# Hidden-charm decays of $X(3915)$ and $Z(3930)$ as the $\mathbf{P}$-wave charmonia 

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#### Abstract

In this work, we investigate the $X(3915)$ and $Z(3930)$ decays into $J / \psi \omega$ with the $\chi_{c 0}^{\prime}(2 P)$ and $\chi_{c 2}^{\prime}(2 P)$ assignments to $X(3915)$ and $Z(3930)$, respectively. The results show that the decay width of $Z(3930) \rightarrow J / \psi \omega$ is at least one order smaller than that of $X(3915) \rightarrow J / \psi \omega$. This observation explains why only one structure, $X$ (3915), has been observed in the $J / \psi \omega$ invariant mass spectrum for the process $\gamma \gamma \rightarrow J / \psi \omega$.


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The $\gamma \gamma$ fusion process is an ideal platform to produce charmonium-like states. In the past, the Belle and BaBar experiments have reported many charmonium-like states in the $\gamma \gamma$ fusion processes. Among these observations, $X(3915)$ has mass $M_{X(3915)}=(3915 \pm 3$ (stat.) $\pm 2$ (syst.)) MeV and width $\Gamma_{X(3915)}=(17 \pm 10$ (stat.) $\pm 3$ (syst.)) MeV. Since $X$ (3915) was observed in the $J / \psi \omega$ invariant mass spectrum of $\gamma \gamma \rightarrow J / \psi \omega$, the possible quantum number should be $J^{P C}=0^{++}$or $J^{P C}=$ $2^{++}$, which results in the corresponding Belle measurement of $\Gamma_{X(3915) \rightarrow \gamma \gamma} \cdot B R(X(3915) \rightarrow$ $J / \psi \omega)=(61 \pm 17$ (stat.) $\pm 8$ (syst.)) eV or $(18 \pm 5$ (stat.) $\pm 2$ (syst.)) eV [1]. As a candidate for charmonium $\chi_{c 2}^{\prime}(2 P)\left(n^{2 s+1} L_{J}=2^{3} P_{2}\right), Z(3930)$ was first observed in the process $\gamma \gamma \rightarrow$ $D \bar{D}$ [2]. The experimental information on $Z(3930)$ gives $M_{Z(3930)}=3929 \pm$ (stat.) $5 \pm 2$ (syst.) $\mathrm{MeV}, \Gamma_{Z(3930)}=29 \pm 10$ (stat.) $\pm 2$ (syst.) MeV , and $\Gamma_{Z(3930) \rightarrow \gamma \gamma} \cdot B R(Z(3930) \rightarrow D \bar{D})=0.18 \pm$ 0.05 (stat.) $\pm 0.03$ (syst.) keV [2]. Later, the BaBar Collaboration also confirmed the observation of $Z(3930)$ in $\gamma \gamma \rightarrow D \bar{D}[3]$.
In Ref. [4], the assignments of $X(3915)$ or $Z(3930)$ as $\chi_{c 0}^{\prime}(2 P)$ or $\chi_{c 2}^{\prime}(2 P)$ charmonium states were proposed by analyzing the mass spectrum and calculating the strong decay of P -wave charmonium. Later, in Ref. [5], the BaBar Collaboration announced that the charmonium-like state $X$ (3915) had been confirmed in the $\gamma \gamma \rightarrow J / \psi$ process with a spin-parity $J^{P}=0^{+}$[5], which is consistent with the prediction in Ref. [4].
If $Z(3930)$ is a $\chi_{c 2}^{\prime}(2 P)$ state, $Z(3930)$ theoretically has the hidden-charm decay channel $J / \psi \omega$ besides its observed open-charm decay $D \bar{D}$. Hence, the signal of $Z(3930)$ should appear in the same $J / \psi \omega$ invariant mass spectrum as $X(3915)$, which was observed by Belle [1]. However,


