

Hierarchical Fuzzy Logic Based Variable Structure Control for Vehicles Platooning¹

Yulin Ma, *IEEE Member*, Zhixiong Li, *IEEE Member*, Reza Malekian, *IEEE Senior Member*, Rui Zhang, Miguel Angel Sotelo, *IEEE Senior Member*

Abstract—This paper proposes a variable structure control approach for vehicles platooning based on a hierarchical fuzzy logic. The leader-follower vehicle dynamics with model uncertainties is discussed from the viewpoint of consensus problem. A practical two-layer fuzzy control for the platooning is designed by employing two common spacing policies to ensure system robustness in different scenarios. The two policies, i.e. constant distance and constant time headway, utilize the predecessor-successor information flow from the immediate predecessor and follower other than controlled vehicles. The first layer of the fuzzy system combines spacing control with velocity-acceleration control to achieve a rapid tracking for the desired control commands, and the second layer uses the sliding mode reach law to adaptively compensate for reducing the state errors caused by parameter uncertainties and disturbances. Shift between different controller parameters is based on performance boundaries to guarantee the stability of individual vehicle and platooning. These Performance boundaries can be determined by using a Lyapunov method with exponential stability. Simulation of a ten-vehicle platooning with two spacing policies shows that the control performance of the newly proposed method is effective and promising.

Index Terms—Vehicles platooning, platoon stability, hierarchical fuzzy control, constant distance, constant time headway, adaptive compensation

¹Research supported in part by the National Key Research and Development Program (2016YFB0100903), and in part by the National Natural Science Foundation of China (61503284 and 51408417) and in part by the National Research Foundation, South Africa (IFR160118156967 and RDYR160404161474).

Dr. Y. Ma is with National Center of ITS Engineering and Technology, Research Institute of Highway, Ministry of Transport, Beijing, 100088 China (author e-mail: myl@itsc.cn).

Dr. Z. Li is with School of Mechatronic Engineering, China University of Mining and Technology, Xuzhou 221000, China; he is also with Department of Mechanical Engineering, Iowa State University, Ames 50010, USA (corresponding author e-mail: zhixiong.li@ieee.org).

Dr. R. Malekian is with Department of Electrical, Electronic & Computer Engineering, University of Pretoria, Pretoria 0002, South Africa (email: reza.malekian@ieee.org).

Dr. R. Zhang, is with School of Automotive and Transportation, Tianjin University of Technology and Education, Tianjin, 300222 China (e-mail: zhangrui@tute.edu.cn).

Prof. M. Sotelo is with the Department of Computer Engineering, University of Alcalá, Alcalá de Henares (Madrid), Spain (email: miguel.sotelo@uah.es)

I. INTRODUCTION

VEHICLES platooning has always been under large field operational tests, from earlier PATH – a California traffic automation project^[1], and Smart Cruise 21 DEMO – a Japanese platooning service^[2], to Energy ITS – a Japanese platooning project^[3], and then to current GCDC/SARTRE/SCANIA – three European platooning projects^[4-6]. The design of vehicle platooning system mainly requires the integrations of spacing policy, information flow and control scheme. The desired safety spacing that the controlled vehicle is expected to keep from its preceding one is called the spacing policy, which typically chooses the velocity of the controlled vehicle. It also can choose a constant or other variable as the control object^[7]. The most common spacing policy uses a constant distance or time headway. When using the constant distance policy, the distance between the inter and controlled vehicles is independent, and applying the single predecessor information may not ensure the string stability^[8]. The term “string stability” is a property whereby the velocities/positions of the controlled vehicles will not be impacted by the fluctuations of the velocity/position of the leading vehicle^[9, 10]. This property is attained through the information flow of the leading vehicle in the constant distance (CD) policy or constant time headway (CTH) policy, where the inter-vehicle communication is utilized to get accurate velocity/position information^[8, 11]. Although the CTH that applies the information flow of the leading vehicle may guarantee the string stability, the string stability is only achieved in small or medium platoon^[12, 13]. If the predecessor-successor information flow is used, the velocities/positions of both the preceding and following vehicles can be employed to achieve the string stability in large platoon^[14-16].

A platooning control system is mainly dependent on the control scheme. The popular model-based control scheme combines the nonlinear vehicle model with various control laws such as PID^[13, 17], sliding mode^[13, 18], adaptive control^[19], linear optimal control^[20], and H ∞ control^[21] to provide expected command tracking performance and string stability. However, some problems still remain with the model-based scheme. Firstly, its control laws are usually obtained by means of strict linearization of the vehicle models and normalization of their input-output behavior to simplify the platoon complexity. Secondly, in order to make sure the string stability, it is well satisfied that the difference of the transfer functions

between the preceding vehicle and controlled one with respect to the H_∞/H_2 norm is less than one, but the specific hints or restrictions on the norms for the platooning system are hard to be understood or convinced [7, 22].

Nowadays, the fuzzy logic has been broadly used for control of longitudinal and lateral vehicle dynamics [23, 24]. It has also been used for following and lane-change maneuvers of vehicles [25, 26]. However, it often builds a fuzzy logic system with the help of prior expert knowledge to define the structure and rules of the fuzzy controller and determine the control parameters. Nevertheless, one laborious task for the fuzzy controller is to minimize the fuzzy rules and reduce the computation effort. To facilitate the design of a platoon controller, a variable structure control approach is proposed based on a hierarchical fuzzy logic system. In this new approach, the spacing policies of both constant distance and constant time headway (CTH) have been adopted with predecessor-successor information flow to control the vehicle platooning. ‘‘Hierarchical’’ herein denotes that the output of one fuzzy controller is treated as the input of another fuzzy controller. The advantage of the hierarchical fuzzy structure is that the fuzzy rules can increase linearly to the input variables, and hence, the fuzzy rule sets can be significantly cut down [27]. ‘‘Variable structure control’’ herein can compensate for the approximate errors of the fuzzy system by varying the sliding surface. Moreover, by means of leader-follower consensus control, a Lyapunov method with exponential stability is used to develop performance boundaries to determine the controller parameters and to ensure the control stability of both individual vehicle and platoon.

The reminders of this paper are organized as follows. Section II describes the mathematical model of a vehicle longitudinal model. The hierarchical fuzzy control approach is presented in Section III. The consensus issues of platoon stability are discussed in Section IV. In Section V, numerical testing results are presented to demonstrate the promising performance of the proposed approach and conclusions thereafter in Section VI.

II. VEHICLE MATHEMATICAL MODEL

This paper assumes horizontal road condition in the platooning control. The dynamic model of i^{th} vehicle is described by Eq. (1) [28, 29]

$$\ddot{x}_i = f_i(\dot{x}_i, \ddot{x}_i) + g_i(\dot{x}_i)u_i \quad (1)$$

where

$$\begin{cases} f_i(\dot{x}_i, \ddot{x}_i) = -\frac{1}{\tau_i(\dot{x}_i)}(\ddot{x}_i + \frac{k_{d,i}}{m_i}\dot{x}_i^2 + \frac{k_{m,i}}{m_i}) - \frac{2k_{d,i}}{m_i}\dot{x}_i\ddot{x}_i + \frac{\dot{d}_1(t)}{m_i} + \frac{d_2(t)}{m_i} \\ g_i(\dot{x}_i) = \frac{1}{m_i\tau_i(\dot{x}_i)} \end{cases}$$

and the model parameters are listed in Table I.

