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Special Article - Tools for Experiment and Theory

HELAS and MadGraph/MadEvent with spin-2 particles

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Abstract Fortran subroutines to calculate helicity amplitudes with massive spin-2 particles (massive gravitons), which couple to the standard model particles via the energy momentum tensor, are added to the HELAS (HELicity Amplitude Subroutines) library. They are coded in such a way that arbitrary scattering amplitudes with one graviton production and its decays can be generated automatically by MadGraph and MadEvent, after slight modifications. All the codes have been tested carefully by making use of the invariance of the helicity amplitudes under the gauge and general coordinate transformations.

1 Introduction

The idea of extra space dimensions has attracted much attention in recent years, since it can give us a novel solution to the hierarchy problem, or an alternative explanation of the hierarchical difference between the Planck scale ($M_{\rm Pl} \sim 10^{19}$ GeV) and the electroweak scale ($m_W \sim 10^2$ GeV).

So far, there have been various extra dimension models, which can be divided into two major classes according to the geometry of the background space-time manifold. The first one includes the ADD (Arkani-Hamed, Dimopoulos, and Dvali) model [1–3] and its variants, which extend the dimension of the total space-time to $D = 4 + \delta$, with a factorizable metric and large size of the compact extra dimensions ($\gg 1/M_{\rm Pl}$). The second one includes the 5-dimensional RS (Randall and Sundrum) model [4, 5] and its variants, in which a warped metric is introduced along the 5-th dimension and the size of the extra dimension needs not to be much larger than the Planck length.

In both classes of extra dimension models, there appear Kaluza-Klein (KK) towers of massive spin-2 gravitons, which can interact with the standard model (SM) fields. The effective interaction Lagrangian is given by [6, 7]

$$\mathcal{L}_{\rm int} = -\frac{1}{\Lambda}\sum_{\vec{n}} T^{(\vec{n})\mu\nu}\mathcal{T}_{\mu\nu}, \tag{1}$$

where $T^{(\vec{n})\mu\nu}$ is the $\vec{n}$-th graviton KK modes, and Λ is the relevant coupling scale. In the ADD model we have

$$\Lambda = \overline{M}_{\rm Pl} \equiv M_{\rm Pl}/\sqrt{8\pi} \sim 2.4\times 10^{18}\ \text{GeV}, \tag{2}$$

where $\overline{M}_{\rm Pl}$ is the 4-dimensional reduced Planck scale, and in the RS model

$$\Lambda = e^{-kr_c\pi}\,\overline{M}_{\rm Pl} \tag{3}$$

is at the electroweak scale, where k is a scale of order of the Planck scale and r_c is the compactification radius.

In (1), $\mathcal{T}_{\mu\nu}$ is the energy-momentum tensor of the SM fields,

$$\begin{aligned}\mathcal{T}_{\mu\nu} &= \left(-\eta_{\mu\nu}\mathcal{L}_{\rm SM} + 2\frac{\delta\mathcal{L}_{\rm SM}}{\delta g^{\mu\nu}}\right)\bigg|_{g^{\mu\nu}=\eta^{\mu\nu}} \\ &= \mathcal{T}^H_{\mu\nu} + \mathcal{T}^F_{\mu\nu} + \mathcal{T}^g_{\mu\nu} + \mathcal{T}^\gamma_{\mu\nu} + \mathcal{T}^Z_{\mu\nu} + \mathcal{T}^W_{\mu\nu} + \cdots,\end{aligned} \tag{4}$$

where $g^{\mu\nu}$ is the metric and $\eta^{\mu\nu} = \text{diag}(1,-1,-1,-1)$ is the Minkowski value, and each energy-momentum tensor is:

$$\begin{aligned}\mathcal{T}^H_{\mu\nu} = -\eta_{\mu\nu}\bigg[&\frac{1}{2}\partial^\rho H\partial_\rho H - \frac{m_H^2}{2}H^2 - \frac{g_W m_H^2}{4m_W}H^3 \\ &- \frac{g_W m_H^2}{32 m_W^2}H^4 - \frac{g_W m_F}{2m_W}\overline{\psi}\psi H \\ &+ \frac{g_Z m_Z}{2}Z_\mu Z^\mu H + \frac{g_Z^2}{8}Z_\mu Z^\mu H^2\end{aligned}$$

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$$
\begin{aligned}
&+ g_W m_W W^+_\mu W^{-\mu} H + \frac{g_W^2}{4} W^+_\mu W^{-\mu} H^2 \Big] \\
&+ \partial_\mu H \partial_\nu H + g_Z m_Z Z_\mu Z_\nu H + \frac{g_Z^2}{4} Z_\mu Z_\nu H^2 \\
&+ \Big[g_W m_W W^+_\mu W^-_\nu H + \frac{g_W^2}{4} W^+_\mu W^-_\nu H^2 \\
&+ (\mu \leftrightarrow \nu) \Big], \qquad (5a)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^F_{\mu\nu} = {}& -\eta_{\mu\nu} \Big[\overline{\psi}(i\gamma^\rho D_\rho - m_F)\psi - \frac{1}{2}\partial^\rho(\overline{\psi} i \gamma_\rho \psi) \\
& - \Big(\frac{g_W}{\sqrt{2}} V_{ij} \overline{\psi}_{u_i} \gamma^\rho P_L \psi_{d_j} W^+_\rho \\
& + \frac{g_W}{\sqrt{2}} U_{ij} \overline{\psi}_{l_i} \gamma^\rho P_L \psi_{\nu_j} W^-_\rho + \text{h.c.} \Big) \Big] \\
& + \Big[\frac{1}{2} \overline{\psi} i \gamma_\mu D_\nu \psi - \frac{1}{4} \partial_\mu (\overline{\psi} i \gamma_\nu \psi) + (\mu \leftrightarrow \nu) \Big] \\
& + \Big[-\frac{g_W}{\sqrt{2}} V_{ij} \overline{\psi}_{u_i} \gamma_\mu P_L \psi_{d_j} W^+_\nu \\
& - \frac{g_W}{\sqrt{2}} U_{ij} \overline{\psi}_{l_i} \gamma_\mu P_L \psi_{\nu_j} W^-_\nu + \text{h.c.} + (\mu \leftrightarrow \nu) \Big], \qquad (5b)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^g_{\mu\nu} = {}& -\eta_{\mu\nu} \Big[-\frac{1}{4} F^{a,\rho\sigma} F^a_{\rho\sigma} + \partial^\rho \partial^\sigma A^a_\sigma A^a_\rho - \frac{1}{2} (\partial^\rho A^a_\rho)^2 \Big] \\
& - F^{a,\rho}_\mu F^a_{\nu\rho} + \partial_\mu \partial^\rho A^a_\rho A^a_\nu + \partial_\nu \partial^\rho A^a_\rho A^a_\mu, \qquad (5c)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^\gamma_{\mu\nu} = {}& -\eta_{\mu\nu} \Big[-\frac{1}{4} F^{\rho\sigma} F_{\rho\sigma} + \partial^\rho \partial^\sigma A_\sigma A_\rho - \frac{1}{2} (\partial^\rho A_\rho)^2 \Big] \\
& - F_\mu^{\ \rho} F_{\nu\rho} + \partial_\mu \partial^\rho A_\rho A_\nu + \partial_\nu \partial^\rho A_\rho A_\mu, \qquad (5d)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^Z_{\mu\nu} = {}& -\eta_{\mu\nu} \Big[-\frac{1}{4} Z^{\rho\sigma} Z_{\rho\sigma} + \frac{m_Z^2}{2} Z^\rho Z_\rho \Big] \\
& - Z_\mu^{\ \rho} Z_{\nu\rho} + m_Z^2 Z_\mu Z_\nu, \qquad (5e)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^W_{\mu\nu} = {}& -\eta_{\mu\nu} \Big[-\frac{1}{2} W^{+\rho\sigma} W^-_{\rho\sigma} + m_W^2 W^{+\rho} W^-_\rho \Big] \\
& - \big[W^{+\rho}_\mu W^-_{\nu\rho} - m_W^2 W^+_\mu W^-_\nu + (\mu \leftrightarrow \nu) \big], \qquad (5f)
\end{aligned}
$$

with $e = g_W \sin\theta_W = g_Z \sin\theta_W \cos\theta_W$ and the projection operator $P_L = \frac{1}{2}(1-\gamma_5)$. Here, the covariant derivative is

$$
\begin{aligned}
D_\mu \equiv {}& \partial_\mu + i g_s T^a A^a_\mu + i e Q_f A_\mu \\
& + i g_Z (T^3 P_L - Q_f \sin^2\theta_W) Z_\mu. \qquad (6)
\end{aligned}
$$

Note that the derivative couplings of the W bosons are written explicitly in (5b). Each field-strength tensor for the gauge bosons is

$$
\begin{aligned}
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + ie\big[W^+_\mu W^-_\nu - (\mu \leftrightarrow \nu)\big], \\
Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu + i g_W \cos\theta_W \\
&\quad \times \big[W^+_\mu W^-_\nu - (\mu \leftrightarrow \nu)\big], \qquad (7) \\
W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\
&\quad \mp i g_W \big[\sin\theta_W W^\pm_\mu A_\nu + \cos\theta_W W^\pm_\mu Z_\nu \\
&\quad - (\mu \leftrightarrow \nu)\big].
\end{aligned}
$$

Notice that as in the standard HELAS package [8], we use the unitary gauge for the massive vector-boson propagators and the Feynman gauge for the massless ones.