Fig. 1. Typical diagrams describing the $X(3915) \rightarrow J / \psi \omega($ a) and $Z(3930) \rightarrow J / \psi \omega((\mathrm{b})-(\mathrm{d}))$ decays. After making the charge conjugate transformation $\left(D^{(*)} \leftrightarrow \bar{D}^{(*)}\right)$ and the isospin transformation $\left(D^{(*) 0} \leftrightarrow D^{(*)+}\right.$ and $\left.\bar{D}^{(*) 0} \leftrightarrow D^{(*)-}\right)$, one gets other diagrams.
the experimental data for the $J / \psi \omega$ invariant mass spectrum show no evidence of $Z(3930)$. This fact urges us to explain why there only exists one signal, $X(3915)$, observed in the process $\gamma \gamma \rightarrow J / \psi \omega$.
In this work, we dedicate ourselves to studying the $X(3915)$ and $Z(3930)$ decays into $J / \psi \omega$ under the $\chi_{c 0}^{\prime}(2 P)$ and $\chi_{c 2}^{\prime}(2 P)$ assignments to $X(3915)$ and $Z(3930)$, respectively. By this study, we wan to answer whether the decay $Z(3930) \rightarrow J / \psi \omega$ is suppressed compared with $X(3915) \rightarrow J / \psi \omega$ under the P -wave charmonium assignments to $X(3915)$ and $Z(3930)$, which can shed light on the above puzzle.
As higher charmonia, the $X(3915)$ and $Z(3930)$ decays into $J / \psi \omega$ occur via hadronic loop effects with the open-charm decay channels as the intermediate state. This mechanism has been studied in Refs. [6-13] when calculating the hidden-charm and open-charm decays of charmonium and other charmonium-like states.
The $X(3915)$ and $Z(3930)$ under discussion are candidates for the first radial excitations of $\chi_{c 0}$ and $\chi_{c 2}$, respectively. Since the masses of $X(3915)$ and $Z(3930)$ are above the thresholds of $D \bar{D}$ and $D \bar{D}^{*}$ and below the $D^{*} \bar{D}^{*}$ threshold, $X(3915)$ and $Z(3930)$ dominantly decay into $D \bar{D}$ and $D \bar{D}^{*}$, which contribute to the total widths of $X(3915)$ and $Z(3930)$ (see Ref. [4] for more details). As the subordinate decay mode, $J / \psi \omega$ is assumed from the rescattering contribution of the dominant decays $X(3915) / Z(3930) \rightarrow D \bar{D}, D \bar{D}^{*}$, which is the reason why we only consider the intermediate $D \bar{D}$ and $D \bar{D}^{*}$ contributions in this work. Under the $\chi_{c 0}^{\prime}(2 P)$ assignment to $X(3915)$, the hiddencharm decay $X(3915) \rightarrow J / \psi \omega$ occurs through the intermediate states $D \bar{D}$ since $X(3915)$ with $J^{P C}=0^{++}$dominantly decays into $D \bar{D}$, as indicated in Ref. [4]. The hadron level descriptions of $X(3915) \rightarrow D \bar{D} \rightarrow J / \psi \omega$ are shown in Fig. 1(a). The expression for the decay amplitude of the hidden-charm decay $X(3915) \rightarrow D \bar{D} \rightarrow J / \psi \omega$ reads

$$
\begin{equation*}
\mathcal{M}[X(3915) \rightarrow J / \psi \omega]=4\left[\mathcal{A}_{(\mathrm{a})}^{D}+\mathcal{A}_{(\mathrm{b})}^{D^{*}}\right] . \tag{1}
\end{equation*}
$$

As the $\chi_{c 2}^{\prime}(2 P)$ state, $Z(3930)$ mainly decays into $D \bar{D}$ and $D \bar{D}^{*}+$ h.c. [4]. Thus, its hidden-charm decay $Z(3930) \rightarrow J / \psi \omega$ is shown in Figs. 1(b)-(d). The amplitude for the processes $Z(3930) \rightarrow$ $D^{(*)} \bar{D}^{(*)} \rightarrow J / \psi \omega$ can be expressed as

$$
\begin{equation*}
\mathcal{M}[Z(3930) \rightarrow J / \psi \omega]=4\left[\mathcal{M}_{(\mathrm{b})}^{D}+\mathcal{M}_{(\mathrm{b})}^{D^{*}}+\mathcal{M}_{(\mathrm{c})}^{D}+\mathcal{M}_{(\mathrm{c})}^{D^{*}}+\mathcal{M}_{(\mathrm{d})}^{D}+\mathcal{M}_{(\mathrm{d})}^{D^{*}}\right] \tag{2}
\end{equation*}
$$

where the factor 4 in Eqs. (1) and (2) results from the charge conjugate and isospin transformations.

Table 1. The values of the coupling constants shown in Eqs. (3)-(5). Here, we take $m_{D}=\left(m_{D^{0}}+m_{D^{ \pm}}\right) / 2, m_{D^{*}}=\left(m_{D^{* 0}}+m_{D^{* \pm}}\right) / 2, g_{V}=m_{\rho} / f_{\pi}, m_{\rho}=0.77 \mathrm{MeV}, \beta=0.9$, $\lambda=0.56 \mathrm{GeV}^{-1}, g=0.59$, and $f_{\pi}=132 \mathrm{MeV}[15-18]$.

| Coupling | Expression | Value | Coupling | Expression | Value |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $g_{J / \psi \mathcal{D D}}$ | - | 7.71 | $g_{\mathcal{D}^{*} \mathcal{D}^{* V}}$ | $\frac{\beta g_{V}}{\sqrt{2}}$ | 3.71 |
| $g_{J / \psi \mathcal{D}^{*} \mathcal{D}}$ | - | $3.98 \mathrm{GeV}^{-1}$ | $f_{\mathcal{D}^{*} \mathcal{D}^{* V}}$ | $\frac{\lambda g_{V} m_{D^{*}}}{\sqrt{2}}$ | 4.64 |
| $g_{J / \psi \mathcal{D}^{*} \mathcal{D}^{*}}$ | $g_{J / \psi \mathcal{D D}}$ | 7.71 | $f_{\mathcal{D}^{*} \mathcal{D}^{*}}$ | $\frac{\lambda g_{V}}{\sqrt{2}}$ | $2.31 \mathrm{GeV}^{-1}$ |

