Mon. Not. R. Astron. Soc. **276,** 1402–1408 (1995)

Helium abundance in the solar envelope

Sarbani Basu[1] and H. M. Antia[2]

[1] *Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark*
[2] *Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India*

Accepted 1995 May 24. Received 1995 April 24

ABSTRACT

The abundance of helium in the solar envelope can be determined using the variation of the adiabatic index of the stellar material in the second helium ionization zone. All techniques for inferring helium abundance from the observed frequencies of solar p modes are known to be sensitive to the equation of state used in the reference models. The sensitivity of inferred helium abundance to the equation of state is studied by using different reference models with MHD and OPAL equations of state. Recent observations of high-degree solar p-mode frequencies yield a helium abundance $Y = 0.246$ when determined using reference models with the MHD equation of state and $Y = 0.249$ using the OPAL equation of state. Further, the models constructed with the OPAL equation of state are found to be in better agreement with the inferred sound speed below the He II ionization zone.

Key words: equation of state – Sun: abundances – Sun: oscillations.

1 INTRODUCTION

Spectroscopic measurements of the abundance of helium in the Sun are very uncertain, hence helioseismology plays a major role in determining the helium abundance in the solar envelope. The abundance is obtained from the variation of the adiabatic index of the solar material in the second helium ionization zone. The change in the adiabatic index depends mainly on the relative abundances, X and Y, of hydrogen and helium, respectively. Of course, there could be some contribution from the heavier elements, some of which could be undergoing ionization in the same region, but this contribution is expected to be small. Various techniques have been used to infer the helium abundance from the observed frequencies (e.g., Gough 1984; Däppen et al. 1991; Christensen-Dalsgaard & Pérez Hernandez 1991; Vorontsov, Baturin & Pamyatnykh 1992; Kosovichev et al. 1992; Antia & Basu 1994a and references therein). The results obtained by different workers have not, however, been fully consistent with each other. It is found that the helioseismic measurement of helium abundance is sensitive to the equation of state of stellar material, which is used in translating the variation of Γ_1 to the difference in Y. Apart from the equation of state, uncertainties in the model of solar surface layers also introduce errors in the measurement of helium abundance (cf. Antia & Basu 1994a).

Recently, the OPAL equation of state has been computed (Rogers 1994) and it would be interesting to test how this equation of state compares with the MHD (Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988; Däppen et al. 1988) equation of state. It also provides a natural test for the sensitivity of helium abundance measurements to the equation of state.

Apart from equations of state, there has also been a significant improvement in modelling the structure of surface layers of the Sun. In particular, it has been demonstrated (Basu & Antia 1994) that if the convective flux is calculated using the prescription of Canuto & Mazzitelli (1991) then the resulting solar models yield frequencies that are in much better agreement with the observed frequencies. Similarly, it has been found that the use of the opacity tables of Kurucz (1991) in the atmosphere also leads to a significant improvement in the frequencies of the solar model. With all these improvements the calculated frequencies of the solar model are generally found to be within 12 μHz of the observed frequencies for all p modes with frequencies less than 5 mHz.

Bachman et al. (1995) have provided a fresh set of observed frequencies of high-degree modes, which are claimed to be substantially better than the frequency tables of Libbrecht, Woodard & Kaufman (1990), which have been extensively used in helioseismic analysis so far. Since the new observations cover only modes with $l \geq 100$ they are not suitable for inferring the internal structure of the Sun, but these should be sufficient to probe the helium ionization zones. This will enable us to test the sensitivity of measurements of helium abundance to systematic errors in observed frequencies.

In the present work, using reference models constructed with the MHD and OPAL equations of state, we attempt to infer the helium abundance in the solar envelope using the techniques described by Antia & Basu (1994a, hereinafter AB) and to study the influence of varying the equation of state on the determination of helium abundance. Further, we also attempt to test which of these equations of state is closer to that of solar material, as inferred from the observed frequencies.

The rest of the paper is organized as follows. In Section 2 we give a brief description of the models used for calibration and the techniques for determining the helium abundance. In Section 3 we discuss the results obtained using the test models, while in Section 4 we present the results using the observed frequencies. Finally, in Section 5 we present our conclusions.

2 THE TECHNIQUE AND THE MODELS USED

The techniques for determining Y from the functions $H_1(w)$, $H_2(\omega)$, and $W(r)$ have been discussed in detail in AB. We briefly recapitulate the main points here.