In this paper, we present new HELAS subroutines [8] for the massive gravitons and their interactions based on the effective Lagrangian of (1), and implement them into MadGraph/MadEvent (MG/ME) [9–11].[1]

The paper is organized as follows: In Sect. 2 we give the new HELAS subroutines. In Sect. 3 we describe how to implement amplitudes with a massive spin-2 graviton into MG/ME. In Sect. 4 we give sample numerical results. Section 5 contains a brief conclusion.

2 HELAS subroutines for spin-2 particles

In this section, we list the contents of all the new HELAS subroutines that are needed to evaluate massive spin-2 graviton production at hadron colliders in association with quark and gluon jets, and its decays into a pair of all the SM particles, or into arbitrary numbers of quarks and gluons.

To begin with, in Sect. 2.1 the subroutine to compute the external lines for a spin-2 tensor particle is presented. Next, in Sects. 2.2 to 2.7, the subroutines to compute the interactions of the tensor boson with the SM particles are explained. The new vertex subroutines are listed in Table 1. Those subroutines which we do not present in this paper are FFST, SSST and SSSST type vertices for the interactions with scalars; VVVT and VVVVT for the interactions with the electroweak gauge bosons. These contribute *e.g.* to the graviton decays into three or more weakly interacting particles. We present the effective Lagrangian of (5a) to (5f) for completeness sake. Finally, we briefly mention how we test our new subroutines in Sect. 2.8.

[1] The Fortran code for simulations of the massive gravitons is available on the web [12].

Table 1 List of the new vertex subroutines in `HELAS` system

Vertex	Inputs	Output	Subroutine
SST	SST	Amplitude	`SSTXXX`
	ST	S	`HSTXXX`
	SS	T	`USSXXX`
FFT	FFT	Amplitude	`IOTXXX`
	FT	F	`FTIXXX`, `FTOXXX`
	FF	T	`UIOXXX`
VVT	VVT	Amplitude	`VVTXXX`
	VT	V	`JVTXXX`
	VV	T	`UVVXXX`
FFVT	FFVT	Amplitude	`IOVTXX`
	FVT	F	`FVTIXX`, `FVTOXX`
	FFT	V	`JIOTXX`
	FFV	T	`UIOVXX`
VVVT	VVVT	Amplitude	`VVVTXX`
	VVT	V	`JVVTXX`
	VVV	T	`UVVVXX`
VVVVT	GGGGT	Amplitude	`GGGGTX`
	GGGT	G	`JGGGTX`
	GGGG	T	`UGGGGX`

2.1 Tensor wavefunction

2.1.1 `TXXXXX`

This subroutine computes the spin-2 `T`ensor particle wavefunction; namely, $\epsilon^{\mu\nu}(p,\lambda)$ and $\epsilon^{\mu\nu}(p,\lambda)^*$, in terms of its four-momentum p and helicity λ, and should be called as

```
CALL TXXXXX(P, TMASS, NHEL, NST , TC).
```

The input `P(0:3)` is a real four-dimensional array which contains the four-momentum p^μ of the tensor boson, `TMASS` is its mass, `NHEL` $(=\pm 2, \pm 1, 0)$ specifies its helicity λ, and `NST` specifies whether the boson is in the final state (`NST = 1`) or in the initial state (`NST = -1`). The output `TC(18)` is a complex 18-dimensional array, among which the first 16 components contain the wavefunction as

$$\mathtt{TC}(4\mu+\nu+1) = \mathtt{T}(\mu+1,\nu+1), \tag{8}$$

namely

$$\mathtt{TC}(\ 1) = \mathtt{T}(1,1),$$

$$\mathtt{TC}(\ 2) = \mathtt{T}(1,2),$$

$$\mathtt{TC}(\ 3) = \mathtt{T}(1,3),$$

$$\mathtt{TC}(\ 4) = \mathtt{T}(1,4),$$

$$\vdots$$

$$\mathtt{TC}(16) = \mathtt{T}(4,4),$$

where

$$\mathtt{T}(\mu+1,\nu+1) = \begin{cases} \epsilon^{\mu\nu}(p,\lambda)^* & \text{for } \mathtt{NST}=1, \\ \epsilon^{\mu\nu}(p,\lambda) & \text{for } \mathtt{NST}=-1, \end{cases} \tag{9}$$

and the last two contain the flowing-out four-momentum

$$(\mathtt{TC}(17), \mathtt{TC}(18)) = \mathtt{NST}(\mathtt{P}(0)+i\,\mathtt{P}(3), \mathtt{P}(1)+i\,\mathtt{P}(2)). \tag{10}$$

The helicity states of the tensor boson can be expressed as

$$\begin{aligned} &\epsilon^{\mu\nu}(p,\pm 2) = \epsilon^{\mu}(p,\pm)\epsilon^{\nu}(p,\pm), \\ &\epsilon^{\mu\nu}(p,\pm 1) = \frac{1}{\sqrt{2}}\big[\epsilon^{\mu}(p,\pm)\epsilon^{\nu}(p,0) + \epsilon^{\mu}(p,0)\epsilon^{\nu}(p,\pm)\big], \\ &\epsilon^{\mu\nu}(p,0) = \frac{1}{\sqrt{6}}\big[\epsilon^{\mu}(p,+)\epsilon^{\nu}(p,-) + \epsilon^{\mu}(p,-)\epsilon^{\nu}(p,+) \\ &\qquad\qquad + 2\epsilon^{\mu}(p,0)\epsilon^{\nu}(p,0)\big], \end{aligned} \tag{11}$$

by using the vector boson wavefunctions $\epsilon^\mu(p,\lambda)$ that obey the relation

$$J_-\epsilon^{\mu}(p,\lambda) = \epsilon^{\mu}(p,\lambda-1), \tag{12}$$

where $J_- = J_x - iJ_y$ is the J_z lowering operator. The spin-1 vector wavefunction in the `HELAS` convention [8] satisfies this relation, and hence we simply use the `HELAS` code to obtain the tensor wavefunction. These tensor wavefunctions are traceless, transverse, orthogonal, and symmetric,

$$\begin{aligned} &\epsilon(p,\lambda)^{\mu}_{\ \mu} = 0, \qquad p_\mu\epsilon^{\mu\nu}(p,\lambda) = p_\nu\epsilon^{\mu\nu}(p,\lambda) = 0, \\ &\epsilon^{\mu\nu}(p,\lambda)\epsilon_{\mu\nu}(p,\lambda')^* = \delta_{\lambda\lambda'}, \qquad \epsilon^{\mu\nu}(p,\lambda) = \epsilon^{\nu\mu}(p,\lambda), \end{aligned} \tag{13}$$

and the completeness relation is

$$\sum_{\lambda=-2}^{+2} \epsilon^{\mu\nu}(p,\lambda)\epsilon^{\alpha\beta}(p,\lambda)^* = B^{\mu\nu,\alpha\beta}(p) \tag{14}$$

with

$$\begin{aligned} &B^{\mu\nu,\alpha\beta}(p) \\ &\quad = \frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}) \\ &\qquad - \frac{1}{2m_T^2}(\eta^{\mu\alpha}p^{\nu}p^{\beta} + \eta^{\nu\beta}p^{\mu}p^{\alpha} + \eta^{\mu\beta}p^{\nu}p^{\alpha} \\ &\qquad + \eta^{\nu\alpha}p^{\mu}p^{\beta}) \\ &\qquad + \frac{1}{6}\left(\eta^{\mu\nu} + \frac{2}{m_T^2}p^{\mu}p^{\nu}\right)\left(\eta^{\alpha\beta} + \frac{2}{m_T^2}p^{\alpha}p^{\beta}\right). \end{aligned} \tag{15}$$

2.2 SST vertex

The SST vertices are obtained from the interaction Lagrangian among the tensor and two scalar bosons:

$$\mathcal{L}_{\mathtt{SST}} = \mathtt{GT} T^{\mu\nu *}\big[-\eta_{\mu\nu}\big\{(\partial^\rho S^*)^*(\partial_\rho S^*) - m_S^2 S S^*\big\} + (\partial_\mu S^*)^*(\partial_\nu S^*) + (\partial_\nu S^*)^*(\partial_\mu S^*)\big] \tag{16a}$$

for the complex scalar field, or

$$\mathcal{L}_{\mathtt{SST}} = \mathtt{GT} T^{\mu\nu *}\left[-\eta_{\mu\nu}\left\{\frac{1}{2}\partial^\rho S^*\partial_\rho S^* - \frac{m_S^2}{2} S^* S^*\right\} + \partial_\mu S^*\partial_\nu S^*\right] \tag{16b}$$

for the real scalar field. Here,

$$\mathtt{GT} = \mathtt{GTS} = -1/\Lambda \tag{17}$$

is the coupling constant.

