Hidden Messages in Heavy Tails: DCT-Domain Watermark Detection Using Alpha-Stable Models

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- Master's Thesis in *Signal and Image Processing Systems: Theory, Implementations, Applications, University of Patras, Greece, 2000*

- Importance of watermarking
- Watermark generation and embedding
- “Blind” watermark detection
- Statistical modeling of DCT coefficients
- Comparison of nearly optimal detectors: generalized Gaussian, Cauchy
- Experimental Results
- Conclusions

Digital information: easily reproduced and distributed without loss of fidelity

- *Watermarking*:

  Embedding of a digital signal specifying legitimate owner/receiver of data *directly in the data*
- Part of a general system, not a complete solution

PROPERTIES

- Secret key known only to legal owner
- Imperceptibility, Robustness
- *Kerkhoff's Law* : The system is secure even if an attacker knows the principles and methods of watermark embedment but not the secret key

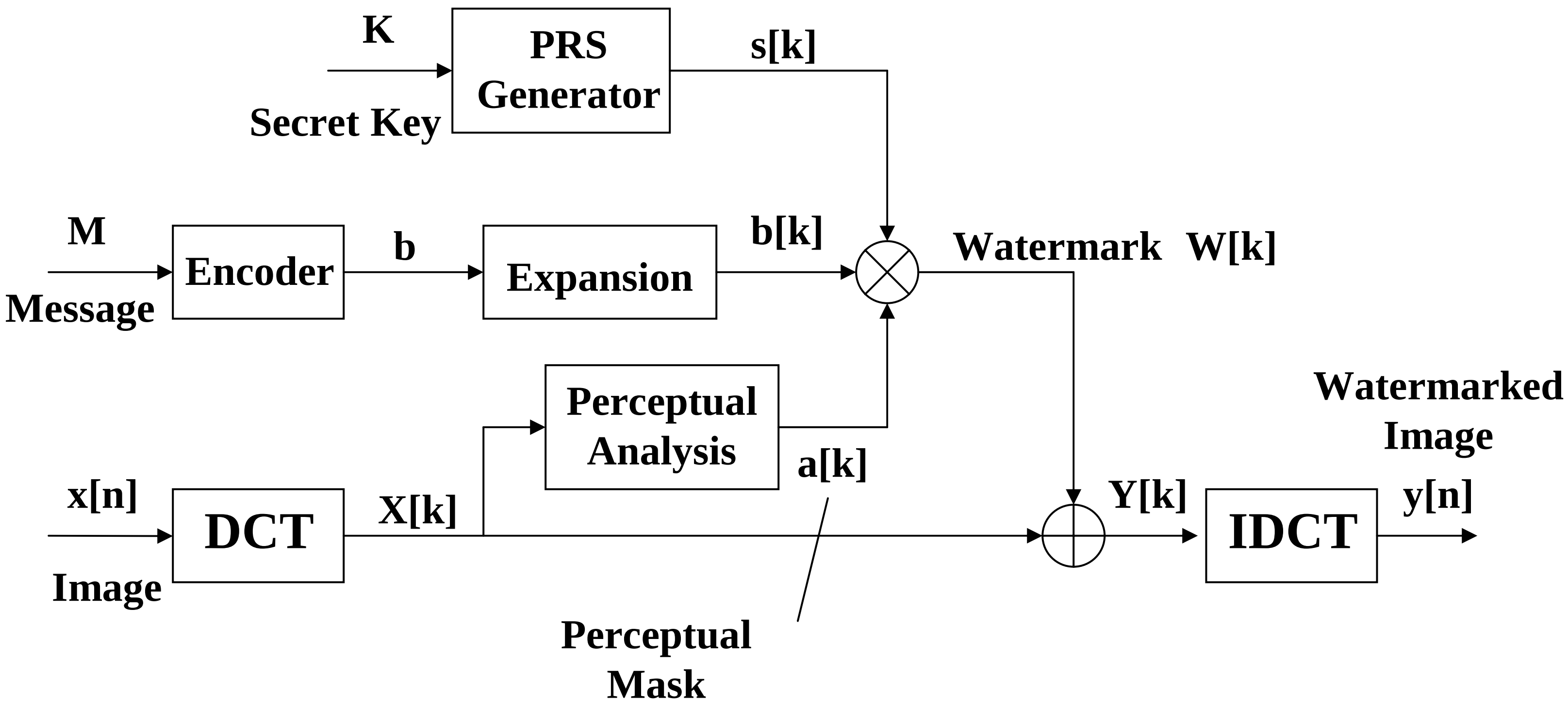
- **WATERMARK DETECTION/EXTRACTION**
- Availability of original data
- Detection = Binary hypothesis test for watermark existence
- Extraction = extraction of message as well (*fingerprinting*)
- Accurate statistical model $\Rightarrow$ efficient watermark detection