The asymptotic description of stellar p modes allows us to write scaled frequency differences between two solar models, or the Sun and a solar model, in the form

$$S(w)\frac{\omega_0-\omega}{\omega_0}=H_1(w)+H_2(\omega), \qquad (1)$$

where

$$S(w)=\int_{r_t}^{R_\odot}\left(1-\frac{c_0^2}{w^2r^2}\right)^{-1/2}\frac{\mathrm{d}r}{c_0} \qquad (2)$$

(cf. Christensen-Dalsgaard, Gough & Thompson 1989). In the above equation, ω_0 is the frequency of a given mode for the reference model, ω is that for the Sun or a test solar model, r_t is the lower turning point of the mode, $w=\omega/(l+1/2)$, c_0 is the speed of sound in the reference model and c is that in the test model or the Sun. The function $H_1(w)$ can be inverted to obtain the relative sound speed difference between the reference and test models or between the reference model and the Sun (Christensen-Dalsgaard et al. 1989). The function $H_1(w)$ contains information about the internal structure of the Sun, while $H_2(\omega)$ is determined by the structure of surface layers.

Using the frequency differences between a large number of modes we can determine the functions $H_1(w)$ and $H_2(\omega)$ by a least-squares solution of equation (1). For this purpose we expand the functions $H_1(w)$ and $H_2(\omega)$ in terms of B-splines in w and ω, respectively. The difference in the abundance of helium between two models manifests itself as a hump in $H_1(w)$ between the two models at the position corresponding to the second helium ionization zone, and the height of the hump depends on the difference in the helium abundance of the two models. AB have shown that this hump can be calibrated to determine the helium abundance of a solar model. Similarly, the function $H_2(\omega)$ also shows a hump at around 2.5 mHz. For envelope models constructed with identical physics and depth of convection zone, but different Y, $H_1(w)$ and $H_2(\omega)$ are functions of Y alone. The effect of differences in other physical properties, like the equation of state, manifests itself as a smooth curve on which the hump due to the difference in helium abundance is superimposed. The differences in the surface layers also produce a steep trend in $H_1(w)$ at low w. If $H_1(w)$ is inverted to determine the sound speed, the function

$$W(r)=\frac{r^2}{Gm}\frac{\mathrm{d}c^2}{\mathrm{d}r} \qquad (3)$$

can be calculated. This function also shows a peak at the helium ionization zone around $r=0.98\ \mathrm{R}_\odot$ (Gough 1984), and the peak height can also be calibrated to determine the helium abundance.

Since we are interested in determining the helium abundance from the structure of the helium ionization zone, it is enough to use solar envelope models, instead of full models for calibration. We thus use only those modes which have their lower turning points inside the convection zone. For the purpose of calibration we have constructed a sequence of solar envelope models with different helium abundance. To avoid any contamination of the signal due to effects of the differing depths of the convection zone, all models have been constructed with the base of the convection zone at a radius of $r=0.71\ \mathrm{R}_\odot$ which is the solar convection zone depth as determined by Christensen-Dalsgaard, Gough & Thompson (1991). All models have been constructed with the OPAL opacities (Rogers & Iglesias 1992) for temperatures above 20 000 K and with opacities from Kurucz (1991) for lower temperatures. The convective energy flux has been calculated using the formulation of Canuto & Mazzitelli (1991), which is known to give better agreement with observations (Paternó et al. 1993; Basu & Antia 1994). The models have been constructed with either the MHD or OPAL equations of state. For each equation of state we have constructed five models, with hydrogen abundances X of 0.68, 0.70, 0.72, 0.74, and 0.76. The models constructed with the OPAL equation of state have a heavy element abundance of $Z=0.019$, while those constructed with the MHD equation of state have $Z=0.02$. For the sake of convenience we have labelled the models by their equation of state and the hydrogen abundance. Apart from these calibration models, we have also constructed two models, MHD73 and OPAL73, with $X=0.73$, as test models to estimate the sensitivity of our techniques to the equation of state.

In order to find the least-squares solution of equation (1), we use 20 knots each, uniformly spaced in $\log(w)$ and ω to represent $H_1(w)$ and $H_2(\omega)$ respectively. We consider only those modes that have the lower turning point within the convection zone, that is with $w<0.1$ mHz. Further, since we wish to use this technique for the observed frequencies, we restrict our mode set to the observed mode set and weigh each point inversely with the quoted observational errors. We do not use the f mode ($n=0$) since the asymptotic relation cannot be expected to hold for this mode. For the observed frequencies we have used the solar frequencies obtained from the Big Bear observatory as listed in Libbrecht et al. (1990, hereinafter BB) or those obtained by the High-L Helioseismometer (Bachman et al. 1995, hereinafter HLH) separately. Our selection criterion gives us 2134 modes from BB in the frequency range $1.0\leq\omega\leq5.0$ mHz, and 4546 modes from HLH in the frequency range $1.8\leq\omega\leq5.0$ mHz. We use singular value decomposition (SVD) to obtain the least-squares solution of equation (1).

In order to estimate the effect of errors in the observed frequencies on the measurement of Y, we use artificial sets of frequencies where random errors with standard deviations as quoted by the observers are added to the calculated model frequencies. For each set of simulated frequencies, we repeat the whole process to determine Y using all three methods and all calibration curves. We can estimate the standard deviation of the results by the distribution of these calculated values. We have used 25 sets of simulated frequencies. This error estimate should include errors due to the fitting process; however, it does not include systematic errors introduced due to differences in equation of state and other physical inputs.

