Onderzoeksrapport N° 7808

HEURISTIC PROCEDURES FOR THE SINGLE OPERATION, MULTI-ITEM LOADING PROBLEM.

by

M.R. LAMBRECHT[*]
H. VANDERVEKEN.

Wettelijk Depot : D/1978/2376/11.

HEURISTIC PROCEDURES FOR THE SINGLE OPERATION,
MULTI-ITFM LOADING PROBLE1.

M.R. LAMBRECHT
H. VANDERVEKEN

I. INTRODUCTION

This paper considers the planning of individual machine groups or
work centers producing many different products such as components, sub-
assemblies or assemblies. This production planning problem typically
arises in a Material Requirements Planning (MRP) context. A central func-
tion of any MRP system is to determine what components, sub-assemblies or
assemblies are required, how many, when and where they are needed. The
net requirements or planned orders for manufactured items generate load
profiles showing the overloads and underloads on each work center. These
load profiles must be adjusted in order to meet the available capacity
which consists of available manpower and limited resources. Since the
input to the work centers is erratic, the machine loads may vary consider-
ably through time making the scheduling problem very difficult. Moreover,
there is usually a high set-up cost and a substantial loss in productive
time by changing production from one item to the other. The loading models
presented in this paper prepare a loading plan that meets the company's
production demand and minimizes production and inventory holding costs
for each time period over a number of future time periods.

The scheduling problem described above is known as the multi-item,
multi-period lot size problem. The problem can be stated more formally
in the following way : consider a single machine shop producing many dif-
ferent items i, i=1, ..., N, so as to meet a known requirement schedule
d_{it}, over a finite planning horizon t=1, ... T and remain within the known

capacity limitations L_{rt} for a number of resources r, r=1, ... R. The per unit capacity absorption of product i, resource type r, is denoted by k_{ir}. Whenever a production run is started at the beginning of period t a set-up charge s_{it} is incurred. Furthermore, there is a linear production cost v_{it} and inventory I_{it} carried from period t to period t+1 is penalized at a rate of h_{it} per unit per period. The number of units produced in period t, product i is denoted by x_{it}. The objective is to minimize total costs subject to the demand and capacity constraints.

Mathematically we obtain :

(P)   Minimize $\sum\limits_{i=1}^{N} \sum\limits_{t=1}^{T} [s_{it} \delta(x_{it}) + v_{it}x_{it} + h_{it}I_{it}]$

Subject to

(1) $I_{i,t-1} + x_{it} - I_{it} = d_{it}$ $i = 1, ... N$
                                                        $t = 1, ... T$

(2) $\sum\limits_{i=1}^{N} k_{ir} x_{it} \leq L_{rt}$ $r = 1, ... R$
                                                        $t = 1, ... T$

$I_{io} = I_{iT} = 0$

$x_{it} \geq 0, I_{it} \geq 0$

(3) $\delta(x_{it}) = 0$ if $x_{it} = 0$
                     $= 1$ if $x_{it} > 0$

Constraint set (1) states that the inventory at the end of period t-1 plus production in period t equals demand in period t plus ending

inventory period t. Constraint set (2) says that the total capacity absorption in any period t must be less than the available capacity for all resources. Constraints (2) may also include a term to allow for losses of productive time due to production change-over. The (0,1) variables $\delta(x_{it})$ are defined as follows : if production is positive $\delta(x_{it})$ is equal to 1 and a set-up cost is incurred, otherwise the variable takes on the value 0 and no set-up costs are included. The resulting model is of the mixed-integer type and from a computational point of view extremely difficult to solve.

In section II, the structure of the optimal solution is characterized and in section III two heuristic procedures are analyzed. The performance of both heuristics for a sample of test problems is examined in section IV.

II. CHARACTERIZATION OF THE OPTIMAL SOLUTION.

The multi-item lot size problem has attracted much attention during the past two decades. Several linear programming models have been suggested to solve the problem. Alan Manne [12] proposed the following procedure : instead of searching for the optimal solution among all feasible production plans, he restricts his attention to those production programs satisfying the following condition :

(4) $x_{it} \cdot I_{i,t-1} = 0$, $i = 1, \ldots N, \; t = 1, \ldots T.$

Condition (4) states that if production in period t, product i (x_{it}) is positive then $I_{i,t-1}$ must be zero and vice versa. For each product there are 2^{T-1} schedules satisfying (4). Without loss of generality the initial inventory (I_{io}) is assumed to be zero.
Table 1 illustrates condition (4) for a 3-period problem.

		d_t 10	20	30
Schedule	1	10	20	30
	2	30	0	30
	3	10	50	0
	4	60	0	0

Table 1

Each of the 2^{T-1} schedules for product i can now be denoted by a variable, for N products, we have a problem with $N.2^{T-1}$ variables. Next an LP problem is formulated in order to select for each product one of the 2^{T-1} production schedules. There are two reasons why the above approach seems impracticable. Firstly, the number of variables can become extremely large and secondly, the solution to the LP problem

may result in "mixed" production schedules that is to say schedules obtained as combinations of two or more production schedules satisfying (4). However, mixed production schedules are unrealistic because fractional set-up costs are meaningless.