2.2.1 SSTXXX

This subroutine computes the amplitude of the SST vertex from two Scalar boson wavefunctions and a Tensor boson wavefunction, and should be called as

```
CALL SSTXXX(S1, S2, TC, GT, SMASS , VERTEX).
```

The inputs S1(3) and S2(3) are complex three-dimensional arrays which contain the wavefunctions of the Scalar bosons, S1(1) and S2(1), and their four-momenta as

$$p_1^\mu = (\Re e \mathtt{S1(2)}, \Re e \mathtt{S1(3)}, \Im m \mathtt{S1(3)}, \Im m \mathtt{S1(2)}),$$

$$p_2^\mu = (\Re e \mathtt{S2(2)}, \Re e \mathtt{S2(3)}, \Im m \mathtt{S2(3)}, \Im m \mathtt{S2(2)}).$$

The input TC(18) is a complex 18-dimensional array which contains the wavefunction of the Tensor boson, and its four-momentum; see the TXXXXX subroutine in Sect. 2.1.1, GT is the coupling constant in (17) in units of GeV^{-1}, and SMASS is the scalar boson mass m_S in GeV units. Note that all the coupling constants in the latest HELAS library are defined as a complex number except for the 3-point and 4-point vector boson couplings. The output VERTEX is a complex number in units of GeV:

$$\mathtt{VERTEX} = \mathtt{GT} T^{\mu\nu}(m_S^2\eta_{\mu\nu} - C_{\mu\nu,\rho\sigma} p_1^\rho p_2^\sigma)\mathtt{S1(1)S2(1)}, \tag{18}$$

where we used the notation

$$T^{\mu\nu} = \mathtt{T}(\mu+1, \nu+1) = \mathtt{TC}(4\mu+\nu+1) \tag{19}$$

in (8) and (9), and

$$C_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}. \tag{20}$$

2.2.2 HSTXXX

This subroutine computes an off-shell scalar current H made from the interactions of a Scalar boson and a Tensor boson by the SST vertex, and should be called as

```
CALL HSTXXX(TC, SC, GT, SMASS, SWIDTH , HST).
```

The inputs TC(18) and SC(3) are the wavefunctions and momenta of the Tensor and Scalar bosons, respectively, and SWIDTH is the scalar boson width Γ_S. The output HST(3) gives the off-shell scalar current multiplied by the scalar boson propagator, which is expressed as a complex three-dimensional array:

$$\mathtt{HST(1)} = i\mathtt{GT}\frac{i}{q^2 - m_S^2 + i m_S \Gamma_S} T^{\mu\nu} \times (m_S^2\eta_{\mu\nu} + C_{\mu\nu,\rho\sigma} p^\rho q^\sigma)\mathtt{SC(1)}, \tag{21}$$

and

$$\mathtt{HST(2)} = \mathtt{TC(17)} + \mathtt{SC(2)}, \tag{22}$$

$$\mathtt{HST(3)} = \mathtt{TC(18)} + \mathtt{SC(3)}. \tag{23}$$

Here the momenta p and q are

$$p^\mu = (\Re e \mathtt{SC(2)}, \Re e \mathtt{SC(3)}, \Im m \mathtt{SC(3)}, \Im m \mathtt{SC(2)}),$$

$$q^\mu = (\Re e \mathtt{HST(2)}, \Re e \mathtt{HST(3)}, \Im m \mathtt{HST(3)}, \Im m \mathtt{HST(2)}).$$

2.2.3 USSXXX

This subroutine computes an off-shell tensor current U made from two flowing-out Scalar bosons by the SST vertex, and should be called as

```
CALL USSXXX(S1, S2, GT, SMASS, TMASS, TWIDTH ,
            USS).
```

The inputs TMASS and TWIDTH are the tensor boson mass and width, m_T and Γ_T. The output USS(18) gives the off-shell tensor current multiplied by the tensor boson propagator, which is expressed as a complex 18-dimensional array:

$$T^{\alpha\beta} = i\mathtt{GT}\frac{i B^{\mu\nu,\alpha\beta}}{q^2 - m_T^2 + i m_T \Gamma_T} \times (m_S^2\eta_{\mu\nu} - C_{\mu\nu,\rho\sigma} p_1^\rho p_2^\sigma)\mathtt{S1(1)S2(1)}, \tag{24}$$

where we used the notation

$$T^{\alpha\beta} = \mathtt{T}(\alpha+1, \beta+1), \tag{25}$$

whose components are assigned into the first 16 component of USS as in (8), and

$$\texttt{USS(17)} = \texttt{S1(2)} + \texttt{S2(2)}, \tag{26}$$

$$\texttt{USS(18)} = \texttt{S1(3)} + \texttt{S2(3)}. \tag{27}$$

Here, p_1 and p_2 are the momenta of the outgoing scalars, and q is that of the off-shell tensor boson given in (26) and (27) as

$$q^\mu = (\Re e\texttt{USS(17)}, \Re e\texttt{USS(18)}, \Im m\texttt{USS(18)}, \\ \Im m\texttt{USS(17)}).$$

Although the effective Lagrangian of (1) does not dictates the off-shell behavior of the gravitons, we allow gravitons to propagate just once in the total amplitude where there are no external gravitons in the initial or final states. This is convenient when studying the correlated decays of the graviton production and its subsequent decays.

We may also note that the order of the GT couplings should be restricted to 1 when there is an external graviton, and 2 when a graviton is exchanged among the SM particles.

Before turning to the FFT vertex, it should be noticed here that the conventional factors of i in the vertices and those in the propagators are both included in the off-shell wavefunctions, such as (21) and (24) above, according to the HELAS convention. The HELAS amplitude, obtained by the vertices, such as (18), gives the contribution to the T matrix element without the factor of i. See more details in the HELAS manual [8].

2.3 FFT vertex

The FFT vertices are obtained from the interaction Lagrangian among the tensor boson and two fermions:

$$\mathcal{L}_{\texttt{FFT}} = 4\texttt{GT}T^{\mu\nu*}\bigg[-\eta_{\mu\nu}\bigg\{\bar f(i\gamma^\rho\partial_\rho - m_F)f \\ -\frac{1}{2}\partial^\rho(\bar f i\gamma_\rho f)\bigg\} \\ +\bigg\{\frac{1}{2}\bar f i\gamma_\mu\partial_\nu f - \frac{1}{4}\partial_\mu(\bar f i\gamma_\nu f) + (\mu\leftrightarrow\nu)\bigg\}\bigg], \tag{28}$$

where the coupling constant is

$$\texttt{GT} = \texttt{GTF} = -1/(4\Lambda). \tag{29}$$

2.3.1 IOTXXX

This subroutine computes the amplitude of the FFT vertex from a flowing-In fermion spinor, a flowing-Out fermion spinor and a Tensor boson wavefunction, and should be called as

CALL IOTXXX(FI, FO, TC, GT, FMASS , VERTEX).

The inputs FI(6) and FO(6) are complex six-dimensional arrays which contain the wavefunctions of the flowing-In and flowing-Out Fermions, and their four-momenta as

$$p_1^\mu = (\Re e\texttt{FI(5)}, \Re e\texttt{FI(6)}, \Im m\texttt{FI(6)}, \Im m\texttt{FI(5)}),$$
$$p_2^\mu = (\Re e\texttt{FO(5)}, \Re e\texttt{FO(6)}, \Im m\texttt{FO(6)}, \Im m\texttt{FO(5)}).$$

The input GT is the coupling constant in (29), and FMASS is the fermion mass m_F. What we compute here is

$$\texttt{VERTEX} = \texttt{GT(FO)}\big[T^{\mu\nu}\big\{-\eta_{\mu\nu}(\not p_1 + \not p_2 - 2m_F) \\ + \gamma_\mu(p_1+p_2)_\nu + (\mu\leftrightarrow\nu)\big\}\big]\texttt{(FI)}, \tag{30}$$

where we use the notation

$$\texttt{(FI)} = \begin{pmatrix}\texttt{FI(1)}\\ \texttt{FI(2)}\\ \texttt{FI(3)}\\ \texttt{FI(4)}\end{pmatrix}, \tag{31}$$

$$\texttt{(FO)} = (\texttt{FO(1)}, \texttt{FO(2)}, \texttt{FO(3)}, \texttt{FO(4)}). \tag{32}$$

2.3.2 FTIXXX

This subroutine computes an off-shell Fermion wavefunction made from the interactions of a Tensor boson and a flowing-In fermion by the FFT vertex, and should be called as

CALL FTIXXX(FI, TC, GT, FMASS, FWIDTH , FTI).

