# Heterogeneous Price Information and the Effect of Competition ${ }^{1}$ 

Saul Lach ${ }^{2} \quad$ José L. Moraga-González ${ }^{3}$

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[^0]
#### Abstract

This paper examines how the distribution of prices and consumer welfare change with the number of competitors in a model where consumers differ in the amount of price information they have. The number of firms affects prices (and welfare) by changing consumers' information about prices. We only assume that an increase in the number of competitors raises the average number of prices observed by a consumer in the market and (weakly) lowers the probability consumers see only one price. Under this assumption, the lower percentiles of the price distribution decrease with an increase in the number of firms. We then derive a necessary and sufficient condition under which all percentiles of the price distribution decrease when competition is intensified. In such a situation, some percentiles fall more than others, which leads to asymmetric welfare gains from increased competition. We also provide a necessary and sufficient condition under which the higher percentiles of the distribution of prices paid increase. When this happens, the probability that a consumer already paying a high price will pay even a higher price increases and it may even be the case that some consumers experience a welfare loss on average. Nevertheless, the weighted consumer surplus always (weakly) raises with increased competition. We illustrate these results by estimating the response of the distribution of prices to competition in the gasoline retail market in the Netherlands. We find that all percentiles of the price distribution decrease with intensified competition but the magnitude of the change, although not large, varies along the distribution in a way similar to that predicted by the model. We also estimate that the less informed consumers benefit more from increased competition than the more informed ones.


## 1 Introduction

Standard Cournot and Bertrand oligopoly models predict that an increase in the number of firms will lower the equilibrium price. When an additional firm enters the market the equilibrium price falls because firms cut their output (price) too little (much) since they do not internalize the externalities their output (or pricing) decisions have on one another. These standard models formalize the widely accepted view in economics that more competition, as measured by an increase in the number of firms, (1) lowers prices and (2) benefits all consumers.

An important assumption behind these models is that all prices are known to consumers. This implies that market equilibrium is characterized by a single price. In real life, however, we observe price dispersion, i.e., identical stores charging different prices for the same product. Price dispersion can be generated as an equilibrium phenomenon when consumers differ in the amount of information they have about prices. ${ }^{1}$ In a seminal paper, Varian (1980) shows that, when some consumers are informed about the prices of all the firms in the market while the rest only know the price of one firm, the Nash-Bertrand symmetric equilibrium is one of mixed strategies: in equilibrium, firms draw prices from a common non-degenerate price distribution thereby producing price dispersion. ${ }^{2}$

In this paper we address the question of how an increase in the number of firms affects the equilibrium price distribution in the presence of heterogeneous price information. This issue has been analyzed in the theoretical literature but we think that what we have learnt so far is limited and incomplete because the models used in this literature are restrictive in at least two ways. First, most papers have focused on one, and only one, mechanism through which price information is gathered and/or distributed in the market. In fact, the typical paper models either consumer search or firm advertising. However, in reality, these two mechanisms operate simultaneously and, in addition, equally important factors such as spatial features of the market (e.g., the location of firms and consumers) and the patterns of consumer social interactions also have a bearing on the effects of increased competition. Second, for technical reasons, most of these papers allow for little heterogeneity in the amount of price information consumers have.

In fact, in the earlier models of Rosenthal (1980) and Varian (1980) some consumers are fully informed and others know only the price of one firm. Both these models predict that the mean price increases with the number of firms. ${ }^{3}$ The canonical sequential search model of Stahl

[^1](1989) assumes that some consumers are fully informed and others have to search to gather price information. In equilibrium, consumers search only once and the mean price also goes up with the number of firms. Janssen and Moraga-González (2004) study a similar model where consumers, instead, search non-sequentially. They find that increased competition results in more search and lower prices when the number of competitors in the market is low to begin with, but in less search and higher prices when the number of competitors is large. Stahl (1994), generalizing the advertising model of Butters (1977), shows that an increase in the number of firms results in lower prices. Roberts and Stahl (1993), in one of the few combined search-and-advertising models, show that when initial marginal advertising costs are positive, entry of a new firm drives prices higher.

In order to give a more complete answer to the question of how prices respond to increased competition when prices are dispersed in equilibrium, one suggestion would be to include all the aforementioned mechanisms (search, advertising, social networking, firms' and consumers' locations) affecting price information into a single model. This is an extremely difficult task. In this paper, instead, we propose to study this issue by using a reduced-form approach proposed by Armstrong, Vickers and Zhou (2009) that allows for arbitrary heterogeneity in the amount of price information consumers have. ${ }^{4}$ In our model a consumer who enters the market receives price information according to an arbitrary probability generating function. The stochastic process by which consumers get exposed to prices can be seen as a reduced-form for the various mechanisms through which price information flows in the market. We show that the effect of a change in the number of firms on the price distribution intimately depends on how the price information available to consumers is affected by this change. We make a relatively weak assumption about this: the entry of an additional firm raises the (weighted) average number of prices observed by a consumer in the market and (weakly) lowers the probability consumers see only one price. Essentially, this assumption states that the first-order effect of entry of a new firm - an increase in price information among consumers - is larger than the potential decrease in price information due to the possible downward adjustments of search and advertising efforts triggered by this entry.

Under this assumption, we show that only the lower percentiles of the price distribution surely decrease with an increase in the number of firms. We then derive a sufficient condition under which all percentiles of the price distribution decrease when competition is intensified. ${ }^{5}$ The condition requires the ratio of the probability that consumers are informed about one price to the

[^2]probability that they are informed about $s$ prices, $s=2, \ldots, N$, to decrease when $N$ (the number of firms) increases. If this condition holds, then we obtain the result that more competition lowers prices and benefits all consumers, exactly as in standard Cournot and Bertrand models. We think this result is relevant. Allowing for an arbitrary distribution of information among consumers, besides being more realistic, can rule out altogether the unappealing result that the mean price increases with the number of firms. On the contrary, under the condition stated above, the mean price (and the mean price paid by all consumers) decreases with the number of firms. ${ }^{6}$

We next explore how the price response to increased competition varies along the price distribution in this case where all percentiles of the price distribution fall with the number of competitors. We show that the magnitude of the impact of increased competition changes from percentile to percentile. In particular, when the distribution of price information in the market follows the truncated binomial distribution with parameters $N$ and $p$ (the probability of observing a price) and $p$ is relatively small, then the lower percentiles of the price distribution fall more than the higher percentiles after the number of competitors increases. Instead, when the parameter $p$ is relatively high then intermediate percentiles fall more than more extreme ones.

This non-uniform impact of increased competition on the percentiles of the price distribution leads to rich welfare implications. The change in consumers' utility depends on changes in the price paid and we show that this price may increase or decrease depending on how the different percentiles of the price distribution are affected by increased competition. In the binomial distribution example, when the probability of observing a price, $p$, is small, welfare gains from increased competition are higher for better informed consumers. By contrast, when $p$ is high, it may well be the case that poorly informed consumers gain more from competition than well informed ones. Similar results obtain for the discrete uniform distribution.

The paper proceeds by providing a necessary and sufficient condition under which the high percentiles of the price distribution increase with the number of firms, whereas the low percentiles decrease. We show that when the ratio of the probability of observing one price only to the probability of observing two prices increases with the number of firms, then the upper percentiles of the price distribution increase. This condition is intuitive when we think again about the equilibrium price distribution as a set of prices intended to attract differently informed consumers. The low prices of the price distribution are meant to attract consumers who are well informed while higher prices are intended for the progressively less informed consumers. Note

[^3]that as we move up in the price distribution, the prices are less and less successful at capturing well informed consumers. At the top of the price distribution, firms, in effect, only care about consumers observing one or two prices because the chance of selling to better informed consumers is negligible. Given this, if the probability of observing one price relative to the probability of observing two prices increases when we move from an $N$-firm to an $N+1$-firm market, then firms prefer to raise their higher prices and therefore the upper percentiles of the price distribution increase.

The welfare implications of increased competition when the frequency of low and high prices both increase with competition are quite interesting. Conditional on the number of prices observed, we prove that the probability that a consumer already paying a high price will pay even a higher price increases. In general, however, expected utility may or may not be higher after increased competition (i.e., consumers may or may not be expected to pay lower prices), conditional on the number of prices observed. We provide a sufficient condition under which the least informed consumers experience welfare losses with increased competition, exactly as it occurs in Varian's model. Interestingly, ex-ante expected utility, i.e., not conditional on the number of prices observed, increases with competition under our assumption (even under a weaker version of it). Because some consumers may experience welfare losses, this implies that the benefits to some consumers more than offset the losses (if any) to other consumers.

In sum, our theoretical results imply that $(i)$ an increase in the number of firms has asymmetric effects on the price distribution: it always decreases the lower percentiles but the higher percentiles may increase or decrease; (ii) the magnitude of the change in prices varies along the price distribution; (iii) welfare gains from increased competition can be positive and negative for different consumers (i.e., differentiated by the number of prices they observe) and when this happens, those who gain benefit (weakly) more than those who lose so that the weighted average impact of competition is (weakly) favorable; and (4) when all consumers gain from increased competition, the gains can be asymmetrically distributed across consumers, not necessarily being the case that better informed consumers gain more than poorly informed ones.

Our final contribution is to assess the empirical relevance of the implications concerning the effect of competition on prices. For this purpose, we use gasoline prices in the Netherlands to estimate how distinct percentiles of the price distribution respond to changes in the number of gas stations (our measure of competition). We deal with the endogeneity of the number of gas stations by controlling for a relatively large number of market-level characteristics and by using Lee's (2007) control function approach for quantile regressions. Our estimates confirm that the response of prices to increased competition varies along the price distribution, as predicted by the
theoretical model. We find that the response of prices to an increase in the number of competitors, though small, is more pronounced in the middle of the distribution. We use these estimates and the model to estimate the welfare benefits to consumers from increased competition. We find that consumers observing two prices benefit the most from increased competition, followed by those observing a single price. Consumers observing many prices - those having more information - benefit the least. This is consistent with our finding that increased competition has a larger impact on the middle part of the price distribution.

Imperfect and heterogeneous price information is prevalent in many markets subject to competition-enhancing policies (telecom and financial services, gasoline, gas, electricity, airlines, etc.). Our paper shows that the price and welfare effects of such policies might not be as straightforward as those implied by standard models. Typical empirical studies focus on the response of the mean price and variance to increased competition. ${ }^{7}$ But, as shown in this paper, increased competition can potentially have unequal effects among consumers, and such focus is too narrow to be able to capture the complexity of the welfare effects generated by increased competition. Our results also emphasize that distributional issues should become a central part of any welfare assessment of competition-enhancing policies (industry deregulation, trade liberalization, transparency laws, etc.). This, again, advocates the importance of taking a broader view where the interaction between competition and consumer policy is taken into consideration (Armstrong, 2008; Armstrong et al., 2009; Waterson, 2003; Wu and Perloff, 2007).

The paper proceeds as follows. In Section 2 we present a model of the distribution of prices in an oligopolistic market where consumers differ in the amount of prices they observe. We analyze the effect of increased competition on prices and on the welfare of consumers observing different number of prices. In Section 3 we confront the predictions of the theoretical model with data on gasoline prices in the Netherlands. We close the paper by offering some concluding remarks and potential avenues for further research. Proofs of Propositions are relegated to the Appendix.