TABLE I
VEHICLE MODEL PARAMETERS

Parameters	Description
m/kg	total mass
τ/s	engine time constant
$k_d/\text{kg}\cdot\text{m}^{-1}$	aerodynamic drag parameter
k_m/N	mechanical drag
x/m	vehicle's position
$\dot{x}/\text{m}\cdot\text{s}^{-1}$	vehicle's velocity
$\ddot{x}/\text{m}\cdot\text{s}^{-2}$	vehicle's acceleration
d_1	external disturbance
d_2	engine transmission variations
u	the throttle/brake input

Let (x_i, x_{i-1}, x_{i+1}) , (v_i, v_{i-1}, v_{i+1}) and (a_i, a_{i-1}, a_{i+1}) be respectively the position, velocity and acceleration of the i^{th} , $(i-1)^{\text{th}}$ and $(i+1)^{\text{th}}$ vehicle. The spacing error δ_i for i^{th} vehicle is defined as

$$\delta_i = \zeta_i - S_{d,i} \quad (2)$$

where $\zeta_i (= x_{i-1} - x_i - L_i)$ denotes the actual spacing between i^{th} and $(i-1)^{\text{th}}$ vehicles, L_i denotes the $(i-1)^{\text{th}}$ vehicle length, $S_{d,i}$ denotes the desired safety distance. Fig. 1 depicts the relationship of the platoon.

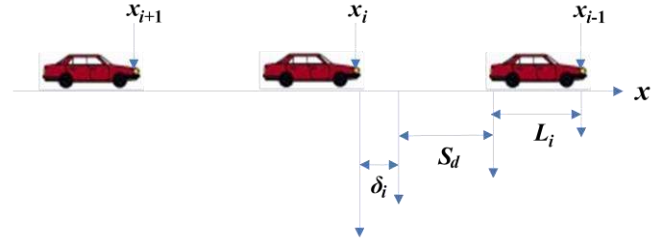


Fig. 1. Platoon configuration.

If the actual predecessor-successor spacing for i^{th} vehicle can be described as

$$\zeta_{\text{ps},i} = \zeta_i - \zeta_{i+1} = x_{i-1} - 2x_i + x_{i+1} \quad (3)$$

Then the desired safety distance $S_{d,i}$ [30] can be obtained for i^{th} vehicle by

$$S_{d,i} = h_1(v_i^2 - v_{i-1}^2) + h_2v_i + h_3 \quad (4)$$

where h_1, h_2, h_3 denote three positive constants, which can be calculated from the human reaction time, vehicle full acceleration and deceleration, and maximum allowable jerk.

When the vehicle platoon performs a steady state maneuver (i.e., v_i is close to v_{i-1}), the desired safety distance can be estimated by Eq. (5)

$$S_{d,i} = h_2v_i + h_3 \quad (5)$$

In Eq. (5), the desired safety distance integrates the policies of both constant distance spacing (h_3) and CTH (h_2v_i). As a result,

the two different spacing policies are combined for the platooning control in this paper. Different controllers will be designed for the vehicles under different scenarios using the predecessor-successor information flow to form a properly running vehicle platoon. It is first assumed that the immediate preceding and following vehicles contribute different effects on the i^{th} vehicle. Prior information about the desired distance, velocity and acceleration of the i^{th} vehicle can be obtained below.

(1) Desired distance:

$$d_{\text{des}} = t_{\text{des}}v_{i-1} + d_0$$

where t_{des} is a time constant, depending on sensor delays due to sampling, d_0 is a safe distance that ensure the anti-collision between two vehicles under some extreme stop situations.

(2) Desired velocity:

$$v_{\text{des}} = q_1v_{i-1} + q_2v_{i+1}$$

where q_1 and q_2 are respectively the designed motion weight factors for $(i-1)^{\text{th}}$ and $(i+1)^{\text{th}}$ vehicles and they are used for both velocity and acceleration.

(3) Desired acceleration:

$$a_{\text{des}} = q_1a_{i-1} + q_2a_{i+1}$$

According to the leader-follower consensus control, the state errors for all vehicles within the platoon in tracking the leader, caused by parameter uncertainties and external disturbances [31, 32], can be eventually reduced to be zero. Nevertheless, the influence of the initial conditions, the attraction domain and the convergence rate on the convergence behavior of the states has to be taken into account. Here, a Lyapunov method with exponential stability is used to develop performance boundaries to determine the controller parameters and to guarantee the control stability of both individual vehicle and platoon.

It should also be noted that, for the last vehicle with no following vehicles, a virtual vehicle is used as follower. Furthermore, the relative velocity/distance between the last vehicle and its virtual follower are assumed to be zero at any time.

The constant speed control is taken for the leading vehicle. Therefore, the control input $u_i(t)$ of the i^{th} vehicle is determined to achieve the following control objectives.

(1) Eliminate the spacing error, i.e.,

$$\lim_{t \rightarrow \infty} |\delta_i| = 0.$$

(2) Regulate the relative velocity of the preceding and following vehicles under the constant speed control of the leading vehicle, i.e.,

$$\lim_{t \rightarrow \infty} |\dot{x}_{i-1} - \dot{x}_i| = 0, \text{ for constant } \dot{x}_i.$$

(3) Control the acceleration of the following vehicle under the constant speed control of the leading vehicle, i.e.,

$$\lim_{t \rightarrow \infty} |\ddot{x}_i| = 0, \text{ for constant } \dot{x}_i.$$

(4) Ensure the platoon exponential stability, i.e.,

$$|\delta_i(t)| \leq |\delta_{i-1}(t)| + e_t,$$

where e_t is an exponentially vanishing term.

III. HIERARCHICAL FUZZY CONTROLLER

Fig. 2 describes a two-layer fuzzy-control structure. The first layer consists of spacing control and velocity/acceleration control, each employing two inputs. The spacing control employs the predecessor-successor spacing error $e_{ps,i}$, i.e., δ_i , and its changing rate $\dot{e}_{ps,i}$ as inputs. The velocity/acceleration control employs the velocity error $e_{v,i}$ (i.e., difference of the actual and reference velocities) and the acceleration error $e_{a,i}$ (i.e., difference of the actual and reference acceleration) as inputs. The second layer shown in the lower part of the Fig. 2 directly employs the outputs of the first layer, and uses the sliding mode reach law. The correction factors of different fuzzy base functions are tuned by the reach law to achieve an adaptive compensation for the approximate error caused by parameter uncertainties and disturbances. The control output in the second layer is the incremental value of u in Eq. (1).

In summary, the structure of the hierarchical fuzzy system includes:

Fuzzy sets: {**NB**: negative big, **NM**: negative medium, **NS**: negative small, **ZO**: zero, **PS**: positive small, **PM**: positive medium, **PB**: positive big}

Fuzzy domains of the hierarchical fuzzy logic system: {1, 2, 3, 4, 5, 6, 7}

Fuzzy rules:

IF E is A_1 and EC is A_2 , THEN U is B

The rule sets of the fuzzy controller in Table II are drawn based on the factors of existing driving experience (refer to [25, 33]) and control objectives. In addition, the rule sets is the same for the two-layer fuzzy controllers.

