Heterogeneous Multi-Channel Wireless Networks: Scheduling and Routing Issues

Technical Report (October 2, 2007)

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Abstract

We consider multi-channel networks where nodes may be equipped with heterogeneous radios, each potentially capable of operation on a limited portion of the total available spectrum. Moreover even the channels may not all be identical; they may possibly have different propagation characteristics, and may support different sets of transmission rates. Much prior research on multi-channel networks has assumed identical channels and radio capabilities. However heterogeneity of channels and radios introduces a host of new issues that must be handled. In recent theoretical work we considered asymptotic transport capacity of multi-channel networks subject to switching constraints. This constitutes a class of instances involving heterogeneous radios, albeit identical channels. We leverage some of the insights obtained from our theoretical results, and now consider a more general model involving heterogeneity of radios and channels for networks of realistic scale. We identify the key issues that differentiate heterogeneous multi-channel networks, and describe a design framework for routing and channel/interface assignment.

I. Introduction

The availability of multiple channels for wireless communication provides an excellent opportunity for performance improvement. However, in a given network, devices may be of varying type, cost and capability. Thus, they may have heterogenous radio capabilities in terms of variable number of available interfaces. Moreover, all interfaces may not be able to switch on all channels, and all channels may not be identical. Much prior work in this domain has considered nodes with identical radios, and channels with identical characteristics. Some recent work [19] has considered routing/channel assignment in the face of heterogeneous radios/channels. Scheduling in multi-channel multi-radio networks has been recently considered in [12] for a model where the data-rate achievable over a link is different for different channels

This research is supported in part by NSF grant CNS 06-27074, US Army Research Office grant W911NF-05-1-0246, and a Vodafone Graduate Fellowship.

A poster summarizing much of the material in this report was presented at ACM Mobicom 2007.

and nodes may have a variable number of radios, but all radios are identical. In [21], a scheme has been proposed to gracefully handle route breakages and improve TCP performance by utilizing secondary 802.11b interfaces in an 802.11a network. However, there are still numerous open problems/issues to be addressed.

In past work, we have studied capacity of randomly deployed networks subject to channel switching constraints [3], [2]. We have introduced some constraint models, viz., adjacent (c, f) assignment and random (c, f) assignment, and studied connectivity and asymptotic transport capacity of such networks. Our results indicate that it may be possible to achieve good throughput characteristics even when individual nodes have limited switching capability. In particular, our capacity construction in [2] for random (c, f) assignment illustrated the existence of a strong coupling between channel and interface selection that arose due to non-interchangeability of interfaces, and complicated the task of scheduling/routing. The impact of this coupling is expected to be more pronounced in small-scale networks, where centralized scheduling is not always feasible, node-density may not be very high, and performance degradation by even small constant factors is significant.

Having obtained useful insights from the asymptotic results, we are now trying to address the problems of routing and channel/interface assignment in multi-channel networks of realistic scale where nodes may possess heterogeneous radio capabilities. We seek to handle scenarios where nodes may be equipped with heterogeneous wireless interfaces, and additionally, all channels may not be identical in terms of supported transmission rates and propagation characteristics. Moreover, we seek solutions where the routing metric/protocol does not make any assumptions about fixing interfaces on channels for long time intervals. This would allow for shorter time-scale link-layer adaptation to local channel conditions. We argue for a protocol design paradigm involving *timescale separation* between the view available to the link-layer and network-layer, whereby the link-layer handles instantaneous decisions about channel-assignment, while the network-layer makes routing decisions based on an aggregate view over a certain window of time. We seek to establish a formal design framework to this effect by articulating clear interfaces for cross-layer information exchange between the link-layer and network-layer, and thereafter obtain suitable routing and channel assignment algorithms within this framework. We identify new issues that may arise due to heterogeneity, and propose a design framework for routing and link-layer protocol design.

II. Related Work

Much attention has been paid in recent years to the use of multiple channels for performance improvement in wireless networks. Asymptotic capacity results for a network with c channels, and $m \leq c$ interfaces per node were established in [9], [10]. The capacity region of multi-channel multi-radio networks in the non-asymptotic regime was studied in [8]. Various channel-assignment, MAC and routing protocols for multi-channel wireless networks have been described in [1], [17], [15], [16], [14], [5], [11], etc. However all of these works assume that all radio-interfaces in the network have identical capabilities. Most of these works also do not explicitly consider heterogeneity in channel characteristics.

Opportunistic channel selection has been considered in MAC protocols such as MOAR [6], DB-MCMAC [4] and OMC-MAC [22]. However none of these have studied the global routing implications of local opportunism in a multi-hop wireless network.

The use of heterogeneous interfaces to handle route breakages has been proposed in [21]. In this work, nodes are equipped with primary 802.11a interfaces and secondary 802.11b interfaces. TCP flows use a primary path comprising the 802.11a interfaces, which is discovered via a reactive routing protocol. A proactive routing protocol is run over the secondary interfaces. When a link-breakage is detected, the TCP traffic can be immediately re-routed over the secondary path while a new primary path is being discovered.

Joint channel assignment and routing in a heterogeneous multi-channel multi-radio wireless network has been considered in [19]. This work targets a situation very similar to what we have considered in this paper. It allows for both heterogeneity in the operational abilities of interfaces, as well as in supported channel data-rates. A joint channel-assignment and routing scheme (JCAR) is proposed. However, this work treats the route for each flow as a sequence of interfaces, and therefore does not consider the possibility of link-layer data-striping. Moreover, it seeks a solution where interfaces switch channels only over substantially long periods of time.

The asymptotic capacity scaling behavior of multi-channel wireless networks with heterogeneous interfaces was studied in [3], [2]. Two switching constraint models were defined, viz., adjacent (c, f) assignment and random (c, f) assignment. It was shown in [2] that for the random (c, f) assignment model, $\sqrt{c}$-switchability yields order-optimal capacity (under the Protocol Model). The optimal capacity construction required synchronized route construction for all flows in the network, and highlighted a coupling between interface selection and channel selection, which led to a strong coupling in decisions made at different hops.

In this paper, we build upon the insights from [2] and examine scheduling/routing issues for multi-channel wireless networks with heterogeneous interfaces/channels in the non-asymptotic regime.

III. Some Notation/Definitions

Denote by $\mathcal{C}$ the set of all possible channels, and by I_v the set of interfaces that node v is equipped with. We use the terms radio and interface interchangeably. Consider interface $m_i \in I_v$. We define an indicator function $switch(c, m_i)$ as follows:

$$switch(c, m_i) = \begin{cases} 1 & \text{if interface } m_i \text{ can switch on channel } c \\ 0 & \text{else} \end{cases} \tag{1}$$

We use M_v^c to denote $|\{m_j : m_j \in I_v, switch(c, m_j) = 1\}|$. Thus:

$$M_v^c > 0 \iff \exists m_i \in I_v, switch(c, m_i) = 1 \tag{2}$$

We differentiate between a node-link and a radio-link. A (directed) node-link is an ordered pair of nodes. A (directed) radio-link is an ordered pair of radios.

Borrowing notation from [12], we denote the endpoints of a node-link l by $b(l)$ and $e(l)$ respectively. Thus, their interface-sets are $I_{b(l)}$ and $I_{e(l)}$ respectively.

