

# Heterogeneous preferences, decision-making capacity, and phase transitions in a complex adaptive system

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There has been a belief that with the directing power of the market, the efficient state of a resource-allocating system can eventually be reached even in a case where the resource is distributed in a biased way. To mimic the realistic huge system for the resource allocation, we designed and conducted a series of economic experiments. From the experiments we found that efficient allocation can be realized despite a lack of communications among the participants or any instructions to them. To explain the underlying mechanism, an extended minority game model called the market-directed resource allocation game (MDRAG) is constructed by introducing heterogeneous preferences into the strategy-building procedures. MDRAG can produce results in good agreement with the experiments. We investigated the influence of agents' decision-making capacity on the system behavior and the phase structure of the MDRAG model as well. A number of phase transitions are identified in the system. In the critical region, we found that the overall system will behave in an efficient, stable, and unpredictable mode in which the market's invisible hand can fully play its role.

biased distribution | invisible hand | economic experiments | minority game | resource allocation

Most of the social, economic, and biological systems involving a large number of interacting agents can be regarded as complex adaptive systems (CAS) (1), because they are characterized by a high degree of adaptive capacities to the changing environment. The interesting dynamics and phase behaviors of these systems have attracted much interest among physical scientists. A number of microscopic CAS models have been proposed (2–6), among which the minority game (MG) (7–9) becomes a representative model. Along with the progress in the research of econophysics (10), MG has been mostly applied to simulate one kind of CAS, namely the stock market (11, 12). Alternatively, MG can also be interpreted as a multiagent system competing for a limited resource (13, 14) that distributes equally in 2 rooms. However, agents in the real world often have to face a competition to the limited resource, which distributes in different places in a biased manner. Examples of such phenomena include companies competing among markets of different sizes (15), drivers selecting different traffic routes (16), people betting on horse racing with the odds of winning a prize, and making decisions on which night to go to which bar (17).

From a global point of view, the ideal evolution of a resource-allocating system would be the following: Although each agent would compete against others only with a self-serving purpose, the system as a whole could eventually reach a harmonic balanced state where the allocation of resource is efficient, stable, and arbitrage-free (which means that no one can benefit from the “misdistribution” of the resource). Note that during the process of evolution to this state, agents could neither have been told about the actual amount of the resources in a specific place nor could they have any direct and full communications, just as if there were an “invisible hand” directing them to cooperate with each other. Then, does this invisible hand always work? In practice, there is plenty of evidence that the invisible hand does

have very strong directing power in places such as financial markets, although sometimes it does fail to work. Such temporary ineffectiveness implies that there must be some basic conditions required for the invisible hand to exert its full power. Through an experimental study and a numerical study with a market-directed resource allocation game (MDRAG, which is an extended version of the MG model), we found that agents equipped with heterogeneous preferences as well as a decision-making capacity that matches with the environmental complexity are sufficient for the spontaneous realization of such a harmonic balanced state.

To illustrate the system behavior, we designed and conducted a series of economic experiments, in collaboration with university students. In the experiments, 89 students from different (mainly physics, mathematics, and economics) departments of Fudan University were recruited and randomly divided into 7 groups (Groups A–G, see Tables 1–4). The number of students in each group was just set for convenience and denoted by  $N$  in Tables 1–4. In the games played in the experiments, students were told that they had to make a choice among a number of rooms, in each round of a session, for sharing the different amounts of virtual money in different rooms. Students who got more than the global average, namely those belonging to the relative minority, would win the payoff. At the beginning of a session, participants were told the number of rooms (2 or 3) and in some cases, the different but fixed amount of virtual money in each room. In the following,  $M_i$  is used to denote the amount of virtual money in room  $i$ . A piece of global information about the payoff in the preceding round in all rooms is announced before a new round starts. In each round, the students must make their own choices without any kind of communication. The payoff per round for a student in room  $i$  is 2 points if  $M_i/N_i > \sum M_i/N$ , and  $-1$  point otherwise. Here,  $N_i$  is the number of the students choosing room  $i$ . The total payoff of a student is the sum of payoffs of all rounds, which will be converted to money payoffs in Renminbi (RMB, or Chinese yuan) with a fixed exchange rate. Because the organizational and statistic procedures were done by a human, one session of 10 rounds took  $\approx 20$  min. More details can be found in the leaflet to the experiments in *Appendix*.