The main objection from a theoretical point of view is the fact that the LP-model of Manne practically always results in approximate solutions. This loss in optimality is due to the fact that dominant schedules i.e., schedules which must be taken into consideration in order to find the optimal production plan constitute in fact a much wider class of production schedules than the ones characterized by (4).

Consider the following small 4 period - 3 product problem, with one capacity constraint $(R = 1)$

d_{it} t i	1	2	3	4
1	20	30	40	10
2	20	10	10	10
3	25	30	30	30

i	1	2	3
$s_{it} = s_i$, ∀t	70	90	200
$v_{it} = v_i$, ∀t	30	40	50
$h_{it} = h_i$, ∀t	3	4	5
k_i	5	4	6

$L_t = \{450, 400, 450, 300\}$

The optimal solution for this problem is given in the table below.

x_{it} t i	1	2	3	4
1	20	42	38	0
2	30	0	20	0
3	25	30	30	30
Capacity absorption	370	390	450	180
Constraint	450	400	450	300

Table 2.

The best (but suboptimal) Wagner-Whitin type of solution is given in the next table.

x_{it} t i	1	2	3	4
1	20	30	50	0
2	40	0	0	10
3	25	30	30	30
Capacity absorption	410	330	430	220
Constraint	450	400	450	300

Table 3.

To shed some light on these issues we characterize in what follows the extreme points of the solution set (P).

The constraints of problem (P) can easily be represented by means of a network. This network representation is given in figure 1 :
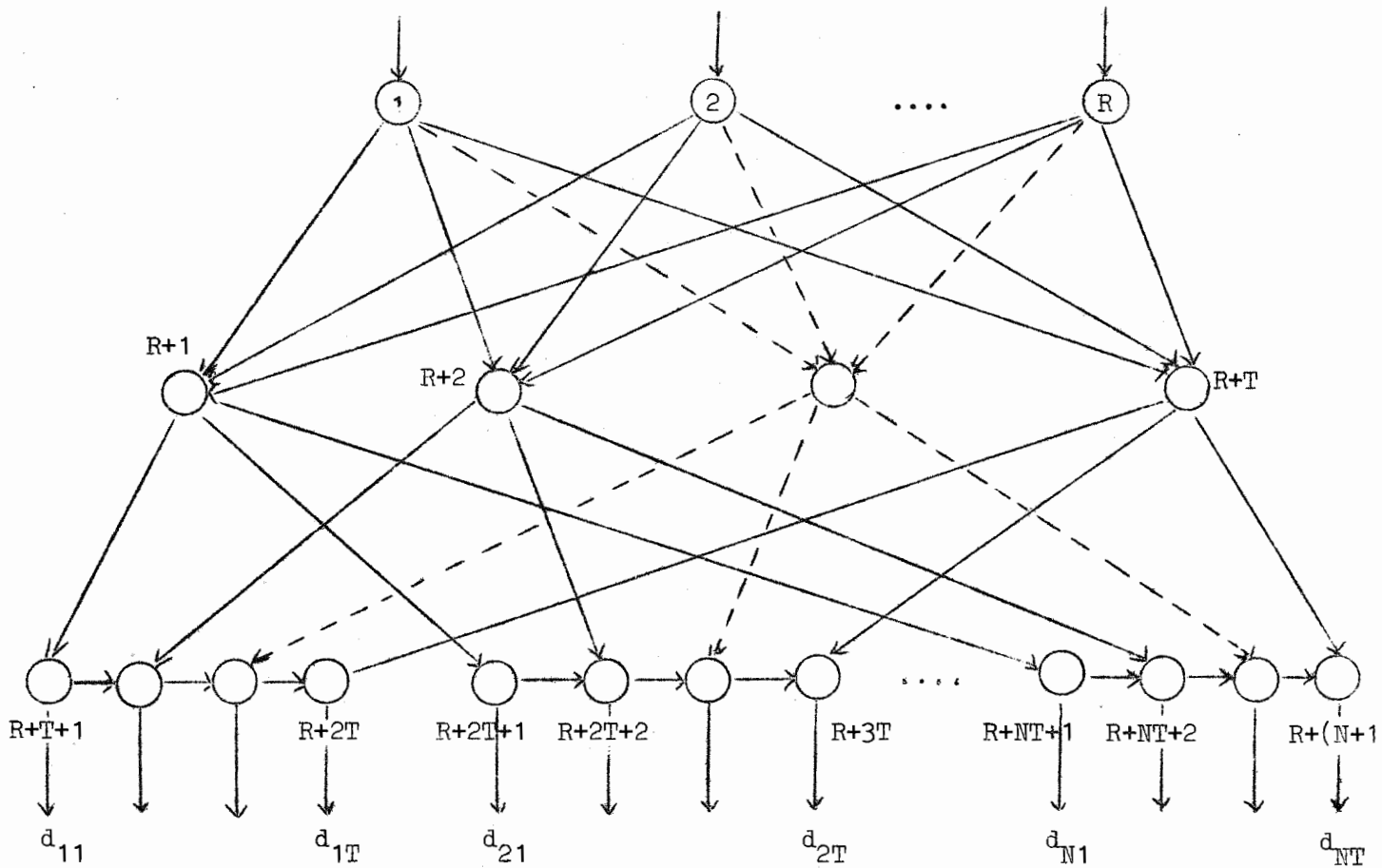
Figure 1.