There are at most $|I_{b(l)}|\cdot|I_{e(l)}|$ potential radio-links associated with link l, corresponding to all possible cross-pairings of interfaces. Denote by $\mathcal{R}(l)$ the set of potential radio-links associated with node-link l.

A radio-link $l_r = (m_s, m_d), m_s \in I_{b(l)}, m_d \in I_{e(l)}$ can be operated on channel c only if:

$switch(c, m_s)\cdot switch(c, m_d) = 1$.

Thus we define another indicator function for $l_r = (m_i, m_j)$:

$$op(c, l_r) = \begin{cases} 1 & \text{if } switch(c, m_i)\cdot switch(c, m_j) = 1 \\ 0 & \text{else} \end{cases} \tag{3}$$

Given an interface m, we denote by $\mathcal{C}(m)$ the set $\{c : c \in \mathcal{C}, switch(c, m) = 1\}$.

We define another indicator function as follows:

$$idle(m, t) = \begin{cases} 1 & \text{if interface } m \text{ is not transmitting/receiving at time } t \\ 0 & \text{else} \end{cases} \tag{4}$$

IV. Forms of Heterogeneity

We briefly summarize the various forms of heterogeneity that we are considering:

A. Interface Heterogeneity

An individual interface may not be capable of operating on all available channels, i.e., it may be subject to *switching constraints*. Thus, the choice of interface may become a non-trivial decision. We examine this issue in great detail in later sections.

B. Heterogeneous Channel Characteristics

It is possible that different channels may have different channel characteristics, if they fall in different parts of the spectrum. Moreover depending on the modulation schemes in use, the supportable data-rates may be different for different channels.

C. Time-varying Channel Conditions

Even if two channels have similar propagation characteristics, and use the same physical-layer technology, they may not have identical channel quality at any instant, because of time-variation in channel conditions due to fading, transient noise sources and other possible factors.

V. Proposed Framework

In this section we describe a high-level architectural framework that we believe is suited to the characteristics of heterogeneous multi-channel wireless networks. We assume that the MAC protocol is pre-specified, and thus our mechanisms lie entirely in the link-layer and network-layer. This allows us to focus on general channel/interface/route selection issues without being tied down to the specific details of a particular MAC scheme. Moreover it allows for the use of different MAC protocols in different channels, so long as there is a unique MAC scheme for any single channel.

Channel Restriction: Taking cognizance of the time-varying nature of the wireless channel, one would like to opportunistically exploit the available channel diversity to improve throughput. However, to do this one needs some mechanism to sample/probe channels. This cost can be significant, especially if the number of available channels is large. Moreover, in a distributed setting, opportunism can have an adverse effect on load-balance, e.g., consider a worst-case scenario where all nodes in a vicinity decide that channel *x* has best quality and start using that channel simultaneously.

One would typically expect that much of benefit of opportunistic exploitation of channel diversity can be obtained by having the choice of a few channels, and thus a reasonable solution lies in restricting the operation of a link to a subset of all possible channels available to it (a *channel pool*). One can then attempt to opportunistically exploit diversity amongst channels in this *channel pool*. In fact, some prior work, e.g., [18], has studies this issue in a single-hop setting and concluded that a few channels indeed provide a good trade-off between diversity-gain and probing cost. The same conclusion is likely to hold even in multi-hop settings. Moreover, we argue that such *channel-restriction* has the potential to provide a degree of a priori load-balance (since different links will have different channel pools), while still retaining the possibility for opportunism.

Timescale Separation: Since the actual channel used on a link can change over the timescale of a few packets, which is substantially shorter than the expected lifetime of a route, it follows that route-selection should not be overly sensitive to instantaneous channel-usage. We argue that the network layer should choose routes with good *expected* characteristics based on a global view, and the link-layer should select the exact channel(s)/interface(s) based on more *instantaneous* local knowledge. This would allow the link-layer to exercise opportunism, while still providing a degree of predictable behavior. Such *timescale separation* is also desirable from the viewpoint of system stability [7].

Routing Approach: We propose the use of single-path routing with link-layer data-striping. Thus a path from source to destination is a single sequence of nodes (and hence also a series of node-links). However each node-link is a set of radio-links and one could exploit this diversity/multiplicity via suitable link-layer strategies.

For the rest of our discussion we assume some form of link-state routing, with a cost associated with each link as well as each pair of adjacent links (i.e., links incident on a common node). Loss of metric isotonicity due to the latter can be compensated for via transformations similar to that described in [20].

A good multi-channel, multi-radio routing metric must do the following:

1) It should spread routing load over many interfaces available in a locality, and thereby try to avoid interface bottlenecks that may arise when the number of radios per node is less than the number of available channels [9].
2) It should attempt to avoid self-interference, which can be an important issue when the number of flows in the network is small.
3) It should attempt to choose routes with good channel and interface diversity (this is especially relevant in case of heterogeneous networks, where available diversity can vary considerably from one route

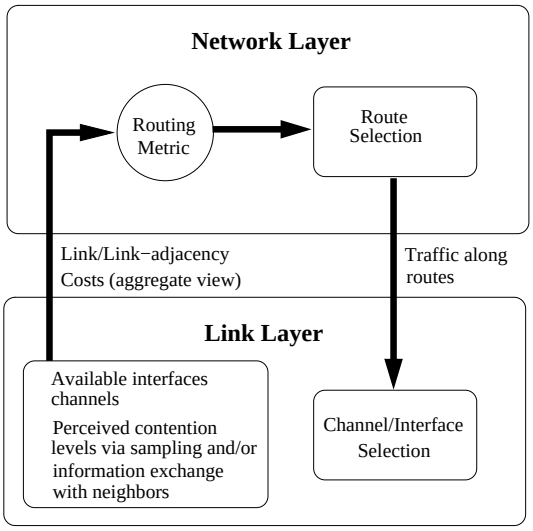

Fig. 1. Block schematic depicting interaction between link-layer and network-layer

to another), so that link-layer can potentially do local adaptation if one channel starts exhibiting poor channel quality.

In light of the above discussion, we can envision a partitioning of roles between the link-layer and the network-layer. Fig. 1 presents a block-schematic of the proposed roles and interactions between the two layers (though there is a coupling between the two layers, we expect that the timescale separation will help avoid excessive undesirable oscillations).

Role of the Link Layer: The link-layer must perform the following functions:

1) Given the current set of routes, decide what channel and interface to use for each packet based on knowledge of neighbors' interface-configuration as well as current (short timescale) channel conditions.
2) Compute link/link-adjacency costs appropriately so that they provide an aggregated (longer-timescale) view of local contention levels and available diversity; this will influence future routes.

Role of the Network Layer: The network-layer must perform the following function:

1) Given the current set of link/link-adjacency costs, select suitable routes as per metric of interest.

Within the proposed framework one can then study channel and interface selection as local decisions to be performed assuming that routes are already given.

VI. On the need for careful interface selection

In this section, we highlight the difference between a network where interfaces have homogeneous switching ability, and one where the interfaces are heterogeneous, and the possibility of severe sub-optimality if the interface-selection logic is oblivious to the fact that interfaces have different operational channel-sets.