3 RESULTS FOR TEST MODELS

Before trying to determine the helium abundance in the solar envelope, it is desirable to make some estimate of the systematic errors involved in the determination of the helium abundance, using models with different equations of state, and to see how close we can get to the actual value in the presence of random errors in the frequencies. To this end, we have tried to determine the helium abundance of the models MDH73 and OPAL73 with both sets of calibration curves after random errors with standard deviations, as given by the observers, were added to the frequencies of these models. The results for MHD73 obtained with the MHD calibration models and those for OPAL73 with OPAL calibration models should indicate the errors introduced due to the fitting processes and random errors in the frequencies. On the other hand, results for MHD73 with the OPAL models and OPAL73 with the MHD models should give an indication about the sensitivity of our techniques to the equation of state.

The results from all three techniques are summarized in Table 1, which gives the helium abundance by mass in terms of percentages. It may be noted that, even for the test models MHD73 and OPAL73, the results are different when the BB set of modes is used instead of the HLH set because, although the model frequencies are the same, the set of modes used, as well as the relative weights, are different. It can be seen that in general there is no difficulty in determining the helium abundance of model MHD73 using MHD reference models and OPAL73 using OPAL reference models and the error in the results is rather small. Thus when the equation of state in the test model is the same as that in reference models, the helium abundance can be determined reliably using any of the techniques considered. $H_1(w)$ appears to give the best estimate, while $H_2(\omega)$ gives results which are different from the exact value, even though the errors due to random variations in the frequencies may be much smaller.

If we consider the results obtained using calibration models with a different equation of state, we see that the helium abundance of MHD73 obtained using the OPAL reference models tends to be higher than the exact abundance, while the helium abundance of OPAL73 using MHD reference models is generally lower than the actual abundance. The difference is most significant for the technique using $W(r)$. This discrepancy arises because the height of the He II hump in $W(r)$ depends on the equation of state, as well as the helium abundance. Fig. 1, shows $W(r)$ for models MHD72, MHD74, OPAL72 and OPAL74 as computed directly from the known sound speed in these models. It is clear from the figure that, for the same value of X, the height of the helium hump in $W(r)$ is significantly different for the two equations of state, and we would expect an error of roughly 1 per cent in estimating the helium abundance using calibration models with different equations of state. The actual error is somewhat different, as other systematic errors are probably also

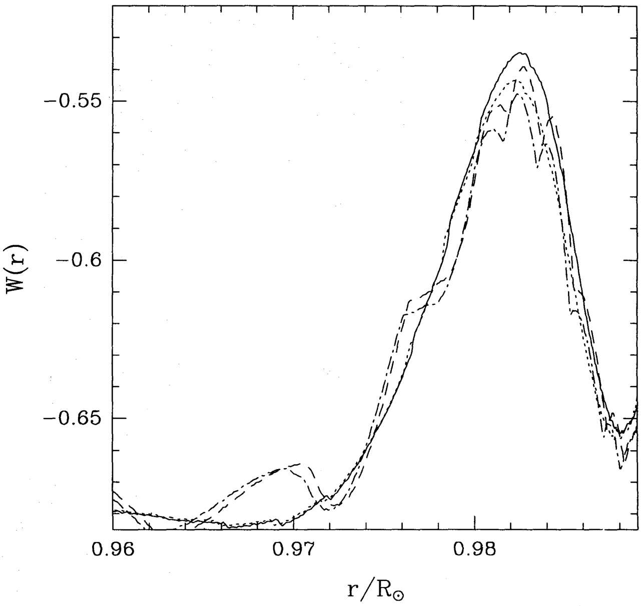

Figure 1. The function $W(r)$ for MHD72 (solid line), MHD74 (dotted line), OPAL72 (dashed line) and OPAL74 (dot-dashed line) reference models.

Table 1. Determining helium abundance.

Model	Y_{exact}	MHD reference models $W(r)$	MHD $H_1(w)$	MHD $H_2(\omega)$	OPAL reference models $W(r)$	OPAL $H_1(w)$	OPAL $H_2(\omega)$
Using BB mode set							
MHD73	25.00	24.95 ± 0.07	25.01 ± 0.13	24.81 ± 0.02	25.62 ± 0.26	25.08 ± 0.16	25.17 ± 0.32
OPAL73	25.10	24.56 ± 0.38	25.11 ± 0.16	24.78 ± 0.32	25.07 ± 0.07	25.16 ± 0.13	25.13 ± 0.02
OBS		24.56 ± 0.09	24.50 ± 0.14	24.31 ± 0.22	25.20 ± 0.29	24.54 ± 0.18	24.57 ± 0.10
Using HLH mode set							
MHD73	25.00	24.97 ± 0.11	25.04 ± 0.05	24.92 ± 0.30	26.17 ± 0.15	25.30 ± 0.22	25.26 ± 0.51
OPAL73	25.10	23.83 ± 0.14	24.89 ± 0.23	24.90 ± 0.30	25.00 ± 0.11	25.11 ± 0.05	25.18 ± 0.37
OBS		23.53 ± 0.47	24.51 ± 0.06	24.84 ± 0.30	24.80 ± 0.49	24.79 ± 0.34	25.04 ± 0.49