The input into nodes 1, ..., R equals $\sum\limits_{i=1}^{N} \sum\limits_{t=1}^{T} k_{ir} d_{it}$, r=1,...R.
The flow over the arcs connecting nodes (1, ..., R) and (R+1, ..., R+T)
represents the aggregate flow in each time period, for each resource type
and equals $\sum\limits_{i=1}^{N} k_{ir} x_{it}$, t=1, ... T, r=1, ... R. This flow is restricted
to be at most L_{rt} units (constraint set (2)).
The flow over the arcs connecting nodes (R+1, ..., R+T) and (R+T+1, ...,
R+(N+1)T) represents x_{it} the amount produced in each time period for each

product. Constraint set (1) is represented in the network by means of
the in and outflows into nodes $(R+T+1, \ldots, R+(N+1)T)$.
We clearly have two types of flows in the network above, first, the aggre-
gate flow between the nodes $(1, \ldots, R)$ and $(R+1, \ldots R+T)$ and second,
the individual product flow between nodes $(R+1, \ldots R+T)$ and $(R+T+1, \ldots,$
$R+(N+1)T)$. The aggregate flow is capacitated whereas the individual pro-
duct flow is not (except for the conditions $x_{it} \geqslant 0$, $I_{it} \geqslant 0$).

The constraint set of (P) is bounded and the objective function is
concave so that its minimum will be achieved at one of the extreme points.
In a network terminology, we shall refer as in [17] to such extreme points
as extremal flows. Roughly speaking, a flow in an uncapacitated network
is extremal if it is constructed in a way that prevents the formation of
positive loops. A loop is defined as a sequence of nodes and arcs ob-
tained by selecting one of the arc orientations and for which the first
and last node coincide. A loop is positive if all of its arcs carry po-
sitive flows. It is also well known [4] that a feasible flow in a
capacitated network is extremal if and only if each of its positive loops
contains at least one saturated arc, i.e. an arc whose flow is at capacity.

In the network of figure 1, loops can be formed in two ways :

(1) between the aggregate and invidual product level, e.g. the loop formed
by the sequence of nodes $(1, R+1, R+T+1, R+T+2, R+2, 1)$ an by the arcs
$(1,R+1)$, $(R+1, R+2T+1)$, $(R+2T+1,R+2T+2)$, $(R+2,R+2T+2)$ and $(1,R+2)$

(2) on the individual product level, e.g. the loop formed by the sequence
of nodes $(R+1, R+T+1, R+T+2, R+2, R+NT+2, R+NT+1, R+1)$ and by the arcs
$(R+1, R+T+1)$, $(R+T+1, R+T+2)$, $(R+2, R+T+2)$, $(R+1, R+NT+1)$, $(R+NT,$
$R+NT+2)$ and $(R+2, R+NT+2)$.

Production programs satisfying the Wagner-Whitin condition $x_{it} \cdot I_{i,t-1} = 0$,
will always be extreme flows. Such production programs always prevent the
formation of positive loops.

If on the other hand $x_{it} \cdot I_{i,t-1} \neq 0$ then two conditions must be satisfied in order to have an extreme flow. We call these conditions respectively the aggregate flow condition and the individual product flow condition.

