_k$, and equate the results to obtain

$$\begin{aligned}&(\mathbf{\Pi}_k - \mathbf{A}_1)^{-1}(\mathbf{H}_1\mathbf{x}_{1k} + \mathbf{v}_{1k} - \boldsymbol{\psi}_k)\\ &= (\mathbf{\Pi}_k - \mathbf{A}_2)^{-1}(\mathbf{H}_2\mathbf{x}_{2k} + \mathbf{v}_{2k} - \boldsymbol{\psi}_k).\end{aligned} \tag{8}$$

This equation imposes restrictions relating the "exogenous" variables $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ and the random terms $\mathbf{v}_{1k}$ and $\mathbf{v}_{2k}$.[8] The implication is that at least some elements of $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ must be correlated with the residuals in (6). As a result, one cannot use all elements of both $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ as instruments to estimate both equations in (6). Hence, the seemingly natural strategy of treating all elements of $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ as exogenous is invalid.

This problem arises because there are $2m$ equations in (6d) and (6s) but only m endogenous variables: the $\mathbf{z}_k$. For those m endogenous variables to be consistent with the $2m$ demand and supply equations, it is necessary that the "exogenous" variables and "random" components be related by equation (8). The functional dependence in (8) among $\mathbf{x}_{1k}$, $\mathbf{x}_{2k}$, $\mathbf{v}_{1k}$, and $\mathbf{v}_{2k}$ arises for the following reason. In the

[8] Let $\mathbf{x}_k$ be the combined vector of demander and supplier characteristics and $\mathbf{v}_k$ be the combined vector of residuals from (6); i.e., $\mathbf{x}_k' = [\mathbf{x}_{1k}', \mathbf{x}_{2k}']$ and $\mathbf{v}_k' = [\mathbf{v}_{1k}', \mathbf{v}_{2k}']$. Let $\mathbf{\Sigma}_k$ be the covariance matrix of $\mathbf{v}_k$. The claim of point 3 may now be stated precisely as follows. It is not possible to choose $\mathbf{x}_k$ and $\mathbf{v}_k$ satisfying (8) such that both of the following are satisfied: $E(\mathbf{x}_k\mathbf{v}_k') = 0$ and $\mathbf{\Sigma}_k$ is nonsingular. To see why, postmultiply (8) by $\mathbf{v}_k'$. Then take expectations assuming that $\mathbf{v}_k$ has mean zero and that $E(\mathbf{x}_k\mathbf{v}_k') = 0$. Use the result to establish that $\mathbf{\Sigma}_k$ is singular. Given that these conditions cannot be satisfied simultaneously, one of them must be relaxed. The assumption that $\mathbf{\Sigma}_k$ is nonsingular must be preserved; the data will reject the assumption of an exact linear dependence among the residuals in the demand and supply equations. Hence, the assumption $E(\mathbf{x}_k\mathbf{v}_k') = 0$ must be forgone, as asserted in point 3. Further details are available on request.

theoretical model reviewed in Section II, buyers are differentiated by characteristics (y, $\boldsymbol{\alpha}$) and sellers are differentiated by characteristics $\boldsymbol{\beta}$. If the type of product that buyers desire is systematically related to (y, $\boldsymbol{\alpha}$) and the type of product that sellers provide is systematically related to $\boldsymbol{\beta}$, then the equilibrium will be one in which the characteristics of buyers will be systematically related to the characteristics of sellers from whom they purchase. For example, it would not be surprising if the income (y) of buyers of stereo equipment is highly correlated with the degree of technical education ($\boldsymbol{\beta}$) of engineers employed by sellers. The correlation arises because of the characteristics (harmonic distortion, wow, flutter, dazzle, etc.) of equipment involved in the exchange.

The results above demonstrate that a more systematic analysis of the sources of error in (5) and (6) is required. The residuals in a hedonic model may, in general, arise from one or more of the following sources:[9] (i) errors in measurement of the product price, (ii) errors in measurement of agent (supplier and demander) characteristics, (iii) errors in measurement of product characteristics, (iv) unobserved agent characteristics, or (v) unobserved product characteristics.