- **SPREAD SPECTRUM WATERMARKING**
- DCT image values x[**k**] at pixels **k** = (i,j) : Noise
- Anti-jamming properties of Spread Spectrum make it robust to some attacks
- Message M encoded to N – D vector **b** that is "spread" over the image (expansion process)

- Secret key K = random generator seed for the pseudorandom sequence s[**k**]
- Watermark strength determined by perceptual mask for DCT data a[**k**] (*Ahumada et. al., Watson*)
- Watermark detection: one bit (b=1, N=1) is *repeated* over pixels – increases *robustness*
- Mask a[**k**] multiplied pixelwise by:
  - pseudorandom sequence s[**k**]
  - bits b[**k**] (b=1 for watermark detection)

  to give watermark W[**k**]=a[**k**]s[**k**]b[**k**]

WATERMARK EMBEDDING



MODELING OF DCT COEFFICIENTS

- LAPLACIAN: tails decay exponentially with x

$$f_X(x) = \frac{b}{2}\exp\left(-b\left|x-\alpha\right|\right) \qquad \begin{cases} mean\ (x) = a \\ \mathrm{var}(x) = 2/b^2 \end{cases}$$

- GENERALIZED GAUSSIAN:

$$f_X(x) = A\exp\left|-\beta x\right|^c \qquad \beta = \frac{1}{\sigma}\left(\frac{\Gamma(3/c)}{\Gamma(1/c)}\right)^{1/2}, A = \frac{\beta c}{2\Gamma(1/c)}$$

- c = 1 Laplacian, c = 2 Gauss
- c can be estimated theoretically for each DCT coefficient
- In practice c = 0.5 is satisfactory (*Hernandez et. al.*)
- Cannot adequately model samples in the tails with high magnitudes

ALPHA– STABLE MODELS :

- Often used to describe heavy-tailed data
- Defined in closed form only by their characteristic function

$$\varphi_X(\omega) = E\left[e^{j\omega X}\right]$$

$$\varphi(\omega) = \exp\left(-j\delta\omega - \gamma|\omega|^{\alpha}\left[1 + j\beta sign(\omega)\phi(\omega,\alpha)\right]\right)$$

$$\phi(t,\alpha) = \begin{cases} \tan\frac{\alpha\pi}{2} & \alpha \neq 1 \\ \frac{2}{\pi}\log|t| & \alpha = 1 \end{cases}$$

- Parameters are estimated from the data (Max Likelihood Estimates)

- Parameters:
  - *location* δ ($-\infty < \delta < \infty$) :
    mean for $1 < \alpha \leq 2$, median for $0 < \alpha \leq 1$
  - *scale* γ ($\gamma > 0$) : equivalent to variance
  - *skewness* β ($-1 \leq \beta \leq 1$) : $\beta = 0$ for symmetric pdf
  - *characteristic exponent* α ($0 < \alpha \leq 2$) : determines distribution shape: *small $\alpha \Rightarrow$ heavy tails*