Three kinds of games, GAME-I, GAME-II, and GAME-III, have been investigated. GAME-II differs from GAME-I in the global information being announced. In GAME-I, both the resource distribution  $M_i$  and the current population  $N_i$  in room  $i$  were announced, whereas only payoffs (2 or  $-1$ ) in each room of the current round were conveyed to players in GAME-II. Note that the environmental complexity was increased in GAME-II, because to win the game, players would have to predict other players' decisions, and, in the meantime, infer the actual amount of virtual money in different rooms. In GAME-III, the global information is the same as that of GAME-II,

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**Table 4. Results of GAME-III**

Session	Group	Round	$M_1$	$M_2$	$\langle N_1 \rangle$	$\langle N_2 \rangle$
1	G ( $N = 10$ )	1 to 5	3	1	5.4	4.6
1	G ( $N = 10$ )	6 to 10	3	1	8.2	1.8
1	G ( $N = 10$ )	11 to 15	3	1	7	3
1	G ( $N = 10$ )	16 to 20	3	1	7	3
1	G ( $N = 10$ )	21 to 25	1	3	7.8	2.2
1	G ( $N = 10$ )	26 to 30	1	3	4.2	5.8
1	G ( $N = 10$ )	31 to 35	1	3	2.8	7.2
1	G ( $N = 10$ )	36 to 40	1	3	2.6	7.4
1	G ( $N = 10$ )	41 to 45	1	3	2.4	7.6

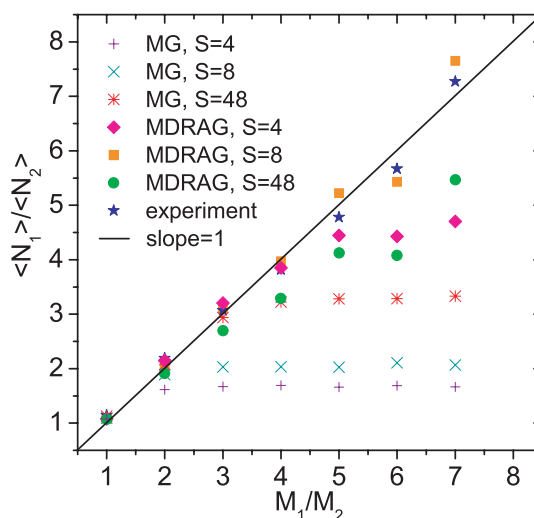
environmental complexity of the games. Note that if  $M_1/M_2 = 1$ , agents need only to worry about other people's decision. Assuming that room 1 always contains more resources, the trivial case will be  $M_1/M_2 > N - 1$ , because all of the agents can easily find out that going to room 1 would be the right choice under this situation. On the other hand, when the ratio is set  $1 < M_1/M_2 \leq N - 1$ , the larger this ratio is, the more difficult it would be for the market to direct the system to the ideal state. Other parameters concern the decision-making capacity, which can be generalized into 3 elements (<http://plato.stanford.edu/entries/decision-capacity/>), namely, (i) the possession of a set of values and goals necessary for evaluating different options; (ii) the ability to communicate and understand information; and (iii) the ability to reason and to deliberate about one's choices. The first element has already been built into both the models as the evaluation of the strategies with the minority-favorable payoff function. The second element relates to the model parameter  $P$ . Because the total number of possible situations depends on the completeness of the perception of the world, we relate it to cognition ability. Finally, more strategies could be helpful if one needs to deliberate his/her choices of decisions, hence the strategy number  $S$  is related to the third element of the decision-making capacity, the ability of choice deliberation.