Still another potential interpretation is that residuals arise as a result of agents' errors in assessing product price or product characteristics. Treatment of this case requires characterization of equilibrium when suppliers and demanders have differing information about product characteristics. While this is a case deserving attention, characterization of hedonic equilibrium under asymmetric information is beyond the scope of this paper. Hence, the case in which agents make errors in assessing product characteristics will not be considered.

A. *Model Specification*

It will be assumed that the true product price and a set of true characteristics of agents and products exist, some of which may be unobserved by the econometrician. The relationships between these unobserved variables and observed prices and characteristics will then be specified.

Let $\mathbf{z}_k^*$ be the vector of true product characteristics, and $\mathbf{x}_{1k}^*$ and $\mathbf{x}_{2k}^*$ the vectors of true characteristics of demanders and suppliers. Then the true hedonic price, demand, and supply equations are

[9] Approximation errors in the choice of functional form might also be considered. Since the true functional form will rarely if ever be known, this interpretation does not appear sufficiently interesting to warrant further consideration.

$$p_k^* = \gamma_k + \boldsymbol{\psi}_k' \mathbf{z}_k^* + \frac{\mathbf{z}_k^{*\prime} \boldsymbol{\Pi}_k \mathbf{z}_k^*}{2}, \quad k = 1, \ldots, K, \tag{9}$$

$$\boldsymbol{\psi}_k + \boldsymbol{\Pi}_k \mathbf{z}_k^* = \frac{\partial p_k^*}{\partial \mathbf{z}_k^*} = \mathbf{A}_1 \mathbf{z}_k^* + \mathbf{H}_1 \mathbf{x}_{1k}^*, \quad k = 1, \ldots, K, \tag{10d}$$

$$\boldsymbol{\psi}_k + \boldsymbol{\Pi}_k \mathbf{z}_k^* = \frac{\partial p_k^*}{\partial \mathbf{z}_k^*} = \mathbf{A}_2 \mathbf{z}_k^* + \mathbf{H}_2 \mathbf{x}_{2k}^*, \quad k = 1, \ldots, K. \tag{10s}$$

To simplify notation, the k subscript will now be dropped, and the discussion should be understood to apply to an arbitrarily chosen market.

Product price will be assumed to be measured with error. Product and agent characteristics will be assumed to be of three types: observed, proxy, and unobserved variables. Observed variables are measured without error, proxy variables are measured with error, and unobserved variables are not measured at all. Observed and proxy variables will be referred to as measured variables. Notation is summarized in table 1. Measured variables are related to the true variables by the following:

$$\begin{vmatrix} \mathbf{z}_o \\ \mathbf{z}_p \\ 0 \end{vmatrix} = \begin{vmatrix} \mathbf{z}_o^* \\ \mathbf{z}_p^* \\ \tilde{\mathbf{z}}^* \end{vmatrix} + \begin{vmatrix} 0 \\ \boldsymbol{\omega} \\ \tilde{\mathbf{z}} \end{vmatrix}, \tag{11a}$$

$$\begin{vmatrix} \mathbf{x}_{io} \\ \mathbf{x}_{ip} \\ 0 \end{vmatrix} = \begin{vmatrix} \mathbf{x}_{io}^* \\ \mathbf{x}_{ip}^* \\ \tilde{\mathbf{x}}_i^* \end{vmatrix} + \begin{vmatrix} 0 \\ \boldsymbol{\delta}_i \\ \tilde{\mathbf{x}}_i \end{vmatrix}, \quad i = 1, 2, \tag{11b}$$

$$p = p^* + \zeta. \tag{12}$$

Note that each demand and supply equation contains a constant term. Thus, $\mathbf{x}_{1o}$ and $\mathbf{x}_{2o}$ always contain at least one element each. By contrast $\mathbf{z}_o$ does not contain a constant.[10]