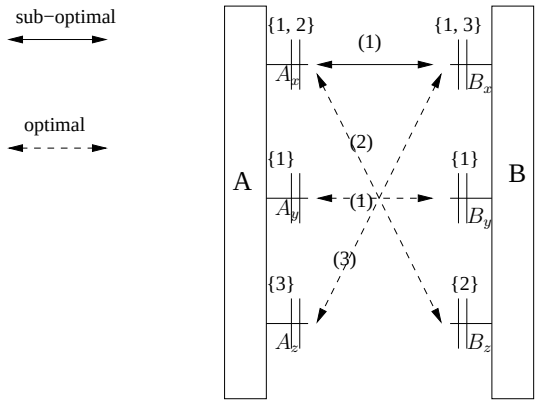

Fig. 2. Example illustrating drawbacks of oblivious interface-selection

Consider the following example (Fig. 2):

There is a single directed link $A \rightarrow B$, and A is always backlogged (i.e., can saturate any capacity available to it). There are three channels $1,2,3$ that all support the same data-rate r over the link. Nodes A,B have three radios each (A_x,A_y,A_z and B_x,B_y,B_z respectively), with channel-sets as shown in Fig. 2. Suppose we have an interface-selection logic that is oblivious of the heterogeneous switching ability. Thus, it might choose to send data over the radio-link $A_x \rightarrow B_x$ using channel 1. As a result of this decidedly sub-optimal scheduling decision, no other concurrent transmission is possible and the throughput is limited to r. Instead the optimal solution is to activate $A_x \rightarrow B_z$ over channel 2, $A_y \rightarrow B_y$ over channel 1, and $A_z \rightarrow B_x$ over channel 3, thereby obtaining a throughput $3r$.

In the above example, it is to be noted that while one would like to have one $A \rightarrow B$ transmission on channel 1, it should be on radio-link $A_y \rightarrow B_y$ and not $A_x \rightarrow B_x$, as the latter would block other transmissions that could otherwise occur concurrently. Thus, the choices of channel and interfaces are both important.

VII. A Taxonomy for Interface Heterogeneity

A clasification of heterogeneous assignments based on identification of some useful structure can be beneficial and may allow for simpler algorithms for specific assignment types. Therefore we have devised the following three category nested classification:

Disjoint Channel-Set (DCS) Assignment: The set of all available channels is $\mathcal{C}$. There is a partition $\{\mathcal{C}_1,\mathcal{C}_2,...,\mathcal{C}_m\}$ of $\mathcal{C}$, such that any radio is assigned one subset $\mathcal{C}_i$, and can then switch on all channels in $\mathcal{C}_i$. The net outcome is that a pair of radios either have no common channel, or else they are capable of operation over exactly the same set of channels.

This is a good model for many situations encountered today, where nodes may be equipped with radios conforming to different standards, e.g., 802.11a, 802.11b, etc. Two radios of the same type have identical operational capabilities, but radios of different types cannot inter-operate.

Resultantly, if a node u is equipped with two radios r_i^u, r_j^u of the same type, they are fully interchangeable, i.e., given a feasible schedule, if we swap r_i^u, r_j^u, we still have a feasible schedule (we assume that for any

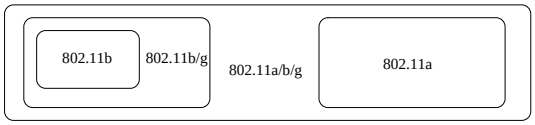

Fig. 3. Example of Inclusive Channel-Set Assignment

other node's radio r^v, the gain between r_i^u and r^v is the same as the gain between r_j^u and r^v; differences in gain between radios on the same node, due to the (small) variation in location is outside the scope of this paper). By extension, all radios of u that can communicate with a certain radio r_v are interchangeable.

Inclusive Channel-Set (ICS) Assignment: The set of all available channel is $\mathcal{C}$. There is a collection of subsets of $\mathcal{C}$: $\{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m\}$, such that $\forall i, j \in \{1, 2, .., m\} : (\mathcal{C}_i \subseteq \mathcal{C}_j) \vee (\mathcal{C}_j \subset \mathcal{C}_i) \vee (\mathcal{C}_i \cap \mathcal{C}_j = \phi)$. Each radio is assigned one of the subsets $\mathcal{C}_i$, and can then switch on all channels in $\mathcal{C}_i$.

This model can capture certain situations where nodes are equipped with a mix of single-mode and multi-mode cards, e.g., current prevalent scenarios involving 802.11b, 802.11b/g, 802.11a, and 802.11a/b/g cards (Fig. 3). In this model, two radios are fully interchangeable iff they are both assigned the same subset, but not if the channel-set of one is a proper subset of the other. However, there exists a *one-way replacability* between any pair of interfaces that have non-disjoint channel-sets.

Arbitrary Assignment: This can involve any arbitrary channel assignment. Thus it includes the prior two assignment classes. The adjacent (c, f) and random (c, f) assignments described in [3] and [2] are instances that do not fall in the previous two categories. Thus different radio-pairs may have a different number of common channels. Two radios are fully interchangeable only if they can operate on exactly the same set of channels.

VIII. DCS Assignments: Interchangeability of Operable Radios

Disjoint channel set assignments can be easily seen to have the property that any two radios are either incapable of inter-operation, or completely interchangeable. Thus the only parameter of interest is the number of radios of each kind that a node is equipped with. The identities of individual radios capable of switching on some channel i are not important.

IX. ICS Assignments: The Least Capable First Policy

The specific structure of the Inclusive Channel Set assignment allows one to formulate some simple rules-of-thumb. Let us define the following simple interface-selection policy:

Least Capable First (LCF) Policy: If a packet needs to be sent over link l on channel c, then at each link-endpoint, amongst all interfaces capable of operating on c, use the interface with least channel-set cardinality.

A schedule is said to be LCF-compliant if the following holds: for any packet transmission that begins at time t on channel c and uses interfaces $m(b(l))$ and $m(e(l))$ respectively at the link-endpoints $b(l)$ and $e(l)$ of node-link l: $|\mathcal{C}(m(x))| = \min_{\substack{m: m \in I_x \\ switch(c,m) \cdot idle(m,t) = 1}} |\mathcal{C}(m)|$, for $x = b(l), e(l)$.

Lemma 1: Given an ICS-assigned network, any feasible schedule can be converted into an equivalent LCF-compliant feasible schedule.

Proof: Suppose we are given a feasible schedule that is not LCF-compliant. Thus there exists at least one time instant t at which a packet is scheduled over some link l on channel c but $|\mathcal{C}(m(x))| > \min_{\substack{m:m\in I_x \\ switch(c,m)\cdot idle(m,t)=1}} |\mathcal{C}(m)|$ for at least one of $x = b(l), e(l)$, i.e., one of the interfaces was not selected as per the LCF policy. Consider the first such instant t. Suppose w.l.o.g. that $x = b(l)$. Thus there exists an $m \in I_x$ such that $m \neq m(b(l))$ and $switch(c,m) = 1$ and $idle(m,t)$ and $|\mathcal{C}(m)| < |\mathcal{C}(m(b(l))|$. Replace the use of $m(b(l))$ at time t by the use of m. If this induces a conflict due to an overlapping usage of interface m at time $t_1 > t$ for sending some packet on channel j, replace that conflicting use of m by $m(b(l))$ (this is always possible since $|\mathcal{C}(m)| < |\mathcal{C}(m(b(l))|$ and $\mathcal{C}(m) \bigcap \mathcal{C}(m(b(l))) \neq \phi \implies \mathcal{C}(m) \subset \mathcal{C}(m(b(l)))$. Repeat the process till an LCF-compliant schedule is obtained. Thus we obtain an equivalent LCF-compliant feasible schedule, which retains exactly the same set of transmissions, but has some interface usages swapped compared to the original schedule. ■

Let us consider any notion of optimality that is dependent only on the set of transmissions that occur (and not on the identity of the individual interfaces that were used). Then:

Lemma 2: There exists an optimal schedule that is LCF-compliant.