    Tail probabilities $$P(X > x) = c_\alpha x^{-\alpha}$$
- Closed form expression of pdf only for:

  $\alpha = 2 \Rightarrow$ GAUSSIAN

  $\alpha = 1 \Rightarrow$ CAUCHY

EXPERIMENTAL MODELING OF DCT COEFFICIENTS

- Model using the Amplitude Probability Density function (APD)

  $$P\left(|X| > a\right)$$

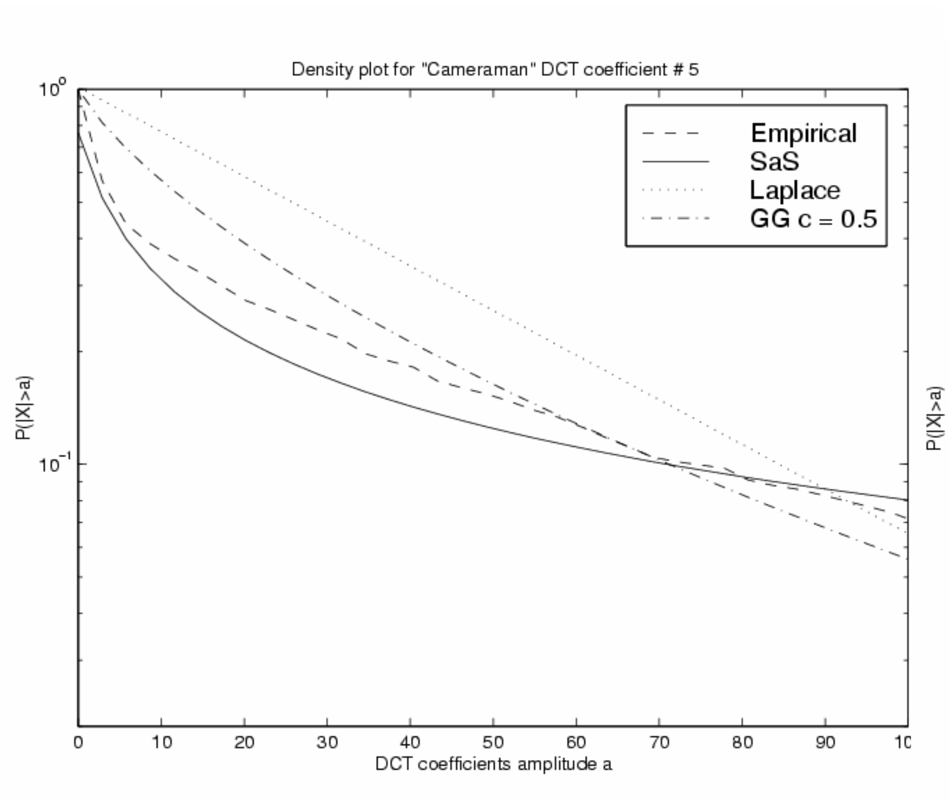
- Consider Symmetric Alpha Stable (SaS, β=0) model
- Theoretical APD :

