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Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data*

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Abstract

This paper analyses the importance of scale economies by means of unbalanced plant-level panel data from three Norwegian manufacturing industries. Focus is on heterogeneous technologies, and unlike most previous work on micro data, the model description includes heterogeneity in both the scale properties (the slope coefficients) and the intercept term, represented by random coefficients in the production function. Three (nested) functional forms are investigated: the Translog, an extended Cobb-Douglas, and the strict Cobb-Douglas. Although constant or moderately increasing returns to scale is found for the average plant, the results reveal considerable variation across plants. Variations in both input and scale elasticities are to a larger extent due to randomness of the production function parameters than to systematic differences in the input mix.

JEL classification: C23, D24, L61, L65, L73

Keywords: panel data, economies of scale, heterogeneity, random coefficients

1. Introduction

The scale properties of production technologies are of vital importance for our understanding of market structure, productivity, and economic growth, and, within the industrial economics literature, economies of scale is put forward as a possible important barrier to entry, see, e.g., Tirole (1989, pp. 305–306) and the references therein. Hence, knowing the scale properties

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may help us understand the evolution of industries. Although there are important exceptions, it is common in empirical analyses of the production process, applying both the primal and the dual approach, to assume a constant returns to scale technology. This is true for analyses using both micro and macro data. One explanation for this restriction is co-movements of the explanatory variables that make it difficult to identify independently the impacts of technical change, capital stock growth, and returns to scale; cf. *inter alia,* Morrison (1988) and Biørn, Lindquist and Skjerpen (2003). However, if the constant returns to scale restriction is false, this is likely to influence conclusions regarding technical change and productivity.

There is a growing number of articles that analyse the production process econometrically using micro data under the assumption that heterogeneity in size, age, management, employees' education, technology, etc., can be represented by a plant specific fixed or random intercept term in the production, cost, or profit function. Most likely, however, such differences will manifest themselves not only as a permanent variation in efficiency across plants, but will also result in heterogeneity in scale properties. In this case, the standard modelling approach, with only fixed or random effects in intercept terms, may lead to inefficient estimation of the slope coefficients and invalid inference.

In this paper we choose a more general approach and analyse the importance of scale economies by estimating a four-factor (*KLEM*) production function with heterogeneous scale properties and no *a priori* restrictions on the returns to scale. Three (nested) functional forms of the production function are investigated: the Translog, an extended Cobb-Douglas, and the strict Cobb-Douglas. Heterogeneity in both the slope coefficients and the intercept term is allowed for. A random coefficient approach, with specific assumptions made about the distribution from which the plant specific coefficients are drawn, is applied. This is a parsimonious and easily interpretable way of representing heterogeneity. The expectation vector in this distribution represents the coefficients of an average plant, while its covariance matrix gives readily interpretable measures of the degree of heterogeneity which is due to the random coefficient variation. In addition, the non-homotheticity of the production function allows for systematic variation in the scale elasticity, i.e., variation with the input quantities. The purpose of this paper is to quantify both the random and the systematic variation of the scale elasticity.

Our primary argument for using the primal approach and not following the alternative dual approach is our focus on heterogeneity in the production function parameters rather than in the parameters of the cost or profit function. Arguments for taking the primal approach, even if the agents follow optimising behaviour, have been given by, *inter alia,* Zellner, Kmenta and Drèze (1966) and Mundlak (1996) in a Cobb-Douglas context; see also Griffiths and Anderson (1982), Mairesse (1990), Mairesse and Griliches (1990), Wan, Griffiths and Anderson (1992), and Griliches and Mairesse (1998, Section 2). Our approach differs from that in the panel data literature on *frontier production functions* and efficiency measurement, dealing with deterministic or stochastic production frontiers in a framework with firm specific heterogeneity; cf. Cornwell and Schmidt (1996).

The panel data set applied is from the Norwegian manufacturing statistics data base of Statistics Norway. It is *unbalanced* and consists of plants from the Pulp and paper industries, the Chemical industries, and the Basic metals industries in Norway. We follow the recommendations in Mátyás and Lovrics (1991) and Baltagi and Chang (1994) and do not omit observations to make the panel balanced. The combination of a random coefficient

model and unbalanced panel data is far from standard, at least in applied econometrics. Mixed regression models with unbalanced design, however, have, to some extent, been discussed in the statistical literature, see, e.g., Amemiya (1994) and Shin (1995). Random coefficients in regression equations in econometrics are treated in the pioneering studies of Swamy (1970, 1971, 1974); see also Hsiao (1975, 1996) and Longford (1995a,b).

A major finding is that substantial improvement in model fit is obtained when allowing for random coefficient heterogeneity. We find constant or moderately increasing returns to scale for a plant with an average technology, but the results reveal important variation across plants, and plants with both increasing and decreasing economies of scale are present.

2. Model and Econometric Method

We assume that the average plant has a four-factor technology, with capital (K), labour (L), energy (E), and materials (M) as inputs and with one output (Y). The most general specification of the technology is assumed to be non-homothetic and is represented by a production function belonging to the Translog class, with a trend, and with some coefficients specified as random variables. This random variation represents non-systematic heterogeneity of the technology. Below we describe the basic elements of our model, for simplicity without explicitly incorporating the unbalancedness of the panel data set. The accommodation of the model to our unbalanced panel data and the Maximum Likelihood estimation procedure is elaborated in Appendix A.

Let subscripts i and t denote the plant and the year (number) of observation, respectively. Our model framework, when we suppress the industry subscript, can be written as

$$y_{it} = c_i + \gamma\tau_t + \frac{1}{2}\gamma^*\tau_t^2 + z_{it}'\alpha_i + \frac{1}{2}z_{it}'Bz_{it} + z_{it}'\delta\tau_t + u_{it}, \tag{1}$$

where $y_{it} = \ln(Y_{it})$, $z_{it} = [\ln(K_{it}), \ln(L_{it}), \ln(E_{it}), \ln(M_{it})]'$, c_i is a plant specific random intercept term, τ_t is a deterministic trend representing the level of the technology in year t, and u_{it} is a genuine disturbance term. The plant dependent vector α_i is specified as random, and the matrix and vector of second-order coefficients, B (symmetric) and δ, as well as γ and γ^*, as constants:[1]

$$\alpha_i = [\alpha_{Ki}, \alpha_{Li}, \alpha_{Ei}, \alpha_{Mi}]', \quad B = [\beta_{jk}], \quad j,k = K, L, E, M, \quad \delta = [\delta_K, \delta_L, \delta_E, \delta_M]'.$$

We also consider simpler models, as will be explained below.