## Results

Results of the economic experiments are compared with the simulation results of the traditional MG and MDRAG in Fig. 1. For each parameter set, we performed the simulation 200 times. In each of these simulations, the code was run over 400 time steps. The first half of the time evolution is for the equilibration of the system, whereas the remaining half is for doing the statistics. With a certain set of parameters ( $S = 8$  and  $P = 16$ ), MDRAG's results perfectly agree with the experimental data under higher environmental complexity. In other words, agents in both experiments and MDRAG can be directed by the market to cooperate with each other so that an efficient allocation of the biasedly distributed resource can be realized even without giving the agents full information or instructions. On the other hand, the traditional MG fails to reproduce the experimental results unless the distribution of resource is biased very weakly up to  $M_1/M_2 = 3$ . Note that MDRAG differs from MG solely in the introduction of heterogeneous preferences in the strategies;

**Table 5. A typical strategy table**

Economic situations	Choices
1	0
2	1
3	1
...	...
P-1	0
P	1



**Fig. 1.**  $\langle N_1 \rangle / \langle N_2 \rangle$  as functions of  $M_1/M_2$ ,  $P = 16$  in MG and MDRAG, and  $N = 24$  for all of the simulations and the experiment. Simulations are run 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). The line with slope = 1 indicates the efficient states:  $\langle N_1 \rangle / \langle N_2 \rangle = M_1/M_2$ .

hence, one may infer that the heterogeneity of agents' preferences is a significant factor to have the invisible hands be effective. This argument is further supported by numerical experiments in the 3-room cases (with parameters  $P = 24$ ,  $N = 120$ , and  $S = 10$ ). Again, here MDRAG is superior to MG in bringing out the directing power of the market. Shown in Table 6, the ratio of  $\langle N_1 \rangle : \langle N_2 \rangle : \langle N_3 \rangle$  converges to  $M_1 : M_2 : M_3$  only in the equilibrium states of MDRAG.

Fig. 1 also shows that the decision-making capacity, in particular, the deliberation of choices (the parameter  $S$ ), would be another factor having an influence on the effectiveness of the invisible hand. Typically, as the environmental complexity ( $M_1/M_2$ ) increases, both MG and MDRAG will deviate from the experimental results. Nevertheless, the problem of MG is much more severe. As shown in the figure, even MG with extremely large  $S$  ( $S = 48$ , a situation that is inconsistent with the real system and will drastically increase the computational cost) can just work at a very low level of environmental complexity. At the same time, the result of MDRAG provides a perfect fit with the experimental data when  $S$  is large enough, but not too large for a given  $P$  value (the reason will be explained in the following discussion of Fig. 3). In a word, MG does not provide a good fit even for large  $S$ , whereas MDRAG can fit the data with a less demanding condition in terms of computational cost.

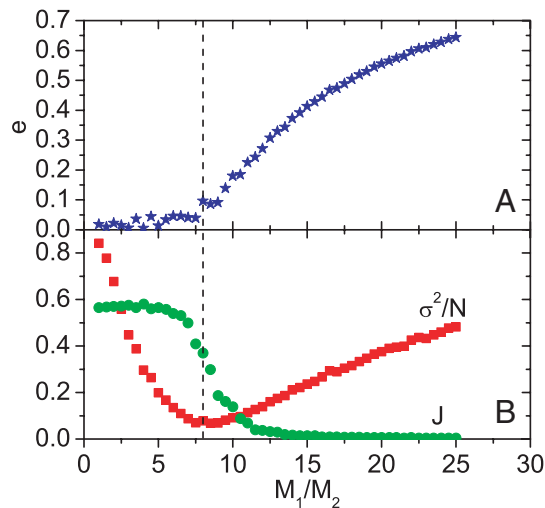
## Discussion

Through a large number of numerical simulations, we have found the dependence of equilibrium states of the system on the model parameters, together with a number of phase transitions in the models. To explore this in more detail, 3 parameters are defined

**Table 6. Performances of MDRAG and MG in 3-room cases**

Resource			MDRAG			MG		
$M_1$	$M_2$	$M_3$	$\langle N_1 \rangle$	$\langle N_2 \rangle$	$\langle N_3 \rangle$	$\langle N_1 \rangle$	$\langle N_2 \rangle$	$\langle N_3 \rangle$
1	1	1	39.9	40.3	39.8	39.2	41.5	39.3
1	2	3	19.7	40.1	60.2	25.1	40.7	54.2
1	4	7	9.6	40	70.4	24.8	40.4	54.8
1	2	9	9.8	19.6	90.6	29.3	33.8	56.9