TABLE II
FUZZY CONTROL RULES

U		E						
		PB	PM	PS	ZO	NS	NM	NB
EC	PB	PB	PB	PB	PB	PM	PS	ZE
	PM	PB	PB	PM	PM	PS	ZE	NS
	PS	PB	PM	PM	PS	ZE	NS	NM
	ZO	PB	PM	PS	ZE	NS	NM	NB
	NS	PM	PS	ZE	NS	NM	NM	NB
	NM	PS	ZE	NS	NM	NM	NB	NB
	NB	ZE	NS	NM	NB	NB	NB	NB

An ordinary fuzzy control system usually applies a singleton fuzzifier, Mamdani inference, a center average defuzzification, and Gauss membership function (GMF).

$$u = f(e, ec) = \frac{\sum_{i=1}^k \theta_i (\mu_E^i(e) \mu_{EC}^i(ec))}{\sum_{i=1}^k \mu_E^i(e) \mu_{EC}^i(ec)} \quad (6)$$

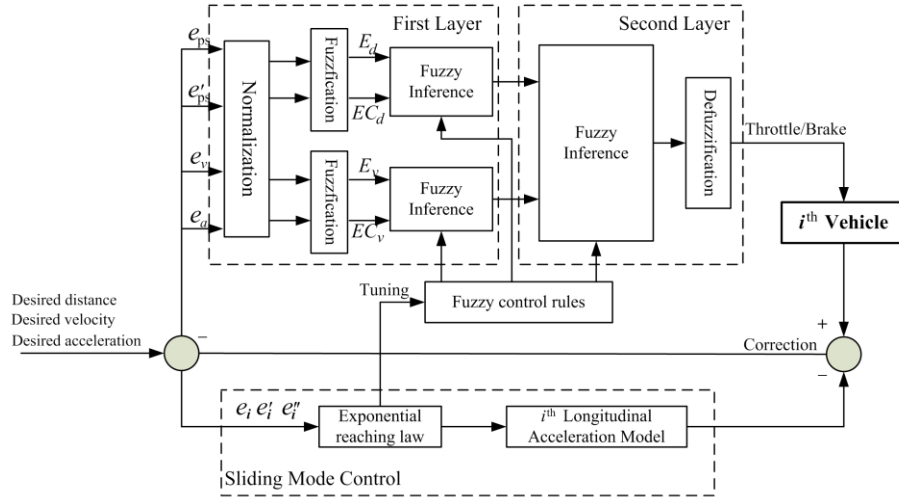


Fig. 2. Hierarchical Fuzzy control system.

where $\mu_E(e)$ and $\mu_{EC}(ec)$ are the GMF of the normal distributions describing fuzzy sets of the error and its changing rate. θ_i is the principal value of i^{th} fuzzy set of u , k is the sequential fuzzy sets, and $k = 1, 2, \dots, 7$.

Define the fuzzy base functions:

$$\varepsilon_i(e, ec) = \frac{\mu_E^i(e) \mu_{EC}^i(ec)}{\sum_{i=1}^k \mu_E^i(e) \mu_{EC}^i(ec)} \quad (7)$$

Obviously, we have

$$\sum_{i=1}^k \varepsilon_i(e, ec) = 1, \quad 0 \leq \varepsilon_i(e, ec) \leq 1$$

and $\varepsilon_i(e, ec)$ could be taken as the weight of the i^{th} rule. Thus, Eq. (6) can be rewritten by Eq. (8).

$$f(e, ec) = \sum_{i=1}^k \theta_i \varepsilon_i(e, ec) \quad (8)$$

Where, ε_i is a group of fuzzy base functions.

Compared with ordinary fuzzy expressions above, the hierarchical fuzzy controller is significantly different. For the sake of better expression and application, we can make some transformation about the ordinary fuzzy structure. Firstly, θ_j is treated as a correction factor of j^{th} fuzzy rule at the first fuzzy layer. Although the outputs of first fuzzy layer need not to be defuzzified and can be directly used for fuzzy reference by the second layer controller, they cannot be employed by the second layer controller until they have merged with each other just as the normal distribution describing the fuzzy set of U .

As an example of controller weights, the weight of the spacing controller output belonging to one of the fuzzy sets $\{\mathbf{PB}\}$ is

$$\mu_{ps,i}^{pb} = \sum_{U_{ps,i} \subset \{PB\}} \theta_{ps,i,j} \varepsilon_{ps,i,j}$$

Thus, the expression of the controller output is

$$u_{ps,i} = \frac{\mu_{ps,i}^{pb}}{PB} + \frac{\mu_{ps,i}^{pm}}{PM} + \frac{\mu_{ps,i}^{ps}}{PS} + \frac{\mu_{ps,i}^{zo}}{ZO} + \frac{\mu_{ps,i}^{ns}}{NS} + \frac{\mu_{ps,i}^{nm}}{NM} + \frac{\mu_{ps,i}^{nb}}{NB}$$

Similarly, the weight of the velocity controller belonging to $\{\mathbf{PB}\}$ is

$$\mu_{v,i}^{pb} = \sum_{U_{v,i} \subset \{PB\}} \theta_{v,i,j} \varepsilon_{v,i,j}$$

and the expression of the velocity controller output is

$$u_{v,i} = \frac{\mu_{v,i}^{pb}}{PB} + \frac{\mu_{v,i}^{pm}}{PM} + \frac{\mu_{v,i}^{ps}}{PS} + \frac{\mu_{v,i}^{zo}}{ZO} + \frac{\mu_{v,i}^{ns}}{NS} + \frac{\mu_{v,i}^{nm}}{NM} + \frac{\mu_{v,i}^{nb}}{NB}$$

Then, the second fuzzy layer can be described as follows:

$$\begin{cases} u_{fz,i} = \sum_{j=1}^k d_{fz,i,j} (\theta_{fz,i,j} \varepsilon_{fz,i,j}) \\ \varepsilon_{fz,i,j} = \min\{\mu_{i,j}(ps), \mu_{i,j}(v)\} \end{cases} \quad (9)$$

where $\theta_{ps,i,j}$, $\theta_{v,i,j}$ and $\theta_{fz,i,j}$ are the correction factors of j^{th} fuzzy rule for the spacing, velocity and second fuzzy layer controller, respectively; $\varepsilon_{ps,i,j}$, $\varepsilon_{v,i,j}$ and $\varepsilon_{fz,i,j}$ are the weights of j^{th} fuzzy rule for the spacing, velocity and second fuzzy layer controller, respectively; $d_{fz,i,j}$ is the center value of the output membership function which is depended by the j^{th} fuzzy rule for the i^{th} vehicle; $\mu_{i,j}(ps)$ and $\mu_{i,j}(v)$ are position and velocity values of the input membership functions of the second fuzzy layer.

As the fuzzy control has a universal approximation performance, it can easily deal with the uncertainties and disturbances of the platooning system. Meanwhile, we use the

sliding mode reach law that tunes the correction factors of different fuzzy base functions to achieve an adaptive compensation for the approximate error.