$g_{\mathcal{D D V}} \quad \frac{\beta g_{V}}{\sqrt{2}} \quad 3.71$

To write out the amplitudes corresponding to the diagrams listed in Fig. 1, we adopt the effective Lagrangian approach. The effective Lagrangian expressing the interactions of $X(3915) / Z(3930)$ with $D \bar{D}$ or $D \bar{D}^{*}+$ h.c. is given by [14]

$$
\begin{align*}
\mathcal{L}_{\chi_{c J}^{\prime} D^{(*)} D^{(*)}}= & g_{\chi_{c 0}^{\prime} D D} \chi_{c 0}^{\prime} \mathcal{D} \mathcal{D}^{\dagger}-g_{\chi_{c 2}^{\prime} D D} \chi_{c 2 \mu \nu}^{\prime} \partial^{\mu} \mathcal{D} \partial^{\nu} \mathcal{D}^{\dagger} \\
& +i g_{\chi_{c 2}^{\prime} D^{*} D} \varepsilon_{\mu \nu \alpha \beta} \partial^{\mu} \chi_{c 2}^{\prime \nu \rho}\left(\partial^{\alpha} \mathcal{D}^{* \beta} \partial_{\rho} \mathcal{D}^{\dagger}+\partial^{\alpha} \mathcal{D}^{* \dagger \beta} \partial_{\rho} \mathcal{D}\right) \tag{3}
\end{align*}
$$

The couplings of charmed mesons with the light vector meson $\omega$ or charmonium $J / \psi$ are constructed in Refs. [15,16], paying attention to the heavy quark symmetry and the chiral $S U(3)$ symmetry, and are given below:

$$
\begin{align*}
\mathcal{L}_{J / \psi D^{(*)} D^{(*)}}= & i g_{J / \psi \mathcal{D D}} \psi_{\mu}\left(\partial^{\mu} \mathcal{D} \mathcal{D}^{\dagger}-\mathcal{D} \partial^{\mu} \mathcal{D}^{\dagger}\right)-g_{J / \psi \mathcal{D}^{*} \mathcal{D}} \varepsilon^{\mu \nu \alpha \beta} \partial_{\mu} \psi_{\nu}\left(\partial_{\alpha} \mathcal{D}_{\beta}^{*} \mathcal{D}^{\dagger}+\mathcal{D} \partial_{\alpha} \mathcal{D}_{\beta}^{* \dagger}\right) \\
& -i g_{J / \psi \mathcal{D}^{*} \mathcal{D}^{*}}\left\{\psi^{\mu}\left(\partial_{\mu} \mathcal{D}^{* \nu} \mathcal{D}_{\nu}^{* \dagger}-\mathcal{D}^{* \nu} \partial_{\mu} \mathcal{D}_{\nu}^{* \dagger}\right)+\left(\partial_{\mu} \psi_{\nu} \mathcal{D}^{* \nu}-\psi_{\nu} \partial_{\mu} \mathcal{D}^{* \nu}\right) \mathcal{D}^{* \mu \dagger}\right. \\
& \left.+\mathcal{D}^{* \mu}\left(\psi^{\nu} \partial_{\mu} \mathcal{D}_{\nu}^{* \dagger}-\partial_{\mu} \psi_{\nu} \mathcal{D}^{* \nu \dagger}\right)\right\}  \tag{4}\\
\mathcal{L}_{\mathcal{D}^{(*)} \mathcal{D}^{(*) \mathbb{V}}}= & -i g_{\mathcal{D D V}} \mathcal{D}_{i}^{\dagger} \stackrel{\leftrightarrow}{\partial}{ }_{\mu} \mathcal{D}^{j}\left(\mathbb{V}^{\mu}\right)_{j}^{i}-2 f_{\mathcal{D}^{*} \mathcal{D} \mathbb{V}} \varepsilon_{\mu \nu \alpha \beta}\left(\partial^{\mu} \mathbb{V}^{\nu}\right)_{j}^{i}\left(\mathcal{D}_{i}^{\dagger \stackrel{\leftrightarrow}{\partial}} \mathcal{D}^{* \beta j}-\mathcal{D}_{i}^{* \beta \dagger} \stackrel{\leftrightarrow}{\partial} \mathcal{D}^{j}\right) \\
& +i g_{\mathcal{D}^{*} \mathcal{D}^{*} \mathbb{V}} \mathcal{D}_{i}^{* \nu \dagger} \stackrel{\leftrightarrow}{\partial}{ }_{\mu} \mathcal{D}_{\nu}^{* j}\left(\mathbb{V}^{\mu}\right)_{j}^{i} 4 i f_{\mathcal{D}^{*} \mathcal{D}^{* V}} \mathcal{D}_{i \mu}^{* \dagger}\left(\partial^{\mu} \mathbb{V}^{\nu}-\partial^{\nu} \mathbb{V}^{\mu}\right)_{j}^{i} \mathcal{D}_{\nu}^{* j} \tag{5}
\end{align*}
$$