## 2 A model of the distribution of prices

In many markets for homogeneous goods the law of one price fails to hold and prices are significantly dispersed. ${ }^{8}$ As shown in a large literature on search and advertising costs an important source of price dispersion is the substantial amount of heterogeneity in the information consumers

[^4]have about prices. ${ }^{9}$
In order to learn how consumer information heterogeneity affects pricing, we use Armstrong, Vickers and Zhou's (2009) model, which allows for a richer information structure than the earlier settings of Varian (1980) and Burdett and Judd (1983). ${ }^{10}$ There are $N \geq 2$ retailers competing in prices to sell a homogeneous good to a large number $L$ of heterogeneous consumers. At a given moment in time, a consumer wishes to purchase at most a single unit of the good. ${ }^{11}$ The maximum willingness to pay for the good of a firm is given by $v$. Letting $c$ denote the unit cost of a firm, define $k \equiv v-c$.

Assume that consumers use a "search" technology such that the probability of observing the prices of $s \leq N$ distinct firms is equal to $\mu_{s}(N)$, with $\sum_{s=1}^{N} \mu_{s}(N)=1$. There are various reasons for why $\mu_{s}(N)$ ought to depend on $N$. Suppose, for example, that consumers observe prices while they move around the city, as when going to work, to school, etc. We could think of this as "passive" search in the sense that consumers do not deliberately take to the streets and search for low prices. In this situation, after entry of a new seller, the chance that a consumer observes exactly $s$ prices will likely change. Alternative mechanisms include "active" consumer search (e.g., Stahl, 1989; Janssen and Moraga-González, 2004), firm advertising (e.g., Butters, 1977; Stahl, 1994), word-of-mouth communication and social networking (Galeotti, 2010), etc. In our paper we will deliberately remain agnostic about the exact mechanisms through which price information is gathered by and/or distributed to consumers and view $\mu_{s}(N)$ as a "reduced-form" for such mechanisms. ${ }^{12}$

For $0 \leq x \leq 1$, define

$$
\begin{equation*}
\alpha_{N}(x) \equiv \sum_{s=1}^{N} \mu_{s}(N) x^{s} \tag{1}
\end{equation*}
$$

as the probability generating function ( $P G F$ ) for the number of prices observed by consumers. Note that the $s^{\text {th }}$ derivative of $\alpha_{N}(x)$ with respect to (wrt) $x$, which we will denote $\alpha_{N}^{(s)}(x)$, evaluated at 0 is equal to $s!\mu_{s}(N)$. We shall assume that the probability of observing exactly one

[^5]price is strictly between 0 and 1, i.e., $\alpha_{N}^{(1)}(0) \equiv \mu_{1}(N) \in(0,1)$.
Firms play a simultaneous-moves game. Let $p_{i}$ be the price of a firm $i$. An individual firm $i$ chooses its price taking the prices of the rival firms as given. There are no pure-strategy equilibria. To see this, consider the position of a firm $i$ and suppose all its rivals were charging a price $\widetilde{p}$, with $c \leq \widetilde{p} \leq v$. Two forces affect price-setting of such firm $i$. First, there is a desire to steal business from its competitors, which pushes this firm to offer better deals than the rivals. This desire arises because the chance consumers see various price offers is strictly positive. Second, the possibility of extracting surplus from consumers who do not compare prices prompts firm $i$ to offer higher prices than its rivals. This desire arises because there is a chance that consumers have no other option than buying at firm $i$. It is easy to see that either of these deviations destabilizes the proposed equilibrium price $\widetilde{p}$. Therefore a single price level cannot accommodate these two incentives. ${ }^{13}$

Denote the mixed strategy of a firm $i$ by a distribution of prices $F_{i}$. We shall only study symmetric equilibria, i.e., equilibria where $F_{i}=F$ for all $i=1,2, \ldots, N .{ }^{14}$ To calculate the expected profit obtained by a firm $i$ offering the good at a price $p_{i} \in[c, v]$ when its rivals choose a price randomly chosen from the cumulative distribution function $F$, we consider the chance that firm $i$ sells to a consumer at random. A consumer will buy from firm $i$ if he observes the offer of firm $i$, which occurs with probability $s \mu_{s}(N) / N$, and the offer of firm $i$ is more attractive than any other offer he receives, which happens with probability $\left(1-F\left(p_{i}\right)\right)^{s-1}$. The expected demand of firm $i$ at price $p_{i}$ is therefore $L \sum_{s=1}^{N} \frac{s \mu_{s}(N)}{N}\left(1-F\left(p_{i}\right)\right)^{s-1}$ which, using (1), is equal to $\frac{L}{N} \alpha_{N}^{(1)}\left(1-F\left(p_{i}\right)\right)$. The expected profit to firm $i$ is

$$
\begin{equation*}
\Pi_{i}\left(p_{i} ; F\right)=\frac{L}{N}\left(p_{i}-c\right) \cdot \alpha_{N}^{(1)}\left(1-F\left(p_{i}\right)\right) \tag{2}
\end{equation*}
$$

In a mixed strategy equilibrium, a firm $i$ must be indifferent between offering any price in the support of $F_{i}$ and offering the upper bound $\bar{p}_{i}$. Therefore, any price $p_{i}$ in the support of $F_{i}$ must satisfy $\Pi_{i}\left(p_{i} ; F_{i}\right)=\Pi_{i}\left(\bar{p}_{i} ; F_{i}\right)$. In symmetric equilibrium, $F_{i}=F, \bar{p}_{i}=\bar{p}$ and $\Pi_{i}=\Pi$. As a result, since $\Pi(\bar{p} ; F)$ is monotonically increasing in $\bar{p}$, it must be the case that $\bar{p}=v$ and $\Pi(\bar{p} ; F)=\frac{L}{N} k \alpha_{N}^{(1)}(0)$. Hence, $F$ must solve

$$
\begin{equation*}
(p-c) \cdot \alpha_{N}^{(1)}\left(1-F\left(p_{i}\right)\right)=k \alpha_{N}^{(1)}(0) \tag{3}
\end{equation*}
$$

for any $p$ in $\left[c+k \alpha_{N}^{(1)}(0) / \alpha_{N}^{(1)}(1), v\right]$, the support of $F \cdot{ }^{15}$

[^6]Unfortunately, equation (3) cannot be solved explicitly for $F$, except in special cases. Existence and uniqueness of an equilibrium price distribution can, however, be easily proven (see Burdett and Judd, 1983). Though it is in general impossible to obtain the equilibrium price distribution analytically, we can easily derive its inverse

$$
\begin{equation*}
q\left(\alpha_{N}(\tau)\right)=c+k \frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)} \tag{4}
\end{equation*}
$$

We note that for $\tau \in[0,1]$, equation (4) gives the $\tau^{t h}$ percentile of the equilibrium price distribution of a firm.

A close look at equilibrium condition (4) serves to make an important point: what truly matters for determining the equilibrium price distribution of a firm is not $N$ - the number of firms - but the distribution of information among consumers. ${ }^{16}$ It is therefore changes in the distribution of information that cause more or less "competitive pressure" in the market; changes in $N$ per se have no effect on prices. We summarize this result in:

Proposition 1 The number of firms $N$ affects the equilibrium price distribution only indirectly through the PGF of price information among consumers $\alpha_{N}(x)$.

The model is, in fact, a model about the effect of consumers' price information on the equilibrium price distribution. Any changes in $\alpha_{N}(x)$ will likely affect equilibrium prices. The focus of this paper, however, is on those changes in $\alpha_{N}(x)$ induced by changes in $N$. These changes will depend on the precise mechanisms through which price information flows into the consumer population. Since we view the PGF $\alpha_{N}(x)$ as a reduced-form for such mechanisms, rather than making assumptions regarding movement patterns of buyers, consumer search protocols, distributions of search and advertising costs, the pattern of social links, etc., we make:

Assumption 1. An increase in the number of firms (i) (weakly) lowers the probability consumers see one price only (i.e. $\left.\alpha_{N+1}^{(1)}(0) \leq \alpha_{N}^{(1)}(0)\right)$ and (ii) raises the (weighted) average number of prices observed in the market (i.e., $\alpha_{N+1}^{(1)}(1)>\alpha_{N}^{(1)}(1)$ ).

Assumption 1 is a rather weak and sensible assumption. In particular, we note that it is weaker than first-order stochastic dominance (FOSD). ${ }^{17}$ Essentially, this assumption states that the first-order effect of entry of a new firm - an increase in price information among consumers - is

[^7]larger than the potential decrease in price information due to the possible downward adjustments of search, advertising efforts, word of mouth communication, etc. triggered by this entry.

### 2.1 Equilibrium price distribution and the number of firms

Our objective is to study the relationship between prices in a market and the number of firms. Typical studies focus on the first and second moments of the price distribution. Here, we take a broader approach and examine the response of all the percentiles of the price distribution to changes in the number of competitors.

To do this, we study the impact of a change in the PGF $\alpha_{N}(x)$, caused by a change in $N$, on the (inverse) price distribution (4). The impact of an increase in $N$ on the percentile $\tau$ of the price distribution is

$$
\begin{equation*}
q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)=k\left[\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}(1-\tau)}-\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}\right] \tag{5}
\end{equation*}
$$

Since $k$ is positive, expression (5) clearly shows that the way an increase in competition affects the different percentiles of the price distribution depends on how $\alpha_{N}(x)$ changes into $\alpha_{N+1}(x)$.

Proposition 2 Suppose that the number of firms increases from $N$ to $N+1$ and that Assumption 1 holds. Then:
(I) There exists $\widehat{\tau} \in(0,1]$ such that all the percentiles of the price distribution below $1-\widehat{\tau}$ decrease.
(II) All the percentiles of the price distribution will decrease if and only if

$$
\begin{equation*}
\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}(1-\tau)}-\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}<0, \text { for all } \tau \tag{6}
\end{equation*}
$$

(III) There exists $\widetilde{\tau} \in[0,1)$ such that all the percentiles $\tau \geq(\leq) \widetilde{\tau}$ of the price distribution will increase (decrease) if and only if 18

$$
\begin{equation*}
\frac{\alpha_{N+1}^{(2)}(0)}{\alpha_{N+1}^{(1)}(0)}-\frac{\alpha_{N}^{(2)}(0)}{\alpha_{N}^{(1)}(0)}<0 \tag{7}
\end{equation*}
$$

This result says that when an increase in the number of competitors does not increase the chance of consumers being informed about only one price and does raise the number of prices they know on average then more competitors in a market always results in a fall in the lower percentiles of the price distribution.

Condition (6) is a necessary and sufficient condition for the equilibrium price distribution with $N$ firms to dominate in a FOSD sense the distribution with $N+1$ firms. In such a case, all

[^8]percentiles of the price distribution fall as we move from an $N$ - to an $N+1$-firm market. This situation accords with the usual intuition that markets with more firms have lower prices and we remark that (6) is violated in Varian's (1980) model. ${ }^{19}$ For an easier-to-interpret sufficient condition, we can state,

Corollary 1 (of Proposition 2) A sufficient condition for all the percentiles of the price distribution to decrease is that the PGF $\alpha_{N}(x)$ satisfies:

$$
\frac{\alpha_{N+1}^{(s)}(0)}{\alpha_{N+1}^{(1)}(0)}-\frac{\alpha_{N}^{(s)}(0)}{\alpha_{N}^{(1)}(0)} \geq 0 \text { for all } s=1,2, \ldots, N
$$

We note that this condition is weaker than the monotone likelihood ratio property (MLRP). ${ }^{20}$ In other words, if the probability distribution of price information satisfies the MLRP then increased competition implies a fall in all the percentiles of the equilibrium price distribution.

The last part of Proposition 2 says that the upper percentiles of the price distribution will increase if and only if condition (7) holds. Note that cutting prices to capture well informed consumers results in lower expected profits for the firms. As a result, firms try to compensate by adjusting the frequency with which they charge higher prices, thereby generating higher profits from the consumers who are less well informed about prices. As we move up in the price distribution, the prices are less and less successful at capturing well informed consumers. In effect, at the top of the price distribution, firms only care about consumers observing one or two prices because the chance of selling to other (better informed) consumers is negligible. Given this, if the probability that consumers observe one price relative to the probability that consumers observe two prices increases when we move from an $N$ - to an $N+1$-firm market, then firms prefer to raise the frequency of the higher prices and so the upper percentiles of the price distribution increase.