The heterogeneity of the coefficient structure is represented as follows. Let θ_i denote the column vector containing all the (random or fixed) coefficients in the model, i.e.,

$$\theta_i = [c_i,\ \alpha_i',\ \gamma,\ \gamma^*,\ \beta',\ \delta']', \tag{2}$$

where $\beta = \text{vech}\, B$ is the half-vectorisation of B, i.e., the lower triangular part of B stacked into a column vector. We assume that all z_{it}, u_{it}, and θ_i's are mutually independent, with

$\mathsf{E}(u_{it}) = 0$, $\text{var}(u_{it}) = \sigma_{uu}$, and

$$\mathsf{E}(\theta_i) = \theta = [c, \alpha', \gamma, \gamma^*, \beta', \delta']', \qquad \mathsf{E}[(\theta_i - \theta)(\theta_i - \theta)'] = \mathbf{\Omega} = \begin{bmatrix} \omega_{cc} & \Omega'_{\alpha c} & 0 \\ \Omega_{\alpha c} & \Omega_{\alpha\alpha} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (3)$$

where $\alpha = \mathsf{E}(\alpha_i)$, $\omega_{cc} = \text{var}(c_i)$, $\Omega_{\alpha\alpha} = \mathsf{E}[(\alpha_i - \alpha)(\alpha_i - \alpha)']$, etc., and the zero sub-matrices of Ω, involving non-random coefficients, have suitable dimensions.

We denote the model with diagonal B, $\delta = 0, \gamma^* = 0$ as the Extended Cobb-Douglas model, $B = 0, \delta = 0, \gamma^* = 0$ giving a strict Cobb-Douglas model. The Extended Cobb-Douglas model implies that the elasticity of output with respect to a specific input (input elasticity, for short) depends on the volume of that input, but is independent of the other inputs. This is in contrast with the more flexible Translog model, in which the input elasticities depend on all inputs. Both the Translog and the Extended Cobb-Douglas technologies are non-homothetic, with variable scale elasticity.

The (column) vector of *input elasticities* of plant i in year t is

$$\eta_{it} = [\eta_{Kit}, \eta_{Lit}, \eta_{Eit}, \eta_{Mit}]' = \frac{\partial y_{it}}{\partial z_{it}} = \alpha_i + B z_{it} + \delta \tau_t, \quad (4)$$

the derivative of log-output with respect to the trend is

$$\eta_{\tau it} = \frac{\partial y_{it}}{\partial \tau_t} = \gamma + \gamma^* \tau_t + \delta' z_{it}, \quad (5)$$

and the *scale elasticity* of plant i in year t is

$$\nu_{it} = e_4' \eta_{it} = \bar{\alpha}_i + \bar{\beta}' z_{it} + \bar{\delta} \tau_t, \quad (6)$$

where e_n is the n vector of ones and $\bar{\alpha}_i = e_4' \alpha_i$, $\bar{\beta}' = e_4' B$, $\bar{\delta} = e_4' \delta$. Conditionally on z_{it}, the random variation of the input elasticities and the scale elasticity is due to the randomness of the first-order coefficients in the production function, α_i, only. The expectation and variance of the scale elasticity of plant i in year t, conditionally on the input vector z_{it}, can be written as, respectively,

$$\mathsf{E}(\nu_{it} \mid z_{it}) = \mathsf{E}(\bar{\alpha}) + \bar{\beta}' z_{it} + \bar{\delta} \tau_t, \quad (7)$$

$$\text{var}(\nu_{it} \mid z_{it}) = \text{var}(\bar{\alpha}), \quad (8)$$

where $\mathsf{E}(\bar{\alpha}) = e_4' \alpha$ and $\text{var}(\bar{\alpha}) = e_4' \Omega_{\alpha\alpha} e_4$. We can interpret $\text{var}(\nu_{it} \mid z_{it})$ as the dispersion of the random heterogeneity of the scale elasticity. Similar expressions can be derived for the input elasticities. Using (7), (8), and the law of iterated expectations, the marginal (unconditional) expectation and variance of the scale elasticity can be written as

$$\mathsf{E}(\nu_{it}) = \mathsf{E}[\mathsf{E}(\nu_{it} \mid z_{it})] = \mathsf{E}(\bar{\alpha}) + \bar{\beta}' \mathsf{E}(z_{it}) + \bar{\delta} \tau_t = e_4' \alpha + e_4' B \mathsf{E}(z_{it}) + e_4' \delta \tau_t, \quad (9)$$

$$\begin{aligned} \text{var}(\nu_{it}) &= \mathsf{E}[\text{var}(\nu_{it} \mid z_{it})] + \text{var}[\mathsf{E}(\nu_{it} \mid z_{it})] = \text{var}(\bar{\alpha}) + \bar{\beta}' \mathsf{V}(z_{it}) \bar{\beta} \\ &= e_4' \Omega_{\alpha\alpha} e_4 + e_4' B \mathsf{V}(z_{it}) B e_4, \end{aligned} \quad (10)$$

where $\mathsf{V}(z_{it})$ is the covariance matrix of z_{it}. Equation (10) represents jointly the heterogeneity in the scale elasticity which is due to the stochastic variation in the first-order coefficients

Table 1. Models.

Model	ω_{cc}	$\Omega_{\alpha\alpha}$	$\Omega_{\alpha c}$	B	$\delta,\ \gamma^*$
TL(c, α)	Unrestricted	Unrestricted	Unrestricted	Unrestricted	Unrestricted
TL(c)	Unrestricted	0	0	Unrestricted	Unrestricted
TL	0	0	0	Unrestricted	Unrestricted
ECD(c, α)	Unrestricted	Unrestricted	Unrestricted	Diagonal	0
ECD(c)	Unrestricted	0	0	Diagonal	0
ECD	0	0	0	Diagonal	0
CD(c, α)	Unrestricted	Unrestricted	Unrestricted	0	0
CD(c)	Unrestricted	0	0	0	0
CD	0	0	0	0	0

(the first term) and the heterogeneity which is due to the variation in the input vector across plants (the second term). Similar expressions can be obtained for the input elasticities.

Nine models, that differ with respect to functional form and the representation of the heterogeneity of the technology, are considered. As abbreviations for Translog, Extended Cobb-Douglas, and strict Cobb-Douglas we use TL, ECD, and CD, respectively—in parenthesis indicating which coefficients are treated as random in each model. The models are specified in Table 1.

The expected coefficient vector θ and the unknown elements of the covariance matrix $\mathbf{\Omega}$, for the different models, are estimated by Maximum Likelihood, using the PROC MIXED procedure in the SAS/STAT software (see Littell et al., 1996). Positive definiteness of the non-zero submatrix of $\mathbf{\Omega}$ (relating to the random coefficients) is imposed as an *a priori* restriction. Details are given in Appendix A.

3. Empirical Results

Goodness of fit. Table 2 reports the goodness of fit of all the estimated models, expressed in terms of the log-likelihood value (LLH),[2] Akaike's Information Criterion (AIC), and Schwarz's Bayesian Criterion (SBC).[3] Within models with the same functional form, these three criteria give identical ranking with respect to the specification of heterogeneity: the models that include heterogeneity in both the intercept term and the first-order coefficients in the production function, i.e., c_i and α_i, give a clearly better fit than the models with heterogeneity only in the intercept term, and a markedly better fit than the models with no coefficient heterogeneity. There is thus evidence that allowing for random heterogeneity in the α_i coefficients improves the fit to our plant panel data.