involved in our calibration procedure, which uses the inverted profiles for $W(r)$ rather than the exact profiles as explained in AB. Thus it appears that, when the equation of state between the test and calibration models are different, $H_1(w)$ and $H_2(\omega)$ yield a more reliable result for the helium abundance.

From Fig. 1 we can see that $W(r)$ for OPAL reference models shows some humps in addition to the main peak due to the He II ionization zone. We believe these humps to be due to interpolation errors in interpolating the equation of state in a rather coarse grid in T, ρ and X. These humps may interfere in our determination of helium abundance using OPAL reference models to some extent. As a result, the helium abundance determined using the OPAL equation of state may not be very reliable. This is particularly true for the HLH data set, where inaccuracies in the equation of state are combined with inadequate frequency coverage at low frequencies.

If we compare the results obtained using two different mode sets, we find that, apart from $W(r)$, the helium abundance using $H_1(w)$ also varies when a different mode set is used. After doing some experiments with different weights we found that the main source of difference is the sudden rise in frequency errors at $l=400$ in the BB set. Apart from the sudden increase in errors the number of modes in the tables also reduces sharply for $l>400$, thus reducing the effective weights of these modes even further. Because of this we get a second hump in the calibration curve around log $w=-2.3$ mHz, as can be seen in Fig. 2. This hump is absent in the calibration curve using the weights as given by errors in HLH frequency tables. By comparing the $H_1(w)$ between two calibration models with the exact value that is expected from the

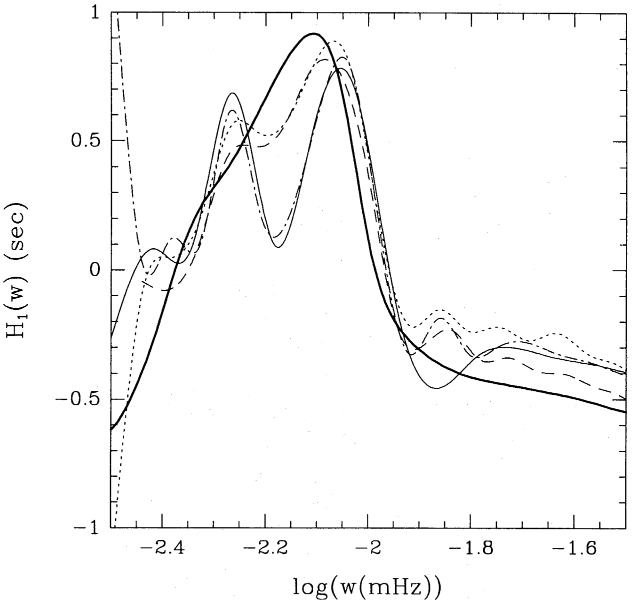

Figure 2. The calibration curves, i.e., $H_1(w)$ between models MHD72 and MHD74 obtained with the BB data set (thin continuous line) and the HLH data set (dashed line). The thick solid line shows $H_1(w)$ obtained from the known sound speed differences between the models, the dotted line shows $H_1(w)$ obtained using a smaller data set containing only models that are common to the two sets and using weights from HLH, and the dot-dashed line is the same but with weights from BB.

known sound speed differences, we find that the $H_1(w)$ obtained using the HLH set is closer to the exact curve. To demonstrate that this is indeed an artefact of weights, we use only those modes that are common to the two data sets and use either of the weights to find the least-squares solution. In that case it turns out that the $H_1(w)$ obtained using the BB weights shows the second hump, while the one obtained using HLH weights does not show that. Nevertheless, looking at the table we find that the helium abundance as determined from $H_1(w)$ (for test models with differing equations of state) appears to be more reliable for the BB data set. Thus it appears that the systematic error introduced by the difference in equation of state is nearly cancelled by the errors introduced because of differences in calibration curves when the BB data set is used. On the other hand, for the HLH data set, where the calibration curves are more reliable, the difference in equation of state leads to an error in the estimate of the helium abundance.

The function $H_2(\omega)$ is not particularly sensitive to this sudden rise in errors at $l=400$, as in that case $H_2(\omega)$ would be essentially determined by the low-degree ($l<400$) modes. As a result, the value of Y as determined using $H_2(\omega)$ is not very different with the two data sets. The function $W(r)$ is essentially determined by $H_1(w)$ and we would expect these results also to be affected by the weights.

