HIERARCHICAL IMAGE ANALYSIS USING IRREGULAR TESSELLATIONS

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I. INTRODUCTION

An image pyramid is a multi-layer data structure in which the input is represented at successively reduced resolutions. The input picture, a N^2 array, is taken as level 0, the bottom of the pyramid. The upper levels are built recursively by a parallel process, usually using a four-fold reduction in resolution. The height of the image pyramid is thus *log N*. Many tasks can be accomplished in *O[log N]* on an image pyramid with a small increase of memory space [Rosenfeld, 1984 ; Uhr, 1987].
The dependence of the low resolution representations on the interaction between the sampling grid and the input image is of importance for image pyramid applications. The rigidity of the sampling structure must be taken into consideration, for example, many segmentation algorithms employ a delineation process in which the weights (parent-child links) are iteratively changed *after* the initial pyramid was built [Hong 84].
We present irregular tessellations to generate an adaptive multiresolution description of the input image ; the hierarchy is built bottom-up adapted to the content of the input image and most of the properties of the "classical" image pyramids are preserved. We employ a local stochastic process which can be associated with different feature fusion criteria to build lower resolution representations. A graph formulation is defined to achieve this target. Applied on labeled pictures, every connected component is reduced to a separate root, and the adjacency graph is simultaneously built. In gray level pictures we perform a segmentation of the initial image. This work is presented in more details in [Montanvert 1989].

II. IRREGULAR TESSELLATIONS AND STOCHASTIC PYRAMIDS

In the image pyramids based on regular tessellations the artifacts caused by the rigidity of the sampling structure are always present. Only a hierarchy of irregular tessellations can be molded to the structure of the input image ; however, the topological relations among cells on the different levels are no longer carried implicitly by the sampling structure. Thus it is convenient to use the formalism of graphs.
The ensemble of cells defining the level l of the pyramid are taken as the vertices V[l] of an undirected graph G[l] = (V[l], E[l]). The edges E[l] of the graph describe the adjacency relations between cells at level l. The graph G[0] describes the 8-connected sampling grid of the input, G[l] is called the *adjacency graph*.
As the image pyramid is to be built recursively bottom-up we must define the procedure of deriving G[$l+1$] from G[l], we are dealing with a *graph contraction* problem. We must design rules for :
- choosing the new set of vertices V[$l+1$] from V[l] ,i.e., the survivors of the decimation process;
- allocating each non-surviving vertex of level l to the survivors, i.e., generating the parent-children links;
- creating the edges E[$l+1$], i.e., recognizing the adjacency relations among the surviving cells.

In order to have any cell in the hierarchy correspond to a compact region of the input, a node of $c[l+1] \in V[l+1]$ must represent a connected subset of $V[l]$ which defines the children of $c[l+1]$. We want to assure that this process can employ only local operations in parallel to build $G[l+1]$. A solution is provided by searching a maximum independant set (MIS) of vertices [Luby, 1985] among $V[l]$. Because of the properties of the MIS, two important constraints will be respected :

(1) any non-surviving cell at level l has at least one survivor in its neighborhood,

(2) two adjacent vertices on level l cannot both survive and thus on level $l+1$ the number of vertices must decrease significantly yielding a pyramidal hierarchy.

The last step in defining the graph of level $l+1$ is the definition of the edges $E[l+1]$: two vertices are connected in $G[l+1]$ iff there exists at least one path between them in $G[l]$ of two or three edges long. The graph $G[l+1]$ of the next level is now completely defined by a parallel and local process.

The survivors will be spread on $V[l]$ in a more flexible way than on a regular pyramid structure. To obtain a multiresolution hierarchy adapted to the input image, this selection of parents and then the allocation of children will depend on the content of the image.

A probabilistic algorithm achieves the graph contraction satisfying the two constraints ; it is analyzed in more details in [Meer 1989, Meer and Connelly 1989]. It is different from other solutions proposed in the literature. The basic principle is to iteratively extract from $V[l]$ some vertices satisfying constraints (1) and (2), which then belong to $V[l+1]$, and so on until no more vertices can be added : each vertice owns a random value (the outcome of a random variable) and is kept iff this value is a local maxima on the subgraph induced by the current iteration. It converges after a few steps. If the process is applied to build a whole structure (until it remains just one vertex), a complete irregular tessellation hierarchy is defined. The power of this algorithm (compared to other algorithms to compute the MIS) comes from the local maxima principle which allows the process to be adapted to an information such as images.

III. LABELED IMAGES

In a labeled image every maximal set of connected pixels sharing the same label is a connected component of the image. Looking at its neighborhood a cell can see which cells share its label (they are said to be in the same class) which define the *similarity graph* $G'[l]=(V[l] , E'[l])$ induced on $G[l]$. In the similarity graph the connected components become maximal connected subgraphs. The techniques describe precedently are adapted in the following way : to become a survivor the outcome of the random variable drawn by the cell must be the local maximum *among* the outcomes drawn by the neighbors in the same class. Thus the subgraphs of the similarity graph are processed independently and in parallel, each subgraph being recursively contracted into one vertex, the *root* of the connected component ; at each iteration a maximum independent set of the similarity graph is extracted to fix the new subgraph.(see Figure 1).