Concentrating on the functional form while using models with the same specification of heterogeneity, the picture is somewhat less clear. According to the AIC criterion, TL$(\cdot)$ outperforms both ECD$(\cdot)$ and CD$(\cdot)$ in all industries. The SBC criterion, which penalizes coefficient-rich models relatively harder than AIC, ranks CD(c, α) first for all industries. The estimates of the genuine disturbance variance, σ_{uu}, support our general conclusion that the fit improves when more heterogeneity is allowed for and/or the flexibility of the functional form is increased.

Table 2. Model fitting information for the various models in the three industries.[a]

	Model								
	TL(c,α)	ECD(c,α)	CD(c,α)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
Pulp and paper									
q^b	37	26	22	23	12	8	22	11	7
LLH	−330.65	−358.77	−364.73	−725.83	−764.85	−779.22	−2250.15	−2368.11	−2409.85
AIC	−367.65	−384.77	−386.73	−748.83	−776.85	−787.22	−2272.15	−2379.11	−2416.85
SBC	−477.65	−462.06	−452.14	−817.20	−812.52	−811.00	−2337.55	−2411.81	−2437.66
MSPE[c]	0.2790	0.2760	0.2757	0.3114	0.3167	0.3165	0.5319	0.5484	0.5494
σ_{uu}	0.0397	0.0406	0.0408	0.0708	0.0729	0.0734	0.2883	0.3134	0.3228
ω_{cc}	5.9590	5.7793	5.9336	0.3789	0.3821	0.4003	0[d]	0[d]	0[d]
φ	0.9034	0.8986	0.9085	0.8426	0.8398	0.8450	0[d]	0[d]	0[d]
Chemicals									
q^b	37	26	22	23	12	8	22	11	7
LLH	−1350.06	−1371.88	−1374.42	−1539.89	−1601.92	−1624.17	−2102.14	−2151.74	−2179.96
AIC	−1387.06	−1387.88	−1396.42	−1562.89	−1613.92	−1632.17	−2124.14	−2162.74	−2186.96
SBC	−1482.19	−1464.73	−1452.98	−1622.02	−1644.77	−1652.74	−2180.70	−2191.02	−2204.96
MSPE[ce]	..	..	..	1.1277	1.1544	1.1549	1.6051	1.6276	1.6485
σ_{uu}	0.2926	0.3019	0.3019	0.5214	0.5711	0.6028	1.6295	1.7626	1.8431
ω_{cc}	23.6710	24.6901	25.0253	1.5646	1.5791	1.5196	0[d]	0[d]	0[d]
φ	0.9082	0.8993	0.9006	0.7501	0.7307	0.7160	0[d]	0[d]	0[d]
Basic metals									
q^b	37	26	22	23	12	8	22	11	7
LLH	−1041.70	−1076.64	−1083.00	−1223.51	−1279.51	−1284.46	−2533.11	−2622.37	−2654.60
AIC	−1078.70	−1102.64	−1099.00	−1246.51	−1291.51	−1292.46	−2555.11	−2633.37	−2661.60
SBC	−1183.02	−1175.95	−1144.11	−1311.36	−1325.35	−1315.02	−2617.14	−2664.39	−2681.34
MSPE[c]	0.4876	0.4907	0.4965	0.5158	0.5280	0.5268	0.8493	0.9002	0.9231
σ_{uu}	0.0968	0.0984	0.0986	0.1409	0.1483	0.1490	0.6704	0.7306	0.7536
ω_{cc}	2.7431	3.3558	3.5973	0.6517	0.7084	0.7109	0[d]	0[d]	0[d]
φ	0.8196	0.8212	0.8293	0.8223	0.8269	0.8267	0[d]	0[d]	0[d]

[a] LLH is the Log likelihood value; AIC is Akaike's Information Criterion; SBC is Schwarz's Bayesian Criterion; MSPE is the mean square prediction error.

[b] q is the number of parameters in the model.

[c] Pulp and paper: Based on 442 predictions for the years 1989–1993; Chemicals: Based on 233 predictions for the years 1989–1993; Basic metals: Based on 376 predictions for the years 1989–1993.

[d] A priori restriction.

[e] MSPE is not available for the random coefficient models due to lack of convergence in reduced samples.

It is also of interest to compare the models with regard to their prediction performance. We predict log-output over the years 1989–1993. For each year we utilise predicted (or estimated in the case of non-random parameters) coefficients, cf. Appendix A, based on observations up to the previous year. Letting $\widehat{y}_{is}^{s-1}$ denote the predicted log-output of plant i in calendar year s based on observations up to year $s-1$ and y_{is} the value realized, we define the mean square prediction error as $\text{MSPE} = [(1/M)\sum_i \sum_{s\in S(i)}(y_{is} - \widehat{y}_{is}^{s-1})^2]^{1/2}$, where $S(i)$ is the set of years for which we make predictions for plant i and M is the number of predictions.[4] The MSPEs are given in Table 2. Due to lack of convergence when estimating

the random coefficent models on reduced samples for Chemicals, which had the smallest number of observations among the three industries, we are unable to report MSPEs for these models. Again we find that adding a random intercept improves the prediction considerably and that further improvement is obtained when also the first order coefficients are random. For the random coefficient models there are small differences with regard to MSPE. In Pulp and paper the ranking of the models agrees with that of the SBC-criterion. In Basic metals the ranking is the opposite and the lowest MSPE is found for Model TL(c, α), which also was the best-fitting model according to the AIC. Our overall conclusions from the prediction experiments thus agree well with those following from the information criteria.

Degree of coefficient heterogeneity. The last row of each panel of Table 2 gives a measure of the overall degree of coefficient (including intercept) heterogeneity. The measure is the estimated share, φ—at the overall sample mean of the inputs,[5] of the variances of the gross disturbance $\psi_{(ip)t}$ [cf. (A.3) and (A.6)]—which is due to coefficient heterogeneity. In the model with random intercept c_i only, it is the estimated value of $\varphi = \omega_{cc}/(\omega_{cc} + \sigma_{uu})$. In the random coefficients model, the share is the estimated value of $\varphi = w'\Omega_w w/(w'\Omega_w w + \sigma_{uu})$, where w is the 5×1 vector with 1 in the first position and the log of the overall mean of the inputs in the remaining positions, and Ω_w is the sub-matrix of $\mathbf{\Omega}$ which corresponds to the random coefficients, cf. (3). A very high share of the total variance is due to coefficient heterogeneity: 72–85 per cent in the models with random intercept term and 82–91 per cent in the models that also include random first-order coefficients.

Mean input and scale elasticities. The complete set of (mean) coefficient estimates in the various models is given in Biørn, Lindquist and Skjerpen (2000a, Tables A2–A4) and is not, for lack of space, reported here. The derived estimates of the expected input elasticities, the expected scale elasticity, and the expected trend effect—all calculated at the overall mean of the inputs—are given in Table 3.[6] The expected scale elasticity is relatively stable across models and clearly indicates weakly increasing or constant returns to scale for Pulp and paper and Basic metals. The estimates for Chemicals are more variable and both Models TL(c, α) and ECD(c, α) show increasing returns to scale, with scale elasticities in the range 1.3–1.4.