In order to clarify these two conditions some additional terminology is needed.

(a) N^O is defined as the set of products for which $x_{it} \cdot I_{i,t-1} = 0$, $\forall t$ and $N^\ast$ is defined as the set of products for which this property is not satisfied ($N^\ast U N^O = N$).

(b) e_{rt} is defined as the excess or slack capacity in period t, resource type r, i.e.,

$$e_{rt} = L_{rt} - \sum_{i=1}^{N} k_{ir} \cdot x_{it} \quad , \qquad \begin{array}{l} t = 1, \ldots T \\ r = 1, \ldots R. \end{array}$$

(c) A regeneration point, k, for product i is defined as a period with zero ending inventory or $I_{ik} = 0$.

(d) Between each pair of successive regeneration points (k,l) for product i, $0 \leqslant k < 1 \leqslant T$, we have that $I_{ik} = I_{il} = 0$ and $I_{ip} > 0$, p = k+1, ..., 1-1. Production between successive regeneration points (k,l) may either be zero or strictly positive. Assume that production is zero in q periods with $0 \leqslant q \leqslant (1-k)$ and positive in the remaining (1-k-q) periods.

Consider a product $i \epsilon N^\ast$.
Wa may say that the <u>aggregate flow condition</u> for product $i \epsilon N^\ast$ is satisfied if for <u>each</u> pair of regeneration points (k,l), with q periods of zero production and (1-k-q) periods of positive production, we have that for <u>at least</u> one resource type r, there is <u>at most</u> one of the (1-k-q) periods with positive excess capacity.

The aggregate flow condition is very similar to the property developed
by Florian and Klein [5], [9], [10] for the single item capacity con-
strained dynamic lot size problem. Since we are dealing with a network
with R sources (resource types), the above condition must hold for at
least one resource type [17].

In the above example $N^* = \{1\}$. For product 1 however, the aggre-
gate flow condition is satisfied, since for the pair of regeneration points
(1,4), with zero production in period 4 (q=1) and positive production in
periods 2 and 3 (1-k-q = 2), there is only one period (namely period 2)
with positive excess capacity.

If there is more than one product in the set N^*, then the aggregate
flow condition is not enough to prevent the formation of positive loops.
A second condition, the individual product flow condition, must be satis-
fied.

Consider products i', i" and i° belonging to N^*. The production and
ventory flow between the pairs of regeneration points for products belonging
to N^* may form positive loops.
The detection of positive loops can be illustrated by means of the fol-
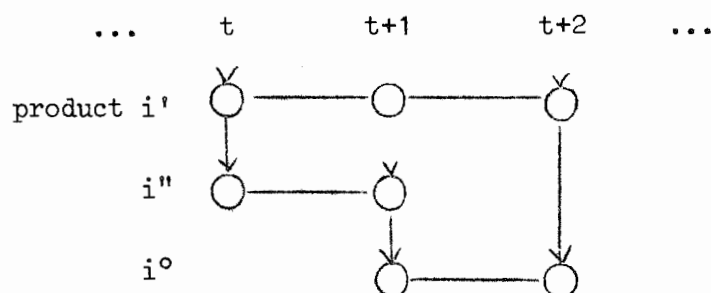lowing graphical aid (see figure 2).

figure 2

Consider the following pairs of regeneration points (t-1, t+2), (t-1, t+1) and (t, t+2) for respectively products i', i" and i°. The horizontal line segments represent positive inventory between the pairs of regeneration points. Nodes labeled ($\breve{0}$) refer to periods with positive production. Vertical lines are drawn only between labeled nodes.

If through this procedure a connected path is formed, then a positive loop will be formed between the regeneration points of the products belonging to $N^{\ast}$. Note that this procedure enables us to detect all positive loops.

We say that the <u>individual product flow</u> condition is satisfied if no such connected path can be formed.

It is obvious that this condition has a great influence on the construction of solution algorithms.

By construction, $D^{\ast}$, the set of feasible production plans $X^{\ast}$:

$$X^{\ast} = \{(x_{it}, I_{it}), \forall i, t \mid x_{it} \cdot I_{i,t-1} = 0, \forall t, i \epsilon N^{\circ} \text{ or } x_{it} \cdot I_{i,t-1} \neq 0$$

for $i \epsilon N^{\ast}$ and for which the aggregate and individual product flow conditions are satisfied \} ,

corresponds to the finite set of extreme points.