Another important difference between the two mode sets is the absence of low-frequency ($\nu<1.8$ mHz) and low-degree ($l<100$) modes in the HLH data set. Because of the lack of modes at the low-frequency end, the hump in the $H_2(\omega)$ calibration curves is not properly defined, leading to significantly higher errors in determining Y using the HLH set. The problem becomes more acute for the OPAL reference models, as in that case the humps due to interpolation errors also contribute to the uncertainties. Nevertheless, the helium abundance as determined using $H_2(\omega)$ is least sensitive to the difference in mode set and weight.

4 RESULTS FOR OBSERVED FREQUENCIES

Having tested our techniques on test models, we turn our attention to the observations. From Table 1 it can be seen that the abundances obtained using $W(r)$ and the two different equations of state are widely different from each other. On the other hand, the results obtained using $H_1(w)$ and $H_2(\omega)$ are reasonably close. The results obtained using the two different sets of observed frequencies are somewhat different from each other. We believe this is a result of systematic differences between the two sets of observed frequencies at high degrees. In particular, from the results on test models using different equations of state, it is clear that the technique involving $H_2(\omega)$ is not particularly sensitive to changes in the mode set or weights, while for observed frequencies with both sets of reference models we find that Y estimated using the HLH set is approximately 0.005 larger than the value estimated using the BB data set.

Since the results obtained using the hump in $W(r)$ appear to be very sensitive to the equation of state in the reference models, we discard these results and take the weighted average of the results obtained using $H_1(w)$ and $H_2(\omega)$ in each case. Thus for the BB set we get $Y=0.2443\pm0.0012$ using MHD reference models and $Y=0.2456\pm0.0009$ using OPAL reference models, while for the HLH set we get

$Y = 0.2456 \pm 0.0007$ and 0.2489 ± 0.0028, respectively, for the MHD and OPAL reference models. If we look at the results obtained using different equations of state we find that, for the HLH data set, the helium abundance obtained using the MHD equation of state is about 0.003 lower than that using the OPAL equation of state. A similar difference is seen in the results using $H_2(\omega)$ for the BB data set also. However, $H_1(w)$ appears to give almost identical result for the two equations of state. This could be a coincidence or a result of cancellation of actual differences by the systematic errors coming from the second hump in the calibration curves (Fig. 2).

If we compare the present results with those obtained by AB, it is clear that, even with the same BB data set and using the MHD calibration models, the results obtained using all three techniques differ by about 0.5–1 per cent. We believe that the difference is due to improvements in the models and hence we expect the current results to be more reliable. The improvement in the models can be clearly seen by comparing the functions $H_1(w)$ and $H_2(\omega)$ between our reference models and the observed frequencies, with those given by AB. Fig. 3 displays $H_1(w)$ and $H_2(\omega)$ for MHD reference models and observed frequencies from the HLH set. If we compare these figures with figs 13 and 16 of AB, it is clear that the steep trend in $H_1(w)$ at the low-w end is considerably reduced and shifted to lower w. Similarly, the variation in $H_2(\omega)$ over the entire frequency range has been reduced from about 20 s in AB to 3 s in the present work. This is due to significant improvement in the treatment of surface layers by using the Canuto–Mazzitelli prescription for calculating the convective flux, and using Kurucz's opacity table in the upper layers. Because of the substantial reduction in the gradient of $H_2(\omega)$ it is much easier to fit the calibration curves, thus giving a more reliable determination of Y.

Recently Kosovichev (1994) has found a much greater difference in the solar helium abundance as estimated using the MHD and OPAL equations of state. This could be because he has used a full non-asymptotic inversion, which may be more sensitive to equation of state variations, or because he used only those modes with $l \leq 140$ which may not be sufficient to resolve the entire He II ionization zone, leading to larger errors.

In order to find out which equation of state is closer to that of the solar material, we can compare the functions $H_1(w)$ and $H_2(\omega)$ between the Sun and reference models obtained using the two equations of state. Fig. 4 shows $H_1(w)$ and $H_2(\omega)$ using the OPAL reference models. These can be compared with Fig 3 with MHD reference models. The variation within the He II ionization zone in $H_1(w)$ will of course depend on the helium abundance in the reference model and it may be difficult to say anything about the equation of state from that variation. However, if we concentrate on the curves for $X = 0.72$ and 0.74, which bracket the solar hydrogen abundance, it is clear that, beyond the helium ionization