**Fig. 2.** The “ $M_1/M_2$  phase transition” in MDRAG, for  $N = 100$ ,  $P = 64$ , and  $S = 8$ . Simulations are run 300 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). The dashed line denotes  $M_1/M_2 = 8$ . (A)  $e$  as a function of  $M_1/M_2$ . (B)  $\sigma^2/N$  and  $J$  versus  $M_1/M_2$ , respectively.

that describe system behaviors in 3 aspects, namely, efficiency, stability, and predictability. First the efficiency of resource allocation can be described as  $e = |\langle N_1 \rangle / \langle N_2 \rangle - M_1/M_2| / (M_1/M_2)$ . Note that  $0 \leq e < 1$  and a smaller  $e$  means a higher efficiency in the allocation of the resource. The stability of a system can be described by  $\sigma^2/N \equiv (1/2N) \sum_{i=1}^P \langle (N_i - \bar{N}_i)^2 \rangle$ , which denotes the population fluctuation away from the optimal state.<sup>†</sup> Here,  $\bar{N}_i = M_i N / \sum M_i$ , and  $\langle A \rangle$  is the average of time series  $A_t$ . The predictability is related to  $H \equiv (1/2NP) \sum_{\mu=1}^P \sum_{i=1}^2 \langle (N_i - \bar{N}_i | \mu)^2 \rangle$ , in which  $\langle A | \mu \rangle$  is the conditional average of  $A_t$ , given that  $\mu_t = \mu$ , one of the  $P$  possible economic situations. If  $\sigma^2/N \neq H$ , it means that agents may take different actions at different times for the same economic situation (namely, the market behavior is unpredictable). For clarity, we describe the predictability of system by defining  $J = 1 - H/\sigma^2$ . It is obvious that  $0 \leq J < 1$  and a smaller  $J$  means a higher predictability.

The variation of system behavior along with the change of environmental complexity  $M_1/M_2$  is shown in Fig. 2. As shown in Fig. 2A, the system changes from an efficient state into an inefficient state at some critical value  $(M_1/M_2)_c \approx S$ . For other values of  $P$ , the system behavior stays the same as long as  $P$  is larger than  $M_1/M_2$ . In Fig. 2B, around the same critical value of  $M_1/M_2$ ,  $\sigma^2/N$  changes from a decreasing function to an increasing function, giving the smallest fluctuation in the population distribution at the critical point. Meanwhile, the order parameter  $J$  also falls into zero at  $(M_1/M_2)_c$ , suggesting that a phase transition, named the “ $M_1/M_2$  phase transition,” occurs at this critical point. To be more illustrative, when the environmental complexity is much smaller than the critical value, the system could reside in an efficient, unpredictable, but relatively unstable state. Getting closer to the phase transition point, the stability of system will be improved until the most stable state is reached. Then, after crossing the critical point, the decision-making capability of the whole system has been exhausted, and it will fall into an inefficient, predictable, and unstable state. At the vicinity of the critical point, as if participants of the game worried about being eliminated from the competition, the market inspires all of its

guiding potential and leads the system to the ideal state for the resource allocation, a state that is both efficient and stable and where no unfair arbitrage chance can exist.

It is important to know that MDRAG and MG have totally different phase structures, which could be analyzed by comparing the  $S - P$  contours of the descriptive parameters for the 2 models, see Fig. 3. From the analysis, we could also know how the decision-making capacity influence the overall performance of the resource allocation system, in case the environmental complexity is fixed ( $M_1/M_2 = 4$ ). Features of the contour maps (Fig. 3) are summarized as the following (different  $M_1/M_2$ s do not change the conclusions):

(i) Compared with the traditional MG as a whole, MDRAG has a much wider range of parameters for the availability of the efficient, stable, and unpredictable states. In particular, there is almost no eligible region in Fig. 3A if we take the criterion of efficiency as  $e < 0.08$ . Also, the predictable region ( $J < 0.02$ ) in Fig. 3F is much smaller than that of the MG’s results in Fig. 3C. These facts indicate that MDRAG has a much better performance than MG as a resource-allocating system.