III. HEURISTIC APPROACHES.

The objective of heuristic approaches is to find "good" solutions
for complex decision problems with less computational effort compared to
the optimal seeking algorithms. Several heuristics have been proposed [2]
[6], [8], [11], [13] for solving the multi-item scheduling problem. All
those heuristics are based on Manne's LP formulation but since the number
of variables usually turns out to be very large for practical problems,
these methods have not been very successful. Moreover, these heuristics,
require input (requirements, capacities, etc...) over the complete planning
horizon which is not desirable in an environment subject to many changes.
Eisenhut [3] on the other hand proposed a heuristic based on marginal
analysis and at the same time lot sizes are determined on a period by period
basis. His approach is basically myopic in the sense that only limited
information is needed to make the current decision. Such sequential ap-
proaches have also disadvantages especially for problems with capacity
considerations. The main drawback of the Eisenhut heuristic lies in the
fact that it can end up with an infeasible solution. We therefore propose
to extend his heuristic by incorporating a feedback mechanism to escape
from infeasibility.

A. The Eisenhut Heuristic.

The Eisenhut approach starts from a requirements matrix whose elements
d_{it}, i=1, ... N, t=1, ... T are the net requirements for the different pro-
ducts over the planning horizon, and then tries to group requirements of
different periods in the same lot without exceeding the capacity constraints.
For each requirement, d_{it}, a coefficient is calculated indicating if cost
reductions are possible by including requirements of later periods in the

current production order. The derivation of this "appreciation factor" goes as follows :

(7) $\quad C_i(t) = \dfrac{s_i + I_i(t)}{t}$, $\quad i = 1, \ldots N$, where

$\quad$ $C_i(t)$ is the combined set-up and inventory holding cost per period for product i, for a production order including production re-quirements for periods 1 up to t, with $t \leqslant T$

$\quad$ s_i denotes the set-up cost for product i

$\quad$ $I_i(t)$ stands for the cumulative inventory holding cost corresponding to an order including requirements for periods 1 up to t, or

(8) $\quad I_i(t) = h_i \displaystyle\sum_{k=1}^{t} (k-1)d_{ik}$ $\quad$ with

$\quad$ h_i as the inventory holding cost per unit, per period for product i.

The objective is to minimize $C_i(t)$. We therefore calculate the first derivative of (7).

(9) $\quad C_i'(t) = \dfrac{I_i'(t)}{t} - \dfrac{I_i(t)}{t^2} - \dfrac{s_i}{t^2}$

$\quad$ where $I_i'(t)$ is the change in inventory holding cost due to incor-porating the requirement d_{it} in the present lot, or

(10) $\quad I_i'(t) = h_i (t-1) d_{it}$

Since $\quad \displaystyle\sum_{k=1}^{t} (k-1) = \dfrac{(t-1)t}{2}$ $\quad$ or $\quad (t-1) = \dfrac{2}{t} [\displaystyle\sum_{k=1}^{t} (k-1)]$

We can rewrite expression (10) as follows :

$$(11) \quad I'_i(t) = \frac{2h_i}{t} \left[\sum_{k=1}^{t} (k-1)d_{it} \right]$$

Assuming that demand is constant through time, expression (11) reduces to

$$(12) \quad I'_i(t) = \frac{2}{t} I_i(t)$$

Substituting (12) in (9), we obtain

$$C'_i(t) = \frac{2}{t^2} I_i(t) - \frac{I_i(t)}{t^2} - \frac{s_i}{t^2}$$

or

$$(13) \quad C'_i(t) = \frac{I_i(t) - s_i}{t^2}$$

Let $C'_i(t)$ approach zero, and since t has discrete values we find $t^{\mathbf{x}}$, the best unconstrained order cycle if

$$(14) \quad I_i(t^{\mathbf{x}}) \leqslant s_i \text{ and } I_i(t^{\mathbf{x}}+1) > s_i$$

Note that criterium (14) is known as the Part Period Balancing criterium.

Instead of using (13) Eisenhut prefers the following coefficient

$$(15) \quad U_i(t) = - \frac{C'_i(t)}{d_{it}} = \frac{s_i - I_i(t)}{t^2 . d_{it}} \qquad i = 1, \ldots N$$

such that for $t < t^{\mathbf{x}}$ the term $U_i(t)$ will be positive, indicating a cost reduction and for $t > t^{\mathbf{x}}$, $U_i(t)$ will be negative indicating a cost increase.

Note that in (15) the coefficient $C_i'(t)$ is devided by d_{it} so that the appreciation factor is now expressed in costs per unit per period (Our computational experience indicate that the devision by d_{it} is of no importance.)

The procedure can be summarized as follows :

Step 1 : Construct the requirement matrix, the rows i, i=1, ... N stand for the different products and the columns t, t=1, ... T for the different time periods, the elements (i,t) indicate requirements d_{it} for product i, period t.