Proof: Consider any optimal schedule. If it is already LCF-compliant we are done. Else from Lemma 1, we can convert this schedule into an equivalent LCF-compliant schedule with exactly the same set of transmissions. Thus this new schedule is also optimal. The result follows. ■

X. Coupling between Channel and Interface Selection

The asymptotic capacity analysis in [2] illustrated that there is a coupling between channel and interface selection. We briefly summarize the key aspects of the obtained insights. In [2], we considered a switching constraint model called random (c, f) assignment, and described a capacity-achieving construction for a randomly deployed network of n nodes. In this model, each node has a single half-duplex interface which is pre-assigned a subset of f channels out of a total of c (where $c = O(\log n)$) uniformly at random from all such possible subsets. The interface can then only switch between these f channels.

Let us consider the implications of this: if we have to choose a route for a flow, then the first hop transmission must necessarily be scheduled on one of the f channels that the source can switch on (since the source will be sending it); the first relay node must also be one that has at least one channel in common with the source node (so that it can receive the transmission); moreover if channel x is chosen, then the relay node must be capable of switching on channel x. Similarly, the choice of channel at each subsequent hop is limited to the channel-subset of the hop-sender, and the choice of next relay is limited to nodes that can switch on such a channel. Thus the choice of relay at hop i determines the channel choices and consequently relay choices available for hop $i+1$. This leads to a coupling across hops of the same route. Moreover, this also leads to a strong coupling across routes. This can be illustrated via a simple example as follows:

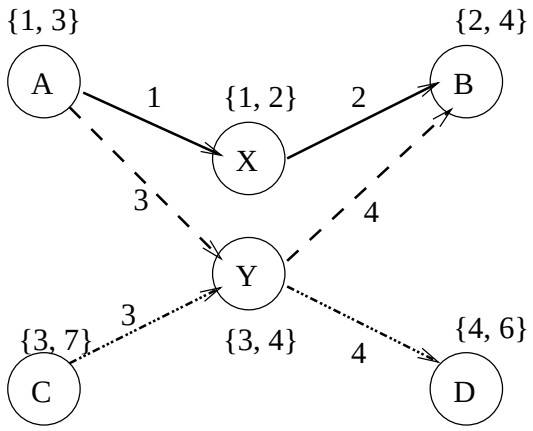

Fig. 4. Example illustrating coupling between routes

Consider nodes A,B,C,D,X,Y, each of which is equipped with a single interface. Consider two flows $A \rightarrow B$ and $C \rightarrow D$. A,B and C,D are not neighbors, but the nodes X,Y are neighbors of all nodes A,B,C,D, and can thus act as relays for the flows. The channel-sets of the nodes are as shown in Fig. 4. The first flow can use the route $A \xrightarrow{1} X \xrightarrow{2} B$ or $A \xrightarrow{3} Y \xrightarrow{4} B$. The second flow has only one choice $C \xrightarrow{3} Y \xrightarrow{4} D$. Suppose we perform route-selection for the two flows sequentially in the order $A \rightarrow B, C \rightarrow D$. If the first flow chooses its route without consideration of the second flow and its constraints, it may end up choosing $A \xrightarrow{3} Y \xrightarrow{4} B$. Since the second flow must necessarily choose $C \xrightarrow{3} Y \xrightarrow{4} D$, this will lead to a bottleneck. The optimal choice is for the first flow to use route $A \xrightarrow{1} X \xrightarrow{2} B$ and for the second flow to use $C \xrightarrow{3} Y \xrightarrow{4} D$. Note that if all interfaces could switch on all channels, this problem would not have arisen, as regardless of which route the first flow chose, the second flow could always choose the node-disjoint route, and use different channels on that route.

The above discussion illustrates that in the presence of interface heterogeneity, the selection of relays, channels and interfaces is much more complicated than in the homogeneous case.

We further illustrate the coupling between channel and interface-selection through a real-world example:

Consider the scenario depicted in Fig. 5: node A is equipped with a 802.11a and a 802.11b/g card. Node B is A's neighbor and is equipped with a multi-mode 802.11a/b/g card. Node C is also A's neighbor and it is equipped with a 802.11a card. Suppose the channel/interface binding logic is oblivious to interface heterogeneity; it simply processes packets in the incoming queue sequentially, determines a channel based on channel-quality and only then does it select an interface to use.

Suppose A is initially sending packets only to B. One of the 802.11a channels has best channel quality, albeit only marginally better than one of the 802.11g channels. Thus the channel/interface binding logic chooses to use the 802.11a channel, and hence must use the 802.11a card. Once a batch of packets has been thus assigned to the card, it receives some packets intended for C. Since C is only reachable via the 802.11a card, these packets must be queued till the already assigned batch of packets are transmitted. Ideally, A would be able to near-immediately switch to using the 802.11g card for communication with

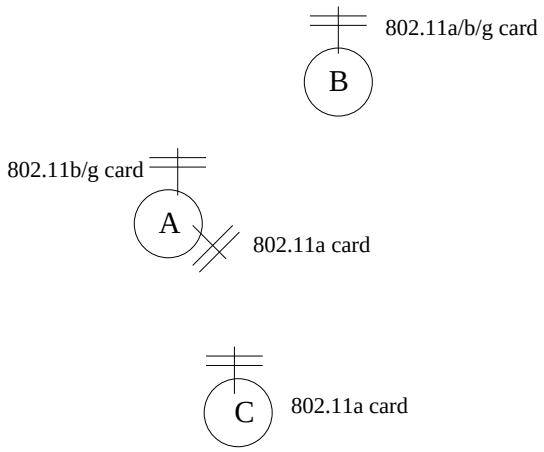

Fig. 5. Example illustrating coupled nature of channel and interface selection

B, and start using the 802.11a card for communication with C.

The implication of the above observation is that channel-selection cannot optimally be decoupled from interface-selection. Moreover it might be desirable to perform the binding as late as possible, and after taking into consideration other packets in the queue.

XI. Scheduling Implications of Interface Heterogeneity: A Useful Theoretical Framework

As mentioned previously, Lin and Rasool [12] have proposed a distributed algorithm for multi-channel multi-radio networks. In their model, it is assumed that a maximal scheduler is used for link-scheduling. A maximal scheduler has a worst-case approximation ratio of $\frac{1}{\kappa}$, where κ is the maximum interference degree of the network, in situations involving a single channel. However, this may not be the case in the face of channel diversity and variable number of radios per node. Thus they propose a queue-loading algorithm that essentially serves to modulate the input to the maximal scheduler. Their algorithm has an approximation-ratio of $\frac{1}{\kappa+2}$.