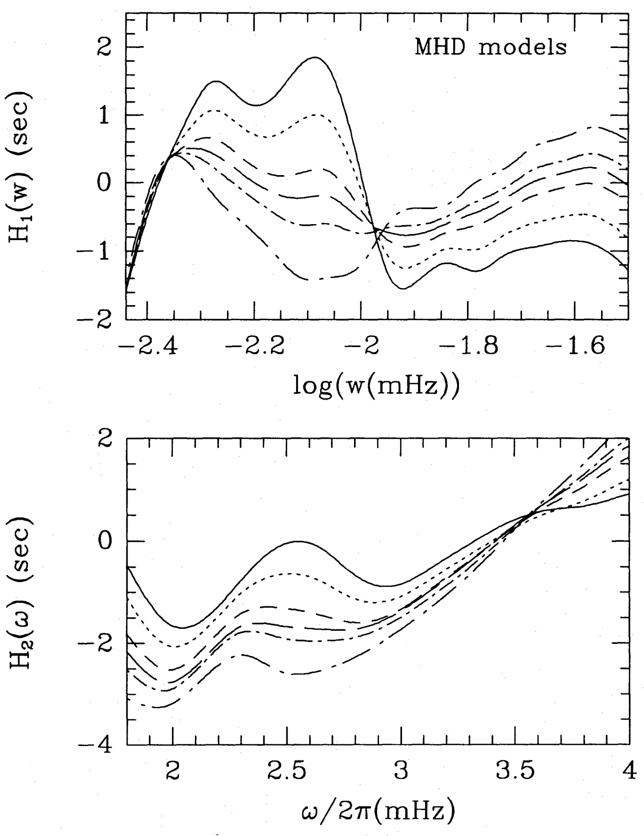

Figure 3. Comparison of the functions $H_1(w)$ and $H_2(\omega)$ between the MHD reference models MHD68 (solid line), MHD70 (dotted line), MHD72 (short dashed line), MHD73 (long dashed line), MHD74 (dot-short dashed line), MHD76 (dot-long dashed line) and the Sun.

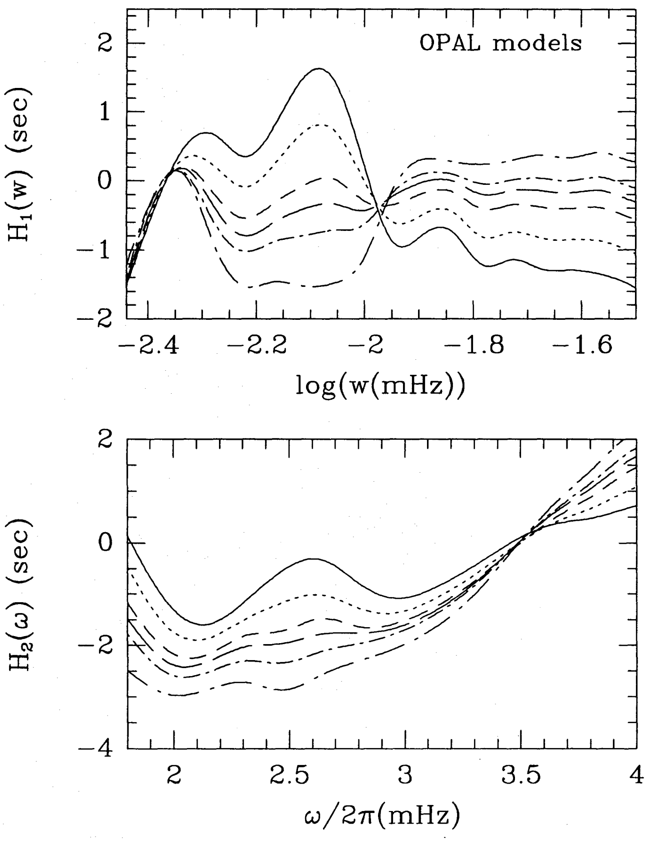

Figure 4. Comparison of the functions $H_1(w)$ and $H_2(\omega)$ between the OPAL reference models OPAL68 (solid line), OPAL70 (dotted line), OPAL72 (short dashed line), OPAL73 (long dashed line), OPAL74 (dot-short dashed line), OPAL76 (dot-long dashed line), and the Sun.