Step 2 : Compute for each product i and each period t (except the first period, who's requirements must be satisfied), the coefficient $U_i(t)$, omitting negative values. If in the current period the net requirements for product i are zero, i.e. $d_{it} = 0$, then product i is disregarded since it will not be advantageous to start a production run for that item. Postponing such a run will result in lower inventory carrying charges.

Step 3 : In order to determine the lot sizes for period t (starting at t=1) the following steps must be carried out. For each product i, the requirements of period t must be satisfied and hence included in the lot. Next, for each product, scan the appreciation factors of the most recent requirements which have not yet been included in the lot of period t. Select the highest positive $U_i(t)$ and add the corresponding requirements to the lot of period t except if insufficient capacity is available. If capaicity should be exceeded, the corresponding product is discarded and the next highest positive coefficient is selected. This step is repeated until capacity is reached for one of the resources, or until no positive coefficients remain. Note that each time that the order size is increased, the slack capacity e_{rt} must be updated.

Step 4 : Update the requirements matrix by deleting the requirements of
period t and all requirements included in the production orders
of period t.

Step 5 : Repeat steps 2 to 4 until the requirements matrix is reduced to
one column, namely the requirements of period T

The above algorithm may best be illustrated by means of the problem example
given in Section II.

Step 1

i \ t	1	2	3	4
1	35	40	30	37
2	24	27	29	22
3	0	50	47	42

Step 2

i \ t	1	2	3	4
1	35^*	$40^{0,375}$	30^0	37^-
2	24^*	$27^{0,611}$	29^-	22^-
3	0^*	50^*	47^*	42^*

. The coefficient for the requirements d_{i1}, i=1,2,3 need not be determined
because they must be included in the production orders of period 1. The
remaining capacities are :

$$e_{11} = 230 - (35 \times 2 + 24 \times 3) = 88$$

$$e_{21} = 170 - (35 \times 1 + 24 \times 2) = 87$$

. $U_1(2) = \dfrac{100 - 40}{2^2 . 40} = 0,375$, $U_2(2) = \dfrac{120 - 27 \times 2}{2^2 . 27} = 0,611$

$U_3(2)$, $U_3(3)$, $U_3(4)$ are not computed because demand in period 1 is zero.

$$U_1(3) = \frac{100 - 40 - 60}{3^2 \text{x} 30} = 0$$

All other coefficients are negative.

Step 3 : $U_2(2)$ is the highest positive coefficient, the corresponding re-
quirements are 27. We increase the lot of product 2 with 27
units and reduce the available capacities correspondingly.

$$e_{11} = 88 - (27\text{x}3) = 7$$

$$e_{21} = 87 - (27\text{x}2) = 33$$

The next highest coefficient is $U_1(2)$, but the production order for pro-
duct 1 cannot be increased by 40 units as there is not enough capacity
available.

Step 4,2

t \ i	2	3	4
1	40^*	$30^{0,583}$	37^-
2	0^*	29^*	22^*
3	50^*	$47^{0,580}$	42^-

$$e_{12} = 200 - (40\text{x}2 + 50\text{x}1) = 70$$

$$e_{22} = 180 - (40\text{x}1 + 50\text{x}1) = 90$$

Step 3 : Highest coefficient : $U_1(3)$, increase the lot size for product
1, period 2 by 30 units; the remaining capacities are

$$e_{12} = 70 - (30\text{x}2) = 10$$

$$e_{22} = 90 - (30\text{x}1) = 60$$

The second highest coefficient is $U_3(3)$, the order size cannot be increased by 47 units since capacity limits will be exceeded for resource 1.

Step 4,2

t i	3	4
1	0^*	37^*
2	29^*	$22^{0,86}$
3	47^*	$42^{0,74}$

$$e_{13} = 270 - (29\text{x}3 + 47\text{x}1) = 136$$

$$e_{23} = 160 - (29\text{x}2 + 47\text{x}1) = 55$$

Step 3 : Highest coefficient : $U_2(4)$, increase lot size for product 2, period 3 with 22 units

$$e_{13} = 136 - (22\text{x}3) = 70$$

$$e_{23} = 55 - (22\text{x}2) = 11$$

The net requirements corresponding to $U_3(4)$, the next highest coefficient, cannot be scheduled because of insufficient capacity.