Overall, the estimated expected input elasticities at the sample mean show larger variability across models than does the scale elasticity. Most estimates have the expected positive sign, the exception is the labour elasticity in six of the nine cases that do not include coefficient heterogeneity. A weakly, although not significantly, negative[7] labour elasticity is also found in Model TL(c, α) in Basic metals, however. Comparing columns 4–6 with columns 1–3 in Table 3, it is clear that when we allow for randomness of the α_i's, the standard deviation estimates of the expected coefficients increase substantially—in most cases to almost the double value. This seems to be a consequence of allowing a less restrictive model specification.

Trend effects. The trend variable τ_t is represented by the calendar year. Excepting the three models with no heterogeneity in Pulp and paper, the estimated (sample mean) trend derivative η_τ (Table 3) is significantly positive in all models. The positive values vary around 0.7 per cent in Pulp and paper, around 3–4 per cent in Chemicals, and around 2 per cent in

Table 3. Estimated scale properties. Standard errors in parentheses.[a]

	Model								
	TL(c,α)	ECD(c,α)	CD(c,α)	TL(c)	ECD(c)	CD(c)	TL	ECD	CD
Pulp and paper									
η_K	0.2677	0.2448	0.2503	0.1275	0.1532	0.1717	0.1093	0.0969	0.0735
	(0.0469)	(0.0455)	(0.0344)	(0.0296)	(0.0277)	(0.0197)	(0.0272)	(0.0264)	(0.0187)
η_L	0.1485	0.1476	0.1717	0.1096	0.1330	0.1863	−0.4189	−0.3036	−0.2314
	(0.0514)	(0.0468)	(0.0381)	(0.0301)	(0.0278)	(0.0215)	(0.0270)	(0.0251)	(0.0206)
η_E	0.1339	0.1487	0.0854	0.1649	0.1582	0.0921	0.4936	0.3995	0.3333
	(0.0256)	(0.0253)	(0.0169)	(0.0189)	(0.0175)	(0.0103)	(0.0190)	(0.0166)	(0.0099)
η_M	0.5136	0.5183	0.5666	0.6367	0.6243	0.6064	0.7074	0.7586	0.7530
	(0.0421)	(0.0406)	(0.0309)	(0.0263)	(0.0239)	(0.0167)	(0.0246)	(0.0226)	(0.0160)
ν	1.0637	1.0595	1.0740	1.0386	1.0687	1.0564	0.8914	0.9514	0.9284
	(0.0426)	(0.0411)	(0.0287)	(0.0302)	(0.0291)	(0.0186)	(0.0139)	(0.0135)	(0.0095)
η_τ	0.0067	0.0065	0.0065	0.0069	0.0074	0.0084	−0.0093	−0.0005	−0.0002
	(0.0017)	(0.0013)	(0.0013)	(0.0016)	(0.0012)	(0.0012)	(0.0026)	(0.0019)	(0.0019)
Chemicals									
η_K	0.5201	0.4490	0.1270	0.5020	0.3986	0.0713	0.7839	0.7557	0.4646
	(0.1954)	(0.1905)	(0.1149)	(0.1176)	(0.1150)	(0.0667)	(0.0846)	(0.0826)	(0.0577)
η_L	0.4215	0.3457	0.3117	0.2057	0.2618	0.4711	−0.2408	−0.1379	0.0537
	(0.2245)	(0.2089)	(0.1605)	(0.1150)	(0.0978)	(0.0763)	(0.1041)	(0.0883)	(0.0733)
η_E	0.1500	0.1698	0.2156	0.1548	0.1916	0.2244	0.2504	0.2247	0.3046
	(0.1089)	(0.0999)	(0.0718)	(0.0646)	(0.0594)	(0.0368)	(0.0618)	(0.0575)	(0.0350)
η_M	0.3389	0.3478	0.3544	0.4548	0.5245	0.2530	0.2341	0.2146	0.1825
	(0.1321)	(0.1250)	(0.0968)	(0.0807)	(0.0725)	(0.0482)	(0.0693)	(0.0597)	(0.0404)
ν	1.4305	1.3123	1.0087	1.3172	1.3763	1.0199	1.0276	1.0570	1.0053
	(0.1918)	(0.1821)	(0.1062)	(0.1085)	(0.1059)	(0.0606)	(0.0435)	(0.0437)	(0.0292)
η_τ	0.0375	0.0323	0.0306	0.0329	0.0384	0.0422	0.0243	0.0253	0.0238
	(0.0067)	(0.0048)	(0.0047)	(0.0066)	(0.0046)	(0.0045)	(0.0098)	(0.0067)	(0.0068)
Basic metals									
η_K	0.1806	0.0270	0.1246	0.1180	0.0619	0.0944	0.1039	0.0103	0.1438
	(0.0830)	(0.0732)	(0.0472)	(0.0587)	(0.0461)	(0.0273)	(0.0564)	(0.0437)	(0.0280)
η_L	−0.0316	0.2400	0.2749	0.0381	0.2257	0.3073	−0.0648	0.1149	0.1629
	(0.0847)	(0.0702)	(0.0550)	(0.0602)	(0.0444)	(0.0351)	(0.0721)	(0.0484)	(0.0360)
η_E	0.4440	0.3970	0.2138	0.3010	0.1734	0.1628	0.3857	0.2478	0.1502
	(0.0635)	(0.0618)	(0.0374)	(0.0437)	(0.0330)	(0.0174)	(0.0386)	(0.0293)	(0.0190)
η_M	0.4262	0.3960	0.4928	0.5521	0.5666	0.5210	0.5411	0.6446	0.6868
	(0.0640)	(0.0598)	(0.0406)	(0.0452)	(0.0356)	(0.0235)	(0.0480)	(0.0324)	(0.0217)
ν	1.0192	1.0600	1.1061	1.0091	1.0276	1.0856	0.9660	1.0176	1.1438
	(0.0570)	(0.0535)	(0.0324)	(0.0510)	(0.0492)	(0.0271)	(0.0999)	(0.0228)	(0.0142)
η_τ	0.0156	0.0220	0.0214	0.0153	0.0215	0.0228	0.0246	0.0211	0.0220
	(0.0035)	(0.0022)	(0.0021)	(0.0034)	(0.0020)	(0.0020)	(0.0059)	(0.0036)	(0.0036)

[a]The elasticity of output with respect to a specific input j (η_j), the scale elasticity (ν) and the derivative of the log of output with respect to time (η_τ) are evaluated at the overall empirical mean and at the expectation of random coefficients.