The model of [12] is quite interesting, and conforms to a situation where the MAC is pre-specified and channel-diversity unaware (in this case a maximal scheduler), and thus the onus of obtaining good performance falls on link-layer mechanisms (in their case, introduction of per-channel queues for every link, and formulation of a rule to load these queues).

However their model assumes that all interfaces are homogeneous, i.e., any interface can be used for operation on any channel. Thus their result is not directly applicable to a scenario with heterogeneous interfaces. We are interested in exploring the case of heterogeneous interfaces.

A. A Variant Algorithm

As a preliminary theoretical investigation into the impact of interface heterogeneity, we focused on the possibility of making a minor modification of [12] to heterogeneous interfaces without affecting the approximation ratio.

We describe this in detail in this section. We have attempted to retain the same notation as [12] as far as possible, to facilitate ease of comparison, and have introduced new notation wherever necessary.

B. Notation/Definitions

We consider a scenario where single-path routing is performed. The route for each flow s is considered given. We define indicator variables $\mathcal{H}_l^s$, such that $\mathcal{H}_l^s = 1$ if the route for flow s traverses link l, and is 0 else. Each flow is assumed to generate traffic at a constant rate λ_s. As in [12], we ignore issues due to multi-hop routing, and assume that each node-link along the route for s gets λ_s new data-units to send in each time-slot.

The total number of node-links in the network is denoted by L. Two node-links are said to *conflict* if they cannot both be simultaneously active on the same channel (if they could potentially use some common channel). Given a node-link l, $\mathbf{I}(l)$ is the set of node-links that have at least one radiolink capable of interfering with at least one radiolink of l, and is formally defined as:

$$\mathbf{I}(l) = \{l' : \exists c \in \mathcal{C}, l_r \in \mathcal{R}(l), l'_r \in \mathcal{R}(l') \text{ such that } op(c,l_r)\cdot op(c,l'_r) = 1, \text{ and } l,l' \text{ conflict } \}$$

Furthermore, we adopt the convention $l \in \mathbf{I}(l)$.

A non-interfering subset of $\mathbf{I}(l)$ is a subset of links in $\mathbf{I}(l)$ such that all links in this subset are mutually non-*conflict*ing. Thereafter interference degree of l over channel c is defined as in [12] as the maximum cardinality of any non-interfering subset in $\mathbf{I}(l)$. The maximum inference degree over all link-channel pairs in the network is denoted by κ.

Implicit in the above is the assumption that conflict is a property of node-links and not individual channels/radios. There may be scenarios where this may not be the case. However, our analysis would continue to be applicable (although it would be over-conservative) if we define two node-links to conflict if there exists at least one pair of conflicting radiolinks, one belonging to each node-link. It is also to be noted that the analysis can be easily generalized to account accurately for the more general scenario where conflict is a property of the type of radio/channel.

Each node-link maintains a queue $\mathbf{q_l}$ of packets that are waiting to be transmitted over node-link l. Furthermore, corresponding to each node-link l, there is a set of queues $\eta_{\mathbf{l_r}}^{\mathbf{c}}$ ($l_r \in \mathcal{R}(l)$ and $op(c,l_r) = 1$) corresponding to each valid radiolink-channel pair associated with l. The queue-lengths at time t are denoted by $q_l(t)$ and $\eta_{l_r}^c(t)$ respectively.

We denote by $\mathcal{E}(m)$ the set of radiolinks (incoming or outgoing) incident on interface m.

We denote by $r_{l_r}^c$, the achievable data-rate over radiolink l if channel c is used. In a scenario where achievable data-rate is the same for all interface-pairs corresponding to a node-pair (e.g., the assumption in [12]), one can simply set $r_{l_r}^c = r_l^c, \forall l_r \in \mathcal{R}(l)$, and the algorithm continues to work. However, it is also capable of handling the more general case where the data-rates may vary.

As in [12], time is considered slotted. In each slot t, a maximal scheduler is used to schedule a set of non-interfering radio-links over each channel. Let the set of radiolinks scheduled in slot t on channel c

be denoted by $\mathcal{M}^c(t)$. Then by definition of a maximal scheduler, for any radiolink-channel pair (l_r, c) where $l_r \in \mathcal{R}(l)$ has interfaces $s(l_r)$ and $d(l_r)$ as endpoints, and $op(c, l_r) = 1$ and radiolink-channel pair $\eta^c_{l_r}(t) \geq r^c_{l_r}$, at least one of the following holds:

- $\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} I_{k_r \in \mathcal{M}^c(t)} \geq 1$ (either l_r or an interfering radiolink is scheduled on c).
- $\sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} I_{k_r \in \mathcal{M}^d(t)} = 1$ (the sending-interface $s(l_r)$ is busy as another radiolink incident on it has been scheduled)
- $\sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} I_{k_r \in \mathcal{M}^d(t)} = 1$ (the receiving-interface $d(l_r)$ is busy as another radiolink incident on it has been scheduled)

We utilize the following stability criterion (from [13]) based on Lyapunov drift:

Let $\overrightarrow{U}^{(c)}(t) = (U_i^{(c)}(t))$ be the backlog matrix, where $U_i^{(c)}(t)$ is the backlog in queue i for commodity c. Let $L(\overrightarrow{U})$ be a non-negative function of $\overrightarrow{U}$.

Lemma 3: (Lyapunov Stability) [13] If the Lyapunov function of unfinished work $L(\overrightarrow{U})$ satisfies:

$$E[L(\overrightarrow{U}(t+1)) - L(\overrightarrow{U}(t)) | \overrightarrow{U}(t)] \leq B - \varepsilon \sum_{i,c} \theta_i^{(c)} U_i^{(c)}(t)$$

for some positive constants $B, \theta_i^{(c)}$, then:

$$\limsup_{M \to \infty} \sum_{i,c} \theta_i^{(c)} \left\{ \frac{1}{M} \sum_{k=0}^{M-1} E[U_i^{(c)}(kT)] \right\} \leq B$$

Furthermore, if there is a nonzero probability that the system will eventually empty, then a steady state distribution for unfinished work exists, with bounded average occupancies $\overline{U}_i^c$ satisfying $\sum_{i,c} \overline{U}_i^c \leq B$.

C. The Algorithm

For each radiolink-channel pair (l_r, c):

$$x^c_{l_r}(t) = \begin{cases} r^c_{l_r} & \text{if } op(c,l_r) \text{ and } \frac{q_l(t)}{\alpha_l} \geq \frac{1}{r^c_{l_r}} \left[\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\eta^c_{k_r}(t)}{r^c_{k_r}} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\eta^d_{k_r}(t)}{r^d_{k_r}} + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\eta^d_{k_r}(t)}{r^d_{k_r}} \right] \\ 0 & \text{else} \end{cases}$$

where α_l is an arbitrarily chosen positive constant.

We load $x^c_{l_r}(t)$ packets into queue $\eta^c_{l_r}$, till either all queues have been loaded or the number of packets left in q_l are less than $x^c_{l_r}(t)$ for all remaining (radiolink, channel) queues, at which point, one of those queues is loaded with less than the full quantum of packets.

Thus at time-step t, $\min\{q_l(t), \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} x_{l_r}^c(t)\}$ packets are removed from q_l. Denote by $y_{l_r}^c(t)$ the actual number of packets transferred to the channel-queue of channel c. Then: $0 \leq y_{l_r}^c(t) \leq x_{l_r}^c(t)$ and $\sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} y_{l_r}^c(t) = \min\{q_l(t), \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} x_{l_r}^c(t)\}$.