The output FTI(6) gives the off-shell fermion wavefunction multiplied by the fermion propagator, which is expressed as a complex six-dimensional array:

$$\texttt{(FTI)} = i\texttt{GT}\frac{i(\not q + m_F)}{q^2 - m_F^2 + im_F\Gamma_F}T^{\mu\nu} \\ \times\big[-\eta_{\mu\nu}(\not p + \not q - 2m_F) \\ + \gamma_\mu(p+q)_\nu + (\mu\leftrightarrow\nu)\big]\texttt{(FI)}, \tag{33}$$

where we use the notation

$$\texttt{(FTI)} = \begin{pmatrix}\texttt{FTI(1)}\\ \texttt{FTI(2)}\\ \texttt{FTI(3)}\\ \texttt{FTI(4)}\end{pmatrix}, \tag{34}$$

and

$$\texttt{FTI(5)} = \texttt{FI(5)} - \texttt{TC(17)}, \tag{35}$$

$$\texttt{FTI(6)} = \texttt{FI(6)} - \texttt{TC(18)}. \tag{36}$$

Here the momenta p and q are

$$p^\mu = (\Re e\mathtt{FI}(5), \Re e\mathtt{FI}(6), \Im m\mathtt{FI}(6), \Im m\mathtt{FI}(5)),$$

$$q^\mu = (\Re e\mathtt{FTI}(5), \Re e\mathtt{FTI}(6), \Im m\mathtt{FTI}(6), \Im m\mathtt{FTI}(5)).$$

2.3.3 FTOXXX

This subroutine computes an off-shell Fermion wavefunction made from the interactions of a Tensor boson and a flowing-Out fermion by the FFT vertex, and should be called as

CALL FTOXXX(FO, TC, GT, FMASS, FWIDTH , FTO).

The output FTO(6) is a complex six-dimensional array:

$$\begin{aligned}(\mathtt{FTO}) &= i\mathtt{GT}(\mathtt{FO})T^{\mu\nu} \\ &\quad\times \big[-\eta_{\mu\nu}(\not p + \not q - 2m_F) \\ &\quad + \gamma_\mu(p+q)_\nu + (\mu \leftrightarrow \nu)\big] \\ &\quad\times \frac{i(\not q + m_F)}{q^2 - m_F^2 + i m_F \Gamma_F},\end{aligned} \tag{37}$$

where we use the notation

$$(\mathtt{FTO}) = (\mathtt{FTO}(1), \mathtt{FTO}(2), \mathtt{FTO}(3), \mathtt{FTO}(4)), \tag{38}$$

and

$$\mathtt{FTO}(5) = \mathtt{FO}(5) + \mathtt{TC}(17), \tag{39}$$

$$\mathtt{FTO}(6) = \mathtt{FO}(6) + \mathtt{TC}(18). \tag{40}$$

Here the momenta p and q are

$$p^\mu = (\Re e\mathtt{FO}(5), \Re e\mathtt{FO}(6), \Im m\mathtt{FO}(6), \Im m\mathtt{FO}(5)),$$

$$q^\mu = (\Re e\mathtt{FTO}(5), \Re e\mathtt{FTO}(6), \Im m\mathtt{FTO}(6), \Im m\mathtt{FTO}(5)).$$

2.3.4 UIOXXX

This subroutine computes the bi-spinor tensor current U made from flowing-In and flowing-Out fermions by the FFT vertex, and should be called as

CALL UIOXXX(FI, FO, GT, FMASS, TMASS, TWIDTH ,
UIO).

The output UIO(18) is a complex 18-dimensional array:

$$\begin{aligned}T^{\alpha\beta} &= i\mathtt{GT}\frac{iB^{\mu\nu,\alpha\beta}}{q^2 - m_T^2 + i m_T \Gamma_T} \\ &\quad\times (\mathtt{FO})\big[-\eta_{\mu\nu}(\not p_1 + \not p_2 - 2m_F) \\ &\quad + \gamma_\mu(p_1+p_2)_\nu + (\mu \leftrightarrow \nu)\big](\mathtt{FI})\end{aligned} \tag{41}$$

for the first 16 components of UIO, and

$$\mathtt{UIO}(17) = -\mathtt{FI}(5) + \mathtt{FO}(5), \tag{42}$$

$$\mathtt{UIO}(18) = -\mathtt{FI}(6) + \mathtt{FO}(6). \tag{43}$$

Here, p_1 and p_2 are the momenta of the flowing-in and flowing-out fermions, respectively, and q is that of the tensor particle.

2.4 VVT vertex

The VVT vertices are obtained from the interaction Lagrangian among the tensor and two vector bosons:

$$\begin{aligned}\mathcal{L}_{\mathtt{VVT}} &= \mathtt{GT}T^{\mu\nu*}\bigg[\eta_{\mu\nu}\bigg\{\frac{1}{2}(\partial^\rho V^{\sigma*} - \partial^\sigma V^{\rho*})^*(\partial_\rho V_\sigma^* \\ &\quad - \partial_\sigma V_\rho^*) - m_V^2 V^\rho V_\rho^*\bigg\} \\ &\quad - \big\{(\partial_\mu V^{\rho*} - \partial^\rho V_\mu^*)^*(\partial_\nu V_\rho^* - \partial_\rho V_\nu^*) \\ &\quad + m_V^2 V_\mu V_\nu^* + (\mu \leftrightarrow \nu)\big\}\bigg]\end{aligned} \tag{44a}$$

for the complex vector bosons, or W bosons, and

$$\begin{aligned}\mathcal{L}_{\mathtt{VVT}} &= \mathtt{GT}T^{\mu\nu*}\bigg[\eta_{\mu\nu}\bigg\{\frac{1}{4}(\partial^\rho V^{\sigma*} - \partial^\sigma V^{\rho*})(\partial_\rho V_\sigma^* - \partial_\sigma V_\rho^*) \\ &\quad - \frac{m_V^2}{2} V^{\rho*} V_\rho^*\bigg\} \\ &\quad - (\partial_\mu V^{\rho*} - \partial^\rho V_\mu^*)(\partial_\nu V_\rho^* - \partial_\rho V_\nu^*) + m_V^2 V_\mu^* V_\nu^* \\ &\quad - \xi^{-1}\eta_{\mu\nu}\bigg(\partial^\rho\partial^\sigma V_\sigma^* V_\rho^* + \frac{1}{2}\partial^\rho V_\rho^* \partial^\sigma V_\sigma^*\bigg) \\ &\quad + \xi^{-1}(\partial_\mu\partial^\rho V_\rho^* V_\nu^* + \partial_\nu\partial^\rho V_\rho^* V_\mu^*)\bigg],\end{aligned} \tag{44b}$$

for the real ones, or gluons, photons, and Z bosons. Here the coupling constant is

$$\mathtt{GT} = \mathtt{GTV} = -1/\Lambda. \tag{45}$$

The ξ terms are the gauge-fixing terms, which vanish for massive vector bosons in the unitary gauge. For massless vector bosons we take $\xi = 1$ in the Feynman gauge.

2.4.1 VVTXXX

This subroutine computes the amplitude of the VVT vertex from two Vector boson polarization vectors and a Tensor wavefunction, and should be called as

CALL VVTXXX(V1, V2, TC, GT, VMASS , VERTEX).