Table 4. The distribution of plant specific coefficients in model ECD(c, α). Variances along the main diagonal and correlation coefficients below.

	c_i	α_{Ki}	α_{Li}	α_{Ei}	α_{Mi}
Pulp and paper					
c_i	5.7793				
α_{Ki}	−0.4419	0.1163			
α_{Li}	−0.7111	−0.0850	0.1501		
α_{Ei}	0.3424	−0.4025	−0.2504	0.0231	
α_{Mi}	0.3791	−0.5748	−0.4264	0.0707	0.1075
Chemicals					
c_i	24.6901				
α_{Ki}	−0.1680	0.5284			
α_{Li}	−0.7909	−0.3138	1.2903		
α_{Ei}	0.4052	0.0071	−0.5199	0.2408	
α_{Mi}	0.1818	−0.3718	−0.2111	−0.3161	0.4423
Basic metals					
c_i	3.3558				
α_{Ki}	−0.0698	0.1611			
α_{Li}	−0.6744	−0.5664	0.1753		
α_{Ei}	0.2091	−0.6226	0.2600	0.1004	
α_{Mi}	0.2153	0.0698	−0.4188	−0.6390	0.1335

Basic metals. This is consistent with industry specific R&D costs, since there probably is a connection between cumulated R&D costs and technical progress. The Chemical industry invests much more in R&D than the other two industries, measured both in NOK and as a share of value added.

Distribution of the random coefficients. Tables 4–6 characterize, in different ways, plant heterogeneity. We give results for the ECD(c, α)-specification; the results for the other functional forms are reported in Biørn, Lindquist and Skjerpen (2000a, Tables 4–6). The results are in general very robust to the form of the average production function. Table 4 reports the covariance matrix of the random coefficients, with variance estimates along the main diagonal and correlation coefficient estimates below. The majority of the correlation coefficients are negative, and in several cases, their absolute values are quite large. Hence, a relatively high coefficient of one input is often matched with a relatively low coefficient of the other inputs.

Predicted input and scale elasticities: Random and systematic heterogeneity. From predictions of plant specific random coefficients, cf. Appendix A, we can obtain plant specific scale elasticities, v_i, and input elasticities, $(\eta_{Ki}, \eta_{Li}, \eta_{Ei}, \eta_{Mi})$. Tables 5 and 6 report descriptive statistics of these plant specific predictions. These tables represent both the *random* heterogeneity, i.e., due to the random coefficient variation, and the *systematic* heterogeneity which is due to differences in the input mix across plants; cf. (10). The range of the predicted scale

Table 5. Descriptive statistics of plant specific predicted elasticities for model ECD(c, α).

Predicted Elasticities	Mean	Std. Error	Coef. of Variation[a]	Minimum Value	Maximum Value	Share of Values < 0[b]
Pulp and paper						
$\hat{\upsilon}$	1.0685	0.1906	0.1784	0.4176	2.0975	0.0
$\hat{\eta}_K$	0.2511	0.2491	0.9922	−0.9079	1.4487	8.0
$\hat{\eta}_L$	0.1667	0.2768	1.6602	−0.8692	1.6395	20.7
$\hat{\eta}_E$	0.0875	0.1088	1.2425	−0.4752	0.4150	15.6
$\hat{\eta}_M$	0.5632	0.2365	0.4199	−0.5824	2.6413	1.3
Chemicals						
$\hat{\upsilon}$	1.0381	0.5750	0.5539	−0.2242	4.4304	3.3
$\hat{\eta}_K$	0.1595	0.5753	3.6073	−1.4167	2.0343	40.0
$\hat{\eta}_L$	0.3065	0.8805	2.8730	−2.4537	4.3556	31.1
$\hat{\eta}_E$	0.2106	0.3708	1.7605	−0.9477	1.8410	21.1
$\hat{\eta}_M$	0.3615	0.4989	1.3800	−2.1927	2.0873	15.6
Basic metals						
$\hat{\upsilon}$	1.0687	0.1315	0.1230	0.7210	1.5817	0.0
$\hat{\eta}_K$	0.0956	0.3085	3.2250	−1.6831	0.8331	34.9
$\hat{\eta}_L$	0.2665	0.2906	1.0904	−0.4398	1.5265	12.7
$\hat{\eta}_E$	0.2238	0.2687	1.2007	−0.9921	1.3472	15.7
$\hat{\eta}_M$	0.4828	0.2631	0.5451	−0.2515	1.5322	3.6

[a]Defined as the standard error divided by the mean.
[b]In percentage of the total number of plants.

Table 6. The distribution of plant specific predicted elasticities from Model ECD(c, α). Variances along the main diagonal and correlation coefficients below.

	$\hat{\upsilon}$	$\hat{\eta}_K$	$\hat{\eta}_L$	$\hat{\eta}_E$	$\hat{\eta}_M$
Pulp and paper					
$\hat{\upsilon}$	0.036				
$\hat{\eta}_K$	0.162	0.062			
$\hat{\eta}_L$	0.778	−0.009	0.077		
$\hat{\eta}_E$	−0.294	−0.447	−0.336	0.012	
$\hat{\eta}_M$	−0.140	−0.707	−0.380	0.167	0.056
Chemicals					
$\hat{\upsilon}$	0.331				
$\hat{\eta}_K$	0.233	0.331			
$\hat{\eta}_L$	0.689	−0.325	0.775		
$\hat{\eta}_E$	−0.474	0.042	−0.556	0.137	
$\hat{\eta}_M$	0.019	−0.341	−0.182	−0.356	0.249
Basic metals					
$\hat{\upsilon}$	0.017				
$\hat{\eta}_K$	−0.003	0.095			
$\hat{\eta}_L$	0.562	−0.663	0.084		
$\hat{\eta}_E$	−0.276	−0.685	0.356	0.072	
$\hat{\eta}_M$	0.165	0.258	−0.411	−0.750	0.069

elasticities is (0.42, 2.10), (−0.22, 4.43) and (0.72, 1.58) in Pulp and paper, Chemicals, and Basic metals, respectively (Table 5, columns 4 and 5). About two thirds of the plants in Pulp and paper and Basic metals have increasing returns to scale when evaluated at the plant specific means of the explanatory variables. The corresponding share in Chemicals is somewhat lower, about 0.55.

The variability of the scale elasticity is much less than the variability of the input elasticities, measured by the coefficients of variation (Table 5, column 3). For all functional forms and all industries, the coefficient of variation is smaller for the scale elasticity than for any of the input elasticities. The coefficient of variation of the scale elasticities is uniquely higher in Chemicals than in the two other industries. This explains why we can find clearly increasing returns to scale at the sample mean in some models in Chemicals only, although this industry has the smallest share of individual plants with increasing returns.