All radiolink-channel pairs (l_r, c) for which $\eta_{l_r}^c(t) \geq r_{l_r}^c$ are included in the input to the maximal scheduler described earlier for the current time-slot t. The output of the maximal scheduler yields the set of links that will be activated during the current time-slot.

Lemma 4: When the above-proposed queue-loading rule is used in conjunction with maximal scheduling, the efficiency-ratio is at least $\frac{1}{\kappa+2}$.

Proof: Using a Lyapunov stability based approach we show that the proposed algorithm stabilizes the network for all load vectors $\overrightarrow{\lambda}$, such that $(\kappa+2)\overrightarrow{\lambda}$ falls within the stability region (a vector $\overrightarrow{\delta}$ falls within the stability region if there exists some algorithm (possibly unknown) that is capable of stabilizing the network when the load-vector is $\overrightarrow{\delta}$).

The full proof is available in the appendix.

D. Discussion

The proposed variant algorithm illustrates that by explicitly taking into account interface heterogeneity in the scheduling algorithm, one can potentially get good performance guarantees. Moreover, it is likely that an algorithm that uses only per-channel queues may suffer further performance degradation. Even when $r_{l_r}^c = r_l^c$ for all $l_r \in \mathcal{R}(l)$ (i.e., the same for all radiolinks corresponding to a node-link, and a given c), an algorithm based on only per-channel queues at each link may perhaps not be able to provide a provable ratio of $\frac{1}{\kappa+2}$.

Let us now consider the amount of information-exchange required by our variant algorithm. Each node-link now needs to maintain a larger number of queues (one for each valid radiolink-channel pair). However, even in the new algorithm, each node-link l requires knowledge of exactly c pieces of information from all links $k \in \mathbf{I}(l)$, viz., the value $\sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\eta_{l_r}^c(t)}{r_{l_r}^c}$, for each c. However now each node-link l needs $|I_{e(l)}|$ piece of information from endpoint $e(l)$, viz. $\sum_{k_r \in \mathcal{I}(m)} \sum_{\substack{d \in \mathcal{C}(m) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d}$ for each interface m at node $e(l)$ (the information for $b(l)$ is available locally). Translated to information exchange between nodes, note that there is likely to be significant overlap in $\mathbf{I}(l)$ for links l having the same $b(l)$, but $e(l)$ is different for all these node-links. Thus each node u needs $|I_v|$ pieces of information from each neighbor v and c pieces of information from the hop-sender node of each node-link that conflicts with some node-link incident on u. Thus the amount of information exchanged between neighbors increases (compared to the algorithm of [12]), but the same amount of information is needed from non-neighboring interferers.

XII. Ongoing Work

In this paper we proposed a high-level framework for the design of routing and link-layer protocols for heterogeneous multi-channel wireless networks. We also discussed some issues in channel/interface selection and link-scheduling that arise as a result of heterogeneity. We are now working on the design and evaluation of a suite of protocols conforming to this framework.

Appendix

Proof of Lemma 3

This proof is obtained by suitable modification of a proof in [12].

At time-step t, $\min\{q_l(t), \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} x_{l_r}^c(t)\}$ packets are removed from q_l. Denote by $y_{l_r}^c(t)$ the actual number of packets transferred to the channel-queue of channel c. Then: $0 \leq y_{l_r}^c(t) \leq x_{l_r}^c(t)$ and $\sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} y_{l_r}^c(t) = \min\{q_l(t), \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} x_{l_r}^c(t)\}$.

At the same time, $\sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s$ new packets are received. This yields the following equation:

$$q_l(t+1) = q_l(t) + \sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s - \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} y_{l_r}^c(t) \tag{5}$$

The evolution of the channel-queues is governed by:

$$\eta_{l_r}^c(t+1) = \eta_{l_r}^c(t) + y_{l_r}^c(t) - r_{l_r}^c I_{l_r \in \mathcal{M}^c(t)} \tag{6}$$

Very similar to [12], we use the following Lyapunov function:

$$V(\overrightarrow{q}, \overrightarrow{\eta}) = V_q(\overrightarrow{q}) + V_\eta(\overrightarrow{\eta}) \tag{7}$$

■

where:

$$V_q(\overrightarrow{q}) = \sum_{l=1}^{L} \frac{(q_l(t))^2}{2\alpha_l} \tag{8}$$

$$V_\eta(\overrightarrow{\eta}) = \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} \frac{\eta_{l_r}^c(t)}{2r_{l_r}^c} \left[\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\eta_{k_r}^c(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d} + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d} \right] \tag{9}$$

Then, as shown in [12], it can be easily seen that:

$$\begin{aligned}
V_q(\overrightarrow{q}(t+1)) - V_q(\overrightarrow{q}(t)) &= \sum_{l=1}^{L} \frac{(q_l(t+1))^2}{2\alpha_l} - \frac{(q_l(t))^2}{2\alpha_l} \\
&= \sum_{l=1}^{L} \frac{(q_l(t+1))^2 - (q_l(t))^2}{2\alpha_l} \\
&= \sum_{l=1}^{L} \frac{(q_l(t+1) + q_l(t))(q_l(t+1) - q_l(t))}{2\alpha_l} \\
&= \sum_{l=1}^{L} \frac{q_l(t)(q_l(t+1) - q_l(t)) + (q_l(t+1)(q_l(t+1) - q_l(t))}{2\alpha_l} \\
&= \sum_{l=1}^{L} \frac{2q_l(t)(q_l(t+1) - q_l(t)) + (q_l(t+1) - q_l(t))^2}{2\alpha_l} \\
&= \sum_{l=1}^{L} \frac{q_l(t)(q_l(t+1) - q_l(t))}{\alpha_l} + \frac{(q_l(t+1) - q_l(t))^2}{2\alpha_l} \\
&\leq \sum_{l=1}^{L} \frac{q_l(t)(\sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s - \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} y_{l_r}^c(t))}{\alpha_l} + C_1
\end{aligned}$$

where $C_1 = \frac{1}{2\alpha_l} \sum_{l=1}^{L} (\sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s)^2$ (since $y_{l_r}^c(t) \geq 0$ for all l_r, c).

Since $\sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} y_{l_r}^c(t) = \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} x_{l_r}^c(t)$ whenever $q_l(t) \geq \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} r_{l_r}^c$, it follows that:

$$V_q(\overrightarrow{q}(t+1)) - V_q(\overrightarrow{q}(t)) \leq \sum_{l=1}^{L} \frac{q_l(t)(\sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s - \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} x_{l_r}^c(t))}{\alpha_l} + C_2 \tag{10}$$

where $C_2 = C_1 + \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} r_{l_r}^c$.