The inputs V1(6) and V2(6) are complex six-dimensional arrays which contain the Vector boson wavefunctions, and their momenta as

$$p_1^\mu = (\Re e\texttt{V1(5)}, \Re e\texttt{V1(6)}, \Im m\texttt{V1(6)}, \Im m\texttt{V1(5)}),$$

$$p_2^\mu = (\Re e\texttt{V2(5)}, \Re e\texttt{V2(6)}, \Im m\texttt{V2(6)}, \Im m\texttt{V2(5)}).$$

The input GT is the coupling constant in (45), and VMASS is the vector boson mass m_V. What we compute here is

$$\begin{aligned}\texttt{VERTEX} = \texttt{GT}T^{\mu\nu}\big[&(m_V^2 + p_1 \cdot p_2)C_{\mu\nu,\rho\sigma} \\ &+ D_{\mu\nu,\rho\sigma}(p_1, p_2) \\ &+ \xi^{-1}E_{\mu\nu,\rho\sigma}(p_1, p_2)\big]V_1^\rho V_2^\sigma,\end{aligned} \tag{46}$$

where we use the notation

$$V_1^\mu = \texttt{V1}(\mu + 1), \tag{47}$$

$$V_2^\mu = \texttt{V2}(\mu + 1), \tag{48}$$

and

$$\begin{aligned}D_{\mu\nu,\rho\sigma}(p_1, p_2) &= \eta_{\mu\nu} p_{1\sigma} p_{2\rho} - \big[\eta_{\mu\sigma} p_{1\nu} p_{2\rho} + \eta_{\mu\rho} p_{1\sigma} p_{2\nu} \\ &\quad - \eta_{\rho\sigma} p_{1\mu} p_{2\nu} + (\mu \leftrightarrow \nu)\big],\end{aligned} \tag{49}$$

$$\begin{aligned}E_{\mu\nu,\rho\sigma}(p_1, p_2) &= \eta_{\mu\nu}(p_{1\rho}p_{1\sigma} + p_{2\rho}p_{2\sigma} + p_{1\rho}p_{2\sigma}) \\ &\quad - \big[\eta_{\nu\sigma} p_{1\mu} p_{1\rho} + \eta_{\nu\rho} p_{2\mu} p_{2\sigma} \\ &\quad + (\mu \leftrightarrow \nu)\big].\end{aligned} \tag{50}$$

2.4.2 JVTXXX

This subroutine computes an off-shell vector current J made from the interactions of a Vector boson and a Tensor boson by the VVT vertex, and should be called as

```
CALL JVTXXX(VC, TC, GT, VMASS, VWIDTH , JVT).
```

The input VC(6) is the wavefunction and momentum of the Vector boson. The output JVT(6) gives the off-shell vector current multiplied by the vector boson propagator, which is expressed as a complex six-dimensional array:

$$\begin{aligned}&\texttt{JVT}(\alpha + 1) \\ &\quad = i\texttt{GT}\frac{i}{q^2 - m_V^2 + im_V\Gamma_V}\left(-\eta^{\sigma\alpha} + \frac{q^\sigma q^\alpha}{m_V^2}\right) \\ &\qquad \times T^{\mu\nu}\big[(m_V^2 - p \cdot q)C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(p, -q)\big]V^\rho\end{aligned} \tag{51}$$

for the massive vector boson, or

$$\begin{aligned}\texttt{JVT}(\alpha + 1) = i\texttt{GT}\frac{-i}{q^2}\eta^{\sigma\alpha}T^{\mu\nu}\big[&-(p \cdot q)C_{\mu\nu,\rho\sigma} \\ &+ D_{\mu\nu,\rho\sigma}(p, -q) + E_{\mu\nu,\rho\sigma}(p, -q)\big]V^\rho\end{aligned} \tag{52}$$

for the massless vector boson, and

$$\texttt{JVT(5)} = \texttt{VC(5)} + \texttt{TC(17)}, \tag{53}$$

$$\texttt{JVT(6)} = \texttt{VC(6)} + \texttt{TC(18)}. \tag{54}$$

Here the momenta p and q are

$$p^\mu = (\Re e\texttt{VC(5)}, \Re e\texttt{VC(6)}, \Im m\texttt{VC(6)}, \Im m\texttt{VC(5)}),$$

$$q^\mu = (\Re e\texttt{JVT(5)}, \Re e\texttt{JVT(6)}, \Im m\texttt{JVT(6)}, \Im m\texttt{JVT(5)}).$$

2.4.3 UVVXXX

This subroutine computes an off-shell tensor current U made from two flowing-out Vector bosons by the VVT vertex, and should be called as

```
CALL UVVXXX(V1, V2, GT, VMASS, TMASS, TWIDTH ,
            UVV).
```

The output UVV(18) is a complex 18-dimensional array:

$$\begin{aligned}T^{\alpha\beta} = i\texttt{GT}&\frac{iB^{\mu\nu,\alpha\beta}}{q^2 - m_T^2 + im_T\Gamma_T} \\ &\times \big[(m_V^2 + p_1 \cdot p_2)C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(p_1, p_2) \\ &+ \xi^{-1}E_{\mu\nu,\rho\sigma}(p_1, p_2)\big]V_1^\rho V_2^\sigma\end{aligned} \tag{55}$$

for the first 16 components of UVV, and

$$\texttt{UVV(17)} = \texttt{V1(5)} + \texttt{V2(5)}, \tag{56}$$

$$\texttt{UVV(18)} = \texttt{V1(6)} + \texttt{V2(6)}. \tag{57}$$

Here, p_1 and p_2 are the momenta of the outgoing vector bosons, and q is that of the tensor boson.

2.5 FFVT vertex

The FFVT vertices are obtained from the interaction Lagrangian among the tensor boson, vector boson and two fermions:

$$\begin{aligned}\mathcal{L}_{\texttt{FFVT}} = -2\texttt{GT}T^{\mu\nu*}&(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}) \\ &\times \bar{f}\gamma^\sigma\big[\texttt{GC(1)}P_L + \texttt{GC(2)}P_R\big]f\,V^{\rho*}\end{aligned} \tag{58}$$

with the chiral-projection operator $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$. The coupling constant GT is

$$\texttt{GT} = \texttt{GTFV} = -1/(2\Lambda), \tag{59}$$

Table 2 List of the coupling constants for each vertex. All the particles and the coupling constants are written in the MG notation. `y` stands for a massive graviton, `f` represents all possible fermions, and `v` is the SM gauge bosons (`g`, `a`, `z`, `w`). `GC` is a SM coupling constant, while `GT` is a non-renormalizable coupling constant defined in each subroutine in Sect. 2

3-point couplings						GT	
SST		h	h	y		GTS	
FFT		f	f	y		GTF	
VVT		v	v	y		GTV	
4-point couplings						GT	GC
FFVT		d	d	a	y	GTFV	GAD
		u	u	a	y	GTFV	GAU
		l	l	a	y	GTFV	GAL
		d	d	z	y	GTFV	GZD
		u	u	z	y	GTFV	GZU
		l	l	z	y	GTFV	GZL
		vl	vl	z	y	GTFV	GZN
		d	u	w-	y	GTFV	GWF
		u	d	w+	y	GTFV	GWF
		l-	vl	w-	y	GTFV	GWF
		vl	l-	w+	y	GTFV	GWF
		q	q	g	y	GTFV	GG
VVVT		g	g	g	y	GTV	G
5-point couplings						GT	GC
VVVVT	g	g	g	g	y	GTV	G

and `GC(1)` and `GC(2)` are the relevant `FFV` left and right coupling constants. The list of the coupling constants is shown in Table 2. For instance, in the case of the interaction with a gluon, the `FFV` couplings are

$$\mathtt{GC}(1)=\mathtt{GC}(2)=-g_s, \tag{60}$$

where the sign of the coupling is fixed by the `HELAS` convention [8].

2.5.1 IOVTXX

This subroutine computes the amplitude of the `FFVT` vertex from a flowing-`In` fermion spinor, a flowing-`Out` fermion spinor, a `Vector` boson polarization vector and a `Tensor` boson wavefunction, and should be called as

```
CALL IOVTXX(FI, FO, VC, TC, GC, GT , VERTEX).
```

What we compute here is

$$\begin{aligned}\mathtt{VERTEX}=&-\mathtt{GT}(\mathtt{FO})\big[T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma}-C_{\mu\nu,\rho\sigma})\\&\times V^\rho\gamma^\sigma(\mathtt{GC}(1)P_L+\mathtt{GC}(2)P_R)\big](\mathtt{FI}).\end{aligned} \tag{61}$$

2.5.2 FVTIXX

This subroutine computes an off-shell fermion wavefunction by the `FFVT` vertex, and should be called as

```
CALL FVTIXX(FI, VC, TC, GC, GT, FMASS, FWIDTH ,
            FVTI).
```

What we compute here is

$$\begin{aligned}(\mathtt{FVTI})=&-i\mathtt{GT}\frac{i(\not{q}+m_F)}{q^2-m_F^2+im_F\Gamma_F}\\&\times T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma}-C_{\mu\nu,\rho\sigma})\\&\times V^\rho\gamma^\sigma\big[\mathtt{GC}(1)P_L+\mathtt{GC}(2)P_R\big](\mathtt{FI})\end{aligned} \tag{62}$$

for the first 4 components of `FVTI(6)`, and

$$\mathtt{FVTI}(5)=\mathtt{FI}(5)-\mathtt{VC}(5)-\mathtt{TC}(17), \tag{63}$$

$$\mathtt{FVTI}(6)=\mathtt{FI}(6)-\mathtt{VC}(6)-\mathtt{TC}(18). \tag{64}$$

Here the momentum q is

$$\begin{aligned}q^\mu=&(\Re e\mathtt{FVTI}(5),\Re e\mathtt{FVTI}(6),\Im m\mathtt{FVTI}(6),\\&\Im m\mathtt{FVTI}(5)).\end{aligned}$$