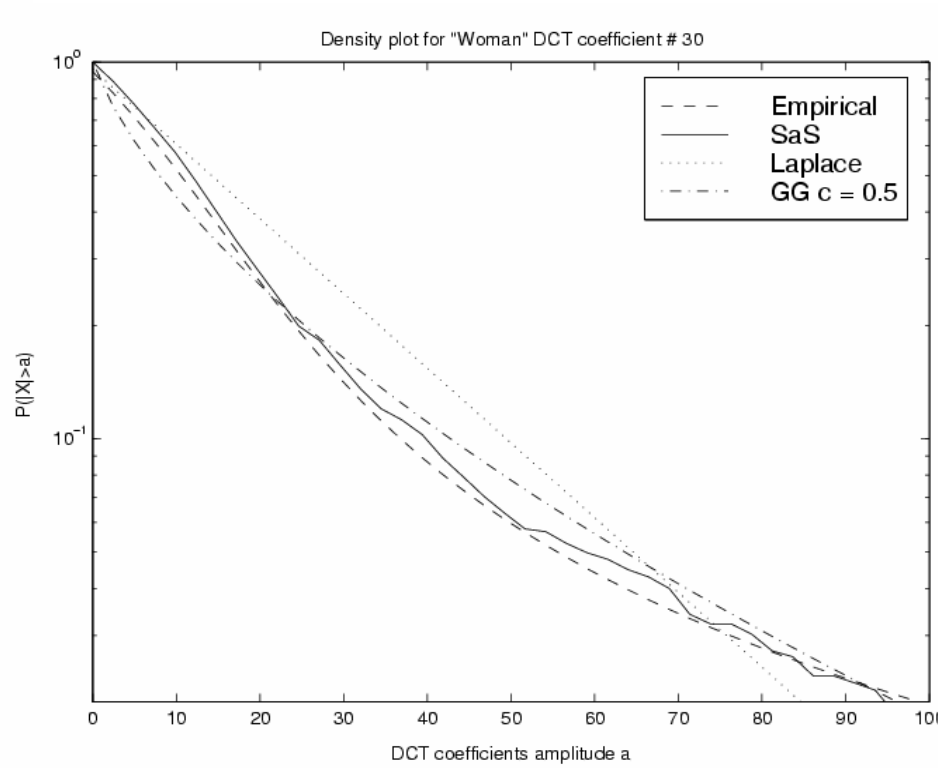
  Uses ML parameter estimates from the data
- Empirical APD :

  Block DCT : distribution of each coefficient over all blocks.

  256x256 images : 1024x1 vector

- Cameraman DCT #5: SaS gave closest fit to empirical APD
- Woman DCT #30: SaS and gen. Gaussian give very good fit to empirical APD





WATERMARK DETECTION

- Binary hypothesis test :

$$H_1 : Y[k] = X[k] + W[k]$$

$$H_0 : Y[k] = X[k]$$

- Log-likelihood ratio test : $l(Y) = \ln\left(\frac{f(Y \mid H_1)}{f(Y \mid H_0)}\right) \overset{H_1}{>} \eta$

- Watermark = signal, image = noise
- Low, mid frequency DCT coefficients
- Original and watermarked images have *similar statistical properties*

- Neyman – Pearson Testing
  Receiver Operating Characteristics :

$$P_{fa} = Q\left(\frac{t - m_0}{\sigma_1}\right), P_{\det} = Q\left(\frac{t - m_1}{\sigma_1}\right) \qquad Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt$$

- Mean and variance of l(Y) : $m_0 = -m_1, \sigma_0 = \sigma_1$
- Threshold : $t = m_0 + \sigma_1 Q^{-1}\left(P_{fa}\right)$

- Generalized Gaussian log – likelihood ratio:

$$l(Y) = \sum_k \beta[k]^{c[k]} \left(|Y[k]|^{c[k]} - |Y[k] - a[k]s[k]|^{c[k]} \right)$$

- Experimental verification of l(Y) mean, variance

GENERALIZED GAUSSIAN (c = 0.5)				
IMAGE	m_0	m_1	σ_0^2	σ_1^2
Lena (th.)	-3.66	3.66	16.55	16.55
Lena (exp.)	-3.66	3.65	17.40	17.40
Woman (th.)	-4.71	4.71	12.41	12.41
Wom. (exp.)	-4.59	4.82	11.81	11.81

Experimental verification of l(Y) mean, variance

CAUCHY				
IMAGE	m_0	m_1	σ_0^2	σ_1^2
Lena (th.)	-2.96	2.90	3.31	3.31
Lena (exp.)	-2.86	2.95	3.16	3.16
Wom. (th.)	-10.05	9.75	58.07	58.07
Wom. (exp.)	10.36	9.71	58.08	58.08