where $\mathcal{D}=\left(D^{0}, D^{+}, D_{s}^{+}\right),\left(\mathcal{D}^{\dagger}\right)^{T}=\left(\bar{D}^{0}, D^{-}, D_{s}^{-}\right)$, and $\stackrel{\leftrightarrow}{\partial}=\vec{\partial}-\overleftarrow{\partial}$. The light vector nonet meson can form the following $3 \times 3$ matrix $\mathbb{V}$ :

$$
\mathbb{V}=\left(\begin{array}{ccc}
\frac{\rho^{0}}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^{+} & K^{*+}  \tag{6}\\
\rho^{-} & \frac{-\rho^{0}}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{* 0} \\
K^{*-} & \bar{K}^{* 0} & \phi
\end{array}\right)
$$

The coupling constants of $\chi_{c 0}^{\prime} \rightarrow D \bar{D}$ and $\chi_{c 2}^{\prime} \rightarrow D \bar{D}, D \bar{D}^{*}+$ h.c. are obtained by fitting the total widths of $X(3915)$ and $Z(3930)$, which will be presented later. The coupling constants of $J / \psi$ interacting with a pair of charmed mesons and a coupling constant of charmed mesons interacting with a light vector meson are given in Table 1 [15-18].

The amplitudes for $\chi_{c 0}^{\prime}\left(p_{0}\right) \rightarrow\left[D\left(p_{1}\right) \bar{D}\left(p_{2}\right)\right] D^{(*)}(q) \rightarrow J / \psi\left(p_{3}\right) \omega\left(p_{4}\right)$ corresponding to Fig. 1(a) are given by

$$
\begin{align*}
\mathcal{A}_{(\mathrm{a})}^{D}= & (i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[g_{\chi_{\chi_{0}^{\prime} \mathcal{D D}}}\right]\left[i g_{J / \psi \mathcal{D D}} \epsilon_{\psi_{\mu}}\left(i q^{\mu}+i p_{1}^{\mu}\right)\right]\left[-i g_{\mathcal{D D V}}\left(-i p_{2 v}+i q_{\nu}\right) \epsilon_{\omega}{ }^{\nu}\right] \\
& \times \frac{i}{p_{1}^{2}-m_{D}^{2}} \frac{i}{p_{2}^{2}-m_{D}^{2}} \frac{i}{q^{2}-m_{D}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \\
\mathcal{A}_{(\mathrm{a})}^{D^{*}}= & (i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[g_{\chi_{c 0}^{\prime} \mathcal{D} \mathcal{D}}\right]\left[-g_{J / \psi \mathcal{D} * \mathcal{D}} \varepsilon^{\mu \nu \alpha \beta}\left(i p_{3 \mu}\right) \epsilon_{\psi_{\nu}}\left(i q_{\alpha}\right)\right] \\
& \times\left[-2 f_{\mathcal{D}^{*} \mathcal{D V}} \varepsilon_{\sigma \lambda \rho \xi}\left(i p_{4}^{\sigma}\right) \epsilon_{\omega}^{\lambda}\left(i p_{2}^{\rho}-i q^{\rho}\right)\right] \frac{i}{p_{1}^{2}-m_{D}^{2}} \frac{i}{p_{2}^{2}-n m_{D}^{2}} \frac{i \tilde{g}_{\beta}^{\xi}(q)}{q^{2}-m_{D^{*}}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \tag{7}
\end{align*}
$$

where $\mathcal{A}_{\text {(a) }}^{D}$ and $\mathcal{A}_{\text {(a) }}^{D^{*}}$ are the amplitudes corresponding to diagram (a) in Fig. 1 with the $D$ and $D^{*}$ meson exchanges, respectively. Similarly, we can easily write out the the expressions for the decay amplitudes of $Z(3930) \rightarrow J / \psi \omega$ corresponding to Figs. 1(b)-(d), which are