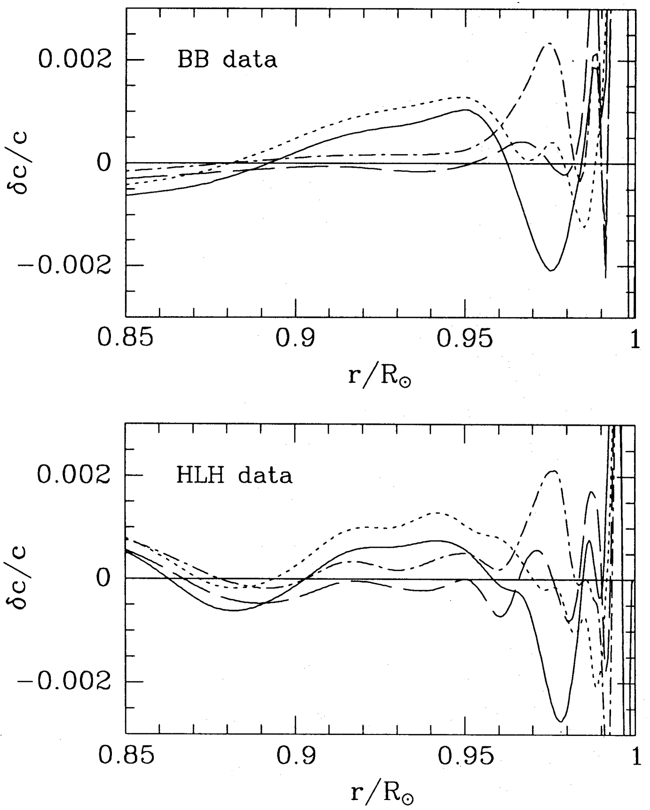

Figure 5. Relative sound speed difference $\delta c/c$ between the Sun and the reference models MHD72 (solid line), MHD74 (dotted line), OPAL72 (dashed line) and OPAL74 (dot-dashed line). The upper panel shows the results for the BB data and the lower panel for the HLH data.

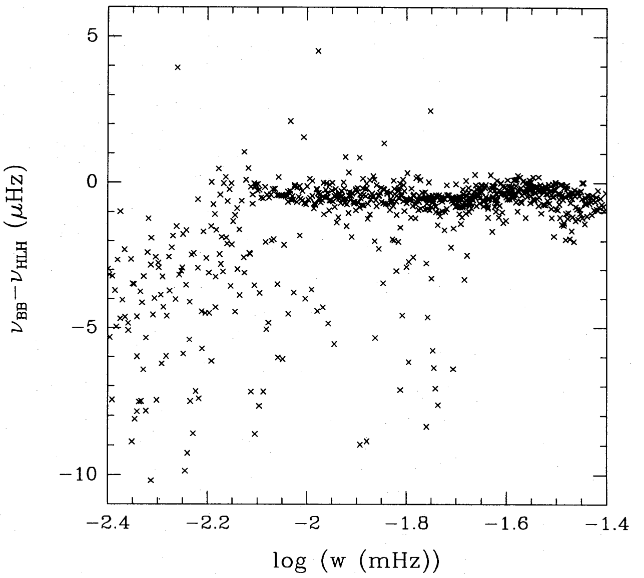

Figure 6. The frequency difference between the BB and HLH data sets is shown as a function of $w=\omega/(l+0.5)$.

zone, the curves using the OPAL equation of state are in general flatter than those using the MHD equation of state. The same is probably true for $H_2(\omega)$. Since the gradient of $H_1(w)$ determines the difference in sound speed between the reference model and the Sun, we can expect the sound speed in the OPAL model to be closer to that in the Sun. This is borne out by the results of sound speed inversion, displayed in Fig. 5. In particular the hump in the relative sound speed difference between the Sun and the solar models, around $r=0.95\ \mathrm{R}_\odot$, which was noticed by Dziembowski, Pamyatnykh & Sienkiewicz (1992) and Antia & Basu (1994b) using MHD reference models, is significantly reduced in OPAL models. It may be noted that we have used different heavy element abundances for the models with MHD and OPAL equations of state. By considering a model with $Z=0.02$ using the OPAL equation of state, we have verified that this hump is not caused by the difference in Z.

By considering the sound speed difference in the He II ionization zone around $r=0.98\ \mathrm{R}_\odot$ it also appears that the OPAL models are closer to the Sun. However, considering the errors in asymptotic inversion in this region and the variation in sound speed with the helium abundance, we are not sure if this result is significant.

5 CONCLUSIONS

We have used three techniques for estimating the helium abundance in the solar envelope using frequencies of p modes. From tests on solar models using MHD and OPAL equations of state it appears that the method based on the height of the hump in $W(r)$ is extremely sensitive to the differences in the equation of state, while the other two methods are not particularly sensitive to small differences in the equation of state. Thus, if the difference between the MHD or OPAL equation of state and that of the solar material is comparable to the difference between the two equations of state, then the determination of helium abundance using $H_1(w)$ and $H_2(\omega)$ can be fairly reliable. The errors in this case would be around 0.3 per cent.

If we consider two different sets of observed frequencies, we find that the systematic differences between these observed frequencies result in a somewhat larger difference in the helium abundance. The Libbrecht et al. (1990) data set gives a helium abundance that is about 0.3 per cent lower than that obtained from the Bachman et al. (1995) data set. Most of this difference appears to be because of systematic differences in the observed frequencies, though some of the difference could also arise from differences in the mode set and relative weights. As has been pointed out by HLH, there is a significant difference between the two sets of observed frequencies for $l>400$ and, further there is a sudden jump in frequency difference at $l=400$, arising from differences in technique used by BB in measuring the frequencies of these modes. To illustrate this difference, in Fig. 6 we give the frequency difference between the two sets of observations as a function of log w. It is clear from this figure that there is a break in this difference at a point that corresponds to modes that have turning points in the upper part of the He II ionization zone. The points lying below the main bunch at higher w are due to the high-frequency modes of high degree, which have substantial errors associated with them. These modes extend throughout the He II ionization zone, as well as some distance below it. Thus we would expect some difference between the results obtained using the two sets of observations.