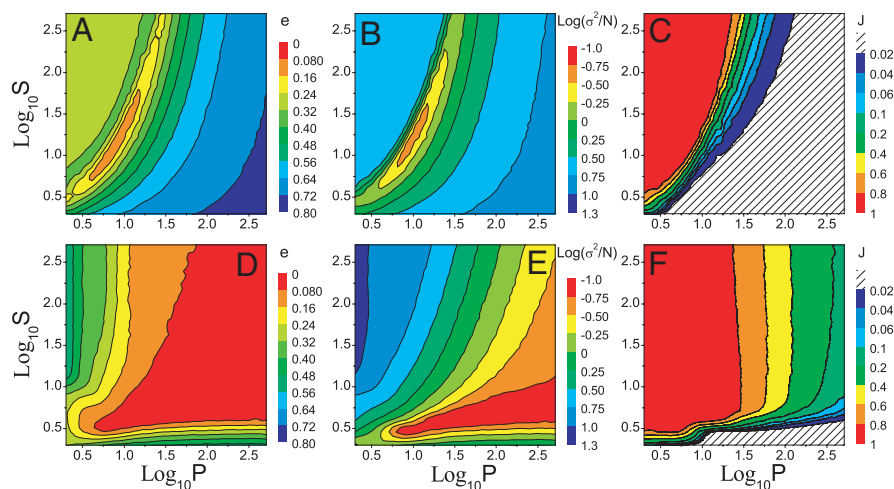
(ii) Patterns of the contour maps suggest that MG and MDRAG have totally different dependency on parameters. Fig. 3A–C indicates that  $P$  and  $S$  are not independent in the traditional MG model, which confirms the previous findings (19). On the other hand, in MDRAG, there is always a region where the system behavior is almost controlled by the parameter  $S$ . In Fig. 3D, for large enough  $P$ , the system can reach the efficient state if  $S$  exceeds a critical value  $S_p$ , where  $S_p$  will converge to the limit value  $M_1/M_2$  with increasing  $P$ . For  $S < S_p$ , the system can never reach the efficient state no matter how  $P$  changes. For a very large  $P$  and  $S < M_1/M_2$ , it can be proved that the probability for agents to enter the richer room is  $S/(S + 1)$ , so that the system stays in the inefficient states ( $\langle N_1 \rangle / \langle N_2 \rangle < M_1/M_2$ ).

(iii) Observing Fig. 3D and F, one may find both an “ $S$  phase transition” and a “ $P$  phase transition.” As mentioned above, for large enough  $P$ , the increase of  $S$  can abruptly bring the system from the inefficient/predictable phase to the efficient/unpredictable phase, and it is named  $S$  phase transition. On the other hand, in the narrow region where  $S < M_1/M_2$ , the increase of  $P$  can also produce of a drastic change from the unpredictable phase to the predictable phase, and it is named  $P$  phase transition. The existence of the  $S$  phase transition can be explained by the fact that the number of available choices in decision making is a key factor for agents to find the right choice from strategies with an adequate heterogeneity of preferences. But for  $S \gg P$ , it will also cause a slight decrease of the efficiency because of the conflicts of the different predictions from the equally good strategies with the same preference. This explains why MDRAG,  $S = 48$  performs worse than MDRAG,  $S = 8$ , when  $P = 16$  in Fig. 1. The  $P$  phase transition reflects that for some incompetent ability of choice deliberation, a critical value of the cognition ability can enhance the decision-making capacity to match the environmental complexity.

(iv) It is also noteworthy that the parameter  $\alpha = 2/PN$ , which is the main control parameter in the MG model (8, 9), no longer controls on the behavior of the MDRAG system. Varying  $N$  while keeping  $M_1/M_2$  as a constant, the basic feature, especially the critical position of the contour maps, will remain unchanged.