Part III of Proposition 2 also says that when condition (7) holds the equilibrium price distributions (before and after an increase in N ) cross each other once.

Proposition 2 has stated conditions under which the percentiles of the price distribution increase or decrease when we move from a market with $N$ retailers to a market with $N+1$ retailers; however, the proposition is silent with respect to whether some percentiles increase (or decrease) more than others. This is a relevant issue because an increase in the number of competitors may be felt more in some percentiles than in others and this opens up the possibility that consumers' gains from an increase in competition be asymmetric in sign and magnitude. To investigate this issue further, we analyze in detail some examples.

[^9]Example 1 (The truncated binomial distribution) ${ }^{21}$ The truncated binomial distribution (TBD) has PGF

$$
\alpha_{N}(x)=\frac{[1-p(1-x)]^{N}-(1-p)^{N}}{1-(1-p)^{N}} .
$$

where $p \in[0,1]$ is the success probability of a Bernoulli experiment. The experiment consists of observing (or not) a price and the binomial distribution gives the probability of observing s prices out of $N$ independent trials. Note that Assumption 1 holds for the TBD. In fact,

$$
\alpha_{N}^{(1)}(0)=\frac{N p(1-p)^{N-1}}{1-(1-p)^{N}}
$$

Taking the derivative of $\alpha_{N}^{(1)}(0)$ wrt $N$ gives

$$
\begin{equation*}
\frac{p(1-p)^{N-1}\left[1-(1-p)^{N}+N \ln (1-p)\right]}{\left[1-(1-p)^{N}\right]^{2}} . \tag{8}
\end{equation*}
$$

The sign of (8) depends on $1-(1-p)^{N}+N \ln (1-p)$, which decreases in $p$. Setting $p=0$ in this expression gives 0 , which implies that (8) is always negative. As a result $\alpha_{N}^{(1)}(0)$ decreases in $N$ (first part of Assumption 1). Moreover, the mean of the TBD is $N p /\left[1-(1-p)^{N}\right]$. Taking the derivative of the mean wrt $N$ gives

$$
\begin{equation*}
\frac{p-p(1-p)^{N}[1-N \ln (1-p)]}{\left[1-(1-p)^{N}\right]^{2}} . \tag{9}
\end{equation*}
$$

The sign of (9) depends on $1-(1-p)^{N}[1-N \ln (1-p)]$, which increases in $N$. Setting $N=2$ in this expression gives $1-(1-p)^{2}[1-2 \ln (1-p)]>0$ for all $p$. Hence (9) is always positive and therefore the mean increases in $N$ (second part of Assumption 1).

We now consider condition (6). We have that

$$
\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}=\frac{\frac{N p(1-p)^{N-1}}{1-(1-p)^{N}}}{\frac{N p(1-p \tau)^{N-1}}{1-(1-p)^{N}}}=\left(\frac{1-p}{1-p \tau}\right)^{N-1},
$$

which clearly decreases in $N$. As a result, when the distribution of price information in the market follows the TBD, an increase in the number of competitors leads to lower (in a FOSD sense) prices.

In order to understand whether lower percentiles fall more in $N$ than higher percentiles, we take the derivative of (5)

$$
\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}(1-\tau)}-\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}=\left(\frac{1-p}{1-p \tau}\right)^{N}-\left(\frac{1-p}{1-p \tau}\right)^{N-1}
$$

wrt $\tau$, which gives

$$
\begin{equation*}
\left(\frac{1-p}{1-p \tau}\right)^{N-2} \frac{p(1-p)}{(1-p \tau)^{3}}[1-N p(1-\tau)-p \tau] . \tag{10}
\end{equation*}
$$

[^10]Inspection of (10) reveals that its sign is always positive provided that $\tau>(N p-1) /[p(N-1)]$. From this we conclude that when $p<1 / N$, (10) is positive for all $\tau$, hence the lower percentiles fall more than higher percentiles. When $p>1 / N$, then there exists a critical $\breve{\tau}$ such that the fall in the percentiles increases in $[0, \breve{\tau}]$ and decreases in $[\breve{\tau}, 1]$. Figure 1 presents two examples.

Example 2 (The discrete uniform distribution) Suppose consumers are equally likely to observe $1,2, \ldots, N$ prices. The probability distribution of observing s prices is therefore a discrete uniform distribution (UD). Its PGF is

$$
\alpha_{N}(x)=\frac{x\left(1-x^{N}\right)}{N(1-x)}
$$

We first note that Assumption 1 also holds for the UD. In fact, $\alpha_{N}^{(1)}(0)=1 / N$, which decreases in $N$. Moreover, the mean of the $U D$ is $(N+1) / 2$, which increases in $N$.

Regarding condition (6), we have

$$
\begin{equation*}
\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}=\frac{\tau^{2}}{1-(1+N \tau)(1-\tau)^{N}} \tag{11}
\end{equation*}
$$

Taking the derivative of (11) wrt $N$ gives

$$
\begin{equation*}
\frac{(1-\tau)^{N} \tau^{2}[\tau+(1+N \tau) \ln (1-\tau)]}{\left[1-(1+N \tau)(1-\tau)^{N}\right]^{2}} \tag{12}
\end{equation*}
$$

The sign of this expression is equal to the sign of $\tau+(1+N \tau) \ln (1-\tau)$, which decreases in $\tau$. Setting $\tau=0$ in this expression gives 0 , hence (12) is always negative. As a result (11) decreases in $N$ and hence condition (6) holds. We conclude that when the distribution of price information in the market follows the discrete uniform distribution, an increase in the number of competitors leads to lower (in a FOSD sense) prices.

In connection with the question which percentiles decrease more, we note that the derivative of the RHS of (5) wrt $\tau$ is rather difficult in this case. However, if we set $N=2$, such derivative gives

$$
\frac{8-6 \tau}{[6+\tau(3 \tau-8)]^{2}}-\frac{2}{(3-2 \tau)^{2}}>0 \text { for all } \tau
$$

Therefore we conclude that when $N=2$, the lower percentiles fall more than the higher percentiles. The top panel in Figure 2 presents an example. We have explored numerically the cases $N \geq 3$ and found that for these cases there exists a critical $\check{\tau}$ such that the fall in the percentiles increases in $[0, \check{\tau}]$ and decreases in $[\check{\tau}, 1]$. See bottom panel of Figure 2 for an example.

Example 3 (Varian's (1980) information structure) Varian's (1980) information structure has PGF

$$
\alpha_{N}(x)=\mu x+(1-\mu) x^{N}
$$

for some $0<\mu<1$. Note that $\alpha_{N}^{(1)}(0)=\mu$, which is constant in $N$, and the mean is $\mu+N(1-\mu)$, which increases in $N$. Therefore, Assumption 1 holds.

Regarding condition (6), we have

$$
\begin{equation*}
\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}=\frac{\mu}{\mu+N(1-\mu)(1-\tau)^{N-1}} \tag{13}
\end{equation*}
$$

Taking the derivative of (13) wrt $N$ gives

$$
\begin{equation*}
-\frac{\mu(1-\mu)(1-\tau)^{N-1}[1+N \ln (1-\tau)]}{\left[\mu+N(1-\mu)(1-\tau)^{N-1}\right]^{2}} . \tag{14}
\end{equation*}
$$

The sign of this expression is the opposite of the sign of $1+N \ln (1-\tau)$, which is positive for $\tau \leq \exp [-1 / N]$ and negative otherwise. As a result, (14) is negative for low $\tau$ and positive for high $\tau$. We conclude that condition (6) is violated.

Consider now condition (7). Since $\alpha_{N+1}^{(s)}(0)=0$ for all $s=2, \ldots, N$, we invoke the $(N+1)^{\text {th }}$ derivative of $\alpha_{N+1}(x)$. Then we have

$$
\frac{\alpha_{N+1}^{(N+1)}(0)}{\alpha_{N+1}^{(1)}(0)}=\frac{(N+1)!(1-\mu)}{\mu}>0
$$

which implies that condition (7) holds. We conclude that when the distribution of price information in the market follows Varian's distribution, an increase in the number of competitors leads to an increase in the high percentiles of the price distribution, and to a decrease in the low percentiles of the price distribution. The top panels in Figure 3 provide an example (for $\mu=0.5$ ).

### 2.2 Consumer welfare and the number of firms

We have seen that the response to an increase in competition differs across the percentiles of the price distribution. In particular, we have shown $(i)$ that some percentiles may increase while others decrease, and (ii) that if they all fall, some may decrease more than others, depending on the properties of the PGF (1). These two results have important implications. Suppose, first, that all percentiles decrease when the number of competitors increases from $N$ to $N+1$. Because some prices may decline more an others, it is possible that consumers observing a given number of prices derive greater benefits from increased competition than other consumers observing a different number of prices. Second, because some prices can actually increase rather than decrease, some consumers may end up paying higher prices, even on average, after an increase in the number of competitors. These implications of the model are in stark contrast to standard full-information oligopoly models. In this subsection we proceed to study the welfare gains that different consumers (i.e., consumers observing different number of prices) will derive from an increase in competition. ${ }^{22}$

[^11]The utility of a consumer who buys from a firm $i$ at a price $p_{i}$ is given by $v-p_{i}$. Denote the utility of a consumer who observes $s$ prices and buys from the cheapest seller by $u_{s}=$ $\max \left\{v-p_{1}, v-p_{2}, \ldots, v-p_{s}\right\}$ where $p_{1}, p_{2}, \ldots, p_{s}$ are i.i.d. random variables drawn from the equilibrium price distribution $F$. The distribution of $u_{s}$ is $(1-F(p))^{s}$. As in Section 2, we derive the inverse of the distribution of $u_{s}$ :

$$
\begin{equation*}
y_{s}\left(\alpha_{N}(\tau)\right)=k\left[1-\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}\left(\tau^{1 / s}\right)}\right] \tag{15}
\end{equation*}
$$

where $\tau \in[0,1]$. In this case, for a given $\tau$, (15) gives the $\tau^{t h}$ percentile of the distribution of the maximum utility received by a consumer who observes $s$ prices. Because $v$ is fixed this distribution provides the same information as the distribution of prices paid and we will also informally refer to (15) as the $\tau^{t h}$ percentile of the distribution of prices paid.

Following the same steps as before, we can study how the distribution of utilities received by a consumer who observes $s$ prices changes when we move from an $N$-firm to an $N+1$-firm market. We have

$$
\begin{equation*}
y_{s}\left(\alpha_{N+1}(\tau)\right)-y_{s}\left(\alpha_{N}(\tau)\right)=k\left[\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}\left(\tau^{1 / s}\right)}-\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}\left(\tau^{1 / s}\right)}\right] \tag{16}
\end{equation*}
$$

Note the similarity between the expression in (16) and that in equation (5). In our empirical section we take advantage of this equation in order to estimate the impact of an increase in $N$ on the utility distribution of a consumer observing $s$ prices. Comparing (16) and (5), it is immediate to see that the change in the percentile $\tau$ of the price distribution is equivalent to the change in the percentile $(1-\tau)^{s}$ of the utility distribution of a consumer observing $s$ prices.

We have,

Corollary 2 (of Proposition 2) Suppose that the number of firms increases from $N$ to $N+1$ and that Assumption 1 holds. Then, for all $s=1,2, \ldots, N$ :
(I) There exists a percentile $\widehat{\tau}_{s} \in[0,1)$ such that all the percentiles $\tau>\widehat{\tau}_{s}$ of the distribution of utilities received by a consumer observing s prices increase
(II) All the percentiles of the distribution of utilities received by a consumer observing s prices increase if and only if condition (6) holds.
(III) There exists $\widetilde{\tau}_{s} \in(0,1]$ such that all the percentiles below (above) $\widetilde{\tau}_{s}$ of the distribution of utilities received by a consumer observing s prices decrease (increase) if and only if condition (7) holds.