Step 4,5

t i	4
1	37
2	0
3	42

The resulting production plan is as follows :

t i	1	2	3	4
1	35	70	0	37
2	51	0	51	0
3	0	50	47	42

Capacity used		1	2	3	4
	r=1	223	190	200	116
	r=2	137	120	149	79

The alert reader should realize that :

a) The Eisenhut heuristic can easily handle problems where set-up times are important, simply by subtracting the set-up times from the available capacity each time that a production run is started.

b) The production programs found by the Eisenhut heuristic always satisfy the condition

$$x_{it} \cdot I_{i,t-1} = 0.$$

That means that the heuristic does not fully exploit the characteristics of optimal solutions as explained in section II.

B. Extended Eisenhut Heuristic.

1. Feedback Mechanism.

The Eisenhut heuristic proceeds uni-directional through the requirements matrix without feed-back capability. As a result, the heuristic can end up with an infeasible solution and this often occurs when the capacity constraints are tight and level from period to period whereas demand has somehow an erratic behavior. To cure this deficiency and thereby enhance the applicability of the Eisenhut heuristic, the following feed-back mechanism is proposed.

If an infeasibility occurs in period t, i.e. if resource require-
ments exceed the available capacity in period t, then we transfer demand
of that period to period t-1 until either one of two conditions occurs :
(1) the capacity limit of one of the resource constraints is attained in
period t-1; (2) the infeasibility disappears in period t. If the in-
feasibility still remains after this phase is completed further trans-
fers of the same or different products must be made to earlier periods
in a similar fashion.

The resulting production programs practically always adhere to the
characteristics of dominant schedules (section II). The aggregate flow
condition is always satisfied, but the individual product flow condition
may be violated for some extreme cases.
Moreover, demand is shifted backward in such a way that no new set-ups are
incurred and inventory holding costs are minimal.

Let INF_{rt} be the amount by which the demand requirements for re-
source r, product i, exceed capacity in period t. The minimum number of
units of product i to be transferred in order to <u>restore feasibility</u> for
resource r is given by :

$$A_{ir} = \left\lceil \frac{\text{INF}_{rt}}{k_{ir}} \right\rceil$$

Where $\lceil a \rceil$ refers to the smallest integer greater than or equal to a.
For practical reasons we always transfer an integer number of units,
although from a theoretical point of view optimal solutions may be non-
integer.

Since feasibility must be restored for all resource constraints
that are violated, we determine

$$B_i = \max_{r \in R^*} \{A_{ir}\}$$

where R^{x} is the set of resources for which the capacity limit is exceeded. It is clear that no more than d_{it} units will be transferred, hence, the number of units that _can_ be transferred for a specific product is given by

$$C_i = \min \{B_i, d_{it}\} \quad , \text{ with } C_i > 0, \text{ otherwise product i is discarded.}$$

If C_i units are transferred to period $t-1$, the inventory holding cost is increased by $C_i.h_i$. To minimize the costs caused by transferring units from t to period $t-1$ we select product q such that :
a) there is a set-up in period $t-1$ (otherwise new set-ups would be created)
b) the increase in inventory holding cost is minimal.

In other words, q is defined by

$$\min_{i \in N^{x}} C_i.h_i, \text{ where } N^{x} = \text{ set of products for which there is a set-up in period } t-1.$$

It is clear that the feed-back procedure may not cause an infeasibility in period $t-1$.

Let
$$D = \min_{\forall r} \left\lfloor \frac{e_{r,t-1}}{k_{qr}} \right\rfloor$$

where $\lfloor a \rfloor$ refers to the greatest integer less than or equal to a.

D denotes the maximum number of units of product q that can be transferred to period $t-1$ without violating the resource constraints for that period.

The _actual_ number of units that will be transferred is then given by E,

where

$$E = \min (D, C_q)$$

We now can end up in one of the following cases :

a) Period t becomes feasible, and the Eisenhut heuristic continues with period t+1.

b) Period t is still infeasible, and
   . either $E = D$, which means that it is impossible to transfer more units of product q to period t-1 without exceeding one of the R constraints of period t-1. We therefore must transfer units to period t-2 using the procedure given above. Note that the infeasibility in period t is reduced by Dxk_{rq} capacity units.

   . or $E = C_q = d_{qt}$ which means that in addition to the demand for product q, units of other products must be transferred to period t-1. The abovementioned procedure must be repeated after disregarding product q.

A detailed flowchart is given in Appendix I.

As we emphasized before, the interesting features of this procedure are twofold : (1) solutions practically always satisfy the characteristics of dominant schedules[x], (2) if the data of the problem allow for a feasible solution the feedback mechanism will always find one.

---

[x] The individual product flow condition is not always satisfied if backtracking is done for more than one product and for more than two periods back in time (this can happen if the capacity constraints are very tight). Minor deviations are also possible because only integral number of units are transferred.