- The mean and variance of the log – likelihood ratio determine the Signal to Noise Ratio SNR:

$$SNR = m_1^2 / \sigma_1^2$$

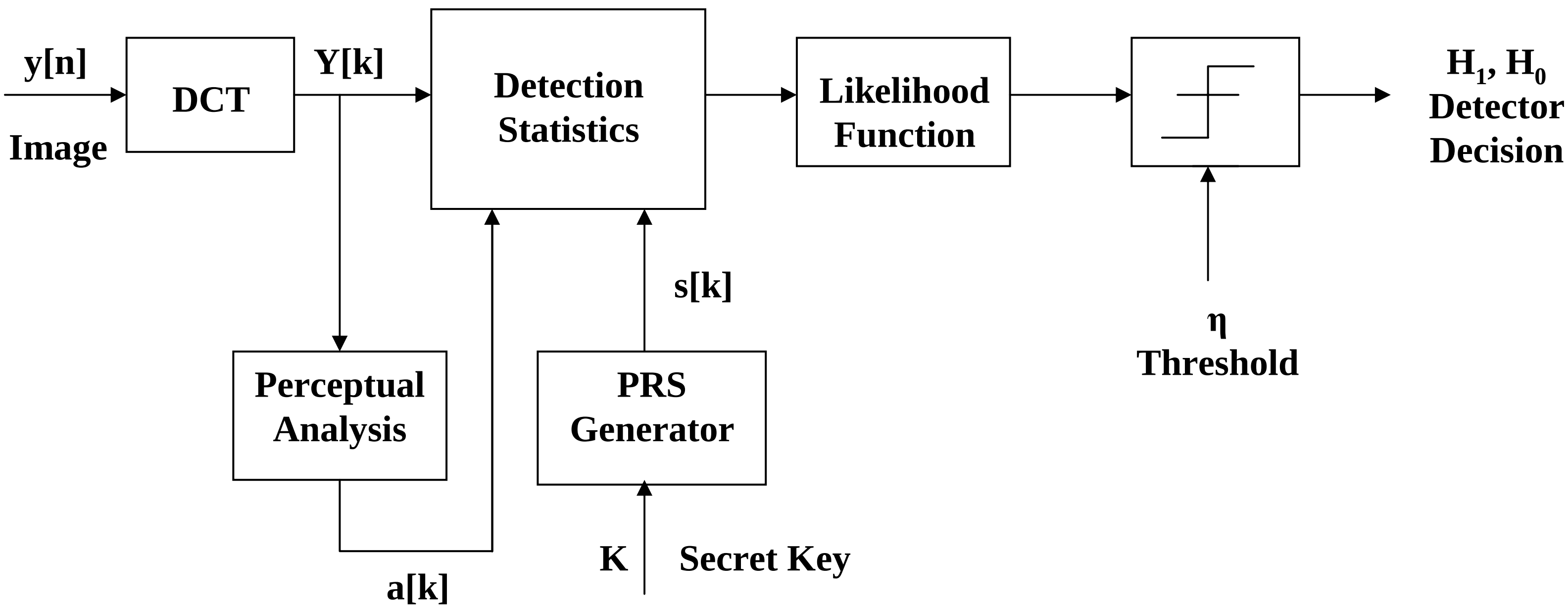
- Detection performance is determined by the ROC curves that depend only on the SNR:

$$P_{\det} = Q\left(Q^{-1}\left(P_{fa}\right) - 2\sqrt{SNR}\right)$$

- High SNR gives better detection performance :