In line with Proposition 2, an increase in the number of firms results in an increase in the upper percentiles of the distribution of utilities derived by all consumers. When condition (6)
holds, then all the percentiles of the utility distribution of a given consumer group increase after the number of firms goes up.

Under condition (7), the distribution of the price paid by any type of consumer when there are $N+1$ firms in the market "crosses-over" the distribution function when there are $N$ firms. In this situation, the probability that consumers who pay a price above the "cross-over" price pay even a higher price next time around increases in $N$. It follows that increased competition raises the probability that these consumers are worse off, despite Assumption 1 being satisfied. ${ }^{23}$

Corollary 2 tells us about how the distribution of the prices paid by a consumer observing $s$ prices changes with the number of firms. To study whether expected consumer welfare increases or decreases, and whether this depends on the number of prices consumers observe, we first compute expected utility conditional on observing $s$ prices, namely

$$
\begin{equation*}
C S_{s N}=k\left[1-\int_{0}^{1} \frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}\left(\tau^{1 / s}\right)} d \tau\right] \tag{17}
\end{equation*}
$$

and then study the sign of $C S_{s N+1}-C S_{s N}$, which is equal to

$$
\begin{equation*}
C S_{s N+1}-C S_{s N}=k \int_{0}^{1}\left[\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}\left(\tau^{1 / s}\right)}-\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}\left(\tau^{1 / s}\right)}\right] d \tau \tag{18}
\end{equation*}
$$

where the last equality follows from changing variables. We then have,

Proposition 3 Suppose that the number of firms increases from $N$ to $N+1$ and that Assumption

## 1 holds. Then:

(I) If (6) holds, all consumers (i.e., for all $s=1,2, \ldots, N$ ) derive greater expected utility conditional on $s$.
(II) If (7) holds and $\alpha_{N}^{(1)}(0)=\alpha_{N+1}^{(1)}(0)$, consumers observing one price only derive lower utility.

This result states that, under condition (6), all consumers will obtain greater expected utilities given the number of prices observed. The proposition however does not inform us about whether some consumers benefit more than others. In fact, it is quite difficult to evaluate analytically how (18) depends on $s$. Using the examples above, however, we can gain some insights into this issue. For example, for the TBD case with $p<1 / N$, the lower percentiles fall more than the higher ones so we expect utility gains from increased competition to rise in $s$. By contrast, when $p>1 / N$, the impact of increased competition is felt more at intermediate percentiles of the equilibrium price distribution and as a result we expect utility gains from increased competition to fall in $s$.

[^12]Example 4 (The truncated binomial distribution (cont'd)) When the price information consumers have follows the TBD, the utility gains from increased competition, equation (18), are as follows

|  | Expected Utility Gains $(p=0.2)$ |  |  |
| :--- | :--- | :--- | :--- |
|  | $N=2$ | $N=3$ | $N=4$ |
| $s=1$ | 0.092 | 0.080 | 0.069 |
| $s=2$ | 0.118 | 0.101 | 0.085 |
| $s=3$ |  | 0.109 | 0.092 |
| $s=4$ |  |  | 0.095 |


|  | Expected Utility Gains $(p=0.8)$ |  |  |
| :--- | :--- | :--- | :--- |
|  | $N=2$ | $N=3$ | $N=4$ |
| $s=1$ | 0.202 | 0.080 | 0.037 |
| $s=2$ | 0.198 | 0.061 | 0.021 |
| $s=3$ |  | 0.051 | 0.015 |
| $s=4$ |  |  | 0.012 |

In this case, gains from increased competition increase in $s$ when $p$ is small and decrease in $s$ when $p$ is high.

Example 5 (The discrete uniform distribution (cont'd)) When the price information consumers have follows the UD, the utility gains from increased competition, equation (18), are as follows

|  | Expected Utility Gains |  |  |
| :--- | :--- | :--- | :--- |
|  | $N=2$ | $N=3$ | $N=4$ |
| $s=1$ | 0.114 | 0.045 | 0.022 |
| $s=2$ | 0.143 | 0.058 | 0.029 |
| $s=3$ |  | 0.064 | 0.032 |
| $s=4$ |  |  | 0.034 |

The table shows that better informed consumers gain more from increased competition.

These two examples show two important points: (i) consumer benefits from increased competition depend on $s$ and therefore utility gains are asymmetrically distributed across the consumer population; and (ii) utility gains need not be increasing in $s$, that is, poorly informed consumers may benefit more that well informed ones.

Proposition 3 also states the remarkable result that some consumers may experience a welfare loss after the number of competitors increases. In fact, when condition (7) holds, $\alpha_{N+1}^{(1)}(0)=\alpha_{N}^{(1)}(0)$ suffices for those consumers observing one price only to experience a welfare loss. This explains why mean prices increase in $N$ in Varian (1980), Rosenthal (1980) and Stahl (1989) (though in the latter the effect is amplified by an increase in the reservation price).

Example 6 (Varian's distribution (cont'd)) When the distribution of price information among consumers follows Varian's distribution then the utility gains from increased competition are negative for the consumers who observe one price only and positive for the consumers who observe all prices in the market. The bottom panel in Figure 3 provides and example for $\mu=0.5$.

Closing the model. We now close the model by looking at the decision of consumers to search (or participate) in this market. Using (17) we can compute the unconditional expected consumer
surplus:

$$
\begin{equation*}
C S_{N}=\sum_{s=1}^{N} \mu_{s}(N) C S_{s N}=k \sum_{s=1}^{N} \mu_{s}(N)\left[1-\int_{0}^{1} \frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(\tau)} s \tau^{s-1} d \tau\right]=k\left[1-\alpha_{N}^{(1)}(0)\right] \tag{19}
\end{equation*}
$$

and close the model by calculating the fraction of consumers $L_{N}^{*}$ who decide to search in the market:

$$
\begin{equation*}
L_{N}^{*}=G\left(k\left[1-\alpha_{N}^{(1)}(0)\right]\right) \tag{20}
\end{equation*}
$$

where $G$ denotes the distribution of search (or participation) costs in the population of consumers (normalized to one).

Using (20) we can write aggregate firm profits as $\Pi=k \alpha_{N}^{(1)}(0) G\left(k\left[1-\alpha_{N}^{(1)}(0)\right]\right)$, aggregate expected consumer surplus as $C=k\left[1-\alpha_{N}^{(1)}(0)\right] G\left(k\left[1-\alpha_{N}^{(1)}(0)\right]\right)$ and therefore total market surplus equals $W=k G\left(k\left[1-\alpha_{N}^{(1)}(0)\right]\right)$.

Proposition 4 Suppose that the number of firms increases from $N$ to $N+1$. Then the surplus of a consumer, aggregate consumer surplus and total market surplus strictly increase (remain constant) if and only if $\alpha_{N+1}^{(1)}(0)<(=) \alpha_{N}^{(1)}(0)$.

Note that for this result we do not need to invoke the second part of Assumption 1, that is the impact on aggregate measures of surplus does not depend on whether consumers see more or fewer prices on average after an increase in the number of competitors.

## 3 Empirical application

The model offers new predictions regarding the sign and magnitude of the impact of increased competition on the distribution of prices and on welfare. In particular, the model predicts that the low percentiles of the price distribution always decrease with competition but that the top percentiles need not. Moreover, the magnitude of the changes may vary along the price distribution so that an increase in competition may have asymmetric effects on prices. In this Section we use data on gasoline prices in the Netherlands to empirically assess these theoretical predictions.

### 3.1 Adapting the model to the gasoline market

The market for gasoline is a good example of a market where price dispersion is observed. ${ }^{24}$ In this market, there are two prominent aspects that contribute to price dispersion. On the one hand, gasoline retailers are typically differentiated in their characteristics. It is reasonable to

[^13]believe that retailers that offer a broader range of side-benefits such as longer opening hours, credit card acceptance, car-washing facilities, etc. incur higher costs and therefore charge higher prices. As a result, observed price differences may, to a certain extent, be the outcome of such retailer differentiation. On the other hand, consumers in these markets are imperfectly informed about prices and, as shown in Section 2, this constitutes another source of price dispersion.

Following Wildenbeest (2011) we extend our theoretical model above to accommodate these two sources of price dispersion in a parsimonious way. Though the basic good sold by firms, gasoline of a given grade, is a perfectly homogeneous product, here we adopt the view that the good is sold bundled with a number of side-services. These services add some value to the basic product and serve to "differentiate" retailers from one another. Let $c_{i}$ be the unit cost of a firm $i$ inclusive of the unit cost of the side-services available at the gas station.

Consumers wish to fill up their gas tank, say, once a week and the maximum willingness to pay for the good of firm $i$ is given by $v_{i}$, which is inclusive of valuation for gasoline as well as for the side-services available at firm $i$. As in Wildenbeest (2011), we assume that $v_{i}-c_{i}$ is constant across firms, and let $k \equiv v_{i}-c_{i}$ for all $i$. This assumption can be rationalized if firms hire factors of production for services in perfectly competitive markets and the production function of services exhibits constant returns to scale. ${ }^{25}$ It is then convenient to view firms as competing in the (net) utilities, $v_{i}-p_{i}$, they offer to the buyers (as in Armstrong and Vickers, 2001). Under the constant valuation-to-cost margin assumption, it turns out that firms have symmetric (utility) strategy spaces and a unique symmetric equilibrium (in utilities) exists. The corresponding inverse of the equilibrium price distribution $F_{i}$ of a firm $i$ is

$$
\begin{equation*}
q_{i}\left(\alpha_{N}(\tau)\right)=c_{i}+k \frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)} \tag{21}
\end{equation*}
$$

where $\tau$ takes values on $[0,1]$.
Because $c_{i}=v_{i}-k$, this equation clearly shows that the prices of different firms reflect their differences in side-benefits. To illustrate this, Figure 4 plots the price distributions corresponding to three different firms when the PGF of price information is the discrete uniform distribution, for $v_{1}=1, v_{2}=1.1, v_{3}=1.2$ and $k=1$.

A desirable feature of this (modified) model is that it has the ability to explain two features typically observed in price data from retail markets: the existence of serial correlation in firm-level prices, and the observation that firms undercut one another (see e.g., Lach, 2002; Lewis, 2008;

[^14]Wildenbeest, 2011). ${ }^{26}$ Serial correlation is the result of the "fixed firm-effect" $v_{i}$ (or $c_{i}$ ): firms offering higher side-benefits draw their prices from supports containing higher prices and, as a result, they typically charge higher prices than firms offering lower side-benefits. This, however, need not occur always because, when side-benefits are somewhat similar across firms, the price supports of different firms will overlap. This implies that, from time to time, firms offering higher side-benefits may be seen charging lower prices than firms offering lower side-benefits.

In the next subsection we present the data that will allow us to estimate equation (21) and test some of the theoretical findings in Section 2.

### 3.2 The data

We use daily prices for Euro 95 gasoline from a large sample of gas stations in the Netherlands. The price data were obtained from Athlon Car Lease Nederland B.V., the largest private car leasing company in the Netherlands with over 129,000 cars as of the end of 2008 (www.athloncarlease.com). ${ }^{27}$ The typical contract between Athlon and its lessees stipulates that Athlon pays for the gasoline consumed (up to a limit) as well as for car maintenance, insurance, etc. In order to do this, Athlon gets the lessees' gas receipts and it is from these receipts that the fuel prices are retrieved. Athlon's lessees do not get special discounts so the prices reported by Athlon are actual prices paid by drivers at the pump.