Define a sliding surface as

$$S_i = c_{i,1}e_{ps,i} + c_{i,2}e_{v,i} + e_{a,i} \quad (10)$$

where $c_{i,1}, c_{i,2} > 0$ and the sliding surface satisfies the following sliding mode reaching conditions

$$\dot{S}_i = -\lambda S_i, \quad \lambda > 0 \quad (11)$$

We can obtain

$$\dot{e}_{a,i} + (\lambda + c_{i,2})e_{a,i} + (\lambda c_{i,2} + c_{i,1})e_{v,i} + \lambda c_{i,1}e_{ps,i} = 0 \quad (12)$$

According to (1), (10), (12), an ideal tracking controller is obtained as follows

$$u_i^* = \frac{1}{g_i(\dot{x}_i)} \left[-f_i(\dot{x}_i, \ddot{x}_i) + \lambda S_i + (c_{i,1}e_{v,i} + c_{i,2}e_{a,i}) + \dot{a}_{i,des} \right] \quad (13)$$

However, since $f_i(\dot{x}_i, \ddot{x}_i)$, $g_i(\dot{x}_i)$ are partially or completely unknown because of time varying vehicle parameters and disturbances, the ideal tracking controller cannot be obtained. Thus, the proposed fuzzy base function controller $u_{fz,i}$ is adopted to estimate the ideal tracking controller in Eq. (13). In the proposed fuzzy controller, the following optimal weight vectors are used to meet the desired tracking performance

$$\begin{aligned} & \{ \theta_{ps,i}^*, \theta_{v,i}^*, \theta_{fz,i}^* \} \\ & = \arg \min_{\theta_{ps,i} \in \Omega_{\theta_{ps,i}}, \theta_{v,i} \in \Omega_{\theta_{v,i}}, \theta_{fz,i} \in \Omega_{\theta_{fz,i}}} \left[\sup |u_i^* - u_{fz,i}| \right] \end{aligned}$$

where $\Omega_{\theta_{ps,i}}, \Omega_{\theta_{v,i}}, \Omega_{\theta_{fz,i}}$ are the feasible regions of the weight vector $\{ \theta_{ps,i}, \theta_{v,i}, \theta_{fz,i} \}$, respectively.

Then, define an approximation error as

$$\omega_i^* = u_{fz,i}^* - u_i^* \quad (14)$$

where $u_{fz,i}^*$ is the optimal controller for the fuzzy base function networks.

Substituting (14) into (12), we have

$$\begin{aligned} \dot{e}_{a,i} &= -(\lambda S_i + c_{i,1}e_{v,i} + c_{i,2}e_{a,i}) \\ &+ g_i(\dot{x}_i)(u_{fz,i}^* - u_{fz,i}) + g_i(\dot{x}_i)(u_i^* - u_{fz,i}^*) \\ &= -(\lambda S_i + c_{i,1}e_{v,i} + c_{i,2}e_{a,i}) + g_i(\dot{x}_i)\varphi_i - g_i(\dot{x}_i)\omega_i^* \end{aligned} \quad (15)$$

where

$$\varphi_i = u_{fz,i}^* - u_{fz,i}$$

If φ_i and ω_i^* have boundaries, the $\dot{e}_{a,i}$ also has boundary.

According to the convergence behavior of exponential stability, the parameter correction of the hierarchical fuzzy structure is not only involved with the distance between the initial position of the state errors and sliding surface, but also the speed that the state errors reach the sliding surface. The adaptive law in Eq. (16) can therefore be used to tune the weight vector.

$$\begin{cases} \tilde{\theta}_{ps,i} = \eta_{ps,i}(\dot{S}_i + \lambda S_i)\varepsilon_{ps,i} \\ \tilde{\theta}_{v,i} = \eta_{v,i}(\dot{S}_i + \lambda S_i)\varepsilon_{v,i} \\ \tilde{\theta}_{fz,i} = \eta_{fz,i}(\dot{S}_i + \lambda S_i)\varepsilon_{fz,i} \end{cases} \quad (16)$$

where $\eta_{ps,i}, \eta_{v,i}, \eta_{fz,i}$ are adaptive parameters.

Finally, state errors of the i^{th} vehicle caused by uncertainties and disturbances can be further reduced by designing an adaptive compensation controller as follows

$$u_i = u_{fz,i} + k_{i,c}(\dot{S}_i + \lambda S_i) \quad (17)$$

where $k_{i,c}$ is the compensation factor.

IV. PLATOON STABILITY

According to the property of string stability, the platooning system should guarantee the string stability to achieve a stable platooning control for the moving vehicles. Hence, the string stability in the platooning control is referred as platoon stability. Since the stability of individual vehicles cannot ensure the platoon stability, it is important to design proper platooning controller to guarantee its stability.

A. Individual vehicle stability

Considering the fact that the parameters and disturbances of the vehicle mathematical model described in Eq. (1) are usually unknown, it is possible to define their boundaries in the platooning controller design.

$$\begin{cases} m_i \leq M \\ k_{d,i} \leq K_D \\ k_{m,i} \leq K_M \\ \tau_i(\dot{x}_i) \leq \Gamma \\ |d_1(t)| \leq D_1, \quad |d_2(t)| \leq D_2 \end{cases} \quad (18)$$

Where, M, K_D, K_M, Γ, D_1 , and D_2 are positive constants.

Let further assume the following constrains

$$0 < b_1 \leq \frac{m_i \tau_i(\dot{x}_i)}{h_2} \leq b_2 \quad (19)$$

Where, b_1 is an unknown positive constant and b_2 is a known positive constant. If the vehicle's jerk has an upper boundary A , i.e.,

$$|\ddot{x}_i| \leq A \quad (20)$$

Then, the following Lyapunov-like function can be used as the design base for the platooning controller.

$$V_1 = \frac{1}{2} \frac{m_i \tau_i(\dot{x}_i)}{h_2} S_i^2 \quad (21)$$

and the derivative of V_1 can be found by substituting (1) and (10) into (21) as follows.

$$\dot{V}_1 = \frac{m_i \tau_i(\dot{x}_i)}{h_2} S_i (\dot{a}_{i,\text{des}} - \ddot{x}_i + c_{i,1} \dot{e}_{\text{ps},i} + c_{i,2} \dot{e}_{\text{v},i})$$

Taking the boundaries in Eqs. (11) and (15) and the control law u_i into accounts, we have:

$$\dot{V}_1 \leq -\frac{1}{2} \frac{kM\Gamma}{h_2} S_i^2 \leq -k \frac{1}{2} \frac{m_i \tau_i(\dot{x}_i)}{h_2} S_i^2 = -kV_1(t)$$

where

$$k > \max \left\{ \frac{2(1+h_2)}{h_2}, k_{i,c} \right\}$$

As a result, an exponential convergence of $S_i(t)$ can be derived by

$$\frac{b_1}{2} S_i^2(t) \leq V_1(t) \leq V_1(0) e^{-kt}, \quad \forall t \in [0, \infty)$$

It is concluded from the above analysis that the vehicle stability is achieved based on the obtained exponential convergence.

B. Platoon stability

The platoon string stability is analyzed in this Section. The L_2 sense^[15-18] is widely used in literature to evaluate the platoon stability by ensuring convergence in the disturbances that transfer from the leading vehicle to its followers. Eq. (22) depicts the evaluation basis.

$$\|\tilde{y}\|_2 \leq \|G\|_\infty \|\tilde{u}\|_2 \quad (22)$$

where \tilde{u} and \tilde{y} denote the input and output of a control system G .

In the platooning control, the spacing errors of i^{th} and $(i-1)^{\text{th}}$ vehicles can be related to

$$\delta_i = F \delta_{i-1}$$

where F is a linear operator.

Here, if $\|F\|_\infty \leq 1$ or $|F(j\omega)| \leq 1, \forall \omega > 0$, then $\|\delta_i\|_2 \leq \|\delta_{i-1}\|_2$, which means that the disturbances propagated upstream along the platoon are not magnified in L_2 sense if the

spacing error of i^{th} vehicle is less than or equal to that of $(i-1)^{\text{th}}$ vehicle.

However, under this circumstance, we can only obtain

$$\int_0^\infty |\delta_i|^2 dt \leq \int_0^\infty |\delta_{i-1}|^2 dt$$

It will be much appreciated if the spacing error of i^{th} vehicle is less than that of $(i-1)^{\text{th}}$ vehicle at any time except a vanishing term. In order to guarantee the platoon stability, the constrain described in Eq. (23) should be satisfied.

$$|\delta_i(t)| \leq |\delta_{i-1}(t)| + e_t \quad (23)$$

where e_t is a vanishing term.