This specification provides a quite general characterization of the sources of error. For example, when the vector of unmeasured variables $(\tilde{\mathbf{z}}', \tilde{\mathbf{x}}_1', \tilde{\mathbf{x}}_2')$ does not appear, a pure measurement error model emerges as an important special case. The distinction between un-

[10] Recall that $y - p(\mathbf{z})$ is included as an element of $\mathbf{x}_1$ and $p(\mathbf{z})$ is included as an element of $\mathbf{x}_2$. Since product price is measured with error, this induces a correlation across these elements of the demand and supply equations. Arbitrary correlation of errors across demand and supply equations is permitted, so this effect can be subsumed in the covariance matrix of measurement errors.

TABLE 1

NOTATION

o	Subscript or superscript denoting observed variables
p	Subscript or superscript denoting variables for which proxy measures are available
$*$	Denotes true value of a variable
$\mathbf{z}_o$, $\mathbf{x}_{1o}$, $\mathbf{x}_{2o}$	Observed product, demander, and supplier characteristics
$\mathbf{z}_p$, $\mathbf{x}_{1p}$, $\mathbf{x}_{2p}$	Proxy product, demander, and supplier characteristics
$\boldsymbol{\omega}$, $\boldsymbol{\delta}_1$, $\boldsymbol{\delta}_2$	Errors in measuring $\mathbf{z}_p$, $\mathbf{x}_{1p}$, and $\mathbf{x}_{2p}$
$\tilde{\mathbf{z}}$, $\tilde{\mathbf{x}}_1$, $\tilde{\mathbf{x}}_2$	Unmeasured product, demander, and supplier characteristics
m_o, m_p, $\tilde{m}$	Dimensions of $\mathbf{z}_o$, $\mathbf{z}_p$, and $\tilde{\mathbf{z}}$
n_{io}, n_{ip}, $\tilde{n}_i$	Dimensions of $\mathbf{x}_{io}$, $\mathbf{x}_{ip}$, and $\tilde{\mathbf{x}}_i$; $i = 1, 2$
$\mathbf{z}' = [\mathbf{z}_o', \mathbf{z}_p']$	Measured product characteristics
$\mathbf{x}_i' = [\mathbf{x}_{io}', \mathbf{x}_{ip}']$	Measured demander ($i = 1$) and supplier ($i = 2$) characteristics
$m = m_o + m_p$	Number of measured product characteristics
$n_i = n_{io} + n_{ip}$	Number of measured demander ($i = 1$) and supplier ($i = 2$) characteristics
p, ζ	Product price and error in measuring product price
γ, $\boldsymbol{\psi}$, $\boldsymbol{\Pi}$	Scalar, $m \times 1$ vector, and $m \times m$ matrix, respectively, of hedonic price function parameters
$\mathbf{A}_i$, $\mathbf{H}_i$	$m \times m$ and $m \times n_i$ matrices of parameters in demand ($i = 1$) and supply ($i = 2$) equations
$n_o = n_{1o} + n_{2o}$	Number of observed agent characteristics
$n_p = n_{1p} + n_{2p}$	Number of proxy measures of agent characteristics

measured variables and measurement errors proves to be an important one. If data from different markets are measured in the same way, the distribution of measurement errors can be assumed to be the same across markets. In contrast, the distribution of unmeasured variables can be expected to differ across markets for the same reason that the distribution of measured variables does.

For identification and estimation to be feasible, orthogonality conditions among the measured, true, and unmeasured elements of the model are required. Moreover, as was demonstrated in the introduction to this section, care must be taken to ensure that the orthogonality conditions are compatible with equilibrium in the model. While it is probably not possible to choose a universally applicable set of orthogonality conditions, the following conditions have the virtues of being both reasonably general and compatible with equilibrium conditions in the model. The methods used to develop identification and estimation procedures with these conditions can also be applied if a priori reasoning or specification tests suggest that alternative orthogonality conditions are more appropriate for a particular application.