Prices were obtained from 3,143 gas stations for the 6 -month period March 1, 2006September 1, 2006. In the Netherlands, gas prices change on a daily basis. Because the price information arrives directly from the lessees, not all stations are sampled every day, which results in an unbalanced panel data of gas stations. ${ }^{28}$ There are 247,962 station-day observations on Euro 95 prices. For illustration, Figure 5 displays the density function of prices in all gas stations in the sample.

The mean and median price of Euro 95 gas in our sample is 142.2 and 142 cents, respectively, and the standard deviation is 5.4 cents. Most of this variation is within markets (municipalities): the average of the within-market standard deviations is 4.8 while the average of the within-market mean prices is 142.3 . The lowest price is 102 cents while the highest price in the sample is 167 . Not surprisingly, there is dispersion in gasoline prices and, although, not very large it has some

[^15]economic significance. As an illustration, we computed the difference between the highest and lowest prices offered by gas stations within the same market and during the same day. The largest difference was 38 cents (in Langedijk on May 2 and in Boxtel on June 4). This implies that a consumer filling a 50 -liter tank at the lowest-priced station instead of at the highest-priced station would have saved 19 euros.

We view our sample of prices from a gas station $i$ as random draws from the distribution $F_{i}$. We are able to assume this because of several reasons. First, Athlon's lessees do not pay themselves for the gas (it is part of the contract) and therefore it is reasonable to assume that they have no incentives to search for gas stations offering the lowest prices. This is important because, otherwise, our sample would have been a sample of the stations with lowest-priced gasoline. ${ }^{29}$ Second, all gas stations in the Netherlands are self-service and therefore there is a single price for gas in each station. Finally, we believe that the extent to which various prices in a given market are set by a single firm (because of joint-ownership) and/or reflect collusive agreements is minor, implying that prices can be viewed as independent draws. ${ }^{30}$

Equilibrium utility (price) distributions are defined for a given market. We define markets as the geographical area comprised by a municipality. There are 439 municipalities in the Netherlands for which we have gasoline price data. The majority of the municipalities are quite small in terms of population: two thirds of the municipalities have less than 30,000 inhabitants. ${ }^{31}$ We have a list of all the gas stations in each market (in August 2007) and we can therefore compute $N$, but we do not have prices for all the $N$ stations since we only observe prices from Athlon's lessees. On average across markets, however, the number of stations in the sample represents 88 percent of all the gas stations.

The mean number of stations by market is 8.2, respectively, and there is a lot of variation across markets - the standard deviation is almost as large as the mean, 7.6 stations. This variation is better seen in Table 1 where the distribution of $N$ per market (municipality) is tabulated. $N$

[^16]ranges between 1 and 80 (Amsterdam has 59 stations and Rotterdam has 80). Note that two thirds of the markets have 8 or less gas stations. In the following empirical analysis we do not use the 15 markets with $N=1$ because the model in Section 2 does not apply to monopolies (we verify than this omission does not affect our conclusions).

### 3.3 Estimating the effect of $\mathbf{N}$ on price percentiles

The $\tau^{t h}$ percentile of the distribution of prices for firm $i$ in market (municipality) $m=1, \ldots, 439$ is given by (21) which we rewrite now as

$$
\begin{equation*}
q_{i}\left(\alpha_{N, m}(\tau)\right)=c_{i m}+g_{m}\left(N_{m}, \tau\right) \quad \tau \in[0,1] \tag{22}
\end{equation*}
$$

where $g_{m}\left(N_{m}, \tau\right)=k_{m} \alpha_{N, m}^{(1)}(0) / \alpha_{N, m}^{(1)}(1-\tau)$, and we allow for market-specific valuations and costs, $k_{m}$, as well as market-specific PGFs for consumer information. Note that we now index the PGF by $(N, m)$ to indicate that it depends on market $m^{\prime} s$ characteristics as well as on $N$.

We wish to estimate the effect of $N$ on the percentiles of the price distribution. In order to derive an empirically tractable model we assume that the variation across markets in $g_{m}\left(N_{m}, \tau\right)$ is captured by a set of market-level covariates $x_{m}$ and $e_{1 m}$ such that

$$
\begin{aligned}
g_{m}\left(N_{m}, \tau\right) & =g\left(N_{m}, \tau\right)+x_{m} \gamma_{\tau}+e_{1 m} \\
& \approx \delta_{\tau} \ln N_{m}+x_{m} \gamma_{\tau}+e_{1 m}
\end{aligned}
$$

where the function $g\left(N_{m}, \tau\right)$ is common to all markets and approximated by $\delta_{\tau} \ln N$, and $x_{m}$ and $e_{1 m}$ are, respectively, observed and unobserved covariates assumed, without loss of generality, to be uncorrelated (this only changes the interpretation of $\gamma_{\tau}$.

The logarithm approximation is a parsimonious way of capturing "diminishing returns" to $N$, which is a sensible assumption when changes in $N$ affect prices by changing the price information of consumers. Note that we allow the effect of the covariates to vary across percentiles.

Because we lack data on retail costs, we treat $c_{i m}$ as an unobserved disturbance and estimate

$$
\begin{equation*}
q_{i}\left(\alpha_{N, m}(\tau)\right)=x_{m} \gamma_{\tau}+\delta_{\tau} \ln N_{m}+c_{i m}+e_{1 m} \tag{23}
\end{equation*}
$$

by quantile regressions methods for the 11 percentiles $\tau=0.05, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95 .{ }^{32}$
Because $N_{m}$ varies only at the municipality level, identification of its effect on prices is based on the cross-sectional variation in the number of stations. ${ }^{33}$ This variation should be exogenous, i.e., not correlated with unobserved marginal costs $c_{i m}$ nor with unobserved marketlevel heterogeneity $e_{1 m}$. To satisfy this requirement we need $x_{m}$ to be rich enough to control

[^17]for any correlation between costs and number of stations in the market, and to capture all market level features that affect prices and are also correlated with $N$. To be precise, by linearly projecting marginal costs on the market level variables we can write $c_{i m}=x_{m} \pi+e_{2 i m}$ where $e_{2 i m}$ is uncorrelated with $x_{m}$ by construction. We then write
\[

$$
\begin{equation*}
q_{i}\left(\alpha_{N, m}(\tau)\right)=x_{m}\left(\gamma_{\tau}+\pi\right)+\delta_{\tau} \ln N_{m}+e_{1 m}+e_{2 i m} \tag{24}
\end{equation*}
$$

\]

and assume that $e_{1 m}$ and $e_{2 i m}$ are independent of $N_{m}$, conditional on $x_{m}$, making $N_{m}$ exogenous in this equation. ${ }^{34}$

In our application, $x_{m}$ captures factors affecting marginal costs, consumers' valuation of gasoline and their shopping (or search) behavior as reflected in the PGF. Changes in retail costs are mainly driven by changes in the wholesale price of gasoline and this price changed frequently during our sample period. We therefore include in $x_{m}$ the daily spot price of gasoline from the Amsterdam-Rotterdam-Antwerp (ARA) spot market which is common to all gas stations in the Netherlands. ${ }^{35}$ Doing this is also a way of controlling for the time-variation in the distribution from which firms draw their prices.

We also include a set of 12 provincial dummies in $x_{m}$ (we cannot, of course, use municipality dummies since these are perfectly collinear with $N_{m}$ ). Thus, identification of the effect of $N$ on prices is based on the variation in $N$ across municipalities within the same provinces, thereby controlling for much of the heterogeneity that arises from geographically distinct markets.

In addition, we also include the average household income and the average value of property in the municipality as measures of income and wealth of the population. We add the share of cars registered to businesses (out of total cars in the municipality). We expect these variables to be positively correlated with the willingness to pay and with the share of non-price sensitive consumers. Thus, income, property value and business cars should positively affect prices. To capture possible effects of income inequality on prices (Frankel and Gould, 2001), we add the shares of households with income in the bottom two and in the top two deciles of the income distribution. We also include the share of the population between 20 and 65 years of age and the share higher than 65 years old to control for differences in shopping behavior over the lifetime. Because consumers' shopping behavior may be different in a geographically small, interconnected municipality than in a large, spatially-spread municipality, the PGF of price information may vary with the geography of the market. We therefore include control variables related to the geographic or spatial characteristics of markets, in particular, the total area of the municipality (in $k m^{2}$ ), the area that is land (also in $k m^{2}$ ), the share of land that is built (urbanized), the

[^18]share that is agrarian (the remainder is land for recreation and forests), and the kilometers of roads within the municipality borders. Some municipalities have missing covariate data and this reduces the number of markets used in estimation from 424 (439-15) to 408 markets.

Figure 6 (blue solid curve) plots $\widehat{\delta}_{\tau}$ against $\tau$ - the solid line - and a 95 percent confidence band. ${ }^{36}$ These estimated coefficients are proportional to the response of the $\tau^{\text {th }}$ price percentile to an increase in the number of firms from $N$ to $N+1$,

$$
q_{i}\left(\alpha_{N+1, m}(\tau)\right)-q_{i}\left(\alpha_{N, m}(\tau)\right)=\delta_{\tau} \ln [N+1 / N]
$$

A flat line in Figure 6 would indicate that the price response does not vary along the distribution of prices. ${ }^{37}$ This is not what we observe. In fact, the competitive response varies, sometimes dramatically, across percentiles. It declines until the middle part of the price distribution and thereafter increases sharply. Overall, the empirical evidence supports a prediction of the model in Section 2, namely that, when the equilibrium is characterized by price dispersion, an increase in competition can have asymmetric effects on prices.

The exogeneity of $N$ may be too strong an assumption because $x_{m}$ may fail to capture all factors affecting marginal costs, as well as other unobserved market level factors, that are correlated with $N_{m}$. For this reason we adopt an approach suggested by Lee (2007) to deal with the endogeneity of regressors in percentile regressions. Lee (2007) develops a control function approach whereby, in a first stage, the endogenous regressor $\ln N$ is regressed on $x_{m}$ and on excluded exogenous regressors (instruments), and residuals $\widehat{v}$ are computed. These residuals are correlated with the unobserved market level factors and the part of the marginal cost not correlated with $x_{m}$. In a second stage, we include a fourth order polynomial in $\widehat{v}$ in equation (24) to control for the correlation between $e_{1 m}+e_{2 i m}$ and $\ln N_{m}$.

In our application we use market size and entry costs to generate exogenous variation in $N$. Free entry and a zero profit condition would predict a positive relationship between the number of stations in the market and market size $L$ (Bresnahan and Reiss, 1991), and a negative relationship between $N$ and entry costs.

We measure market size by population size and entry costs by the level of municipality taxes imposed on business real estate. ${ }^{38}$ Most theoretical models of price dispersion have prices being independent of market size. Our model is no exception: $L$ does not affect prices directly even though it is endogenously determined by (20). We note that a potential channel through which market size could affect prices is through returns to scale in retailing. In this case, marginal

[^19]cost would decrease with market size. In gasoline retailing, however, the assumption of a common marginal cost is justified because variable costs are mostly driven by the cost of gasoline. The typical brand in the Netherlands buys its gasoline from the Amsterdam-Rotterdam-Antwerp (ARA) spot market (this is true even for Shell which sells much more gasoline than it produces). The ARA market is a centralized marketplace where price discrimination mechanisms such as quantity discounts are unfeasible due to the anonymity of the traders. Therefore, it is reasonably safe to assume that most gas stations in the Netherlands face similar wholesale gasoline prices irrespective of the population level in their markets.