First, we define another sliding surface as

$$S_i = c \delta_i + \dot{\delta}_i$$

where $c > 0$, satisfying

$$c = \frac{1+h_2}{h_2} + \frac{1}{2k_{i,c}}$$

Since

$$\begin{aligned} \dot{\delta}_{i-1} &= -c \delta_{i-1} + S_{i-1} \\ \dot{\delta}_i &= -c \delta_i + S_i \end{aligned}$$

Define

$$z = \delta_i - \delta_{i-1}$$

we have

$$\dot{z} = -cz + S_i - S_{i-1} \quad (24)$$

Thus, Eq. (24) demonstrates the stable dynamics with input term of $(S_i - S_{i-1})$, whose boundary will eventually converge to zero.

To determine the boundary of z , we define the following positive definite function:

$$V_2 = \frac{1}{2} z^2 + \frac{1}{2} \frac{V_i(0)}{b_1} e^{-k_1 t} + \frac{1}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \quad (25)$$

where

$$\begin{aligned} V_i(t) &= \frac{1}{2} \frac{m_i \tau_i(\dot{x}_i)}{h_2} S_i^2(t) \\ V_{i-1}(t) &= \frac{1}{2} \frac{m_{i-1} \tau_{i-1}(\dot{x}_{i-1})}{h_2} S_{i-1}^2(t) \end{aligned}$$

Then the time derivative of V_2 according to Eq. (24) is

$$\begin{aligned}
\dot{V}_2 &= z[-cz + S_i(t) - S_{i-1}(t)] \\
&\quad - \frac{k_i}{2} \frac{V_i(0)}{b_1} e^{-k_i t} - \frac{k_{i-1}}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \\
&\leq -cz^2 + \frac{1}{2} z^2 + S_i^2(t) + \frac{1}{2} z^2 + S_{i-1}^2(t) \\
&\quad - \frac{k_i}{2} \frac{V_i(0)}{b_1} e^{-k_i t} - \frac{k_{i-1}}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \\
&= (-c+1)z^2 + \frac{1}{2} \frac{V_i(0)}{b_1} e^{-k_i t} - \frac{k_i}{2} \frac{V_i(0)}{b_1} e^{-k_i t} \\
&\quad + \frac{1}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} - \frac{k_{i-1}}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t}
\end{aligned}$$

where we denote

$$\begin{aligned}
1 - k_i &= -k_{i,1} & k_{i,1} &> 0 \\
1 - k_{i-1} &= -k_{(i-1),1} & k_{(i-1),1} &> 0
\end{aligned}$$

Thus,

$$\begin{aligned}
\dot{V}_2 &\leq (1-c)z^2 - k_{i,1} \frac{1}{2} \frac{V_i(0)}{b_1} e^{-k_i t} - k_{(i-1),1} \frac{1}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \\
&= -lV_2(t)
\end{aligned}$$

Since $c = (1 + h_2) / h_2 + 1 / 2k_{i,c}$, $c - 1 > 0$, we have

$$\dot{V}_2 = -lV_2(t)$$

where $l = \min\{2(1-c), k_{i,1}, k_{(i-1),1}\}$ which implies

$$V_2 \leq V_2(0)e^{-lt}$$

or

$$\begin{aligned}
&\frac{1}{2} z^2(t) + \frac{1}{2} \frac{V_i(0)}{b_1} e^{-k_i t} + \frac{1}{2} \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \\
&\leq \left[\frac{1}{2} z^2(0) + \frac{1}{2} \frac{V_i(0)}{b_1} + \frac{1}{2} \frac{V_{i-1}(0)}{b_1} \right] e^{-lt}
\end{aligned}$$

Therefore,

$$\begin{aligned}
&|\delta_i(t) - \delta_{i-1}(t)| \\
&\leq \left\{ \left[\frac{1}{2} (\delta_i(0) - \delta_{i-1}(0))^2 + \frac{1}{2} \frac{V_i(0)}{b_1} + \frac{1}{2} \frac{V_{i-1}(0)}{b_1} \right] e^{-lt} \right. \\
&\quad \left. - \frac{V_i(0)}{b_1} e^{-k_i t} - \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \right\}^{1/2}
\end{aligned} \quad (26)$$

Again,

$$\begin{aligned}
|\delta_i(t)| &\leq |\delta_{i-1}(t)| \\
&+ \left\{ \left[\frac{1}{2} (\delta_i(0) - \delta_{i-1}(0))^2 + \frac{1}{2} \frac{V_i(0)}{b_1} + \frac{1}{2} \frac{V_{i-1}(0)}{b_1} \right] e^{-lt} \right. \\
&\quad \left. - \frac{V_i(0)}{b_1} e^{-k_i t} - \frac{V_{i-1}(0)}{b_1} e^{-k_{i-1} t} \right\}^{1/2}
\end{aligned}$$

and hence, the platoon stability constrained by Eq. (23) is automatically satisfied if the exponential stability of the spacing error is achieved.

V. SIMULATION RESULTS

In this paper a series of simulation tests are carried out by control a platoon of 10 vehicles. Table III lists the parameters in the tests.

TABLE III
VEHICLE AND CONTROL PARAMETERS

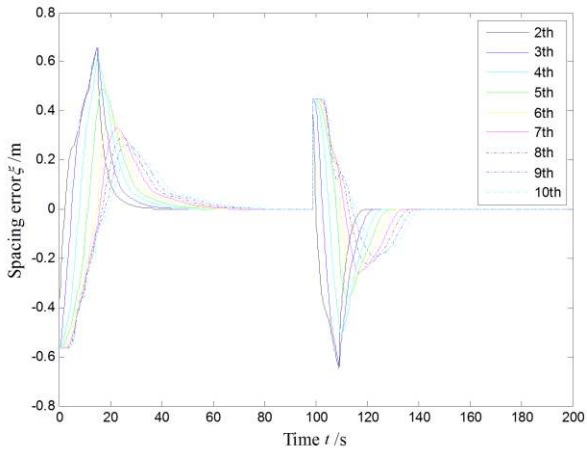
parameters	value	parameters	value
m	1500	q_2	0.2
k_d	0.3	λ	0.6
k_m	140	c_1	2
τ	0.2	c_2	3
t_{des}	0.02	η_{ps}	0.8
L	5	η_v	0.3
d_0	7	η_{fc}	0.1
q_1	0.8	k_c	0.1

In the first simulation scenario, the spacing policy of CTH that employs the predecessor-successor information flow from both the immediate predecessor and follower of the controlled vehicle is employed. Each vehicle in the platoon is identical and has the same parameters in Table III, and the disturbance is ignored in each vehicle. The platoon leader firstly accelerates from 0 to 30 m/s at 2 m/s². After the velocity reaches 30 m/s, the leader begins to decelerate to 10 m/s at -2 m/s². Fig. 3 describes simulation results. It can observe expected velocity tracking ability of the proposed hierarchical approach in the figure. Meanwhile, the platoon stability can also be observed.