It will be assumed that the vector of measurement errors $[\boldsymbol{\omega}', \boldsymbol{\delta}_1', \boldsymbol{\delta}_2', \zeta]$ has zero mean and is uncorrelated with the true values of all variables. The measurement errors may all be mutually correlated. However, restrictions on the covariances among these errors can contribute to identification and estimation, as will be indicated in Section III*B*. It will be assumed that unmeasured product characteristics, $\tilde{\mathbf{z}}$, have zero mean and are uncorrelated with all measured agent characteristics. Unmeasured agent characteristics $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ will be assumed to have zero mean and to be uncorrelated with measured agent characteristics but correlated with both measured and unmeasured product characteristics.[11]

The unobserved variables and measurement errors constitute the error terms in the model. When (11) and (12) are substituted into (9) and (10), the error terms in the demand system must have full rank as must the error terms in the supply system. This implies (details available on request)

$$m_p + n_{ip} + \tilde{n}_i \geqslant m, \quad i = 1, 2. \tag{13}$$

When both demand and supply are endogenous, the error terms must also satisfy the equilibrium condition (analogous to [8]) that is obtained by solving the demand and supply equations for $\mathbf{z}$ and equating the results. This condition and the requirement that the covariance matrix of errors have full rank imply (details available on request)

$$m_p + n_{1p} + n_{2p} + \min[m, \tilde{n}_1, \tilde{n}_2] \geqslant 2m. \tag{14}$$

Equations (13) and (14) limit the simplifying assumptions that can be imposed when specifying the structure of unobservables in the model. When supply or demand is exogenous, (14) does not apply, and considerable flexibility is permitted in specifying unobservables. For example, when supply is exogenous, residuals in the demand equations could be due only to errors in measuring product characteristics, only to errors in measuring agent characteristics, only to unobserved agent characteristics, or to a combination of these. Less flexibility is permitted when both supply and demand are endogenous. Since $m_p \leqslant m$, equation (14) rules out specifications in which the only source of error is in measuring product characteristics. Similarly, (14) rules out specifications in which errors arise only from unobserved characteristics of agents and products: at least m variables must be measured with error!

[11] These orthogonality conditions do not violate any equilibrium conditions in the model. The strongest of these conditions are those regarding $\tilde{\mathbf{z}}$. With the assumptions regarding $\tilde{\mathbf{z}}$, parameters of interest can be identified and estimated without estimating the coefficients of $\tilde{\mathbf{z}}$ in the hedonic price, demand, and supply equations. Under weaker assumptions, it may be necessary to estimate the coefficients of $\tilde{\mathbf{z}}$ in the hedonic price function and the demand and supply functions. For this one must estimate the implicit prices of unobserved variables as well as the demand for and supply of those variables. This would require that considerable additional structure be placed on the model and would place substantial demands on the data.

The hedonic price function can be expressed in terms of measured variables and random components by substituting (11a) and (12) into (9). When this is done, the error term u in the resulting equation impounds unmeasured product characteristics and errors in measuring product price and product characteristics:

$$u = u(\zeta, \boldsymbol{\omega}, \tilde{\mathbf{z}}). \tag{15}$$

If there are product characteristics $\mathbf{z}_p$ that are measured with error, then those characteristics will be correlated with $\boldsymbol{\omega}$ and hence with u. For specifications in which such correlations between elements of $\mathbf{z}$ and the error term u are present, these correlations must be taken into account in identification and estimation of the model. In particular, if there are product characteristics that are measured with error or unmeasured product characteristics, the hedonic price function cannot be consistently estimated by ordinary least squares.