A share of the predicted input elasticities is negative (Table 5, column 6), which means that the predicted functions for some of the plants do not fulfill the usual regularity conditions of a production function, as representing the technically efficient combinations of inputs and output. This makes economic interpretation more difficult. It should be remembered, however, that outliers—according to some pre-defined rules—are not excluded from our data set. Such data cleaning, which reduces heterogeneity and hence probably also the problem with theory inconsistent results, is rather common in analyses of micro data. Furthermore, if our data set is affected by variation in factor utilisation over the business cycle, this may also explain the presence of negative predicted input elasticities. Generally, negative predicted input elasticities seem to be somewhat more pronounced for Models TL(c, α) and ECD(c, α) than for CD(c, α). Stated otherwise, increased flexibility of the functional form intensifies the problem of violation of the regularity conditions. Negativity of the predicted input elasticities occurs least frequently for materials. The occurrence of negative predicted input elasticities as well as negative estimates of average input elasticities (cf. Table 3) may suggest that some kind of constrained estimation procedure, or other distributional assumptions for the random coefficients, should have been applied. Such modifications, however, may require computer software which is presently unavailable, and/or may enhance the numerical problems.

Differences between the values for Model ECD(c, α) in Table 3—calculated at the estimated expected values of the random coefficients and the *overall* input means—and the values in Table 5—calculated at the *plant specific* random coefficients and plant specific input means—reflect both random and systematic heterogeneity. In Pulp and paper, the differences are modest for the scale elasticities, the largest difference, 0.06, occurring for the energy elasticity. For the other two industries, we find in general larger discrepancies. In Chemicals, the discrepancy in the scale elasticity is as high as 0.27. The main contribution to these discrepancies comes from the capital elasticity. In Basic metals, the discrepancy in the scale elasticity is modest, but both the energy and the materials elasticities deviate significantly.

Predicted input and scale elasticities: Correlation pattern. The empirical covariance pattern of the predicted plant specific input elasticities, given in the last four columns of Table 6 (variances along the diagonal, correlation coefficients below), shows that most of the correlations are negative. To a considerable extent this reflects the pattern in Table 4 for the (random) first-order coefficients $(\alpha_{Ki}, \alpha_{Li}, \alpha_{Ei}, \alpha_{Mi})$. The empirical variances of

the predictions are far smaller than the estimated population variances of the random input coefficients.

High estimates (in absolute value) of corr(α_{mi}, α_{ni}) (m and n denoting two arbitrary inputs) seem to be accompanied by high empirical correlations between predicted input elasticities of inputs m and n. Since parameter heterogeneity accounts for an important part of the dispersion in the predicted input elasticities, this is not surprising. Reproducing the calculations in Table 6 with the predicted plant specific coefficients replaced by their estimated means confirms that most of the dispersion reflects the randomness of the coefficients. For example, the empirical variances of the scale elasticities are reduced to less than one third.

Finally, the predicted plant specific scale elasticity is strongly positively correlated with the predicted labour input elasticity in all the three industries (correlation coefficient 0.56 or more) (Table 6, column 1). On the other hand, it is clearly negatively correlated with the predicted energy input elasticity (correlation coefficient -0.28 or below).

4. Concluding Remarks

In this paper, the importance of heterogeneity in economies of scale is analysed using an unbalanced plant-level panel data set from Norwegian Manufacturing Statistics in Pulp and paper, Chemicals, and Basic metals industries. A random coefficient approach is chosen, and unlike most previous work on panel data, we allow for heterogeneity in the slope coefficients representing the scale properties as well as in the intercept term. Nine specifications of a four-factor (*KLEM*) production function are estimated: the Translog, an Extended Cobb-Douglas and the strict Cobb-Douglas, each with three representations of the heterogeneity.

We find constant or moderately increasing returns to scale for a plant with an average technology, but the results reveal considerable variation across plants, and plants with both increasing and decreasing economies of scale are present. The input elasticities at the sample mean are even more variable than the scale elasticity. In general, the input elasticity of materials is largest. Variations in the input elasticities across plants seem to a larger extent to be due to randomness of the production function parameters than to systematic differences in the input mix. Including heterogeneity in slope coefficients, in addition to heterogeneous intercept terms, improves the fit substantially. Among the models with heterogeneity in slope coefficients, the fit does not seem to differ much across functional forms. However, according to the predicted input elasticities, the Cobb-Douglas model yields plant specific production functions which to a less degree than the two other functional forms violate the regularity conditions regarding technical efficiency. Hence, it may be advisable to choose a relatively restrictive functional form if heterogeneity in technology is a major concern and is represented by random coefficients.

The lesson we learn from this analysis is that one should work carefully with the representation of the plant specific heterogeneity when analysing production technologies from micro data. This supports the findings of Mairesse and Griliches (1990), who use a simpler description of the average technology than we do. In two of the three industries, our estimated scale elasticity for the average plant is very robust to the choice of the model.

An interesting issue for future research would be to analyse the distribution of scale properties, and their aggregate implications, in more detail. This may be important since knowledge about systematic variation in characteristics of plants with either increasing or decreasing returns to scale, such as age, size, growth performance, etc., could be crucial for our understanding of the evolution of an industry.

Appendix A: Details on Estimation Method and Coefficient Prediction

Consider a data set from an unbalanced panel, in which the plants are observed in at least 1 and at most P years. We assume that the selection rules for the unbalanced panels are ignorable, i.e., the way in which the plants enter or exit is not related to the endogenous variables in the model; see Verbeek and Nijman (1996, Section 18.2). The plants are arranged in groups according to the number of years the plants are observed. Let N_p be the number of plants which are observed in p years (not necessarily the same and not necessarily consecutive), let (ip) index the i'th plant among those observed in p years ($i = 1, \ldots, N_p$; $p = 1, \ldots, P$), and let t index the observation number ($t = 1, \ldots, p$). The total number of plants in the panel is $N = \sum_{p=1}^{P} N_p$ and the total number of observations is $n = \sum_{p=1}^{P} N_p p$. The regression equation, i.e., the production function (1), can be written compactly as

$$y_{(ip)t} = \boldsymbol{x}_{(ip)t}\boldsymbol{\theta}_{(ip)} + u_{(ip)t}, \quad p = 1, \ldots, P; \;\; i = 1, \ldots, N_p; \;\; t = 1, \ldots, p, \tag{A.1}$$

where $\boldsymbol{\theta}_{(ip)}$ is the coefficient vector of plant (ip). The regressand of plant (ip), observation t, is $y_{(ip)t}$, the corresponding $(1 \times H)$ regressor vector is $\boldsymbol{x}_{(ip)t}$, and the disturbance is $u_{(ip)t}$. The $(H \times 1)$ coefficient vector of plant (ip), cf. (2), is

$$\boldsymbol{\theta}_{(ip)} = \boldsymbol{\theta} + \boldsymbol{\epsilon}_{(ip)}, \tag{A.2}$$

where $\boldsymbol{\theta}$ is the common expectation vector of $\boldsymbol{\theta}_{(ip)}$ for all plants, and $\boldsymbol{\epsilon}_{(ip)}$ is a zero mean random vector specific to plant (ip). Inserting (A.2) in (A.1), we get

$$y_{(ip)t} = \boldsymbol{x}_{(ip)t}\boldsymbol{\theta} + \psi_{(ip)t}, \qquad \psi_{(ip)t} = \boldsymbol{x}_{(ip)t}\boldsymbol{\epsilon}_{(ip)} + u_{(ip)t}, \tag{A.3}$$

where we interpret $\psi_{(ip)t}$ as a 'gross disturbance.' We assume that all $\boldsymbol{x}_{(ip)t}$, $u_{(ip)t}$, and $\boldsymbol{\epsilon}_{(ip)}$ are all independent, and that

$$u_{(ip)t} \sim \mathsf{IIN}(0, \sigma_{uu}), \qquad \boldsymbol{\epsilon}_{(ip)} \sim \mathsf{IIN}(\boldsymbol{0}, \boldsymbol{\Omega}), \tag{A.4}$$

where IIN signifies independently, identically, normally distributed. The matrix $\boldsymbol{\Omega}$ is singular, reflecting that some of the coefficients are fixed, cf. (3).