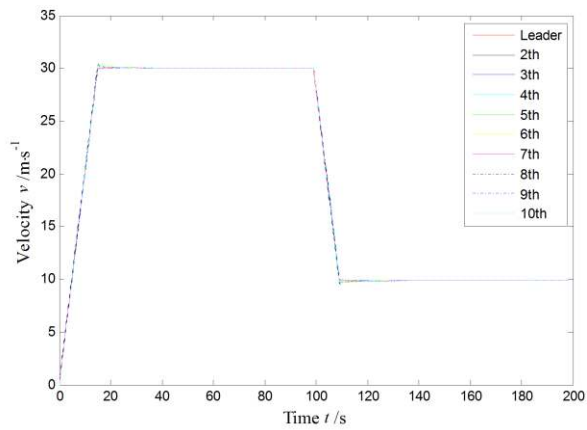
In the second simulation scenario, the CTH is used again. Additionally, to evaluate the platoon stability in the presence of model uncertainties and disturbance, different vehicle parameters ($m_i, k_{d,i}, k_{m,i}, \tau_i$) are adopted in the following vehicles, and the disturbance in the simulations is expressed as

$$d_i(t) = \begin{cases} 0 & t < 0 \\ 400[1 - e^{-0.2t}] & 0 \leq t \leq 36 \\ 0 & 36 \leq t \leq 99 \\ 400[1 - e^{-0.2(t-99)}] & 99 \leq t \leq 135 \\ 0 & t > 135 \end{cases} \quad (27)$$

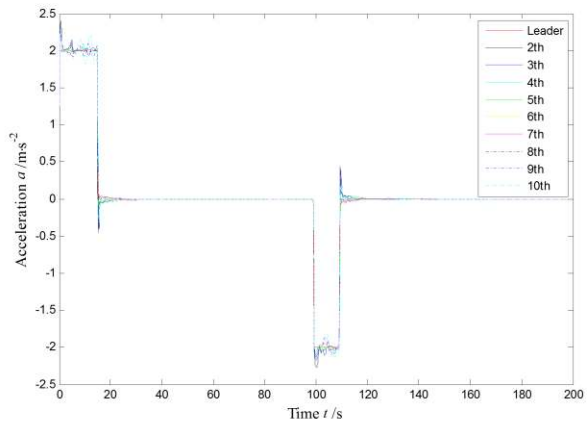
where the subscript i is the driving conditions. The accelerating, decelerating, and disturbance is shown in Fig. 4.



(a) Spacing error profiles



(b) Velocity profiles



(c) Acceleration profiles

Fig. 3. A ten-vehicle platooning simulation with CTH spacing policy.

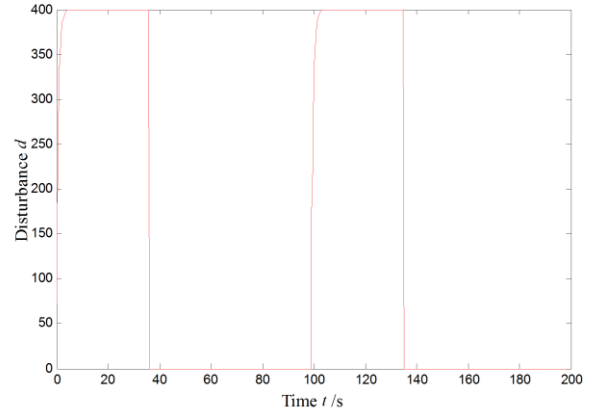
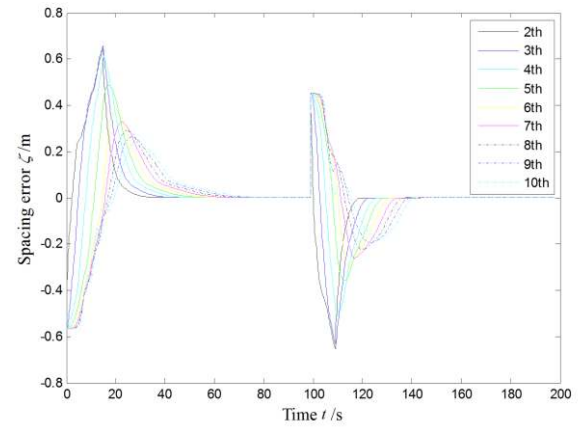
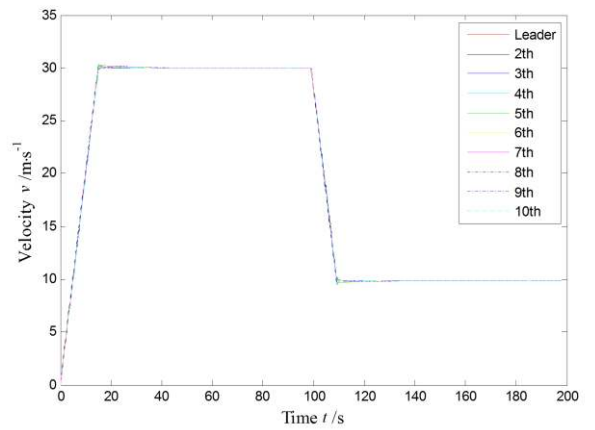


Fig. 4. Schematic diagram of the external disturbance.

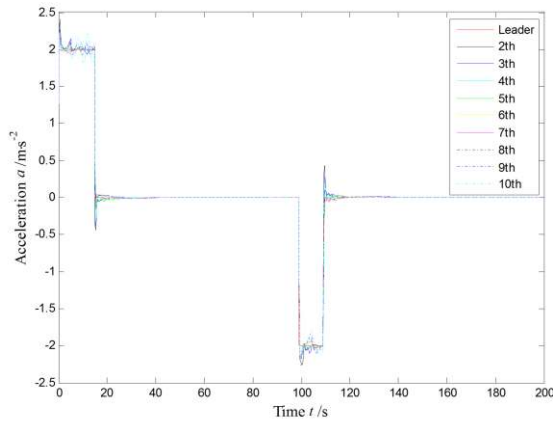
The simulation results under the external disturbance are shown in Fig. 5.



(a) Spacing error profiles



(b) Velocity profiles



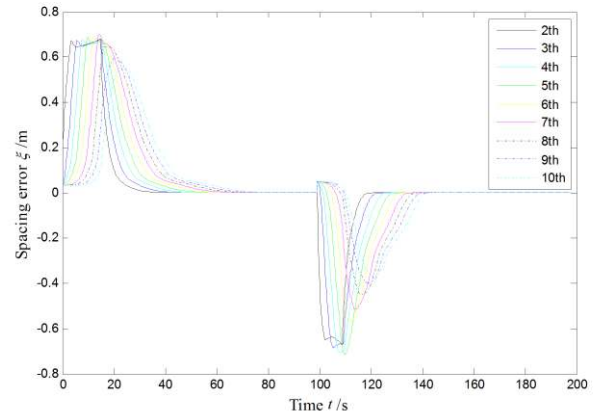
(c) Acceleration profiles

Fig. 5. A ten vehicle platooning simulation with CTH under the external disturbance.

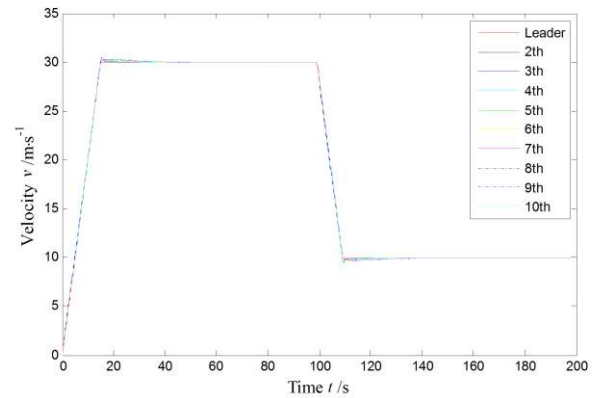
It observes from Fig. 5 that even exposed to model uncertainties and external disturbances, CTH can still guarantee the platoon stability by the proposed hierarchical approach.