The lack of correlation between population size and $e_{1 m}$ is more difficult to justify and its validity depends, again, on what is included in $x_{m}$. Essentially, we assume $x_{m}$ is rich enough to justify this assumption. Tax rates are more likely to be exogenous in this model and are therefore included in the first-stage regressions even though their effect on $N$ is not significant. Table 2 presents results of mean regressions of $\ln N$ on $x_{m}$, population size and tax rates. Because the variables are constant within municipalities, the regression is estimated using one observation per market (this generates the correct standard errors). The $F$-statistic for the joint significance of the population and tax instruments is $94 .{ }^{39}$

The red (dashed) curve in Figure 6 presents estimates of $\widehat{\delta}_{\tau}$ when endogeneity is controlled for. As can be seen, these estimates are more negative than those obtained when endogeneity is not accounted for. Note that unobserved demand-driven factors should affect the number of gas stations in the market in the same direction as they affect prices. Unobserved cost-driven factors, however, would affect $N$ and prices in opposite directions. Thus, because we are better at controlling cost shifters than at controlling demand shifters, it is reasonable to expect the quantile estimator that ignores endogeneity to be biased upwards, as displayed in Figure 6.

Recall that correlation between $e_{1 m}+e_{2 i m}$ and $\ln N$ in (24) biases the usual quantile estimator of $\delta_{\tau}$. Note, however, that if this correlation does not vary across percentiles of the distribution of prices then $\widehat{\delta}_{\tau}$, although inconsistent, is biased in the same way for any percentile $\tau$. This means that differences in $\widehat{\delta}_{\tau}$ across $\tau$ will consistently estimate $\delta_{\tau}-\delta_{\tau^{\prime}}$, for any $\left(\tau, \tau^{\prime}\right)$. That is, we can estimate changes in the response of prices to an increases in $N$ along the price distribution, but not the level of that response. In other words, under the assumption of quantile independence, the blue (solid) curve gives the correct shape of the price response to increased competition (but not its location). ${ }^{40}$ The correct location is given by the orange curve, which

[^20]controls for the endogeneity of $N$ (note the similarity in the shape of both curves).
We perform a number of robustness tests summarized in Figure 7. First, we add to the sample the 15 municipalities having a single gas station and we observe that the estimated $\delta_{\tau}^{\prime} s$ do not change much. Second, we exclude 41 municipalities having 16 or more gas stations so that the sample comprises "small" markets only and, again, the estimated $\delta_{\tau}^{\prime} s$ are not much affected, except for the top percentiles where the estimated effect is stronger. Finally, we identified markets with municipalities and there might be some concern that this administrative definition is not appropriate for delimiting markets. It may well be that the relevant number of stations affecting prices in a given municipality includes the stations in neighboring municipalities. In order to examine this possibility we computed, for each municipality $m$, the number of stations in all the municipalities sharing a border with $m$ and added this new variable to the baseline model (treated as an exogenous regressor). The estimated $\delta_{\tau}^{\prime} s$ are somewhat smaller at the bottom of the distribution but otherwise are quite similar. Importantly, none of these changes to the basic specification changes our qualitative conclusion that an increase in competition has asymmetric effects on prices.

Figure 6 also reveals that the magnitude of the estimated effect of increased competition on prices is not very large: an increase from 2 to 3 gas stations decreases the median price by almost a fifth of a cent $(0.48 \times \ln 1.5)$ which is a small effect, even given the small variation in prices across gas stations (about 5 cents). The equilibrium price distribution, however, is not the same as the distribution of prices actually paid by consumers. This distribution depends on the information consumers have and, for consumers observing $s$ prices, it is given by the distribution of the minimum among the $s$ prices they observe.

As mentioned before, the change in expected utility of a consumer observing $s$ prices when there is an additional gas station in the market is given by equation (18). We can use the estimated effects in Figure 6 to compute such a change. In the empirical model, the change in the percentile $\tau$ of the equilibrium price distribution is $\delta_{\tau} \ln [N+1 / N]$ and, using equation (5), note that the integrand in the top line of (18) is, in fact, $-\delta_{(1-\tau)^{s}} \ln [N+1 / N]$ so that we can compute the change in expected utility by integrating over the estimated $\delta_{\tau}^{\prime} s$. To do this, we first reestimate the model for the 99 percentiles $\tau=1,2, \ldots, 99$ and each estimated coefficient $\delta_{\tau}$ is assigned to the $(1-\tau)^{s}$ quantile of the utility distribution (after multiplying it by -1 ). We then multiply these coefficients by the difference in consecutive values of $(1-\tau)^{s}$ (i.e., $d \tau$ ) and add up (as in Figure 6, we do not multiply by $\ln [N+1 / N]$ ). The results, appearing in Table 3, indicate

[^21]that consumers observing two prices benefit the most from increased competition, followed by those observing a single price. Those consumers observing many prices - those having more information - benefit the least. As with the TBD example in the theoretical section, these effects are driven by our previous finding that the intermediate percentiles of the price distribution fall the most with competition. This is also an example of an interesting finding reflecting the asymmetric effects of competition.

## 4 Summary and conclusions

Consumers differ in the amount of price information they have. In homogeneous product markets, a large literature has shown that this implies that prices are typically dispersed in equilibrium. This literature, however, has used restrictive assumptions on the distribution of price information in the market. In this paper, we use Armstrong, Vickers and Zhou's (2009) model - which allows for arbitrary distributions of price information- to study the price and welfare effects of an increase in the number of firms. We have shown that this generalization generates new results that have important welfare implications.

In our model an increase in the number of competitors affects prices only indirectly through changes in the amount of price information consumers have. That is, increasing competition affects prices via an informational channel because the number of firms in a market changes the amount and distribution of price information among consumers. We assume that entry of an additional firm raises the average number of prices observed by a consumer in the market and (weakly) lowers the probability consumers see only one price.

We have provided three important results on how the price distribution changes when the number of competitors increases. First, the lower percentiles of the price distribution always decrease with an increase in the number of firms because firms have to cut prices in order to capture better informed consumers. Second, we have put forward a necessary and sufficient condition under which the distribution of prices with $N$ competitors dominates the distribution of prices with $N+1$ competitors in a first-order stochastic sense. This implies that the mean price can decrease with $N$, which is in contrast to Varian's (1980) model and other models on imperfect price information and competition. In such a situation, however, the responsiveness of prices varies along the percentiles of the price distribution. Finally, we have also shown that when the number of consumers observing one price relative to those observing two prices increases when we move from an $N$ - to an $N+$ 1-firm market, then firms prefer to raise the upper percentiles of the price distribution. The intuition is that, at the top of the price distribution, firms, in effect, only care about consumers observing one or two prices because the chance of selling to others is
negligible. Given this, the trade-off goes against the uninformed consumers and the top prices of the price distribution go up.

Because the utility received by consumers will depend on the price actually paid, these three results have important welfare implications. In particular, we have noted three. First, we have observed that even if all prices decrease some percentiles decrease more than others and this actually leads to asymmetric consumer gains from increased competition. That is, some consumers gain more than others when the number of firms increases and it may be the case that poorly informed consumers benefit more from competition than well informed ones. Second, we have shown that, because (the frequency of) high prices can increase as a result of an increase in the number of firms, the probability that some consumers experience welfare losses increases when the number of competitors goes up. Finally, we have provided a sufficient condition under which the least informed consumers lose out on average after the number of firms increases. Nevertheless, expected (or average) consumer surplus always (weakly) increases with increased competition.

We have illustrated the theoretical results using disaggregated gasoline price data for the Netherlands. Our estimates indicate that the magnitude and sign of the change in gasoline prices due to an increase in the number of firms is not uniform across the price distribution and is more pronounced towards the middle of the distribution. We estimate that less informed consumers benefit more from increased competition than more informed ones.

We believe the results of this paper are important since in markets where the structure of information changes with the number of competitors, the price effects of competition-enhancing policies are not as straightforward as one may have initially expected. The novelty of our approach has been to model general structures of information. In this respect, understanding how these general structures of information arise endogenously and change with policy measures is an important and promising area for further research.

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## Appendix A: Proofs

Proof of Proposition 2. First we note that

$$
\begin{equation*}
q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)=k\left[\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}(1-\tau)}-\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(1-\tau)}\right]=\frac{k}{h(\tau)}\left[\frac{\alpha_{N}^{(1)}(1-\tau)}{\alpha_{N}^{(1)}(0)}-\frac{\alpha_{N+1}^{(1)}(1-\tau)}{\alpha_{N+1}^{(1)}(0)}\right] \tag{25}
\end{equation*}
$$

where

$$
h(\tau) \equiv \frac{\alpha_{N+1}^{(1)}(0) \alpha_{N}^{(1)}(0)}{\alpha_{N+1}^{(1)}(1-\tau) \alpha_{N}^{(1)}(1-\tau)}>0
$$

(I) Let us set $\tau=0$ in this expression. It follows that the sign of $q\left(\alpha_{N+1}(0)\right)-q\left(\alpha_{N}(0)\right)$ equals the sign of

$$
-\frac{1}{\alpha_{N}^{(1)}(1) \alpha_{N+1}^{(1)}(1)}\left[\alpha_{N+1}^{(1)}(1) \alpha_{N}^{(1)}(0)-\alpha_{N}^{(1)}(1) \alpha_{N+1}^{(1)}(0)\right]<-\frac{1}{\alpha_{N+1}^{(1)}(1)}\left[\alpha_{N}^{(1)}(0)-\alpha_{N+1}^{(1)}(0)\right] \leq 0,
$$

where the two inequalities follow from Assumption 1 . Since (25) is strictly negative when $\tau=0$, by continuity of the function $q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)$ in $\tau$, we conclude that the low percentiles of the price distribution will always decrease.
(II) If condition (6) holds, then it is straightforward to see that all percentiles will fall.
(III) To prove this, we first note that setting $\tau=1$ in (25) gives 0 . Now, let us take the derivative of (25) wrt $\tau$. We get:

$$
\frac{\partial}{\partial \tau}\left(q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)\right)=k\left\{\frac{\alpha_{N+1}^{(1)}(0) \alpha_{N+1}^{(2)}(1-\tau)}{\left[\alpha_{N+1}^{(1)}(1-\tau)\right]^{2}}-\frac{\alpha_{N}^{(1)}(0) \alpha_{N}^{(2)}(1-\tau)}{\left[\alpha_{N}^{(1)}(1-\tau)\right]^{2}}\right\}
$$

Setting $\tau=1$ gives

$$
\begin{equation*}
\frac{\partial}{\partial \tau}\left(q\left(\alpha_{N+1}(0)\right)-q\left(\alpha_{N}(0)\right)\right)=k\left[\frac{\alpha_{N+1}^{(2)}(0)}{\alpha_{N+1}^{(1)}(0)}-\frac{\alpha_{N}^{(2)}(0)}{\alpha_{N}^{(1)}(0)}\right] \tag{26}
\end{equation*}
$$

When condition (7) holds, this expression is negative. This implies that the difference $q\left(\alpha_{N+1}(\tau)\right)-$ $q\left(\alpha_{N}(\tau)\right)$ is decreasing in a neighborhood of $\tau=1$. Since it is zero when $\tau=1$, by continuity we conclude that $q\left(\alpha_{N+1}(\tau)\right)>q\left(\alpha_{N}(\tau)\right)$ for sufficiently large percentiles. ${ }^{41}$

Moreover, when condition (7) holds there exists a unique $\widetilde{\tau}$ for which $q\left(\alpha_{N+1}(\widetilde{\tau})\right)=$ $q\left(\alpha_{N}(\widetilde{\tau})\right)$. In fact, from (25) it follows that $q\left(\alpha_{N+1}(\tau)\right)=q\left(\alpha_{N}(\tau)\right)$ if and only if

$$
\frac{\alpha_{N}^{(1)}(1-\tau)}{\alpha_{N}^{(1)}(0)}-\frac{\alpha_{N+1}^{(1)}(1-\tau)}{\alpha_{N+1}^{(1)}(0)}=\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N)}{\mu_{1}(N)}-\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}\right](1-\tau)^{\ell-1}=0
$$

[^22]Since, by condition $(7), \mu_{2}(N) / \mu_{1}(N)>\mu_{2}(N+1) / \mu_{1}(N+1)$ we can write this expression as

$$
\sum_{\ell=3}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right](1-\tau)^{\ell-1}=2\left[\frac{\mu_{2}(N)}{\mu_{1}(N)}-\frac{\mu_{2}(N+1)}{\mu_{1}(N+1)}\right](1-\tau)
$$

Dividing by $1-\tau$, we obtain

$$
\sum_{\ell=3}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right](1-\tau)^{\ell-2}=2\left[\frac{\mu_{2}(N)}{\mu_{1}(N)}-\frac{\mu_{2}(N+1)}{\mu_{1}(N+1)}\right]
$$

Now note that the RHS of this expression is positive and constant in $\tau$. By contrast, the LHS is monotonically decreasing in $\tau$ and takes value zero at $\tau=1$. As a result, if the LHS at $\tau=0$ is greater than the RHS then it follows that $\widetilde{\tau}$ is unique. But

$$
\sum_{\ell=3}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right]>2\left[\frac{\mu_{2}(N)}{\mu_{1}(N)}-\frac{\mu_{2}(N+1)}{\mu_{1}(N+1)}\right]
$$

is true from Assumption 1. The proof is now complete.