We stack the p realisations from plant (ip) in $\boldsymbol{y}_{(ip)} = [y_{(ip)1}, \ldots, y_{(ip)p}]'$, $\boldsymbol{X}_{(ip)} = [\boldsymbol{x}'_{(ip)1}, \ldots, \boldsymbol{x}'_{(ip)p}]'$, $\boldsymbol{u}_{(ip)} = [u_{(ip)1}, \ldots, u_{(ip)p}]'$, and $\boldsymbol{\psi}_{(ip)} = [\psi_{(ip)1}, \ldots, \psi_{(ip)p}]'$, and can

then write (A.3) as

$$\boldsymbol{y}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\theta} + \boldsymbol{\psi}_{(ip)}, \qquad \boldsymbol{\psi}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\epsilon}_{(ip)} + \boldsymbol{u}_{(ip)}. \tag{A.5}$$

It follows from (A.3) and (A.4) that all $\boldsymbol{\psi}_{(ip)} \mid \boldsymbol{X}_{(ip)}$ are independent and

$$\boldsymbol{\psi}_{(ip)} \mid \boldsymbol{X}_{(ip)} \sim \mathsf{N}\big(\boldsymbol{0}, \boldsymbol{\Omega}_{(ip)}\big), \qquad \boldsymbol{\Omega}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\Omega}\boldsymbol{X}'_{(ip)} + \sigma_{uu}\boldsymbol{I}_p. \tag{A.6}$$

The joint log-density function of $\boldsymbol{y}_{(ip)}$ conditional on $\boldsymbol{X}_{(ip)}$, is

$$\mathcal{L}_{(ip)} = -\frac{p}{2}\ln(2\pi) - \frac{1}{2}\ln\left|\boldsymbol{\Omega}_{(ip)}\right| - \frac{1}{2}\left[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}\right]'\boldsymbol{\Omega}^{-1}_{(ip)}\left[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}\right],$$

so that by utilising the ordering of the observations in the P groups, we can write the log-likelihood function of all observations on the $\boldsymbol{y}$'s conditional on all observations on the $\boldsymbol{X}$'s as

$$\begin{aligned}\mathcal{L} = \sum_{p=1}^{P}\sum_{i=1}^{N_p} L_{(ip)} &= -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}\ln\left|\boldsymbol{\Omega}_{(ip)}\right| \\ &\quad - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}\left[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}\right]'\boldsymbol{\Omega}^{-1}_{(ip)}\left[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\theta}\right].\end{aligned} \tag{A.7}$$

The *Maximum Likelihood* (*ML*) estimators of $(\boldsymbol{\theta}, \sigma_{uu}, \boldsymbol{\Omega})$ are obtained by maximising $\mathcal{L}$ with respect to (the unknown elements of) these parameter matrices. The solution conditions may be simplified by concentrating $\mathcal{L}$ over $\boldsymbol{\theta}$ and maximising the resulting function with respect to σ_{uu} and the unknown elements of $\boldsymbol{\Omega}$. For a further discussion, see Biørn (1999).

The coefficient vector of plant (ip), $\boldsymbol{\theta}_{(ip)}$, can be predicted as follows:

$$\boldsymbol{\theta}^*_{(ip)} = \widehat{\boldsymbol{\theta}} + \widehat{\boldsymbol{\Omega}}\boldsymbol{X}'_{(ip)}\left(\boldsymbol{X}_{(ip)}\widehat{\boldsymbol{\Omega}}\boldsymbol{X}'_{(ip)} + \widehat{\sigma}_{uu}\boldsymbol{I}_p\right)^{-1}\left(\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\widehat{\boldsymbol{\theta}}\right), \tag{A.8}$$

where $\widehat{\boldsymbol{\theta}}$ is the ML (strictly, the Feasible GLS) estimator of the expected coefficient vector $\boldsymbol{\theta}$ (cf. Lee and Griffiths, 1979, Section 4; and Hsiao, 1986, p. 134), and $\widehat{\boldsymbol{\Omega}}$ and $\widehat{\sigma}_{uu}$ are the corresponding estimates of $\boldsymbol{\Omega}$ and σ_{uu}. Apart from the fact that $\boldsymbol{\Omega}$ and σ_{uu} have been estimated, this is the best linear unbiased predictor (BLUP) of $\boldsymbol{\theta}_{(ip)}$. It can be shown that this expression can be rewritten as a matrix weighted average of the overall (GLS) estimator of $\boldsymbol{\theta}$ and the OLS estimator of $\boldsymbol{\theta}_{(ip)}$, based on observations from plant (ip), i.e., $\widehat{\boldsymbol{\theta}}_{(ip)} = (\boldsymbol{X}'_{(ip)}\boldsymbol{X}_{(ip)})^{-1}(\boldsymbol{X}'_{(ip)}\boldsymbol{y}_{(ip)})$, in the following way

$$\boldsymbol{\theta}^*_{(ip)} = \left[\widehat{\boldsymbol{\Omega}}^{-1} + \widehat{\sigma}^{-1}_{uu}\boldsymbol{X}'_{(ip)}\boldsymbol{X}_{(ip)}\right]^{-1}\left[\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\boldsymbol{\theta}} + \widehat{\sigma}^{-1}_{uu}\boldsymbol{X}'_{(ip)}\boldsymbol{X}_{(ip)}\widehat{\boldsymbol{\theta}}_{(ip)}\right], \tag{A.9}$$

cf. Judge et al. (1985, pp. 540–541). The latter expression, however, is only valid when $p > H$, since otherwise $\widehat{\boldsymbol{\theta}}_{(ip)}$ does not exist.

Appendix B: Data and Empirical Variables

We use an unbalanced plant-level panel data set that covers the period 1972–1993. The primary data source is the Manufacturing Statistics database of Statistics Norway. Our initial data set includes all large plants, generally defined as plants with five or more employees (ten or more employees from 1992 on), classified under the Standard Industrial Classification (SIC)-codes 341 Manufacture of paper and paper products (Pulp and paper, for short), 351 Manufacture of industrial chemicals (Chemicals, for short) and 37 Manufacture of basic metals (Basic metals, for short). Both plants with contiguous and non-contiguous time series are included.