$$
\begin{align*}
& \mathcal{M}_{(\mathrm{b})}^{D}=(i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[-g_{\chi_{c 2}^{\prime} \mathcal{D} \mathcal{D}} \epsilon_{\chi_{c 2}^{\prime}}^{\mu \nu}\left(i p_{1 \mu}\right)\left(i p_{2 v}\right)\right]\left[i g_{J / \psi \mathcal{D D}} \epsilon_{\psi_{\rho}}\left(i q^{\rho}+i p_{1}^{\rho}\right)\right] \\
& \times\left[-i g_{\mathcal{D D} \omega}\left(-i p_{2 \tau}+i q_{\tau}\right) \epsilon_{\omega}^{\tau}\right] \frac{i}{p_{1}^{2}-m_{D}^{2}} \frac{i}{p_{2}^{2}-m_{D}^{2}} \frac{i}{q^{2}-m_{D}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \\
& \mathcal{M}_{(\mathrm{b})}^{D^{*}}=(i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[-g_{\chi_{c 2}^{\prime} \mathcal{D} \mathcal{D}} \epsilon_{\chi_{c 2}^{\prime}}^{\mu \nu}\left(i p_{1 \mu}\right)\left(i p_{2 v}\right)\right]\left[-g_{J / \psi \mathcal{D}^{*} \mathcal{D}} \varepsilon_{\theta \rho \alpha \beta}\left(i p_{3}^{\theta}\right) \epsilon_{\psi}^{\rho}\left(i q^{\alpha}\right)\right] \\
& \times\left[-2 f_{\mathcal{D}^{*} \mathcal{D} \omega} \varepsilon_{\sigma \tau \lambda \phi}\left(i p_{4}^{\sigma}\right) \epsilon_{\omega}^{\tau}\left(i p_{2}^{\lambda}-i q^{\lambda}\right)\right] \frac{i}{p_{1}^{2}-m_{D}^{2}} \frac{i}{p_{2}^{2}-m_{D}^{2}} \frac{i \tilde{g}^{\beta \phi}(q)}{q^{2}-m_{D^{*}}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \\
& \mathcal{M}_{(\mathrm{c})}^{D}=(i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[i g_{\chi_{c 2}^{\prime} D^{*} D^{\delta} \delta \mu \theta \phi}\left(-i p_{0}^{\delta}\right) \epsilon_{\chi_{c 2}^{\prime}}^{\mu \nu}\left(i p_{2}^{\theta}\right)\left(i p_{1 v}\right)\right]\left[i g_{J / \psi \mathcal{D}} \epsilon_{\psi}^{\rho}\left(i q_{\rho}+i p_{1 \rho}\right)\right] \\
& \times\left[-2 f_{\mathcal{D}^{*} \mathcal{D}_{\omega}} \varepsilon_{\sigma \tau \lambda \alpha}\left(i p_{4}^{\sigma}\right) \epsilon_{\omega}^{\tau}\left(-i p_{2}^{\lambda}+i q^{\lambda}\right)\right] \frac{i}{p_{1}^{2}-m_{D}^{2}} \frac{i \tilde{g}^{\phi \alpha}\left(p_{2}\right)}{p_{2}^{2}-m_{D^{*}}^{2}} \frac{i}{q^{2}-m_{D}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \\
& \mathcal{M}_{(\mathrm{c})}^{D^{*}}=(i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[i g_{\chi_{c 2}^{\prime} D^{*} D^{\varepsilon} \delta \mu \theta \phi}\left(-i p_{0}^{\delta}\right) \epsilon_{\chi_{c 2}^{\prime}}^{\mu \nu}\left(i p_{2}^{\theta}\right)\left(i p_{1 \nu}\right)\right]\left[-g_{J / \psi \mathcal{D}^{*} \mathcal{D} \varepsilon_{\lambda \rho \alpha \beta}}\left(i p_{3}^{\lambda}\right) \epsilon_{\psi}^{\rho}\left(i q^{\alpha}\right)\right] \\
& \times\left[i g_{\mathcal{D}^{*} \mathcal{D}^{*} \omega}\left(-i p_{2 \tau}+i q_{\tau}\right) \epsilon_{\omega}^{\tau} g_{\zeta \sigma}+4 i f_{\mathcal{D}^{*} \mathcal{D}^{*} \omega} \epsilon_{\omega}^{\tau}\left(i p_{4 \zeta} g_{\sigma \tau}-i p_{4 \sigma} g_{\tau \zeta}\right)\right] \\
& \times \frac{i}{p_{1}^{2}-m_{D}^{2}} \frac{i \tilde{g}^{\phi \sigma}\left(p_{2}\right)}{p_{2}^{2}-m_{D^{*}}^{2}} \frac{i \tilde{g}^{\zeta \beta}(q)}{q^{2}-m_{D^{*}}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \\
& \mathcal{M}_{(\mathrm{d})}^{D}=(i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[i g_{\chi_{c 2}^{\prime} D^{*} D^{2} \delta \mu \theta \phi}\left(-i p_{0}^{\delta}\right) \epsilon_{\chi_{c 2}^{\prime}}^{\mu \nu}\left(i p_{1}^{\theta}\right)\left(i p_{2}^{\nu}\right)\right]\left[-g_{J / \psi D^{*} D} \varepsilon_{\lambda \rho \alpha \beta}\left(i p_{3}^{\lambda}\right) \epsilon_{\psi}^{\rho}\left(-i p_{1}^{\alpha}\right)\right] \\
& \times\left[i g_{D D \omega} \epsilon_{\omega}^{\tau}\left(i p_{2 \tau}-i q_{\tau}\right)\right] \frac{i \tilde{g} \phi \beta}{p_{1}^{2}-m_{D^{*}}^{2}} \frac{i}{p_{2}^{2}-m_{D}^{2}} \frac{i}{q^{2}-m_{D}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \\
& \mathcal{M}_{(\mathrm{d})}^{D^{*}}=(i)^{3} \int \frac{d^{4} q}{\left(2 \pi^{4}\right)}\left[i g_{\chi_{c 2}^{\prime} D^{*} D^{\varepsilon}} \varepsilon_{\delta \mu \theta \phi}\left(-i p_{0}^{\delta}\right) \epsilon_{\chi_{c 2}^{\prime}}^{\mu \nu}\left(i p_{1}^{\theta}\right)\left(i p_{2}^{\nu}\right)\right] \\
& \times\left[-i g_{J / \psi D^{*} D^{*}} \epsilon_{\psi}^{\rho}\left(g_{\alpha \beta}\left(-i p_{2 \rho}+i q_{\rho}\right)+g_{\beta \rho}\left(i p_{3 \alpha}+i p_{1 \alpha}\right)+g_{\alpha \rho}\left(-i q_{\beta}-i p_{3 \beta}\right)\right)\right] \\
& \times\left[-2 f_{D^{*} D \omega} \varepsilon_{\sigma \tau \lambda \zeta}\left(i p_{4}^{\sigma}\right) \epsilon_{\omega}^{\tau}\left(-i q^{\lambda}+i p_{2}^{\lambda}\right)\right] \frac{i \tilde{g}^{\phi \beta}\left(p_{1}\right)}{p_{1}^{2}-m_{D^{*}}^{2}} \frac{i}{p_{2}^{2}-m_{D}^{2}} \frac{i \tilde{g}^{\alpha \zeta}(q)}{q^{2}-m_{D^{*}}^{2}} \mathcal{F}^{2}\left(q^{2}\right), \tag{8}
\end{align*}
$$