In aspects of the competition for resources, the feed of global information, and the inductive optimization of strategies, both MG and MDRAG may be regarded as eligible models for the economic experiments. However, MG fails to reproduce the experimental results in most cases. By simply accommodating a broader preference distribution of the strategies, MDRAG fits the experimental results without any coordinating capability of the agents. This enables us to comment on the possible mech-

<sup>†</sup>Large fluctuations in populations can cause a higher dissipation in the system. Hence, an efficient and stable state means an optimal state with a low waste in the resource allocation.



**Fig. 3.**  $S - P$  contour maps for different parameters on log-log scales with  $N = 80$  and  $M_1/M_2 = 4$ . For each data point, simulations are run 50 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). (A–C)  $e$ ,  $\sigma^2/N$ , and  $J$  as functions of  $S$  and  $P$  in MG, respectively. (D–F)  $e$ ,  $\sigma^2/N$ , and  $J$  as functions of  $S$  and  $P$  in MDRAG, respectively. Regions filled with diagonal shading denote the predictable states.

anism of the invisible hand, and conditions under which the complex adaptive systems will spontaneously converge to the efficient states. The most important thing for the invisible hand to work is that different players of the economic games should have different preferences, just like the agents in the MDRAG games who have heterogeneous preferences in their strategies. Next, the players should also have an adequate capacity of decision making that matches the complexity of the environment. From the  $M_1/M_2$  phase transition in the MDRAG simulations, we could infer that there would be a failure in achieving the balanced efficient state if the game of the experiment were designed in too complicated a way, e.g., too many rooms or a too-biased distribution of the virtual money. Nevertheless, for the MDRAG model itself, because the model parameters can be tuned freely, we believe that the market directing power can always be brought out completely in this paradigm as long as there is enough computational power. To put it another way, when the experiment happens to be set at the critical range of players' decision-making capacity, just like a finely tuned MDRAG where parameters are set to be critical values of the phase transitions mentioned above, an idealized state of the resource allocating system can be realized, namely, the system is efficient, stable, and unpredictable; see the overlapped regions for small  $e$ , small  $\sigma^2/N$ , and finite  $J$  in Fig. 3 D–F.

Finally, although these intriguing conclusions are supported by the results of MDRAG simulations, there are still some important effects in the real world not included in the model, such as the difference in the decision-making capacities among the agents and the agents' responses to changes of the environment. One challenging task is to consider a suitable relation between agents' behavior and the distribution of resources ( $M_1/M_2$ ), which may have an influence on the dynamic behavior of the whole system.

### Appendix: Leaflet to the Economic Experiment

- A group of persons are taking part in this experiment. The game situation is the same for each participant. In the experiments, any kinds of communication are not allowed.

- At the beginning of the game, all of you will be told the kind of game (GAME-I, GAME-II, or GAME-III) as well as the total number of the players ( $N$ ), rooms (2 or 3), and play rounds.
- In each round of the game, you have to choose and enter 1 of the rooms. The amount of virtual money in each room is different but fixed, represented by  $M_i$  ( $i = 1, 2, \dots$ ).
- You will be told each  $M_i$  (only in GAME-I).
- In each round, you can choose a room to share the virtual money in it and get your quota,  $M_i/N_i$ , if you select room  $i$ . Here,  $N_i$  denotes the total number of players in room  $i$ .
- You may make a new room choice in every round.
- Your payoff per round: After the statistics of each round are done, you will receive a payoff that depends on the relation between your quota and the global average:

$$\text{payoff per round} = \begin{cases} 2 \text{ points,} & \text{if } M_i/N_i > \sum M_i/N \\ -1 \text{ point,} & \text{otherwise.} \end{cases}$$

- Your information per round:
  - The current round number.
  - $N_i$  of each room in the preceding round (only in GAME-I, announced by the game organizer).
  - Payoff (2 or –1) of each room in the preceding round (announced by the game organizer).
  - Your rooms chosen and payoffs got in the preceding game rounds (recorded by yourself).
  - Your cumulated payoffs (calculated by yourself).
- The initial capital of each participant is 0 point. The exchange rate is 1 RMB per (positive) point.

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