B. *Identification*

Two sets of results are summarized in this subsection. The first set is conditions for *unique* identification of the coefficients of the model: the coefficients of the hedonic price functions ($\boldsymbol{\psi}_k$, $\boldsymbol{\Pi}_k$) for $k = 1, \ldots, K$ and the demand ($\mathbf{A}_1$, $\mathbf{H}_1$) and supply ($\mathbf{A}_2$, $\mathbf{H}_2$) equations.[12] The second set is conditions for *local* identification of all parameters of the model: the coefficients as well as the covariance structure of the unobservables and measurement errors.[13]

The sufficient conditions for both unique and local identification are in the form of rank conditions on some matrix. The rank conditions determine unique identification of the parameters if the elements of the relevant matrices do not include functions of the true values of the parameters; otherwise the conditions determine local identification. A detailed analysis is presented in Epple (1985). Selected results are discussed below.

When the conditions in part A of table 2 are satisfied, the parameters of the hedonic price function are identified in that equation alone.[14] There are m_p product characteristics measured with error. The identification conditions in table 2, part A, confirm that one

[12] When some or all product characteristics are measured with error, the constant term, γ_k, may not be identified, and it may not be consistently estimated by the strategies discussed below. This is normally not of concern since the value of the constant term in the hedonic price equation is typically not of interest.

[13] See Fisher (1966) for a discussion of both unique and local identification. Geraci (1977) provides an illuminating discussion of local identification.

[14] More precisely, the equation will be identified if the rank conditions underlying the order conditions in table 2, pt. A, are satisfied. A similar qualifier holds for the entire discussion in this subsection.

TABLE 2

A. Order Conditions for Separate Identification of the Hedonic Price Equation When $\tilde{m} = 0$*

Endogenous Demand	Endogenous Supply	$E(\boldsymbol{\omega\delta}') = 0$ and $E(\boldsymbol{\zeta\delta}') = 0$	Order Conditions
Yes	No	No	$n_{1o} \geqslant m_p + 1$
Yes	No	Yes	$n_1 \geqslant m_p + 1$
Yes	Yes	No	$n_o \geqslant m_p + 2$
Yes	Yes	Yes	$n \geqslant m_p + 2$

B. Order Conditions for Unique Identification of Demand When the Price Function Is Identified†

Endogenous Supply	$\tilde{n}_1 = 0$ and $\tilde{m} = 0$	Order Conditions
No	Yes	$K(m_o + n_{1o}) \geqslant m + n_1$
No	No	$Kn_{1o} \geqslant m + n_1$
Yes	Yes	$K(m_o + n_o - 1) \geqslant m + n_1$
Yes	No	$K(n_o - 1) \geqslant m + n_1$

* If $\tilde{m} \neq 0$, replace m_p with m in the order conditions.
† To obtain comparable conditions for supply, change the first column heading to Endogenous Demand and replace subscript 1 with 2 in the last column.

instrument other than the constant is required for each variable measured with error. The number of agent characteristics available as instruments depends on whether errors in measuring product characteristics are correlated with errors in measuring price and agent characteristics (the covariances $E(\boldsymbol{\zeta\delta}_i')$ and $E(\boldsymbol{\omega\delta}_i')$) and on whether both supply and demand are endogenous.[15]

When the parameters of the hedonic price equation are identified in that equation alone, conditions for identification of the demand (supply) model are determined by investigating whether a representative demand (supply) equation is identified. The identification conditions reported in part B of table 2 require that the number of instruments per market multiplied by the number of markets be no less than the number of parameters in a representative demand (supply) equation. These results assume that no exclusion restrictions have been employed when specifying the demand (supply) system.[16] When both demand and supply are endogenous, there may be sufficient instruments to uniquely identify a typical demand (supply) equation with data for a single market without employing exclusion restric-

[15] Other cases are possible. If any elements of $\boldsymbol{\delta}_i$ are uncorrelated with ζ and $\boldsymbol{\omega}$, the corresponding elements of $\mathbf{x}_{ip}$ may be used as instruments when estimating the price function.