In the third scenario, the constant distance (CD) spacing policy is used that employs the predecessor-successor information flow from both immediate predecessor and follower of the controlled vehicle. The vehicles in the platoon are the same with the absence of disturbance. The platoon leader accelerates from 0 to 30 m/s at 2 m/s². After the leader's velocity reaches 30 m/s, it begins to decelerate to 10 m/s at -2 m/s². Fig. 6 depicts the analysis results. Again, the velocity tracking ability and platoon stability of the proposed hierarchical approach is achieved.

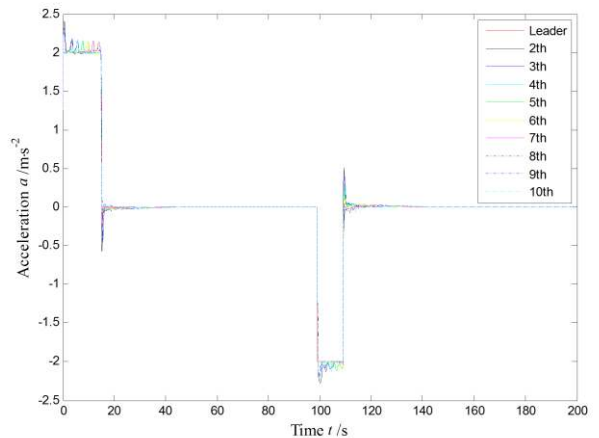
In the last simulation scenario, the CD spacing policy and model uncertainties/external disturbance (see Eq. (27)) are used. Fig. 7 shows the control results.



(a) Spacing error profiles

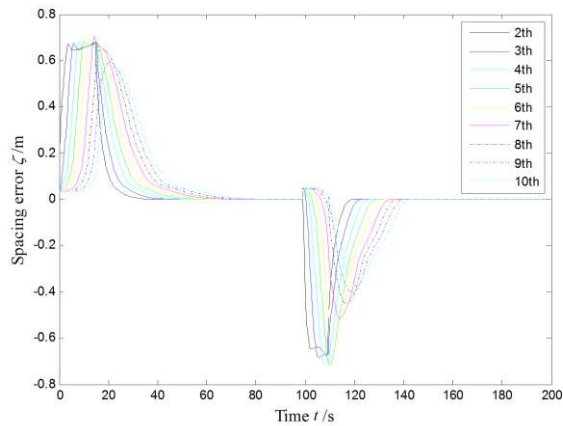


(b) Velocity profiles

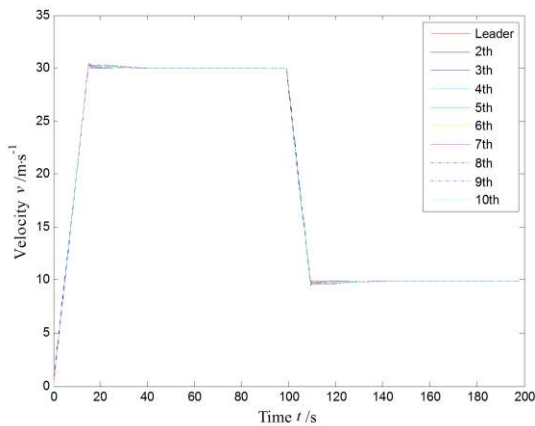


(c) Acceleration profiles

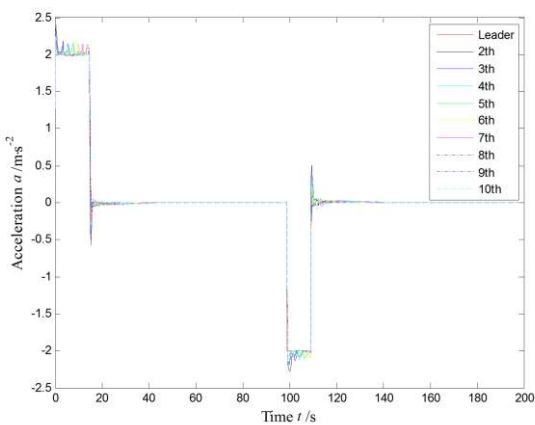
Fig. 6. A ten vehicle platooning simulation with CD spacing policy.



(a) Spacing error profiles



(b) Velocity profiles



(c) Acceleration profiles

Fig. 7. A ten vehicle platooning simulation with CD under the external disturbance.

It is found from Fig. 7 that the CD spacing policy can achieve the platoon stability under the scenario of model uncertainties and disturbances.

VI. CONCLUSIONS AND FUTURE WORK

A practical hierarchical fuzzy controller is presented that employs the predecessor-successor information flow from their immediate predecessor and follower other than both the controlled vehicles. It is composed of a tracking controller at the first fuzzy layer and a compensation controller at the second

fuzzy layer, which can deal with parameter uncertainties and disturbances well. It has clearly revealed that the proposed controller applies not only the CTH spacing policy but also the CD policy to guarantee the stability of individual vehicles and platoon in the presence of uncertainties and disturbances. Because the controller design just relies on “local” measurement by on-board sensors, it is more convenient for the next field operational tests of vehicles platooning.

REFERENCES

- [1] R. Rajamani, H. S. Tan, B. K. Law and W. B. Zhang. “Demonstration of Integrated Longitudinal and Lateral Control for the Operation of Automated Vehicles in Platoons,” *IEEE Transactions on Control Systems Technology*, vol. 8, no. 4, pp. 695-708, 2000.
- [2] S. Kato, S. Tsugawa, K. Tokuda, T. Matsui and H. Fujii. “Vehicle Control Algorithms for Cooperative Driving with Automated Vehicles and Intervehicle Communications,” *IEEE Transactions on Control Systems Technology*, vol. 3, no. 3, pp. 155-161, 2002.
- [3] S. Tsugawa, S. Kato and K. Aoki. “An Automated Truck Platoon for Energy Saving,” in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2011, pp. 4109-4114.
- [4] T. H. A. van den Broek, B. D. Netten, M. Hoedemaeker and J. Ploeg. “The Experimental Setup of a Large Field Operational Test for Cooperative Driving Vehicles at the A270,” in *Proceedings of 13th IEEE International Conference on Intelligent Transportation Systems*, 2010, pp. 198-203.
- [5] E. Chan, P. Gilhead, P. Jelinek, P. Krejčí and T. Robinson. “Cooperative Control of SARTRE Automated Platoon Vehicles,” in *Proceedings of 19th ITS World Congress*, 2012, pp. 1-9.
- [6] A. Alam, J. M^oartensson and K. H. Johansson. “Look-Ahead Cruise Control for Heavy Duty Vehicle Platooning,” in *Proceedings of 16th IEEE International Conference on Intelligent Transportation Systems*, 2013, pp. 928-935.
- [7] K. Santhanakrishnan and R. Rajamani. “On Spacing Policies for Highway Vehicle Automation,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 4, no. 4, pp. 198-204, 2003.
- [8] J. Zhou and H. Peng. “Range Policy of Adaptive Cruise Control Vehicle for Improved Flow Stability and String Stability,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 6, no. 2, pp. 229-237, 2005.
- [9] H. G. Tanner, G. J. Pappas and V. Kumar. “Leader-to-formation stability,” *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 443-455, 2004.
- [10] L. Y. Xiao and F. Gao. “Practical String Stability of Platoon of Adaptive Cruise Control Vehicles,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 12, no. 4, pp. 1184-1194, 2011.
- [11] S. Darbha and K. Rajagopal. “A Review of Constant Time Headway Policy for Automatic Vehicle Following,” in *Proceedings of IEEE International Conference on Intelligent Transportation Systems*, 2001, pp. 65-69.
- [12] S. Huang and W. Ren. “Design of Vehicle Following Control Systems with Actuator Delays,” *International Journal of Systems Science*, vol. 28, no.2, pp. 145-151, 1997.
- [13] L. Y. Xiao, F. Gao and J. F. Wang. “On Scalability of Platoon of Automated Vehicles for Leader-Predecessor Information Framework,” in *Proceedings of IEEE Intelligent Vehicles Symposium*, 2009, pp. 1103-1108.
- [14] P. Seiler, A. Pant and K. Hedrick. “Disturbance Propagation in Vehicle Strings,” *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1835-1841, 2004.
- [15] L. Y. Xiao and F. Gao. “On String Stable Control of Platoon of Automated Vehicles with Predecessor-successor Information Framework,” *Chinese Journal of Mechanical Engineering*, vol. 23, no. 5, pp. 627-634, 2010.
- [16] D. B. Ren, J. M. Zhang, S. M. Cui and J. Y. Zhang. “Vehicle Following Control Using Front and Back Information Based on Vector Lyapunov Function,” *Journal of Central South University(Science and Technology)*, vol. 41, no. 6, pp. 2195-2220.
- [17] P. Xavier and Y. J. Pan. “A Practical PID-based Scheme for the Collaborative Driving of Automated Vehicles,” in *Proceedings of Joint*