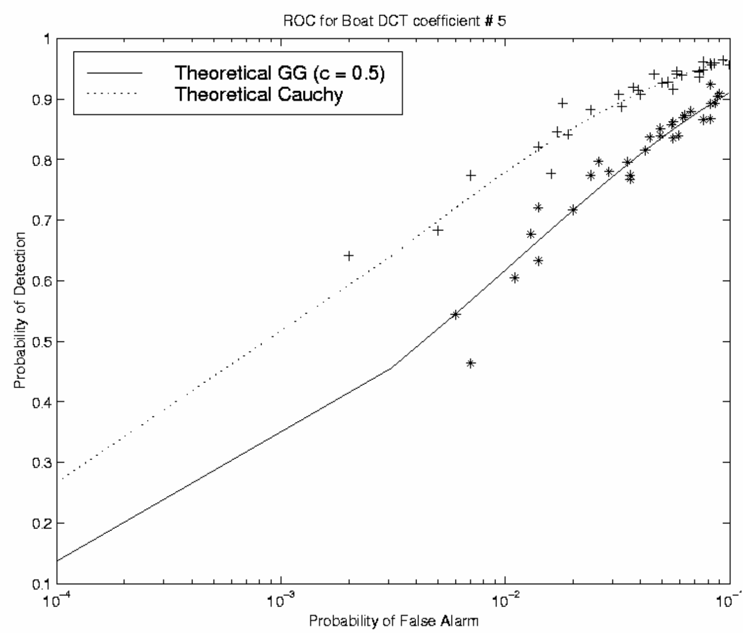
SNR (dB)		
IMAGE coefficient	Cauchy	G.G. (c=0.5)
Boat (#10)	1.31	5.00
Cam. (#5)	5.60	3.75
Lena (#5)	4.21	2.54
Woman (#30)	3.96	2.41