[16] The effect of imposing exclusion restrictions is not reported in the table but is easily established. The number of instruments times the number of markets must be no less than the number of parameters in the equation being estimated.

tions. When supply (demand) is exogenous, data for more than one market will be required to uniquely identify a typical demand (supply) equation if no exclusion restrictions are imposed.

When the hedonic price equation parameters are not separately identified, unique identification must be studied in the context of the entire system of equations (Epple 1985). To conserve space, results for these cases are not discussed here.

If conditions for unique identification are not satisfied, it is possible that conditions for local identification will be. This may occur because the covariance matrix of measurement errors is assumed to be the same across markets—a natural restriction if the data are collected in a similar way in each market. For example, suppose that supply is exogenous and that there are no unobserved characteristics ($\tilde{m} = 0, \tilde{n}_1 = 0$), no errors in measuring product characteristics ($m_p = 0$), and errors in measuring m demander characteristics ($n_{1p} = m$). Then the condition for local identification (Epple 1985) requires $(K - 1)(m + n_1) \geqslant m$ whereas the comparable condition for unique identification (table 2) requires $K(m + n_{1o}) \geqslant m + n_1$. Since $n_1 = n_{1p} + n_{1o}$, a larger number of markets, K, will be required for unique than for local identification.

C. *Estimation*

When the conditions in table 2 are satisfied, instrumental variable estimation procedures are available.[17] In applying these procedures, it is convenient to eliminate the starred variables from the model by substituting from (11) and (12) into (9) and (10). The error terms in the resulting equations are then a composite of measurement errors and unmeasured variables.[18] For each equation in each market, variables available as instruments are all variables in the market that are uncorrelated with the composite error term for that market.[19] This determination can be made from the orthogonality conditions enumerated in Section IIIA. Squares and cross-products of variables

[17] I thank Raymond Palmquist for suggesting that instrumental variable estimation procedures would sometimes be available. Bartik (this issue) also proposes the use of instrumental variables, and he provides an application.

[18] Care must be taken to ensure that the model to be estimated is one in which the covariance matrix of residuals has full rank. That is, eq. (13) is satisfied, and if both demand and supply are endogenous, eq. (14) is also satisfied. Recall that when both supply and demand are exogenous, these conditions rule out use of all characteristics of both demanders and suppliers as instruments for both demand and supply.

[19] If additional orthogonality conditions are imposed (e.g., by assuming that errors in measuring agent characteristics are uncorrelated with errors in measuring product characteristics, $E(\boldsymbol{\omega\delta}_i') = 0$), then additional instruments may be available. Further details concerning instrumental variable estimation procedures are available on request.

that are uncorrelated with the error term in the hedonic price equation can also be used as instruments for estimating that equation.

When the conditions for unique identification are not satisfied but those for local identification are, an estimation procedure must be employed that takes account of the restrictions across markets on the covariance matrices of unobservables. An obvious estimation strategy is to estimate the parameters of the entire model by the method of maximum likelihood. For special cases, the model to be estimated will be relatively simple. Perhaps the most common special case is that in which the distribution of products supplied is taken to be exogenous—a common assumption, for example, in hedonic studies of housing markets. If, in addition, the only sources of error arise from errors in measuring product price and product characteristics, the likelihood function assumes a relatively simple form. In specifying special cases, the restrictions in (13) and (14) on the dimensionality of unobservables should be kept in mind.

For some specifications, estimation of the entire model by maximum likelihood will be cumbersome because the product characteristics enter nonlinearly in the hedonic price function. The following two-step procedure is an attractive alternative. While the hedonic price function is nonlinear, equations (10)–(12) are a linear structural equation model. A two-step estimation procedure is first to estimate the parameters of the hedonic price function separately for each market by instrumental variables using $[\mathbf{z}_o', \mathbf{x}_{1o}', \mathbf{x}_{2o}']$ and the squares and cross-products of these variables as instruments. Then use these consistent estimates of $\boldsymbol{\psi}$ and $\boldsymbol{\Pi}$ as constraints and estimate the model in (10)–(12) simultaneously for all regions using the iterative procedures implemented by Jöreskog and Sörbom (1984).[20] This procedure will yield consistent estimates for all parameters.[21]