2.5.3 FVTOXX

This subroutine computes an off-shell fermion wavefunction by the `FFVT` vertex, and should be called as

```
CALL FVTOXX(FO, VC, TC, GC, GT, FMASS, FWIDTH ,
            FVTO).
```

What we compute here is

$$\begin{aligned}(\mathtt{FVTO})=&-i\mathtt{GT}(\mathtt{FO})T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma}-C_{\mu\nu,\rho\sigma})\\&\times V^\rho\gamma^\sigma\big[\mathtt{GC}(1)P_L+\mathtt{GC}(2)P_R\big]\\&\times\frac{i(\not{q}+m_F)}{q^2-m_F^2+im_F\Gamma_F}\end{aligned} \tag{65}$$

for the first 4 components of `FVTO(6)`, and

$$\mathtt{FVTO}(5)=\mathtt{FO}(5)+\mathtt{VC}(5)+\mathtt{TC}(17), \tag{66}$$

$$\mathtt{FVTO}(6)=\mathtt{FO}(6)+\mathtt{VC}(6)+\mathtt{TC}(18). \tag{67}$$

Here the momentum q is

$$\begin{aligned}q^\mu=&(\Re e\mathtt{FVTO}(5),\Re e\mathtt{FVTO}(6),\Im m\mathtt{FVTO}(6),\\&\Im \mathrm{m}\mathtt{FVTO}(5)).\end{aligned}$$

2.5.4 JIOTXX

This subroutine computes an off-shell vector current by the FFVT vertex, and should be called as

CALL JIOTXX(FI, FO, TC, GC, GT, VMASS, VWIDTH , JIOT).

What we compute here is

$$\begin{aligned}\mathtt{JIOT}(\alpha+1) &= -i\mathtt{GT}\frac{i}{q^2-m_V^2+im_V\Gamma_V}\\ &\times\left(-\eta^{\rho\alpha}+\frac{q^\rho q^\alpha}{m_V^2}\right)\\ &\times T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma}-C_{\mu\nu,\rho\sigma})\\ &\times(\mathtt{FO})\big[\gamma^\sigma\{\mathtt{GC}(1)P_L+\mathtt{GC}(2)P_R\}\big](\mathtt{FI})\end{aligned}\tag{68}$$

for the massive vector boson, or

$$\begin{aligned}\mathtt{JIOT}(\alpha+1) &= -i\mathtt{GT}\frac{-i}{q^2}\eta^{\rho\alpha}T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma}-C_{\mu\nu,\rho\sigma})\\ &\times(\mathtt{FO})\big[\gamma^\sigma\{\mathtt{GC}(1)P_L+\mathtt{GC}(2)P_R\}\big](\mathtt{FI})\end{aligned}\tag{69}$$

for the massless vector boson, and

$$\mathtt{JIOT(5)}=-\mathtt{FI(5)}+\mathtt{FO(5)}+\mathtt{TC(17)},\tag{70}$$

$$\mathtt{JIOT(6)}=-\mathtt{FI(6)}+\mathtt{FO(6)}+\mathtt{TC(18)}.\tag{71}$$

Here q is the momentum of the vector boson.

2.5.5 UIOVXX

This subroutine computes an off-shell tensor current by the FFVT vertex, and should be called as

CALL UIOVXX(FI, FO, VC, GC, GT, TMASS, TWIDTH , UIOV).

What we compute here is

$$\begin{aligned}T^{\alpha\beta} &= -i\mathtt{GT}\frac{iB^{\mu\nu,\alpha\beta}}{q^2-m_T^2+im_T\Gamma_T}(\eta_{\mu\nu}\eta_{\rho\sigma}-C_{\mu\nu,\rho\sigma})\\ &\times(\mathtt{FO})\big[V^\rho\gamma^\sigma\{\mathtt{GC}(1)P_L+\mathtt{GC}(2)P_R\}\big](\mathtt{FI})\end{aligned}\tag{72}$$

for the first 16 components of UIOV(18), and

$$\mathtt{UIOV(17)}=-\mathtt{FI(5)}+\mathtt{FO(5)}+\mathtt{VC(5)},\tag{73}$$

$$\mathtt{UIOV(18)}=-\mathtt{FI(6)}+\mathtt{FO(6)}+\mathtt{VC(6)}.\tag{74}$$

Here q is the momentum of the tensor boson.

2.6 VVVT vertex

The VVVT vertices are obtained from the interaction Lagrangian among the tensor and three vector bosons:

$$\begin{aligned}\mathcal{L}_{\mathrm{VVVT}} &= \mathtt{GT}\,\mathtt{GC}\,f^{abc}T^{\mu\nu*}\bigg[\frac{1}{2}\eta_{\mu\nu}(\partial_\rho V_\sigma^a-\partial_\sigma V_\rho^a)V^{b,\rho}V^{c,\sigma}\\ &\quad-(\partial_\mu V^{a,\rho}-\partial^\rho V_\mu^a)V_\nu^b V_\rho^c\\ &\quad-(\partial_\nu V_\rho^a-\partial_\rho V_\nu^a)V^{b,\rho}V_\mu^c\bigg]\end{aligned}\tag{75}$$

with the structure constant f^{abc} and the coupling constant, as in (45),

$$\mathtt{GT}=\mathtt{GTV}=-1/\Lambda.\tag{76}$$

In this paper we concentrate on the interactions with gluons for the VVVT vertex, so in this case GC is the strong coupling constant,

$$\mathtt{GC}=g_s,\tag{77}$$

and f^{abc} is the structure constants of the group SU(3), which can be handled by the MG automatically. As in the original subroutines for the VVV vertex [8], the following subroutines, VVVTXX, JVVTXX, and UVVVXX, can be used for the electroweak gauge bosons without any modifications.

2.6.1 VVVTXX

This subroutine computes the amplitude of the VVVT vertex from three Vector boson polarization vectors and a Tensor boson wavefunction, and should be called as

CALL VVVTXXX(VA, VB, VC, TC, GC, GT , VERTEX).

What we compute here is

$$\begin{aligned}\mathtt{VERTEX} &= -\mathtt{GT}\,\mathtt{GC}T^{\mu\nu}\big[C_{\mu\nu,\rho\sigma}(p_{a\lambda}-p_{b\lambda})\\ &\quad+C_{\mu\nu,\sigma\lambda}(p_{b\rho}-p_{c\rho})+C_{\mu\nu,\lambda\rho}(p_{c\sigma}-p_{a\sigma})\\ &\quad+F_{\mu\nu,\rho\sigma\lambda}(p_a,p_b,p_c)\big]V_a^\rho V_b^\sigma V_c^\lambda\end{aligned}\tag{78}$$

with

$$\begin{aligned}&F_{\mu\nu,\rho\sigma\lambda}(p_a,p_b,p_c)\\ &\quad=\eta_{\mu\rho}\eta_{\sigma\lambda}(p_b-p_c)_\nu+\eta_{\mu\sigma}\eta_{\rho\lambda}(p_c-p_a)_\nu\\ &\qquad+\eta_{\mu\lambda}\eta_{\rho\sigma}(p_a-p_b)_\nu+(\mu\leftrightarrow\nu).\end{aligned}\tag{79}$$

Here, the vector bosons (gluons in this paper) VA, VB, and VC have the momentum p_a, p_b, and p_c, and the color a, b, and c, respectively.