Let us now focus on the channel-queues:

$$
\begin{aligned}
&V_{\eta}(\overrightarrow{\eta}(t+1)) - V_{\eta}(\overrightarrow{\eta}(t)) = \\
&\sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta^c_{l_r}(t+1)}{2r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\eta^c_{k_r}(t+1)}{r^c_{k_r}}\right.\right. \\
&\left.\left.+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{\eta^d_{k_r}(t+1)}{r^d_{k_r}}+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{\eta^d_{k_r}(t+1)}{r^d_{k_r}}\right)\right. \\
&\left.-\sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\frac{\eta^c_{l_r}(t)}{2r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\eta^c_{k_r}(t)}{r^c_{k_r}}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{\eta^d_{k_r}(t)}{r^d_{k_r}}+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{\eta^d_{k_r}(t)}{r^d_{k_r}}\right)\right] \\
&=\sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta^c_{l_r}(t)}{2r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{y^c_{k_r}(t)-r^c_{l_r}I_{k_r\in\mathcal{M}^c(t)}}{r^c_{k_r}}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{y^d_{k_r}(t)-r^d_{k_r}I_{k_r\in\mathcal{M}^d(t)}}{r^d_{k_r}}\right.\right. \\
&\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{y^d_{k_r}(t)-r^d_{k_r}I_{k_r\in\mathcal{M}^d(t)}}{r^d_{k_r}}\right)\right] \\
&+\frac{y^c_{l_r}(t)-r^c_{l_r}I_{l_r\in\mathcal{M}^c(t)}}{2r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\eta^c_{k_r}(t)}{r^c_{k_r}}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{\eta^d_{k_r}(t)}{r^d_{k_r}}+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{\eta^d_{k_r}(t)}{r^d_{k_r}}\right) \\
&+\frac{y^c_{l_r}(t)-r^c_{l_r}I_{l_r\in\mathcal{M}^c(t)}}{2r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{y^c_{k_r}(t)-r^c_{l_r}I_{k_r\in\mathcal{M}^c(t)}}{r^c_{k_r}}\right. \\
&\left.+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{y^d_{k_r}(t)-r^d_{k_r}I_{k_r\in\mathcal{M}^d(t)}}{r^d_{k_r}}+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{y^d_{k_r}(t)-r^d_{k_r}I_{k_r\in\mathcal{M}^d(t)}}{r^d_{k_r}}\right)\Bigg]
\end{aligned}
$$

$$\leq \sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{y_{k_r}^c(t)-r_{l_r}^c I_{k_r\in\mathcal{M}^c(t)}}{r_{k_r}^c}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)-r_{k_r}^d I_{k_r\in\mathcal{M}^d(t)}}{r_{k_r}^d}\right.\right.$$
$$\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)-r_{k_r}^d I_{k_r\in\mathcal{M}^d(t)}}{r_{k_r}^d}\right)\right.$$
$$\left.+\frac{y_{l_r}^c(t)-r_{l_r}^c I_{l_r\in\mathcal{M}^c(t)}}{2r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{y_{k_r}^c(t)-r_{l_r}^c I_{k_r\in\mathcal{M}^c(t)}}{r_{k_r}^c}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)-r_{k_r}^d I_{k_r\in\mathcal{M}^d(t)}}{r_{k_r}^d}\right.\right.$$
$$\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)-r_{k_r}^d I_{k_r\in\mathcal{M}^d(t)}}{r_{k_r}^d}\right)\right]$$
$$\leq \sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{y_{k_r}^c(t)}{r_{k_r}^c}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)}{r_{k_r}^d}+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)}{r_{k_r}^d}-\mu_{l_r}^c(t)\right)\right.$$
$$\left.+\frac{r_{l_r}^c}{2r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{r_{k_r}^c}{r_{k_r}^c}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{r_{k_r}^d}{r_{k_r}^d}+\sum_{k\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{r_{k_r}^d}{r_{k_r}^d}\right)\right]$$
$$\leq \sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{y_{k_r}^c(t)}{r_{k_r}^c}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)}{r_{k_r}^d}\right.\right.$$
$$\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{y_{k_r}^d(t)}{r_{k_r}^d}-\mu_{l_r}^c(t)\right)\right]+C_3$$
$$\leq \sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{x_{k_r}^c(t)}{r_{k_r}^c}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{x_{k_r}^d(t)}{r_{k_r}^d}\right.\right.$$
$$\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{x_{k_r}^d(t)}{r_{k_r}^d}-\mu_{l_r}^c(t)\right)\right]+C_3$$

where:

$$\mu_{l_r}^c(t) = \sum_{k\in\mathbf{I}(l)} \sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}} I_{k_r\in\mathcal{M}^c(t)} + \sum_{k_r\in\mathcal{E}(s(l_r))} \sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}} I_{k_r\in\mathcal{M}^d(t)} + \sum_{k_r\in\mathcal{E}(d(l_r))} \sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}} I_{k_r\in\mathcal{M}^d(t)} \tag{11}$$

and $C_3 = \frac{1}{2}\left(\sum_{k\in\mathbf{I}(l)} \sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}} 1 + \sum_{k_r\in\mathcal{E}(s(l_r))} \sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}} 1 + \sum_{k_r\in\mathcal{E}(d(l_r))} \sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}} 1\right)$

Thus the drift is given by:

$$E[\Delta V(t)|\overrightarrow{q}(t),\overrightarrow{\eta}(t)] \le \sum_{l=1}^{L} \frac{q_l(t)}{\alpha_l}\left[\sum_{s=1}^{S}\mathcal{H}_l^s\lambda_s - \sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}} x_{l_r}^c(t))\right]$$

$$+\sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(\mathcal{R}(l))\\ op(c,l_r)=1}}\left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{x_{k_r}^c(t)}{r_{k_r}^c} + \sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{x_{k_r}^d(t)}{r_{k_r}^d}\right.\right. \tag{12}$$

$$\left.\left. + \sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{x_{k_r}^d(t)}{r_{k_r}^d} - \mu_{l_r}^c(t)\right)\right] + C_4$$

where $C_4 = C_2 + C_3$.

By assumption, there exists some scheduling algorithm that achieves stability with load vector $(\kappa+2)\overrightarrow{\lambda}$. Similar to [12], we can argue that this implies existence of $\widetilde{x_{l_r}^c}$ for all l_r, c satisfying the following:

$$(1+\varepsilon)^2(\kappa+2)\sum_{s=1}^{S}\mathcal{H}_l^s\lambda_s \le \sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\widetilde{x_{l_r}^c} \text{ for all links } l \tag{13}$$

$$\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\widetilde{x_{k_r}^c}}{r_{k_r}^c} \le \kappa \text{ for all links } l \text{ and channels } c \tag{14}$$

$$\sum_{k_r\in\mathcal{E}(m)}\sum_{\substack{c\in\mathcal{C}(m)\\ op(c,k_r)=1}}\frac{\widetilde{x_{k_r}^c}}{r_{k_r}^c} \le 1 \text{ for all interfaces } m \tag{15}$$

Set $\overline{x_{l_r}^c} = \frac{\widetilde{x_{l_r}^c}}{(1+\varepsilon)(\kappa+2)}$. Then from Eqn. (13), Eqn. (14) and Eqn. (15), we obtain that:

$$(1+\varepsilon)\sum_{s=1}^{S}\mathcal{H}_l^s\lambda_s \le \sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\overline{x_{l_r}^c} \text{ for all links } l \tag{16}$$

$$\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\overline{x^c_{k_r}}}{r^c_{k_r}}\leq\frac{\kappa}{(1+\varepsilon)(\kappa+2)}\text{ for all links }l\text{ and channels }c \tag{17}$$

$$\sum_{k\in\mathcal{E}(m)}\sum_{\substack{c\in\mathcal{C}(m)\\ op(c,k_r)=1}}\frac{\overline{x^c_{k_r}}}{r^c_{k_r}}\leq\frac{1}{(1+\varepsilon)(\kappa+2)}\text{ for all interfaces }m \tag{18}$$