IV. Estimation When Demand and Supply Functions Are Nonlinear in Product Characteristics

Not surprisingly, the same issues that are encountered in the model in Section III arise in specification of nonlinear models. For example, when both supply and demand are endogenous, a condition analo-

[20] The LISREL VI program of Jöreskog and Sörbom (1984) does not permit imposition of parameter restrictions that arise in some specifications of the model of this section. In such instances, the Hotztran program developed by Avery and Hotz (1985) may be used.

[21] Standard error estimates must be corrected to account for the fact that estimated rather than actual values of the hedonic price function parameters are used in the second step of the estimation process.

gous to (8) will also apply in nonlinear models. Sources of error in nonlinear models may, in principle, be the same as those specified for the model in Section III. The appropriate estimation methods will, of course, depend on which specification is chosen. When variables are measured with error, standard instrumental variable procedures may not yield consistent estimates (Amemiya 1985).

In applying the strategy of specifying potential sources of error and choosing orthogonality conditions consistent with that specification, one is assuming that the hedonic model of Section II is an appropriate characterization of the market under study. An alternative characterization is provided by Heckman and Sedlacek (1984) in their two-sector model of the labor market in which job tasks and worker skills are differentiated. While theirs is a model of a market for differentiated products, the hedonic structure does not fit their model because they assume that workers' skills cannot be unbundled and priced separately (i.e., a given worker cannot sell some of his or her skill attributes in the manufacturing sector and others in the nonmanufacturing sector). When product attributes cannot be separately priced and there are multiple sectors in which the product may potentially be sold, Heckman and Sedlacek show that selection bias issues arise. This suggests the possibility that selection bias problems may also arise when hedonic models are estimated with contemporaneous data from separate geographic markets among which trade occurs. This is an important area for future research.

V. A Review of Selected Applications

Witte et al. (1979) use Rosen's procedure to estimate demand and supply functions for housing characteristics. They introduced the functional specification studied in Section III of this paper and the strategy of using multiple markets to achieve identification.[22] Witte et al. use three-stage least squares to estimate the entire set of equations with $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ as instruments for product characteristics. As explained in Section III and illustrated in equation (8), the assumption that both $\mathbf{x}_{1k}$ and $\mathbf{x}_{2k}$ are uncorrelated with the residuals in both (6d) and (6s) is untenable. As a result, the estimation procedure of Witte et al. will not yield consistent parameter estimates.

Harrison and Rubinfeld (1978) apply Rosen's procedure to estimate the demand for clean air. Their intent is to provide a prototype procedure for evaluating the benefits of air quality improvement.

[22] Witte et al. do not include income net of product price as a right-hand-side variable in their demand equations. As explained in Sec. II, $y - p(\mathbf{z})$, not y alone, should generally be included on the right-hand side of the demand equations.

While certain aspects of their empirical methodology are criticized here, the strategy they propose is both ingenious and useful. They first estimate a hedonic housing price function using housing and neighborhood characteristics, accessibility, and pollution as their vector of $\mathbf{z}$ variables. The derivative of this equation with respect to the pollution variable is then regressed against the pollution level and income to estimate the demand for pollution reduction. Identification is obtained by exclusion restrictions in the demand equation coupled with nonlinear restrictions imposed through the choice of functional forms for the hedonic price function and demand function. As explained in Sections II and III and illustrated in figure 1, ordinary least squares estimation of hedonic demand equations will not yield consistent parameter estimates.