An example is used to illustrate the feedback procedure. Consider
the following 4 product, 4 period problem with one capacity constraint.

d_{it} t	1	2	3	4
i				
1	40	35	32	30
2	30	35	37	34
3	55	60	55	50
4	0	30	29	24
Capacity Constraint L_t	270	290	220	280

i	h_i	s_i	k_i
1	3	100	2
2	1	100	2
3	2	150	1
4	1	50	3

Applying the Eisenhut heuristic for the first 2 periods results in

t i	1	2
1	40	35
2	65	0
3	55	115
4	0	30

The third period however is infeasible since the capacity absorption is

32x2 + 37x2 + 29x3 = 225 and the capacity limit is 220.

Units must be transferred from period 3 to period 2. As there is only one capacity constraint, index r is omitted.

$$INF_3 = 225 - 220 = 5, \quad N^* = \{1,3,4\} \text{, for product 2 there is no set-up in period 2.}$$

$$A_1 = \left\lceil \frac{5}{2} \right\rceil = 3$$

$$A_3 = \left\lceil \frac{5}{1} \right\rceil = 5$$

$$A_4 = \left\lceil \frac{5}{3} \right\rceil = 2$$

Since there is only one constraint we have that $A_i = B_i$

$$C_i = \min \{B_i, d_{it}\} \qquad C_1 = \min \{3, 32\} = 3$$
$$C_3 = \min \{5, 0\} = 0, \text{ excluded because no transfers are possible.}$$
$$C_4 = \min \{2, 29\} = 2$$

$$\min_{i \in N^*} C_i h_i = \min \{3 \times 3 \; ; \; 2 \times 1\} \qquad \text{selected product } q = 4$$

$$D = \left\lceil \frac{e_2}{k_4} \right\rceil = \left\lceil \frac{15}{3} \right\rceil = 5 \; ; \quad e_2 = 290 - (35 \times 2 + 115 \times 1 + 30 \times 3) = 15;$$

$$E = \min (D, C_4) = \min (5, 2) = 2.$$

The result of the preceeding steps is that 2 units of product 4 are transferred to period 2. After this step feasibility is restored and the Eisenhut heuristic continues with period 4.

The resulting production program is

x_{it} \\ i	t	1	2	3	4
1		40	35	32	30
2		65	0	37	34
3		55	115	0	50
4		0	32	27	24

2. A New Appreciation Factor.

The Eisenhut appreciation factor is derived under the assumption that requirements are constant through time. This assumption was only necessary to replace the term $I_i'(t)$ by $\frac{2}{t} I_i(t)$ (see section III) whereas the exact derivative assuming fluctuating demand is $h_i(t-1)d_{it}$.

Substituting this exact term in expression (9) results in :

$$C_i'(t) = \frac{h_i(t-1)d_{it}}{t} - \frac{I_i(t)}{t^2} - \frac{s_i}{t^2}$$

Substituting for $I_i(t) = I_i(t-1) + h_i(t-1) d_{it}$

We obtain

$$C_i'(t) = \frac{1}{t^2} [h_i(t-1)^2 d_{it} - I_i(t-1) - s_i]$$

Let $C_i'(t) \rightarrow 0$, and since t has discrete values we find the unconstrained order cycle t^* if

$$(16) \quad h_i(t^*-1)^2 d_{it} \leqslant I_i(t^*-1) + s_i$$

and

$$h_i(t^*)^2 d_{i,t^*+1} > I_i(t^*) + s_i$$

We now prove that condition (16) is equivalent to the Silver-Meal criterium [14].

Silver-Meal [14] suggest to evaluate $C_i(t)$ successively for increasing values of t until the following inequality is satisfied :

$$(17) \quad C_i(t+1) > C_i(t)$$

In [14] it is shown that (17) is equivalent to

$$(18) \quad t^2 d_{i,t+1} > \frac{s_i}{h_i} + \sum_{k=1}^{t} (k-1)\, d_{ik}$$

After multiplying both sides by h_i and substituting for

$$h_i \sum_{k=1}^{t} (k-1) d_{ik} = I_{it}$$

we obtain

$$h_i t^2 D_{i,t+1} > I_i(t) + s_i$$

thereby establishing the equivalence of the condition (16) and the Silver and Meal heuristic.

In the multi-item capacitated lot size problem we propose the following appreciation factor

$$(19) \quad U_i(t) = -\frac{C_i'(t)}{d_{it}} = \frac{I_i(t-1) + s_i - h_i(t-1)^2 d_{it}}{d_{it} \cdot t^2}$$