with $\tilde{g}^{\alpha \beta}(p)=-g^{\alpha \beta}+p^{\alpha} p^{\beta} / m_{D^{*}}^{2}$, where $\mathcal{F}\left(q^{2}\right)$ is the form factor, which is introduced not only to compensate the off-shell effects of the charmed meson but also to describe the structure effects of the vertex of a charmed meson pair interacting with $J / \psi$ or $\omega$. In this work, we adopt the form factor in the form

$$
\mathcal{F}\left(q^{2}\right)=\left(\frac{m_{\mathrm{E}}^{2}-\Lambda^{2}}{q^{2}-\Lambda^{2}}\right)^{N}, \quad \begin{cases}N=1, & \text { monopole form }  \tag{9}\\ N=2, & \text { dipole form }\end{cases}
$$

where $q$ and $m_{\mathrm{E}}$ are the momentum and the mass of the exchanged charmed meson, respectively. Furthermore, $\Lambda$ can be parameterized as $\Lambda=m_{\mathrm{E}}+\alpha \Lambda_{\mathrm{QCD}}$ with a dimensionless parameter $\alpha$ and $\Lambda_{\mathrm{QCD}}=220 \mathrm{MeV}$. The parameter $\alpha$ is of order unity and depends on the specific process $[13,18]$.
With the above elaborate expressions for the amplitudes, one can obtain the partial decay width for $\chi_{c J}^{\prime} \rightarrow J / \psi \omega(J=0,2)$ as
where the overline indicates the sum over the polarizations of the vector meson $J / \psi, \omega$ and tensor meson $\chi_{c 2}^{\prime}$, and $\vec{p}$ indicates the three-momentum of $J / \psi$ in the initial state at rest.

If $X(3915)$ is a $\chi_{c 0}^{\prime}(2 P)$ state, $D \bar{D}$ is its dominant decay. Hence, we can use the experimental width of $X$ (3915) [1] to determine the coupling constant of $\chi_{c 0}^{\prime} \rightarrow D \bar{D}$ interaction, i.e., $g_{\chi_{c 0}^{\prime} D \bar{D}}=2.37$ GeV. However, for $Z(3930)$, there exist two main decay modes $D \bar{D}$ and $D \bar{D}^{*}+$ h.c. Since experiments have so far not given the ratio of $B R(Z(3930) \rightarrow D \bar{D})$ to $B R\left(Z(3930) \rightarrow D \bar{D}^{*}+\right.$ h.c. $)$, we must determine these corresponding coupling constants from the theoretical results estimated by the quark pair creation model. In Ref. [4], the wave functions of $\chi_{c J}^{\prime}$ are simulated by a simple harmonic oscillator wave function with a parameter $R$, which means a root-mean-square radius of the wave function. The partial and total decay widths of $\chi_{c J}^{\prime}$ are dependent on this unique parameter $R$. One can determine the parameter value $R \simeq 1.9 \mathrm{GeV}^{-1}$ in the spatial wave function from the partial decay width of $\chi_{c 0}^{\prime}$ under the assumption $\Gamma_{\chi_{c 0}^{\prime} \rightarrow D \bar{D}} \simeq \Gamma_{\chi_{c 0}^{\prime}}^{\mathrm{tot}}$. With the parameter $R$ estimated by the center value of $\Gamma_{\chi_{c 0}^{\prime}}^{\mathrm{tot}}$, we obtain $\left|g_{\chi_{c 2}^{\prime} D D}\right|=11.69 \mathrm{GeV}^{-1}$ and $\left|g_{\chi_{c 2}^{\prime} D^{*} D}\right|=7.83 \mathrm{GeV}^{-2}$.
For $Z$ (3930) with the assignment of $\chi_{c 2}^{\prime}$, it dominantly decays into $D \bar{D}$ and $D^{*} \bar{D}+$ h.c. The absolute values of the coupling constants between $\chi_{c 2}^{\prime}$ and the charmed meson pairs are evaluated by the quark pair creation model. However, the relative sign of the coupling constants $g_{\chi_{c 2}^{\prime} D D}$ and $g_{\chi_{c 2}^{\prime} D^{*} D}$ in Eq. (3) can be either positive or negative, which corresponds to the subscripts ++ and +- shown in Fig. 2, respectively. Thus, we discuss two cases for $Z(3930) \rightarrow J / \psi \omega$.