Since the HLH data set is claimed to be more accurate than the BB data set we would naturally expect the results obtained using this data set to be closer to the actual value. Further, the quoted errors in the HLH data set are roughly smooth function of degree and frequency, while those in the BB data set show a sharp increase at $l=400$ giving rise to a dip in the $H_1(w)$ calibration curves around low $w=-2.2$ mHz. As a result $H_1(w)$, as calculated using weights determined by the HLH data set, is closer to the exact value than that determined using weights from the BB data set. On the other hand, although the HLH data set has a much larger number of modes in the relevant range, the low-frequency and low-degree modes are missing in this data set. This affects the determination of Y using $H_2(\omega)$, as it is not possible to define the main hump in the calibration curves properly in the absence of low-frequency data.

Using the HLH data set we obtain a helium abundance $Y=0.246 \pm 0.001$, which is the weighted average of the values obtained using $H_1(w)$ and $H_2(\omega)$ with the two equations of state. It is not clear if it is meaningful to combine results with different systematic errors by taking the weighted average, but since the difference between these four measurements is not substantial, we believe that the resulting average would not be too far from the actual value. Compared to this, the BB data set yields $Y=0.245 \pm 0.001$. The difference between our current results and those of AB is mainly due to improvements in the reference models which are significantly closer to the Sun, because of better treatment of convective flux and atmospheric opacities. The frequency difference between the best reference model and the Sun has reduced from about 180 μHz for models used by AB to about 13 μHz in the present case.

Further, by comparing $H_1(w)$ between observed frequencies and the two sets of reference models, it appears that the OPAL equation of state is closer to that of the solar material. This conclusion is also borne out by sound speed inversion, which shows that the hump in relative sound speed difference around $r=0.095$ $R_\odot$, which is present in the MHD reference models, reduces significantly when the OPAL equation of state is used.

ACKNOWLEDGMENTS

We thank F. Rogers for supplying the OPAL equation of state tables, and R. L. Kurucz for supplying the opacity tables. This work was supported in part by the Danish National Research Foundation through its establishment of the Theoretical Astrophysics Centre.

REFERENCES

Antia H. M., Basu S., 1994a, ApJ, 426, 801 (AB)
Antia H. M., Basu S., 1994b, A&AS, 107, 421
Bachman K. T., Duvall T. L. Jr., Harvey J. W., Hill F., 1995, ApJ, 443, 837
Basu S., Antia H. M., 1994, JA&A, 15, 143
Canuto V. M., Mazzitelli I., 1991, ApJ, 370, 295
Christensen-Dalsgaard J., Gough D. O., Thompson M. J., 1989, MNRAS, 238, 481
Christensen-Dalsgaard J., Gough D. O., Thompson M. J., 1991, ApJ, 378, 413
Christensen-Dalsgaard J., Pérez Hernández F., 1991, in Gough D. O., Toome J., eds, Challenges to Theories of the Structure of Moderate-mass Stars. Springer, Heidelberg, p. 43
Däppen W., Gough D. O., Kosovichev A. G., Thompson M. J., 1991, in Gough D. O., Toomre J., eds, Challenges to Theories of the Structure of Moderate-mass Stars. Springer, Heidelberg, p. 111
Däppen W., Mihalas D., Hummer D. G., Mihalas B. W., 1988, ApJ, 332, 261
Dziembowski W. A., Pamyatnykh A. A., Sienkiewicz R., 1992, Acta Astron., 42, 5.
Gough D. O., 1984, Mem. Soc. Astron. Ital., 55, 13
Hummer D. G., Mihalas D., 1988, ApJ, 331, 794
Kosovichev A. G., 1994, Adv. Space Res., in press
Kosovichev A. G., Christensen-Dalsgaard J., Däppen W., Dziembowski W. A., Gough D. O., Thompson M. J., 1992, MNRAS, 259, 536
Kurucz R. L., 1991, in Crivellari L., Hubeny I., Hummer D. G., eds, NATO ASI Series, Stellar Atmospheres: Beyond Classical Models. Kluwer, Dordrecht, p. 441
Libbrecht K. G., Woodard M. F., Kaufman J. M., 1990, ApJS, 74, 1129
Mihalas D., Däppen W., Hummer D. G., 1988, ApJ, 331, 815
Paternò L., Ventura R., Canuto V. M., Mazzitelli I., 1993, ApJ, 402, 733
Rogers F. J., 1994, in Chabrier G., Schatzman E., eds, IAU Colloq. 147. The Equation of State in Astrophysics, Cambridge Univ. Press, Cambridge
Rogers F. J., Iglesias C. A., 1992, ApJS, 79, 507
Vorontsov S. V., Baturin V. A., Pamyatnykh A. A., 1992, MNRAS, 257, 32