Two recent applications (Bartik, this issue; Palmquist 1984) recognize the identification and simultaneity problems inherent in estimation of hedonic models and employ instrumental variables with more than one market to address these problems. Bartik uses data from the U.S. Department of Housing and Urban Development's Experimental Housing Allowance Program. He points out the difficulties associated with using supplier characteristics as instruments. He argues that both supplier and demander characteristics other than income will be correlated with the error term in the demand equation. In contrast to the orthogonality conditions adopted in Section III of this paper, Bartik assumes that observed supplier and demander characteristics other than income will be correlated with unobserved demander characteristics. He also assumes that the distribution of unobserved demander characteristics does not vary across markets. Thus his treatment of observed and unobserved agent characteristics is qualitatively similar to the treatment of variables measured with error in Section III of this paper.

Palmquist (1984) estimates the demand for housing characteristics using data from multiple markets. He estimates the price equation in each market by ordinary least squares. He uses instrumental variables to estimate the demand equations with income and socioeconomic variables (y and $\mathbf{x}_1$ in the notation of Sec. II of this paper) as instruments for product characteristics and functions of product characteristics appearing on the right-hand side of the demand equations.

Palmquist's estimation procedure is appropriate for a special case of the model in Section III. The hedonic price equation will be estimable by ordinary least squares if there are no product characteristics measured with error ($m_p = 0$) and no unmeasured product characteristics ($\tilde{m} = 0$). Variables y and $\mathbf{x}_1$ are appropriate as instruments if there are no demander characteristics that are measured with error ($n_{1p} = 0$). Finally, for the covariance matrix of error terms in the demand equations to be nondegenerate, (13) must be satisfied for $i = 1$. With $m_p = 0$ and $n_{1p} = 0$, (13) requires $\tilde{n}_1 \geqslant m$; there must be at least m unobserved buyer characteristics. If these assumptions are satisfied, Palmquist's estimation procedure will yield consistent parameter estimates.[23]

VI. Summary and Conclusions

Equilibrium conditions in hedonic models impose restrictions on the relationships among measured variables and random components. Some seemingly natural specifications of the stochastic structure of hedonic models prove to be incompatible with these equilibrium conditions. By carefully specifying sources of error and orthogonality conditions, it is possible to obtain stochastic structures that are compatible with the equilibrium conditions and that permit identification and estimation of the parameters of the model. The requisite orthogonality conditions prove to be relatively strong. For these conditions to be satisfied in practice, one must measure a relatively exhaustive set of product, demander, and supplier characteristics. If important characteristics are unmeasured and they are correlated with measured characteristics, the coefficients on measured characteristics will be biased. This truism applies to all econometric models, but it has particular force in hedonic models. The equilibrium conditions imply functional relationships among the characteristics of demanders, suppliers, and products. This in turn reduces the likelihood that important excluded variables will be uncorrelated with included variables of the model.

In the specification of unobservables proposed in this paper, random components arise because of measurement error and/or unmeasured characteristics of demanders, suppliers, and products. Relatively standard assumptions were made regarding measurement errors. Somewhat stronger assumptions were made about unmeasured characteristics. These assumptions are consistent with the equilibrium structure of the model, but they can reasonably be satisfied only when unmeasured characteristics are truly idiosyncratic (e.g., in housing market studies such unmeasured variables might be color of wallpaper or carpeting but presumably not size of lot or number of rooms). An important problem for future research is to explore the potential of specification tests to investigate the appropriateness of alternative orthogonality conditions.

[23] It is interesting to note, however, that if supply were also endogenous, supply could not be treated symmetrically. With the conditions above and the additional restriction that $n_{2p} = 0$, (14) would not be satisfied.

The analysis here has focused on a static environment without uncertainty or asymmetric information. Extending hedonic analysis to dynamic environments in which information and uncertainty play a role is an important problem for future research.

The objective of this paper has been to exhibit conditions under which identification and estimation of hedonic equilibrium models are feasible. Because marginal prices are implicit rather than explicit, hedonic models raise identification and estimation issues beyond those normally confronted in simultaneous models. A great deal of care in specification and estimation is required if valid inferences are to be obtained from such models.

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