48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, 2009, pp. 967-971.

- [18] Y. J. Pan. "Decentralized Robust Control Approach for Coordinated Maneuvering of Vehicles in Platoons," IEEE Transactions on Intelligent Transportation Systems, vol. 10, no. 2, pp. 346-354, 2009.
- [19] D. Swaroop, J. K. Hedrick and S. B. Choi. "Direct Adaptive Longitudinal Control of Vehicle Platoons," IEEE Transactions on Vehicular Technology, vol. 50, no. 1, pp. 150-161, 2001.
- [20] G. Guo and W. Yue. "Hierarchical Platoon Control with Heterogeneous Information Feedback," IET Control Theory and Applications, vol. 5, no. 15, pp. 1766-1781, 2011.
- [21] Y. Yamamura, Y. Seto, H. Nishira and T. Kawabe. "An ACC Design Method for Achieving Both String Stability and Ride Comfort," Journal of System Design and Dynamics, vol. 2, no. 4, pp. 979-990, 2008.
- [22] E. Shaw and J. K. Hedrick. "String Stability Analysis for Heterogeneous Vehicle Strings," in Proceedings of American Control Conference, 2007, pp. 3118-3125.
- [23] R. Muller and G. Nocker. "Intelligent cruise control with fuzzy logic," in Proceedings of IEEE Intelligent Vehicles Symposium, 1992, pp. 173-178.
- [24] T. Hessburg and M. Tomizuka. "Fuzzy Logic Control for Lateral Vehicle Guidance," in Proceedings of IEEE International Conference on Control Applications, 1993, pp. 13-16.
- [25] G. D. Lee and S. W. Kim. "A Longitudinal Control System for a Platoon of Vehicles Using a Fuzzy Sliding Mode Algorithm," Mechatronics, vol. 12, no. 1, pp. 97-118, 2002.
- [26] José E. Naranjo, C. González, R. García and T. de Pedro. "Lane-Change Fuzzy Control in Autonomous Vehicles for the Overtaking Maneuver," IEEE Transactions on Intelligent Transportation Systems, vol. 9, no. 3, pp. 438-450, 2008.
- [27] G. V. S. RAJU, J. ZHOU and R. A. KISNER. "Hierarchical Fuzzy Control," International Journal of Control, vol. 54, no. 5, pp. 1201-1216, 1991.
- [28] A. Stotsky, C. C. Chien and P. Ioannou. "Robust Platoon-stable Controller Design for Autonomous Intelligent Vehicles," Mathematical and computer modelling, vol. 22, no. 4-7, pp. 287-303, 1995.
- [29] Y. L. Ma, Q. Wu, R. Zhang, L. Ye and X. P. Yan. "Speed Tracking Control for a Simulated Intelligent Vehicle Platoon under Variable Road Conditions," Journal of Mechanical Engineering, vol. 47, no. 7, pp. 140-147, 2011.
- [30] P. Ioannou and C. C. Chien. "Autonomous Intelligent Cruise Control," IEEE Transactions on Vehicular Technology, vol. 42, no. 4, pp. 657-672, 1993.
- [31] U. Munz, A. Papachristodoulou, and F. Allgower. "Robust Consensus Controller Design for Nonlinear Relative Degree Two Multi-Agent Systems with Communication Constraints," IEEE Transactions on Automatic Control, vol. 56, no. 1, pp. 145-151, 2011.
- [32] W. W. Yu, G. R. Chen, and M. Cao. "Consensus in Directed Networks of Agents With Nonlinear Dynamics," IEEE Transactions on Automatic Control, vol. 56, no. 6, pp. 1436-1441, 2011.
- [33] Q. Wu, Z. W. He, X. M. Chu and D. Lu. "An Application of the Adaptive Fuzzy Control in the Longitudinal Control of the Platoon," in Proceedings of IEEE Pacific-Asia Workshop on Computational Intelligence and Industrial Application, 2008, pp. 344-348.



Dr. Yulin Ma (M'13) received his PhD in Transportation Engineering from Wuhan University of Technology, China. He was a Post Doctor with Academy of Military Transportation, China. Currently he is an Associate Professor with National Center of ITS Engineering and Technology, Research Institute of Highway, Ministry of Transport, China. His research interests include

intelligent vehicles and intelligent transportation systems. He is currently a reviewer for the IEEE Transactions on Intelligent Transportation Systems, International Journal of Intelligent Transportation Systems Research.



Dr. Zhixiong Li (M'16) received his PhD in Transportation Engineering from Wuhan University of Technology, China. Currently he is a Senior Lecture with China University of Mining and Technology, China, and a research associate in Department of Mechanical Engineering, Iowa State University, USA. His research interests include mechanical system modeling and control. He is an associate editor for the Journal of IEEE Access.



Dr. Reza Malekian (M'10, SM'17) is an Associate Professor with the Department of Electrical, Electronic, and Computer Engineering, University of Pretoria, Pretoria, South Africa. His current research interests include advanced sensor networks, Internet of Things, and mobile communications. Prof. Malekian is also a Chartered Engineer and a Professional Member of the British Computer Society. He is an associate editor for the Journal of IEEE Internet of Things.



Dr. Rui Zhang received her PhD in Transportation Engineering from Wuhan University of Technology, China. Currently she is a lecturer with the School of Automotive and Transportation, Tianjin University of Technology and Education, China. Her main research interests include intelligent transportation systems and intelligent vehicles.



Dr. Miguel Angel Sotelo received the Dr. Eng. degree in Electrical Engineering in 1996 from the Technical University of Madrid, the Ph.D. degree in Electrical Engineering in 2001 from the University of Alcalá (UAH), Alcalá de Henares, Madrid, Spain, and the Master degree in Business Administration (MBA) from the European Business School in 2008.

He is the Editor-in-Chief of IEEE Intelligent Transportation Systems Magazine (2014-2017), Associate Editor of IEEE Transactions on Intelligent Transportation Systems (2008-2015), and Editor-in-Chief of IEEE Intelligent Transportation Systems Society Newsletter (Jan 2013 – Jan 2014). Currently, he is President-Elect of the IEEE Intelligent Transportation Systems Society (starting Jan 2017).