In the description of the empirical variables below, MS indicates that the data are from the Manufacturing Statistics, and the data are plant specific. NNA indicates that the data are from the Norwegian National Accounts. In this case, the data are identical for plants classified in the same National Account industry. While the plants in our unbalanced panel mainly are collected from 18 different industries at the 5-digit SIC-code level, the plants are classified in 14 different National Account industries. We use price indices from NNA to deflate total material costs, gross investments and fire insurance values. The two latter variables are used to calulate data on capital stocks, applying a variant of the perpetual inventory method.

Y: Output, 100 tonnes (MS)
$K = KB + KM$: Total capital stock (buildings/structures plus machinery/transport equipment), 100 000 1991-NOK (MS, NNA)
L: Labour input, 100 man-hours (MS)
E: Energy input, 100 000 kWh, electricity plus fuels (excl. motor gasoline) (MS)
$M = CM/QM$: Input of materials (incl. motor gasoline), 100 000 1991-NOK (MS, NNA)
CM: Total material cost (incl. motor gasoline) (MS)
QM: Price of materials (incl. motor gasoline), 1991 = 1 (NNA)

Output: The plants in the Manufacturing Statistics are in general multi-output plants and report output of a number of products measured in both NOK and primarily tonnes or kg. The classification of products follows The Harmonized Commodity Description and Coding System (HS), and assigns a 7-digit number to each specific commodity. For each plant, an aggregate output measure in tonnes is calculated. Hence, rather than representing output in the three industries by deflated sales, which may be affected by measurement errors (see Klette and Griliches, 1996), our output measures are actual output in physical units, which are in several respects preferable.

Capital stock: The calculations of capital stock data are based on the perpetual inventory method assuming constant depreciation rates. We combine plant data on gross investment with fire insurance values for each of the two categories Buildings and structures and Machinery and transport equipment from the Manufacturing statistics. The data on investment and fire insurance are deflated using industry specific price indices of investment goods from the Norwegian National Accounts (1991 = 1). The depreciation rate for Buildings and structures is 0.020 in all industries. For Machinery and transport equipment, the depreciation rate is set to 0.040 in Pulp and paper and Basic metals, and 0.068 in Chemicals.

For further documentation of the data and the calculations, see Biørn, Lindquist and Skjerpen (2000b, Section 4, and 2003).

Other Inputs: From the Manufacturing Statistics we get the number of man-hours used, total electricity consumption in kWh, the consumption of a number of fuels in various denominations, and total material costs in NOK for each plant. The different fuels, such as coal, coke, fuelwood, petroleum oils and gases, and aerated waters, are transformed to the common denominator kWh by using estimated average energy content of each fuel (Statistics Norway, 1995, p. 124). This enables us to calculate aggregate energy use in kWh for each plant. For most plants, this energy aggregate is dominated by electricity. Total material costs is deflated by the price index (1991 = 1) of material inputs (incl. motor gasoline) from the Norwegian National Accounts. This price is identical for all plants classified in the same National Account industry.

We have removed observations with missing values of output or inputs. This reduced the number of observations by 4–8 per cent in the three industries. The number of plants per year varies from 81 to 179 in Pulp and paper, from 46 to 66 in Chemicals, and from 71 to 111 in Basic metals. There is a clear negative trend in the number of plants from the mid-seventies in all three industries. The unbalance in our data set is shown in Table B1,

Table B1. Number of plants classified by number of replications.

Industry	Pulp & Paper		Chemicals		Basic Metals	
p	N_p	$N_p p$	N_p	$N_p p$	N_p	$N_p p$
22	60	1320	29	638	44	968
21	9	189	0	0	2	42
20	5	100	3	60	4	80
19	3	57	0	0	5	95
18	1	18	2	36	2	36
17	4	68	4	68	5	85
16	6	96	9	144	5	80
15	4	60	6	90	4	60
14	3	42	1	14	5	70
13	4	52	3	39	3	39
12	7	84	1	12	10	120
11	10	110	2	22	7	77
10	12	120	3	30	6	60
09	10	90	2	18	5	45
08	7	56	2	16	2	16
07	15	105	2	14	13	91
06	11	66	3	18	4	24
05	14	70	3	15	5	25
04	9	36	2	8	6	24
03	18	54	3	9	3	9
02	5	10	3	6	6	12
01	20	20	7	7	20	20
Sum: N, n	237	2823	90	1264	166	2078

Note: $p =$ no. of observations per plant, $N_p =$ no. of plants observed p times, $N = \sum N_p$, $n = \sum N_p p$

Table B2. Overall mean and standard deviation of basic variables.

	Pulp & Paper			Chemicals			Basic Metals		
Industry Variable	Mean	Log of Mean	Std. Dev.	Mean	Log of Mean	Std. Dev.	Mean	Log of Mean	Std. Dev.
$\ln(Y)$	4.117		2.079	4.750		2.444	3.586		2.658
Y		5.697			7.351			5.861	
$\ln(K)$	6.691		1.787	7.217		1.942	6.643		2.244
K		7.971			8.777			8.512	
$\ln(L)$	6.836		1.297	6.886		1.496	7.060		1.694
L		7.568			7.889			8.277	
$\ln(E)$	4.417		2.599	5.332		2.481	4.808		2.807
E		6.575			7.440			7.809	
$\ln(M)$	5.393		1.792	5.452		2.014	5.404		2.254
M		6.603			6.983			7.285	
τ	9.940		6.205	10.952		6.188	10.794		6.259

which gives the number of plants sorted by the number of observations. Table B2 gives the overall mean and standard deviation of the basic variables.

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Notes

1. Attempts were made to solve the Maximum Likelihood estimation problem (under normality of the random coefficients and the disturbance terms) with random B, γ, γ^*, and δ. As this turned out to raise numerical problems, we decided to consider only models in which these coefficients are constants.
2. Likelihood Ratio test statistics can be easily calculated from the tables. These statistics are, however, not asymptotically χ^2-distributed under the null hypothesis of full coefficient homogeneity, because the parameters in $\mathbf{\Omega}$ then are on the border of the admissible parameter space, see Shin (1995, p. 321). Thus, for making formal inference of coefficient heterogeneity versus homogeneity, other test procedures may be needed, see the recent papers by Khuri, Mathew and Sinha (1998) and Andrews (1999). We have not followed up these ideas in the present paper.
3. The two latter criteria are defined, for a model, m, by, respectively, $AIC_m = l_m - q_m$ and $SBC_m = l_m - 0.5q_m \ln(N_m)$, where l_m is the log-likelihood value of model m, q_m is its number of parameters, and N_m is its number of observations.
4. We do not consider prediction for a plant which enters the sample in year s.
5. Defined as the logarithms of their arithmetic means; cf. Table B2 in Appendix B.
6. Note that the standard deviation estimates given in parenthesis refer to the uncertainty of the estimated parameters and hence is conceptually different from the standard deviation of the random parameter, i.e., the square root of the diagonal elements of $\Omega_{\alpha\alpha}$.
7. A 5 per cent significance level is used throughout.

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