In Fig. 2, we give the ratio of the width of $X(3915) \rightarrow J / \psi \omega$ to that of $Z(3930) \rightarrow J / \psi \omega$. This result shows that the width of $X(3915) \rightarrow J / \psi \omega$ is at least one order of magnitude larger than that of $Z(3930) \rightarrow J / \psi \omega$ in two different cases (see Fig. 2 for more details). Although the decay width for $\chi_{c 0}^{\prime} / \chi_{c 2}^{\prime} \rightarrow J / \psi \omega$ calculated in this work strongly depends on the parameter $\alpha$, the ratio of the width of $X(3915) \rightarrow J / \psi \omega$ to that of $Z(3930) \rightarrow J / \psi \omega$ has a very large value and is weakly dependent on the parameter $\alpha$, as shown in Fig. 2. Such a large ratio could explain why Belle only reported one enhancement structure, $X(3915)$, in the $J / \psi \omega$ invariant mass spectrum of the $\gamma \gamma \rightarrow J / \psi \omega$ process.

In addition, the $\alpha$ dependence of the $\chi_{c J}^{\prime} \rightarrow J / \psi \omega$ partial decay widths is presented in Fig. 3. Here, we take the monopole form factor as an example. As one finds in Fig. 3, the partial decay widths are strongly dependent on the parameter $\alpha$. As for $\chi_{c 0}^{\prime} \rightarrow J / \psi \omega$, the partial decay width varies from $3.5 \times 10^{-3} \mathrm{MeV}$ to 0.15 MeV in the range $1<\alpha<4$, while, for $\chi_{c 2}^{\prime}$, the partial decay


Fig. 2. $\alpha$ dependence of the ratio of the width of $X(3915) \rightarrow J / \psi \omega$ to that of $Z(3930) \rightarrow J / \psi \omega$. Here, we define $R_{++}=\Gamma[X(3915) \rightarrow J / \psi \omega] / \Gamma[Z(3930) \rightarrow J / \psi \omega]_{++} \quad$ and $R_{+-}=\Gamma[X(3915) \rightarrow J / \psi \omega] / \Gamma[Z(3930) \rightarrow J / \psi \omega]_{+-}$. In addition, we use the superscripts "Monopole" and "Dipole" to distinguish different results by taking monopole and dipole form factors in the calculation, respectively. The calculated results are $R_{++}^{\text {Monopole }}=850-1400, R_{++}^{\text {Dipole }}=239-558, R_{+-}^{\text {Monopole }}=87-130$, and $R_{+-}^{\text {Dipole }}=27-59$.


Fig. 3. $\alpha$ dependence of the partial decay widths of $\chi_{c J}^{\prime} \rightarrow J / \psi \omega$, where $X(3915)$ and $Z(3930)$ are assigned as $\chi_{c 0}^{\prime}$ and $\chi_{c 2}^{\prime}$, respectively. The loop integrals in Eqs. (7) and (8) are evaluated by the Cutkosky cutting rules. The subscripts ++ and +- are the same as those in Fig. 2.
width of $\chi_{c 2}^{\prime} \rightarrow J / \psi \omega$ varies from $4.1 \times 10^{-6} \mathrm{MeV}$ to $1.1 \times 10^{-4} \mathrm{MeV}$ or $4.0 \times 10^{-5} \mathrm{MeV}$ to $1.1 \times 10^{-3} \mathrm{MeV}$ depending on the relative sign between $g_{\chi_{c 2}^{\prime} D D}$ and $g_{\chi_{c 2}^{\prime} D^{*} D}$. In the present calculations, the loop integrals in Eqs. (7) and (8) are evaluated by the Cutkosky cutting rules, where only the imaginary part of the amplitudes is considered. As for the case of the dipole form factor, the magnitudes of the partial decay widths are at least one order smaller than the corresponding ones estimated by the monopole form factor. The calculations in Ref. [26] also indicate that the monopole form factor is more suitable to estimate the partial decay widths of the $\chi_{c J}^{\prime} \rightarrow J / \psi \omega$ process.