Proof of Corollary 1. Using (25), we have that $q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)<0$ if and only

$$
\begin{aligned}
\frac{\alpha_{N}^{(1)}(1-\tau)}{\alpha_{N}^{(1)}(0)}-\frac{\alpha_{N+1}^{(1)}(1-\tau)}{\alpha_{N+1}^{(1)}(0)} & =\sum_{s=1}^{N} s \frac{\mu_{s}(N)}{\mu_{1}(N)}(1-\tau)^{s-1}-\sum_{s=1}^{N+1} s \frac{\mu_{s}(N+1)}{\mu_{1}(N+1)}(1-\tau)^{s-1} \\
& =\sum_{s=1}^{N} s\left[\frac{\mu_{s}(N)}{\mu_{1}(N)}-\frac{\mu_{s}(N+1)}{\mu_{1}(N+1)}\right](1-\tau)^{s-1}-(N+1) \frac{\mu_{N+1}(N+1)}{\mu_{1}(N+1)}(1-\tau)^{N}<0
\end{aligned}
$$

From this expression, it is clear that

$$
\frac{\alpha_{N}^{(s)}(0)}{\alpha_{N}^{(1)}(0)}-\frac{\alpha_{N+1}^{(s)}(0)}{\alpha_{N+1}^{(1)}(0)}=s!\left[\frac{\mu_{s}(N)}{\mu_{1}(N)}-\frac{\mu_{s}(N+1)}{\mu_{1}(N+1)}\right] \leq 0 \text { for all } s \text { suffices. }
$$

Proof of Corollary 2. Follows straightforwardly from Proposition 2.

Proof of Proposition 3. (I) Comparing the expected utility of a consumer who observes $s$ prices when there are $N$ firms and when there are $N+1$ firms gives:

$$
\begin{equation*}
C S_{s N+1}-C S_{s N}=k \int_{0}^{1}\left[\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}\left(\tau^{1 / s}\right)}-\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}\left(\tau^{1 / s}\right)}\right] d \tau=k \int_{0}^{1}\left[\frac{\alpha_{N}^{(1)}(0)}{\alpha_{N}^{(1)}(\tau)}-\frac{\alpha_{N+1}^{(1)}(0)}{\alpha_{N+1}^{(1)}(\tau)}\right] s \tau^{s-1} d \tau \tag{27}
\end{equation*}
$$

When condition (6) holds, the integrand in the above expression is positive and therefore $C S_{s N+1}-$ $C S_{S N}>0$.
(II) When (7) holds, the integrand of (27) is positive (negative) for all $\tau \geq(\leq) \widetilde{\tau}(\widetilde{\tau}$ as defined in the proof of Proposition 2). To deal with that situation, we follow Janssen and

Moraga-González (2004). First, we write

$$
\begin{equation*}
C S_{s N+1}-C S_{s N}=k \int_{0}^{1} \frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right] \tau^{\frac{\ell-1}{s}}}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\ell-1} s\right.} d \tau \tag{28}
\end{equation*}
$$

and split the integral in (28) as follows:

$$
\begin{align*}
& \int_{0}^{1} \frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right] \tau^{\frac{\ell-1}{s}}}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} d \tau \\
& =-\int_{0}^{\tau} \frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N)}{\mu_{1}(N)}-\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}\right] \tau^{\frac{\ell-1}{s}}}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} d \tau \\
& +\int_{\widetilde{\tau}}^{1} \frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right] \tau^{\frac{\ell-1}{s}}}{\left(\sum_{\ell=1}^{N} \ell_{\ell} \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} d \tau \tag{29}
\end{align*}
$$

Notice that the denominator of these integrals increases in $\tau$. Therefore, (29) is lower than

$$
\begin{align*}
& -\int_{0}^{\tilde{\tau}} \frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N)}{\mu_{1}(N)}-\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}\right] \tau^{\frac{\ell-1}{s}}}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} \\
& +\int_{\tilde{\tau}}^{1} \frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right] \tau^{\frac{\ell-1}{s}}}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} d \tau \\
& =\frac{\int_{0}^{1} \sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right] \tau^{\frac{\ell-1}{s}} d \tau}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} \\
& =\frac{\sum_{\ell=1}^{N+1} \ell\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right] \int_{0}^{1} \tau^{\frac{\ell-1}{s}} d \tau}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} \\
& =\frac{\sum_{\ell=1}^{N+1} \frac{\ell s}{s+\ell-1}\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right]}{\left(\sum_{\ell=1}^{N} \ell \frac{\mu_{\ell}(N)}{\mu_{1}(N)} \tau^{\frac{\ell-1}{s}}\right)\left(\sum_{\ell=1}^{N+1} \ell \frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)} \tau^{\frac{\ell-1}{s}}\right)} \tag{30}
\end{align*}
$$

The sign of (30) is equal to the sign of the numerator. Setting $s=1$ in the numerator of (30) gives

$$
\sum_{\ell=1}^{N+1}\left[\frac{\mu_{\ell}(N+1)}{\mu_{1}(N+1)}-\frac{\mu_{\ell}(N)}{\mu_{1}(N)}\right]=\frac{1}{\mu_{1}(N+1)}-\frac{1}{\mu_{1}(N)}
$$

which is equal to zero if the condition $\mu_{1}(N)=\mu_{1}(N+1)$ is satisfied. As a result, we conclude that $C S_{s N+1}-C S_{s N}<0$ for $s=1$.

Proof of Proposition 4. The surplus of a consumer is given by $C S_{N}=k\left[1-\alpha_{N}^{(1)}(0)\right]$ and therefore the first part of the Proposition is obvious. Aggregate consumer surplus is equal to $C_{N}=k\left[1-\alpha_{N}^{(1)}(0)\right] G\left(k\left[1-\alpha_{N}^{(1)}(0)\right]\right)$. Defining $z_{N} \equiv k\left[1-\alpha_{N}^{(1)}(0)\right]$ and comparing $C_{N+1}$
and $C_{N}$ gives $C_{N+1} \geq C_{N}$ if and only if $z_{N+1} G\left(z_{N+1}\right) \geq z_{N} G\left(z_{N}\right)$. Since $z_{N+1} \geq z_{N}$ and $z G(z)$ increases in $z$, the inequality holds. Total market surplus equals $W=k G\left(k\left[1-\alpha_{N}^{(1)}(0)\right]\right)$, which clearly goes up if $\alpha_{N+1}^{(1)}(0)<\alpha_{N}^{(1)}(0)$.

Table 1. Distribution of the number of gas stations (N) by market and by price observations


Table 2. First Stage Mean Regression

| (log) Population | $\begin{gathered} \hline 0.687 * * * \\ (0.0511) \end{gathered}$ |
| :---: | :---: |
| (log) Tax rate | $\begin{gathered} 0.0327 \\ (0.0610) \end{gathered}$ |
| Mean income per household | $\begin{aligned} & -0.00509 \\ & (0.0278) \end{aligned}$ |
| Share of business cars | $\begin{gathered} 1.551^{* * *} \\ (0.565) \end{gathered}$ |
| Area | $\begin{aligned} & 0.0000374 \\ & (0.000367) \end{aligned}$ |
| Land | $\begin{aligned} & 0.00132^{* *} \\ & (0.000619) \end{aligned}$ |
| Share of guilt land | $\begin{gathered} -0.00230 \\ (0.00227) \end{gathered}$ |
| Share of agrarian land | $\begin{gathered} -0.00231 \\ (0.00150) \end{gathered}$ |
| Roads (km) | $\begin{gathered} 0.000264 \\ (0.000182) \end{gathered}$ |
| Spot price of gasoline | $\begin{gathered} 0.000137 \\ (0.000844) \end{gathered}$ |
| Mean value of property | $\begin{aligned} & -0.0000128 \\ & (0.000836) \end{aligned}$ |
| Share of poor households | $\begin{gathered} -0.00249 \\ (0.00593) \end{gathered}$ |
| Share of rich households | $\begin{aligned} & 0.00197 \\ & (0.0119) \end{aligned}$ |
| Share 20-65 years old | $\begin{gathered} -0.750 \\ (1.328) \end{gathered}$ |
| Share older 65 years | $\begin{gathered} 0.767 \\ (1.142) \end{gathered}$ |
| Observations R-squared | 408 0.820 |

[^23]Table 3. Gains in expected utility from increased competition

| $s$ | Gains |
| :---: | :---: |
|  |  |
| 1 | 0.375 |
| 2 | 0.380 |
| 3 | 0.364 |
| 4 | 0.345 |
| 5 | 0.329 |

Need to multiply by $\ln (N+1 / N)$ to obtain the actual gain when moving from $N$ to $N+1$

## Figure 1. Truncated Binomial distribution

Equilibrium CDF of prices ( $p=0.2$ )

$$
q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right) \quad \mathfrak{p}=0.2
$$


comp. response

comp. response

|  |  | 1 |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -0.05 |  | 0.2 | 0.4 | 0.6 | 0.8 |
| -0.1 |  |  |  |  |  |
| -0.15 |  |  |  |  |  |
| -0.2 |  |  |  |  |  |
| -0.25 |  |  |  |  |  |

Figure 2. Discrete Uniform distribution

Equilibrium CDF of prices



$$
q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)
$$





Figure 3. Varian's distribution
(mu=0.5)
Equilibrium CDF of prices

$$
q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)
$$


comp. response



Figure 4. Equilibrium price distributions for 3 firms
Discrete Uniform distribution (k=1)


Figure 5: Density of Euro 95 prices (cents)


Figure 6: Estimates of delta across percentiles
Figure 6: Estimates of delta across quantiles


Figure 7. Robustness Tests

Figure 7: Robustness Checks



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    ${ }^{2}$ The Hebrew University and CEPR. E-mail: [saul.lach@huji.ac.il](mailto:saul.lach@huji.ac.il).
    ${ }^{3}$ VU University Amsterdam. E-mail: [j.l.moragagonzalez@vu.nl](mailto:j.l.moragagonzalez@vu.nl). Moraga is also affiliated with the University of Groningen, the CEPR, the Tinbergen Institute and the SPSP Research Center of the IESE Business School.