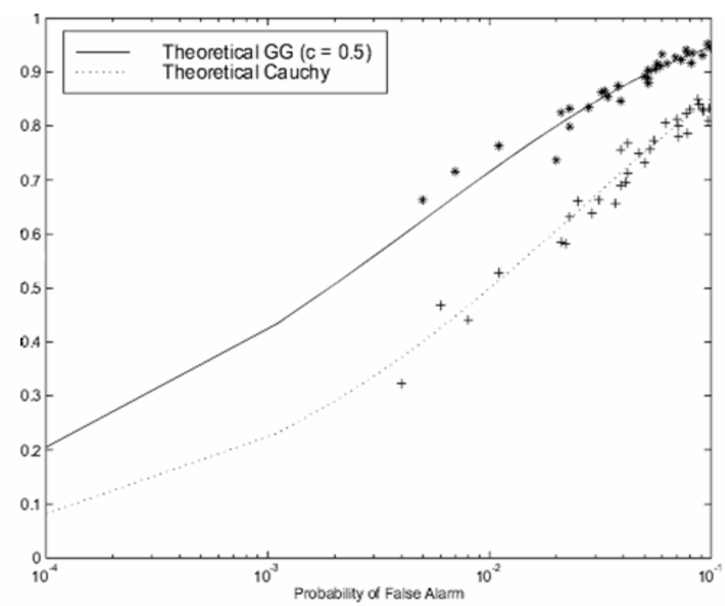
WATERMARK DETECTION



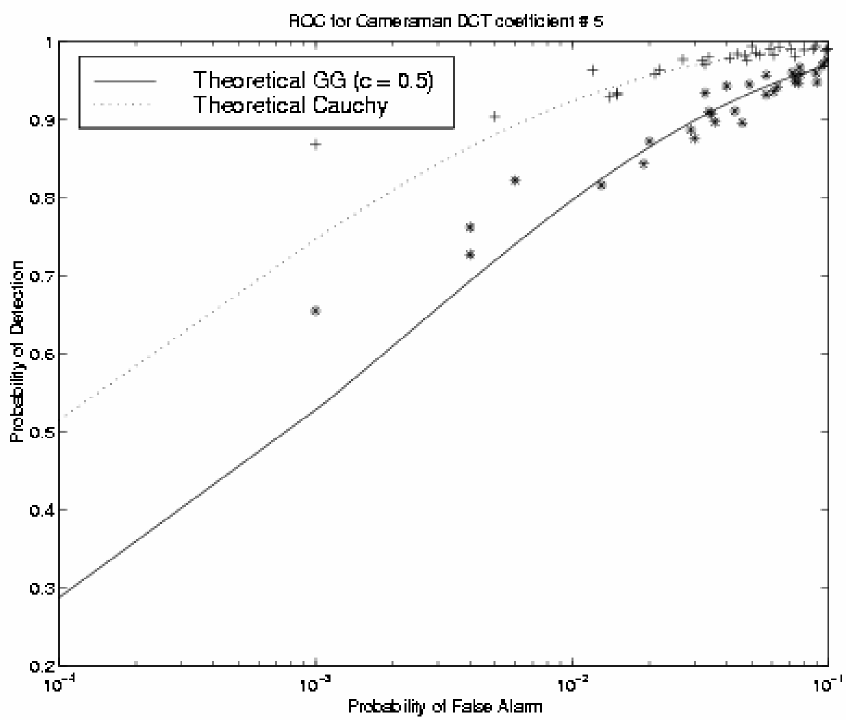
EXPERIMENTAL RESULTS

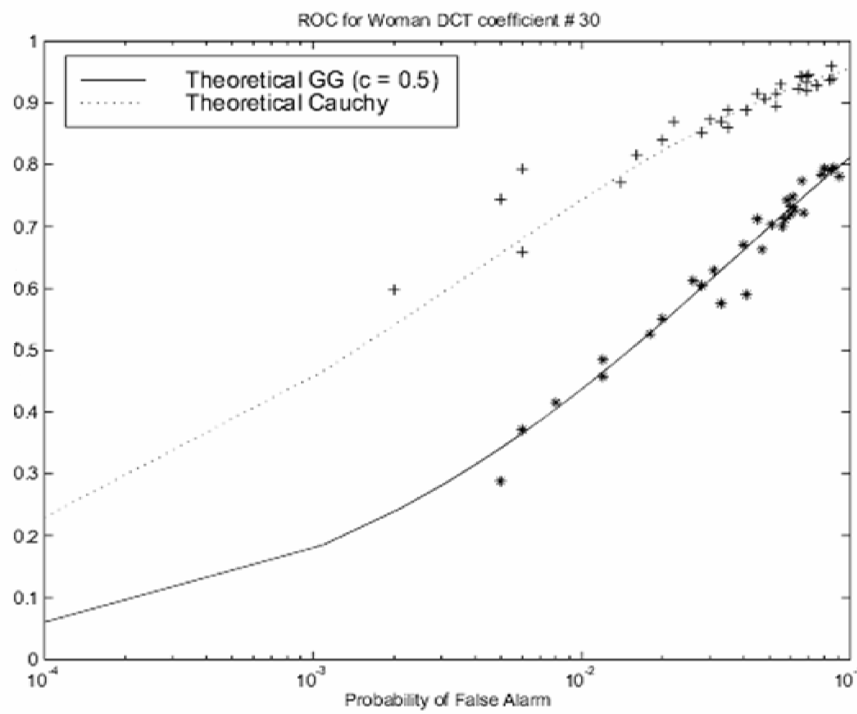
- Experimental results (Monte Carlo) verify theoretical ones.
- Boat DCT #5, #10: Cauchy detector is expected to be:
  - Better for #5 because of better modeling results
  - Worse for #10 – not so heavy tails, closer to Laplacian distribution





- Cameraman DCT #5, woman DCT #30:
  Cauchy gave more accurate modeling and a higher SNR





CONCLUSIONS

- Blind watermark detector
- Improved statistical model for the data – alpha stable model
- Cauchy detector is in closed form
- *Cauchy detectors are in general very robust:*

  their performance remains nearly optimal even for data that deviates from the Cauchy distribution