As indicated above, the absolute decay widths of $X(3915) \rightarrow J / \psi \omega$ and $Z(3930) \rightarrow J / \psi \omega$ are strongly dependent on the parameter $\alpha$, which means that there exists uncertainty in the prediction of these decay widths. In addition, we notice that extracting the decay widths of $X(3915) \rightarrow J / \psi \omega$ from that of $Z(3930) \rightarrow J / \psi \omega$ via the experimental data depends on our understanding of the twophoton decay width of $X(3915)$ and $Z(3930)$, where its predicted two-photon decay width varies with different models. In Refs. [20-23], the two-photon decay width is about $1-2 \mathrm{keV}$ in the relativistic quark model, while the Salpeter method indicates that the decay width for $\chi_{c 0}^{\prime}$ can be larger than

3 keV in a relativistic form and about 5.47 keV in a non-relativistic form [24]. If the center values of the total decay width of $\chi_{c 0}^{\prime}$ and the measured branching ratio $\Gamma_{\chi_{c 0}^{\prime} \rightarrow \gamma \gamma} \mathcal{B}\left(\chi_{c 0}^{\prime} \rightarrow J / \psi \omega\right)$ are adopted, the partial decay width of $\chi_{c 0}^{\prime} \rightarrow J / \psi \omega$ can be less than two hundred keV to 1 MeV , depending on the choice of $\Gamma_{\chi_{c 0}^{\prime} \rightarrow \gamma \gamma}$. In the present work, the evaluated partial decay width can reach 150 keV for $\alpha=4$, which is consistent with the experimental measurements [1].
In summary, $X(3915)$, reported by the Belle Collaboration, is the second enhancement observed in the $\gamma \gamma$ fusion process. As indicated in Ref. [4], $X(3915)$ is a good candidate for $\chi_{c 0}^{\prime}(2 P)$, i.e., the first radial excitation of $\chi_{c 0}(3414)$. Besides its open-charm decay, study of the hidden-charm decay of $X(3915)$ will provide a key hint to understanding the properties of $X(3915)$ and further test the P-wave charmonium explanation of $X(3915)$ in Ref. [4]. Since the mass of $X(3915)$ is above the threshold of $D \bar{D}$ and dominantly decays into $D \bar{D}$, hadronic loop effects [6-13] will play an important role in the hidden-charm decay $X(3915) \rightarrow J / \psi \omega$, which, in fact, results from the coupled-channel effects. In this work, we have performed the calculation of the $X(3915) \rightarrow J / \psi \omega$ processes.
Before the observation of $X$ (3915), Belle reported a state named $Z(3930)$ in $\gamma \gamma$ fusion [2,3], which is also a P-wave charmonium state of the first radial excitation. $Z(3930)$ should decay into $J / \psi \omega$, which seems to indicate that there should exist two peaks close to each other in the $J / \psi \omega$ invariant mass spectrum given by Belle [1]. However, currently, only one structure corresponding to $X(3915)$ has been observed [1]. In order to explain this contradiction, in this work we have further studied $Z(3930) \rightarrow J / \psi \omega$ by the intermediate states $D \bar{D}$ and $D \bar{D}^{*}+$ h.c. The results illustrated in Fig. 2 show that the partial decay width of $Z(3930) \rightarrow J / \psi \omega$ is suppressed when compared with that of $X(3915) \rightarrow J / \psi \omega$, which explains why $Z(3930)$ cannot be observed in the $J / \psi \omega$ invariant mass spectrum.
As more charmonium-like states are observed in the $\gamma \gamma$ fusion process [1-3,25], they provide us with a better chance to explore the properties of these states, especially P-wave charmonium states [4]. The study of the hidden-charm decay of $X(3915)$ in this work supports the proposal of $\chi_{c 0}^{\prime}(2 P)$ assignment to $X(3915)$ in Ref. [4]. Besides applying the hidden-charm and open-charm decays of $X(3915)$ to test the $\chi_{c 0}^{\prime}$ assignment to $X(3915)$, we suggest that an angular distribution analysis of $X(3915)$ in future experiments will be valuable to test the $\chi_{c 0}^{\prime}(2 P)$ explanation of $X(3915)$, since the $J^{P C}$ quantum number of $X(3915)$ must be $0^{++}$. Although $Z(3930)$ is well established as a $\chi_{c 2}^{\prime}(2 P)$ state [2,3], its hidden-charm decay behavior was unclear before this work. Performing the calculation of $Z(3930) \rightarrow J / \psi \omega$ by the hadronic loop mechanism, we further learn that the branching ratio of $Z(3930) \rightarrow J / \psi \omega$ is at least one order smaller than that of $X(3915) \rightarrow$ $J / \psi \omega$, which not only successfully explains the appearance of only one enhancement, $X$ (3915), in the $J / \psi \omega$ invariant mass spectrum but also tests the hadronic loop effects, which is an important non-perturbative mechanism in the decays of charmonium or charmonium-like states [6-13].

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