2.6.2 JVVTXX

This subroutine computes an off-shell vector current by the VVVT vertex, and should be called as

```
CALL JVVTXX(VA, VB, TC, GC, GT, VMASS, VWIDTH ,
                    JVVT).
```

What we compute here is

$$\begin{aligned}
&\mathtt{JVVT}(\alpha+1)\\
&\quad=-i\mathtt{GT}\,\mathtt{GC}\frac{i}{q^2-m_V^2+im_V\Gamma_V}\left(-\eta^{\lambda\alpha}+\frac{q^\lambda q^\alpha}{m_V^2}\right)\\
&\qquad\times T^{\mu\nu}\big[C_{\mu\nu,\rho\sigma}(p_{a\lambda}-p_{b\lambda})+C_{\mu\nu,\sigma\lambda}(p_{b\rho}+q_\rho)\\
&\qquad+C_{\mu\nu,\lambda\rho}(-q_\sigma-p_{a\sigma})+F_{\mu\nu,\rho\sigma\lambda}(p_a,p_b,-q)\big]V_a^\rho V_b^\sigma
\end{aligned}\tag{80}$$

for the massive vector boson, or

$$\begin{aligned}
&\mathtt{JVVT}(\alpha+1)\\
&\quad=-i\mathtt{GT}\,\mathtt{GC}\frac{-i}{q^2}\eta^{\lambda\alpha}T^{\mu\nu}\big[C_{\mu\nu,\rho\sigma}(p_{a\lambda}-p_{b\lambda})\\
&\qquad+C_{\mu\nu,\sigma\lambda}(p_{b\rho}+q_\rho)+C_{\mu\nu,\lambda\rho}(-q_\sigma-p_{a\sigma})\\
&\qquad+F_{\mu\nu,\rho\sigma\lambda}(p_a,p_b,-q)\big]V_a^\rho V_b^\sigma,
\end{aligned}\tag{81}$$

for the massless vector boson, and

$$\mathtt{JVVT(5)}=\mathtt{VA(5)}+\mathtt{VB(5)}+\mathtt{TC(17)},\tag{82}$$

$$\mathtt{JVVT(6)}=\mathtt{VA(6)}+\mathtt{VB(6)}+\mathtt{TC(18)}.\tag{83}$$

Here the momenta p_a, p_b and q are

$$\begin{aligned}
p_a^\mu&=(\Re e\mathtt{VA(5)},\Re e\mathtt{VA(6)},\Im m\mathtt{VA(6)},\Im m\mathtt{VA(5)}),\\
p_b^\mu&=(\Re e\mathtt{VB(5)},\Re e\mathtt{VB(6)},\Im m\mathtt{VB(6)},\Im m\mathtt{VB(5)}),\\
q^\mu&=(\Re e\mathtt{JVVT(5)},\Re e\mathtt{JVVT(6)},\\
&\qquad\Im m\mathtt{JVVT(6)},\Im m\mathtt{JVVT(5)}).
\end{aligned}$$

Note that the off-shell gluon JVVT has the color c.

2.6.3 UVVVXX

This subroutine computes an off-shell tensor current by the VVVT vertex, and should be called as

```
CALL UVVVXX(VA, VB, VC, GC, GT, TMASS, TWIDTH ,
                    UVVV).
```

What we compute here is

$$\begin{aligned}
T^{\alpha\beta}&=-i\mathtt{GT}\,\mathtt{GC}\frac{iB^{\mu\nu,\alpha\beta}}{q^2-m_T^2+im_T\Gamma_T}\big[C_{\mu\nu,\rho\sigma}(p_{a\lambda}-p_{b\lambda})\\
&\quad+C_{\mu\nu,\sigma\lambda}(p_{b\rho}-p_{c\rho})+C_{\mu\nu,\lambda\rho}(p_{c\sigma}-p_{a\sigma})\\
&\quad+F_{\mu\nu,\rho\sigma\lambda}(p_a,p_b,p_c)\big]V_a^\rho V_b^\sigma V_c^\lambda
\end{aligned}\tag{84}$$

for the first 16 components of UVVV(18), and

$$\mathtt{UVVV(17)}=\mathtt{VA(5)}+\mathtt{VB(5)}+\mathtt{VC(5)},\tag{85}$$

$$\mathtt{UVVV(18)}=\mathtt{VA(6)}+\mathtt{VB(6)}+\mathtt{VC(6)}.\tag{86}$$

Here p_a, p_b and p_c are the momenta of the outgoing vector bosons, and q is that of the tensor boson.

2.7 VVVVT vertex

The VVVVT vertices are obtained from the interaction Lagrangian among the tensor and four vector bosons:

$$\begin{aligned}
\mathcal{L}_{\mathtt{VVVVT}}&=-\mathtt{GT}\,\mathtt{GC}^2 f^{abe}f^{cde}T^{\mu\nu*}\\
&\quad\times\left[\frac{1}{4}\eta_{\mu\nu}V^{a,\rho*}V^{b,\sigma*}V_\rho^{c*}V_\sigma^{d*}\right.\\
&\quad\left.-V^{b,\rho*}V_\mu^{a*}V_\nu^{c*}V_\rho^{d*}\right]
\end{aligned}\tag{87}$$

with the coupling constants GT = GTV in (45) and GC = g_s for the interactions with gluons.

We should note that the 5-point vertex cannot be generated by MG, and thus we must add the following subroutines by hand, GGGGTX, JGGGTX, or UGGGGX, to the amplitudes which have the corresponding color structures.

2.7.1 GGGGTX

This subroutine computes the portion of the amplitude of the VVVVT vertex from four Gluon polarization vectors and a Tensor boson wavefunction corresponding to the color structure $f^{abe}f^{cde}$, and should be called as

```
CALL GGGGTX(VA, VB, VC, VD, TC, GC, GT , VERTEX).
```

The output is

$$\mathtt{VERTEX}=-\mathtt{GT}\,\mathtt{GC}^2T^{\mu\nu}G_{\mu\nu,\rho\lambda\sigma\delta}V_a^\rho V_b^\sigma V_c^\lambda V_d^\delta\tag{88}$$

with

$$\begin{aligned}
G_{\mu\nu,\rho\sigma\lambda\delta}&=\eta_{\mu\nu}(\eta_{\rho\sigma}\eta_{\lambda\delta}-\eta_{\rho\delta}\eta_{\sigma\lambda})\\
&\quad+\big[\eta_{\mu\rho}\eta_{\nu\delta}\eta_{\lambda\sigma}+\eta_{\mu\lambda}\eta_{\nu\sigma}\eta_{\rho\delta}-\eta_{\mu\rho}\eta_{\nu\sigma}\eta_{\lambda\delta}\\
&\quad-\eta_{\mu\lambda}\eta_{\nu\delta}\eta_{\rho\sigma}+(\mu\leftrightarrow\nu)\big].
\end{aligned}\tag{89}$$

To obtain the complete amplitude, this subroutine must be called three times (once for each color structure) with the following permutations:

CALL GGGGTX(VA, VB, VC, VD, TC, GC, GT , VERTEX1),

CALL GGGGTX(VA, VC, VD, VB, TC, GC, GT , VERTEX2),

CALL GGGGTX(VA, VD, VB, VC, TC, GC, GT , VERTEX3),

corresponding to the color structure $f^{abe}f^{cde}$, $f^{ace}f^{dbe}$, and $f^{ade}f^{bce}$, respectively.

2.7.2 JGGGTX

This subroutine computes the portion of the off-shell gluon current by the VVVVT vertex, corresponding to the color structure $f^{abe}f^{cde}$, and should be called as

CALL JGGGTX(VA, VB, VC, TC, GC, GT , JGGGT).

What we compute here is

$$\mathtt{JGGGT}(\alpha+1) = -i\,\mathtt{GT}\,\mathtt{GC}^2\frac{-i}{q^2}\eta^{\delta\alpha}T^{\mu\nu}G_{\mu\nu,\rho\lambda\sigma\delta} \times V_a^\rho V_b^\sigma V_c^\lambda, \tag{90}$$

and

$$\mathtt{JGGGT}(5) = \mathtt{VA}(5)+\mathtt{VB}(5)+\mathtt{VC}(5)+\mathtt{TC}(17), \tag{91}$$

$$\mathtt{JGGGT}(6) = \mathtt{VA}(6)+\mathtt{VB}(6)+\mathtt{VC}(6)+\mathtt{TC}(18). \tag{92}$$

Note that the off-shell gluon JGGGT has the color d and the momentum q.

2.7.3 UGGGGX

This subroutine computes the portion of the off-shell tensor current by the VVVVT vertex, corresponding to the color structure $f^{abe}f^{cde}$, and should be called as

CALL UGGGGX(VA, VB, VC, VD, GC, GT, TMASS, TWIDTH, UGGGG).

What we compute here is

$$T^{\alpha\beta} = -i\,\mathtt{GT}\,\mathtt{GC}^2\frac{iB^{\mu\nu,\alpha\beta}}{q^2-m_T^2+im_T\Gamma_T}G_{\mu\nu,\rho\lambda\sigma\delta} \times V_a^\rho V_b^\sigma V_c^\lambda V_d^\delta \tag{93}$$

for the first 16 components of UGGGG(18), and

$$\mathtt{UGGGG}(17) = \mathtt{VA}(5)+\mathtt{VB}(5)+\mathtt{VC}(5)+\mathtt{VD}(5), \tag{94}$$

$$\mathtt{UGGGG}(18) = \mathtt{VA}(6)+\mathtt{VB}(6)+\mathtt{VC}(6)+\mathtt{VD}(6). \tag{95}$$

Here q is the momentum of the tensor boson.