[^1]:    ${ }^{1}$ For a recent survey of models that generate price dispersion, see Baye et al. (2006).
    ${ }^{2}$ In the absence of the possibility to price discriminate across different types of consumers, mixing in prices arises as the optimal way to resolve the tension between charging low prices and obtaining profits from the more informed consumers and charging higher prices and profiting from the less informed consumers.
    ${ }^{3}$ Varian himself did not prove this result. For a proof see Janssen and Moraga-González (2004) and Morgan et al. (2006).

[^2]:    ${ }^{4}$ See also Nermuth et al. (2009), who, using a similar setting, focus on how the Internet affects the distribution of information and prices.
    ${ }^{5}$ We also provide a necessary and sufficient condition

[^3]:    ${ }^{6}$ This prediction is unappealing because it is counterintuitive and at odds with what is usually found in empirical studies (e.g., Barron et al., 2004; Hosken et al., 2008; Haynes and Thompson, 2008). In fact, this distinct prediction has often been used to discriminate among possible explanations of observed price dispersion and Varian-type models have been dismissed as a plausible explanation on this ground. Our paper shows that such prediction is not generic to models where consumers have imperfect and heterogeneous information about prices, but a consequence of these papers' special assumptions about the price information consumers have.

[^4]:    ${ }^{7}$ See, for example, Borenstein and Rose (1994), Barron, Taylor and Umbeck (2004), Baye et al. (2004), Lewis (2008), and Gerardi and Shapiro (2009).
    ${ }^{8}$ Recent empirical studies documenting price dispersion in various markets for homogeneous products include Lach (2002) and Wildenbeest (2011) for grocery products, Hortaçsu and Syverson (2004) for mutual funds, Barron, Taylor and Umbeck (2004), Hosken et al. (2008), and Lewis (2008) for gasoline and Baye, Morgan and Scholten (2004) for products sold online.

[^5]:    ${ }^{9}$ See the seminal papers by Stigler (1961), Butters (1977), Varian (1980), Rosenthal (1980), Burdett and Judd (1983) and Stahl (1989).
    ${ }^{10}$ Varian's (1980) model of sales is isomorphic to an all-pay auction where firms bid by cutting prices in order to win a prize consisting in the additional demand stemming from the fully informed customers (Baye et al., 1992; Moldovanu and Sela, 2001). Allowing for an arbitrary distribution of price information in the market, as we do in this paper, sets our model apart from the all-pay auction literature. First, we do have multiple heterogeneous prizes as in Barut and Kovenock (1998) but in our game a single player can win many, even all, prizes at a time. Second, since poorly informed consumers only see a few prices, a firm bidding for these consumers is only in competition with a subset of other rivals; in this sense our game is better seen as one where players participate in multiple simultaneous all-pay auctions with different number of rival players and heterogeneous prizes. To the best of our knowledge, this situation has not been studied so far.
    ${ }^{11}$ This assumption is inconsequential. All our results extend to the case where consumers have downward sloping demand functions. We assume inelastic demands to ease the exposition only.
    ${ }^{12}$ Note that this formulation accomodates cases where consumers attention to prices is limited. For example, if consumers observe a maximum of 3 prices then $\mu_{j}=0$ for $j=4,5, \ldots, N$.

[^6]:    ${ }^{13}$ As in Varian (1980), when $\mu_{1}(N)=1$, then all firms offering $p=v$ is a pure-strategy equilibrium. When $\mu_{1}(N)=0$ instead, then all firms offering $p=c$ is a pure-strategy equilibrium.
    ${ }^{14}$ Standard derivations, which can be readily adapted from, e.g., Varian (1980), show that the support of $F$ must be a convex set and that $F$ cannot have atoms.
    ${ }^{15}$ Notice that the lower bound of the price distribution is always above marginal cost, which reflects the fact that firms have market power.

[^7]:    ${ }^{16}$ We note that this relies on the assumption of constant returns to scale. With economies or diseconomies of scale, the decrease in quantities caused by an increase in the number of firms would have cost, and by implication, price consequences.
    ${ }^{17}$ To be sure, for our results to hold we need that either of the inequalities $\alpha_{N+1}^{(1)}(0) \leq \alpha_{N}^{(1)}(0)$ and $\alpha_{N+1}^{(1)}(1) \geq$ $\alpha_{N}^{(1)}(1)$ is strict. We have chosen to work with the second inequality being strict but our proofs can easily be adapted to the similar case where the first inequality is strict instead.

[^8]:    ${ }^{18}$ In special cases, it may happen that $\alpha_{N+1}^{(2)}(0)=0$. In those cases, we can invoke higher order derivatives. In particular, the condition would involve the lowest $s \geq 2$ for which $\alpha_{N+1}^{(s)}(0)>0$ (see the Appendix).

[^9]:    ${ }^{19}$ In fact, in Varian's model $\mu_{1}(N)=\alpha_{N}^{(1)}(0)=\alpha_{N+1}^{(1)}(0)=\mu_{1}(N+1)$ and $\mu_{N}(N)=1-\alpha_{N}^{(1)}(0)=1-\alpha_{N+1}^{(1)}(0)=$ $\mu_{N+1}(N+1)$. In this case, condition (6) requires $N-(N+1)(1-\tau)<0$, which can never be satisfied for all $\tau \in[0,1]$.
    ${ }^{20}$ The MLRP requires that the ratio $\alpha_{N}^{(s)}(0) / \alpha_{N+1}^{(s)}(0)$ decreases in $s$ (see Milgrom, 1981).

[^10]:    ${ }^{21}$ We refer here to the zero-truncated distribution, since our random variable -the number of prices consumers observe- has support $\{1,2, \ldots, N\}$.

[^11]:    ${ }^{22}$ Note that we ignore non-price effects of competition such as better quality of service, shorter distances to retailers, etc., so that "welfare effects" here refers exclusively to price effects.

[^12]:    ${ }^{23}$ Anecdotal evidence tells us that many consumers report not to have felt the "supposed gains" from increased competition in liberalized markets such as airlines, gasoline, telecoms, etc. This might be related to this fact in combination with consumers remembering bad news (about prices) more readily than good news.

[^13]:    ${ }^{24}$ Price dispersion in gasoline markets has been widely documented. Recent papers on this topic are, for example, Barron, Taylor and Umbeck (2004), Chandra and Tappata (2008), Hosken et al. (2008), and Lewis (2008).

[^14]:    ${ }^{25}$ That is, potential gains from offering higher service levels end up being competed away in the market for input factors. The result basically follows from an application of Euler's theorem (see Wildenbeest, 2011). With homogeneous functions of degree 1, and when factors are paid their marginal productivity, the total cost of producing a given level of services equals that level of services.

[^15]:    ${ }^{26}$ See also our working paper, Lach and Moraga-González (2009), for empirical evidence consistent with the use of mixed strategies by gas stations in the Netherlands.
    ${ }^{27}$ The data used in this paper are part of the data collected and analyzed by Soetevent, Heijnen and Haan (2008). We are indebted to them for providing us with the gasoline price data and the list of gas stations operating in the Netherlands. They study whether ownership changes in highway gas stations originating from a government program of auctions and divestitures enhances competition. For details on the data collection, see their Appendix B.
    ${ }^{28}$ The number of days or, equivalently, the number of price quotations per gas station in the sample ranges from 1 to 115 days with an average of 92 days and a median of 101 days. We have one price of Euro 95 per station per day. Prices do not change during the day.

[^16]:    ${ }^{29}$ On the other hand, these consumers may be choosing to buy from gas stations offering higher service values and therefore higher prices. We do not control for this type of selection but we do not think it is much of a problem since, as mentioned below, our sample covers 88 percent of all gas stations in The Netherlands
    ${ }^{30}$ Although we do not have information on the gas stations' owners, according to the Dutch Competition Authority, about 62 percent of the gas stations are owned and operated by independent dealers (NMa, 2006). The remaining stations belong to the main oil producers: BP, Esso, Shell, Texaco and Total. But even among these branded stations, most are dealer-operated. For example, Shell serves fewer than 15 percent of the gas stations and about $2 / 3$ of the Shell-branded gas stations are operated by dealers who are free to set their own prices. This suggests that joint ownership of gas stations is not such a prevalent phenomenon as one may be led to believe from casual observation (although we have no data on joint ownership of gas stations by independent owners). An exception to this characterization was the highway market, where most gas stations, 63 percent, were owned and operated by the large oil producers (NMa, 2006). However, starting in 2002, the Dutch government has forced divestitures of highway stations in order to increase competition.
    ${ }^{31}$ This definition of the market ignores stations that may be geographically close (or in the way to work) but located in different municipalities. However, it is not necessary for the gas stations to be located in a given municipality, provided every gas station in a given municipality factors into its pricing strategy the same distribution of information $\mu(N)$. This would approximately be true as long as neighboring municipalities do not differ much.

[^17]:    ${ }^{32}$ We do not estimate more extreme quantiles because the asymptotic distribution of estimators of extreme quantiles is non-standard (see Chernozhukov, 2005).
    ${ }^{33}$ The length of the sample period, 6 months, is too short to observe much entry and, in fact, we do not have records of entry episodes in our data.

[^18]:    ${ }^{34} \gamma_{\tau}$ cannot, of course, be consistently estimated but this is not our main focus.
    ${ }^{35}$ The estimates are not affected if we use a distributed lag of the spot price instead of the contemporaneous price. To minimize the computation burden we use only the latter.

[^19]:    ${ }^{36}$ Standard errors are obtained by bootstrapping the sample 500 times.
    ${ }^{37}$ The factor $\ln [N+1 / N]$ starts at 0.41 when $N=2$ and declines very rapidly, e.g. $\ln \left[\frac{11}{10}\right]=0.10$.
    ${ }^{38}$ Tax rates vary between 1.5 and 18 percent across municipalities with an average of 7.1 percent. We also eperimented with multiplying the tax rate by the average value of land in the municipality to generate another proxy for entry costs but this did not change the results.

[^20]:    ${ }^{39}$ We use a mean regression in the first stage since it is better suited to handle cases where the endogenous regressor is discrete and bounded, as suggested, and applied, by Lee in the 2004 working paper version of Lee (2007). In any case, results are similar when using the median or other quantiles in the first stage.
    ${ }^{40}$ Quantile independence is not a strong assumption in the present context. For example, it means that the mean retail cost at stations pricing at, say, the high end of the distribution does not differ from the mean cost at stations pricing at the low end. This is reasonable since gas prices are not driven by idiosyncratic station-level

[^21]:    cost but rather by the wholesale price which is common to all gas stations. Moreover, gas stations sometimes post high prices and other times post low prices as implied by the use of mixed strategies so that it is the same set of stations observed with high and low prices. The same applies to other unobserved market characteristics.

[^22]:    ${ }^{41}$ It could be the case that (26) is zero. In that case, it is straightforward to see that higher order derivatives can be invoked. For example, we can take the second order derivative of (25) and evaluate it at $\tau=1$, which gives

    $$
    k\left[\frac{\alpha_{N+1}^{(3)}(0)}{\alpha_{N+1}^{\prime}(0)}-\frac{\alpha_{N}^{(3)}(0)}{\alpha_{N}^{\prime}(0)}\right]
    $$

    If this expression is negative, then the derivative of (25) decreases in a neighborhood of $\tau=1$. As a result $q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)$ must be concave in a neighborhood of $\tau=1$ and since it itself and its derivative are equal to zero at $\tau=1$, we conclude $q\left(\alpha_{N+1}(\tau)\right)-q\left(\alpha_{N}(\tau)\right)>0$ in a neighborhood of $\tau=1$.

[^23]:    Standard errors robust to heteroskedsticity in parentheses. 11 provincial dummies included. $\quad{ }^{*} \mathrm{p}<0.10^{* *} \mathrm{p}<0.05^{* * *} \mathrm{p}<0.01$