This yields:

$$\begin{aligned}(1+\varepsilon)\left[\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\overline{x^c_{k_r}}}{r^c_{k_r}}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{\overline{x^c_{k_r}}}{r^c_{k_r}}+\sum_{k\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{\overline{x^c_{k_r}}}{r^c_{k_r}}\right]&\\ \leq\frac{\kappa}{(\kappa+2)}+\frac{1}{(\kappa+2)}+\frac{1}{(\kappa+2)}&\\ \leq 1\text{ for all links }l\text{ and channels }c&\end{aligned} \tag{19}$$

Rewriting Eqn. (12), we obtain:

$$\begin{aligned}&E[\Delta V(t)|\overrightarrow{q}(t),\overrightarrow{\eta}(t)]\\ &\leq\sum_{l=1}^{L}\frac{q_l(t)}{\alpha_l}\left[\sum_{s=1}^{S}\mathcal{H}_l^s\lambda_s-\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\overline{x^c_{l_r}}(t)\right]+\sum_{l=1}^{L}\frac{q_l(t)}{\alpha_l}\left[\sum_{c=1}^{C}\sum_{\substack{c\in\mathcal{C}\\ op(c,l_r)=1}}\overline{x^c_{l_r}}(t)-\sum_{c=1}^{C}\sum_{\substack{c\in\mathcal{C}\\ op(c,l_r)=1}}x^c_{l_r}(t)\right]\\ &+\sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta^c_{l_r}(t)}{r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{\overline{x^c_{k_r}}(t)}{r^c_{k_r}}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{\overline{x^d_{k_r}}(t)}{r^d_{k_r}}\right.\right.\\ &\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{\overline{x^d_{k_r}}(t)}{r^d_{k_r}}-\mu^c_{l_r}(t)\right)\right]\\ &+\sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{\substack{l_r\in\mathcal{R}(l)\\ op(c,l_r)=1}}\left[\frac{\eta^c_{l_r}(t)}{r^c_{l_r}}\left(\sum_{k\in\mathbf{I}(l)}\sum_{\substack{k_r\in\mathcal{R}(k)\\ op(c,k_r)=1}}\frac{x^c_{k_r}(t)-\overline{x^c_{k_r}}(t)}{r^c_{k_r}}+\sum_{k_r\in\mathcal{E}(s(l_r))}\sum_{\substack{d\in\mathcal{C}(s(l_r))\\ op(d,k_r)=1}}\frac{x^d_{k_r}(t)-\overline{x^d_{k_r}}(t)}{r^d_{k_r}}\right.\right.\\ &\left.\left.+\sum_{k_r\in\mathcal{E}(d(l_r))}\sum_{\substack{d\in\mathcal{C}(d(l_r))\\ op(d,k_r)=1}}\frac{x^d_{k_r}(t)-\overline{x^d_{k_r}}(t)}{r^d_{k_r}}\right)\right]+C_4\end{aligned}$$

Since we are using maximal scheduling, we can say that for all radiolinks l_r, $\mu_l^c(t) \geq 1$ whenever $\eta_{l_r}^c(t) \geq r_{l_r}^c$. Then, substituting from Eqn. (16) and Eqn. (19), we have that:

$$
\begin{aligned}
&E[\Delta V(t) | \overrightarrow{q}(t), \overrightarrow{\eta}(t)] \\
&\leq \sum_{l=1}^{L} \frac{q_l(t)}{\alpha_l} \left[-\varepsilon \sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s \right] + \sum_{l=1}^{L} \frac{q_l(t)}{\alpha_l} \left[\sum_{c=1}^{C} \sum_{\substack{c \in \mathcal{C} \\ op(c,l_r)=1}} \overline{x_{l_r}^c}(t) - \sum_{c=1}^{C} \sum_{\substack{c \in \mathcal{C} \\ op(c,l_r)=1}} x_{l_r}^c(t) \right] \\
&+ \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} \left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c} \left(1 - \varepsilon \left(\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\overline{x_{k_r}^c}(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right. \right. \right. \\
&\qquad \left. \left. \left. + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right) - \mu_{l_r}^c(t) \right) \right] \\
&+ \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} \left[\frac{\eta_{l_r}^c(t)}{r_{l_r}^c} \left(\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{x_{k_r}^c(t) - \overline{x_{k_r}^c}(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{x_{k_r}^d(t) - \overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right. \right. \\
&\qquad \left. \left. + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{x_{k_r}^d(t) - \overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right) \right] + C_4 \\
&\leq -\varepsilon \sum_{l=1}^{L} \frac{\sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s}{\alpha_l} q_l(t) - \varepsilon \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} \left[\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\overline{x_{k_r}^c}(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right. \\
&\qquad \left. + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right] \frac{\eta_{l_r}^c(t)}{r_{l_r}^c} \\
&+ \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} \left[\frac{\overline{x_{l_r}^c}(t) - x_{l_r}^c(t)}{r_{l_r}^c} \left(\frac{r_{l_r}^c q_l(t)}{\alpha_l} - \left(\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\eta_{k_r}^c(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d} \right. \right. \right. \\
&\qquad \left. \left. \left. + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d} \right) \right) \right] + C_5
\end{aligned}
$$

where $C_5 = C_4 + \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(l) \\ op(c,l_r)=1}} 1$

Since $0 \leq \overline{x_{l_r}^c} \leq r_{l_r}^c$. and since $x_{l_r}^c(t) = r_{l_r}^c$ only if:

$$\frac{r_{l_r}^c q_l(t)}{\alpha_l} - \left(\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\eta_{k_r}^c(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d} + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\eta_{k_r}^d(t)}{r_{k_r}^d} \right) \geq 0$$

and 0 else, we obtain that:

$$\begin{aligned} E[\Delta V(t) | \overrightarrow{q}(t), \overrightarrow{\eta}(t)] \leq & -\varepsilon \sum_{l=1}^{L} \frac{\sum_{s=1}^{S} \mathcal{H}_l^s \lambda_s}{\alpha_l} q_l(t) \\ & - \varepsilon \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{\substack{l_r \in \mathcal{R}(\mathcal{R}(l)) \\ op(c,l_r)=1}} \frac{1}{r_{l_r}^c} \left[\sum_{k \in \mathbf{I}(l)} \sum_{\substack{k_r \in \mathcal{R}(k) \\ op(c,k_r)=1}} \frac{\overline{x_{k_r}^c}(t)}{r_{k_r}^c} + \sum_{k_r \in \mathcal{E}(s(l_r))} \sum_{\substack{d \in \mathcal{C}(s(l_r)) \\ op(d,k_r)=1}} \frac{\overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right. \\ & \left. + \sum_{k_r \in \mathcal{E}(d(l_r))} \sum_{\substack{d \in \mathcal{C}(d(l_r)) \\ op(d,k_r)=1}} \frac{\overline{x_{k_r}^d}(t)}{r_{k_r}^d} \right] \eta_{l_r}^c(t) + C_5 \end{aligned}$$

We can thus invoke Lemma 3 to obtain the proof of stability.

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