2.8 Checking for the new HELAS subroutines

The new HELAS subroutines are tested by using the QCD gauge invariance and the general coordinate transformation invariance of the helicity amplitudes. In particular, we use the following processes;

$q\bar{q}\to gT$	for IOTXXX, FTIXXX, FTOXXX, IOVTXX,
$gg\to gT$	for VVTXXX, JVTXXX, VVVTXX,
$q\bar{q}\to q\bar{q}(gg)T$	for FVTIXX, FVTOXX, JIOTXX,
$gg\to ggT$	for JVVTXX, GGGGTX,
$q\bar{q}(gg)\to T\to gg$	for UIOXXX, UVVXXX.

More explicitly, we express the helicity amplitudes of the above processes as

$$\mathcal{M}_{\lambda_T\lambda_g} = T_{\mu\nu\rho}\epsilon^{\mu\nu}(p_T,\lambda_T)^*\epsilon^\rho(p_g,\lambda_g)^* \tag{96}$$

with an external tensor and a gluon wavefunction. The identities,

$$p_T^\mu/p_T^0 T_{\mu\nu\rho}\epsilon^\rho(p_g,\lambda_g)^* = p_T^\nu/p_T^0 T_{\mu\nu\rho}\epsilon^\rho(p_g,\lambda_g)^* = 0 \tag{97}$$

for the general coordinate transformation symmetry and

$$p_g^\rho/p_g^0 T_{\mu\nu\rho} = 0 \tag{98}$$

for the SU(3) gauge invariance, test all the above subroutines thoroughly.

The subroutines UIOVXX, UVVVXX, JGGGTX and UGGGGX have been checked in such a way that the above subroutines are used rewriting the helicity amplitudes of the processes

$$q\bar{q}\to T\to q\bar{q}g(ggg),$$

$$gg\to ggT\to ggZZ,$$

$$gg\to ggggT.$$

We test the agreement of the amplitudes between the expressions that use the above subroutines and those without them for all helicity combinations and at arbitrary Lorentz frame.

3 MadGraph/MadEvent implementing for spin-2 gravitons

In this section, we would like to describe how we implement spin-2 gravitons into MG/ME.

First, using the User Model framework in MG [11], we make our new model directories for both the ADD and RS models, including the massive gravitons (particles.dat) and their interactions with the SM particles

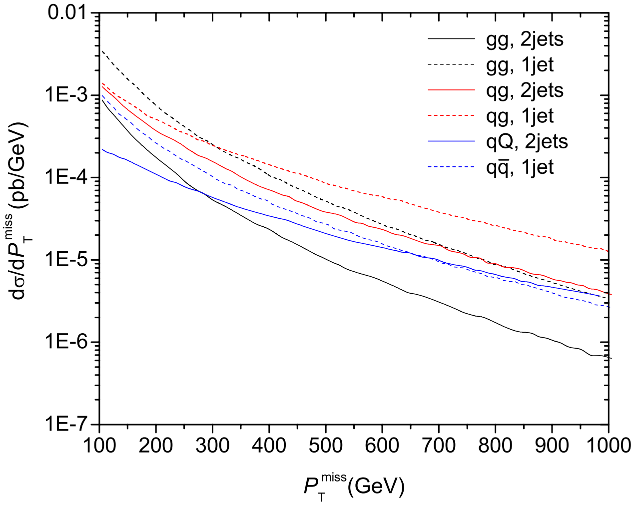

Fig. 1 $P_T^{\rm miss}$ dependence of the total cross sections for the graviton productions with 1-jet (via the gg, qg and $q\bar{q}$ channels) and 2-jets (via the gg, qg and qQ channels) in the ADD model at the LHC, where $q, Q = u, d, s, c$

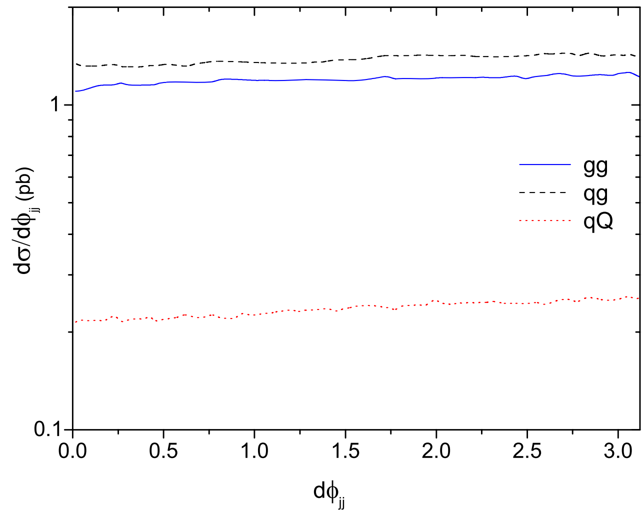

Fig. 2 Distributions of the azimuthal angle separation between the two jets for the first KK graviton plus di-jet productions in the RS model at the LHC, via the gg, qg and qQ channels ($q, Q = u, d, s, c$)

(`couplings.f` and `interactions.dat`); see also Table 2. Then we insert all the new `HELAS` subroutines for spin-2 tensor bosons into the `HELAS` library in `MG`. Since the present `MG` does not handle external spin-2 particles, we further modify the codes in `MG` to tell it how to generate the `SST`, `FFT` and `FFVT` type of vertices and helicity amplitudes (for `VVT` and `VVVT` type, it has already been done for the Higgs effective field theory (`HEFT`) model), and how to deal with the helicity of the spin-2 tensor bosons when they are external. Moreover, since `MG` can only generate Feynman diagrams with up to 4-point vertices, the amplitudes and their `HELAS` codes with the 5-point vertex, `GGGGTX`, `JGGGTX`, or `UGGGGX`, have been added by hand; see more details in Sect. 2.7.

Finally, we note that since in the ADD model the gravitons are densely populated and we should sum over their contributions by modifying the phase space integration in `ME`. In the ADD model, the spectrum of KK graviton modes can be treated as continuous for $\delta \le 6$ [1–3], and the mass density function is given by [6]

$$\rho(m) = S_{\delta-1}\frac{\overline{M}_{\rm Pl}^2}{M_s^{2+\delta}}m^{\delta-1} \quad \text{with } S_{\delta-1} = \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}, \tag{99}$$

where M_s is the ADD model effective scale. Thus, we modify the phase space generating codes in `ME` to add one more random number for graviton mass generating and implement the above graviton mass integration.

4 Sample results

In this section, we present some sample numerical results for the graviton plus mono-jet and di-jet productions at the LHC, using the new `HELAS` subroutines and the modified `MG/ME`.

We use the following jet definition criteria

$$\begin{aligned} &P_T^j > 20\ \text{GeV}, \quad |\eta_j| < 5, \\ &R_{jj} = \sqrt{\Delta\eta^2 + \Delta\phi^2} > 0.6, \end{aligned} \tag{100}$$

where η is the pseudorapidity of the jets and ϕ is the azimuthal angle around the beam direction, and further require

$$\eta_{j_1}\cdot\eta_{j_2} < 0, \qquad |\eta_{j_1} - \eta_{j_2}| > 4.2. \tag{101}$$

The CTEQ6L1 parton distribution functions [14] are employed with the factorization scale chosen as $\mu_f = \min(P_T^j)$ of the jets which satisfy the above cuts. For the QCD coupling, we set it as the geometric mean value, $\alpha_s = \sqrt{\alpha_s(P_T^{j_1})\cdot\alpha_s(P_T^{j_2})}$.

Figure 1 shows the $P_T^{\rm miss}$ distributions for the graviton productions with the mono-jet (via the gg, qg and $q\bar{q}$ channels) and with the di-jet (via the gg, qg and qQ channels) in the ADD model at the LHC, with $\Lambda = 5$ TeV and $\delta = 4$. Note that the unitarity criterion $M_{T_n} < \Lambda$ is used. See more details in Ref. [13].

Figure 2 shows the ϕ_{jj} distributions for the first KK graviton productions with two jets in the RS model at the LHC, via the gg, qg and qQ channels. Here we set $\Lambda = 4$ TeV and $M_{T_1} = 1$ TeV. The total cross sections are 0.74, 3.74 and 4.33 pb for the qQ, gg and qg channels, respectively.

5 Summary

In this paper, we have added new HELAS subroutines to calculate helicity amplitudes with massive spin-2 particles (massive gravitons) to the HELAS library. They are coded in such a way that arbitrary scattering amplitudes with one graviton production and its decays can be generated automatically by MG and ME, after slight modifications. All the codes have been tested carefully by making use of the invariance of the helicity amplitudes under the gauge and general coordinate transformations.

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