All the artifacts of rigid multiresolution hierarchies are eliminated. From each connected component a pyramidal hierarchy based on irregular tessellations is built in log(component_size) steps (the component_size of a connected component is its intrinsic diameter). Since random processes are involved in the construction of the irregular tessellations the location of the roots depends on the outcomes of local processes. Nevertheless, always the same root level adjacency graph is obtained at the top of the hierarchy.The famous connectedness puzzle of Minsky and Papert (Figure 1) can be solved in parallel with the help of the root level adjacency graph.

IV. GRAY SCALE IMAGES

In gray level images we have to analyse the difference between the values of two adjacent pixels. We have seen that in our technique to build the hierarchies the pixels in a neighborhood must be arranged into classes. The class membership induces the similarity graph on which the stochastic decimation takes place. Similar to the labeled images, class membership can be defined based on the gray level difference between the center cell c_0 and one of its neighbors c_i , $i = 1,\ldots,r$, let g_i be their gray levels. Employing an absolute threshold T , c_0 decides that its neighbor c_i is in the same class iff $\delta_i = |g_0 - g_i| \leq T$.
Like in the labeled case this criterion is symmetrical. See Figure 2 for an illustration of a similarity graph on a gray scale picture. The stochastic decimation algorithm selects the survivor cells and the non-survivors are allocated to their most similar surviving neighbor. The survivors (parents) compute a new gray level value g based on their children. Hence the hierarchy is not built on some well-defined subgraphs of G'[l] as it was the case on labeled pictures. Now the similarity subgraphs and then their meaning on the input picture evolved when we build the pyramid, since g_i values are recomputed. We conclude that a symmetric class membership criterion strongly influences the structure of the hierarchy and therefore the final representation of the image ; some artefacts can be created (see Figure 2).

To achieve satisfactory results in our irregular tessellations based multiresolution gray level image analysis a *non-symmetric* class membership criterion must be used. Let the cell c_0 have r neighbors. In this neighborhood we define the local threshold $S[c_0]$ such that $0 \leq S[c_0] \leq T$. Thus the cell c_0 declares as similar to itselve only its neighbors c_i for which $|g_0 - g_i| \leq S[c_0]$. The threshold $S[c_0]$ is specific to the neighborhood of cell c_0 and therefore the criterion is not symmetric. Indeed, in general $|g_0 - g_i| \leq S[c_0]$ does not imply $|g_0 - g_i| \leq S[c_i]$ since the two thresholds are computed based on only partially overlapping neighborhoods. As a consequence of the non-symmetrical membership criterion the similarity graphs become *directed.* The neighborhood dependent local threshold $S[c_0]$ assures that every cell connects first to its neighbors with the most similar gray level values. Thus the individual rows in the image shown in Figure 2.b are reduced to single cells *before* two cells belonging to adjacent rows can become neighbors on the similarity graph.
The value of the local threshold $S[c_0]$ is computed based on a subset of cells in the neighborhood of the cell c_0 . The extreme case $S[c_0] = 0$ corresponds to connected component recognition. The other extreme, $S[c_0] = T$ yields the symmetric class membership criterion. Several approaches are available to determine the $S[c_0]$ value best dichotomizing the neighborhood into two classes. We will employ only gray level information in computing $S[c_0]$.
Let $\delta_{[i]}$, $i = 0,1,\ldots,r$ be the ordered sequence of absolute gray level differences $\delta_i = |g_0 - g_i|$.
Thus $\delta_{[0]} = 0 \leq \delta_{[1]} \leq \ldots \delta_{[s]} \leq S[c_0] < \delta_{[s+1]} \ldots \leq \delta_{[t]} \leq T < \delta_{[t+1]} \ldots \leq \delta_{[r]}$.
We compute k as the *maximum averaged contrast* method in which the threshold $S[c_0]$ is set to the most significant step in the sequence of $\delta_{[i]}$ For all the t neighbors we compute

$$A_j = \frac{\sum_{i=1}^{j} \delta_{[i]}}{j} \qquad B_j = \frac{\sum_{i=j+1}^{t} \delta_{[i]}}{t-j} \qquad 1 \leq j \leq t-1$$

Let u = min arg (max$_j$ (B_j - A_j)) and s = max arg ($\delta_{[i]} = \delta_{[u]}$). The threshold $S[\, c_0 \,] = \delta_{[s]}$ is set by the first occurrence of the maximum averaged contrast between the two classes; it is automatically adjusted to the local gray level configuration.The gray level value of a parent is computed as the weighted average of the children's gray levels. See Figure 3 for an illustration of the result on an aerial image. Different outcomes for the employed random variables cause changes in the hierarchy structure : by changing the set of survivor cells the values attributed to these cells may also change slightly yielding changes in the similarity graphs of subsequent levels. As expected, regions with sharp boundaries in the input image achieve very similar representations (see Figure 3). Lastly we can notice that if in a labeled picture a root (an isolated vertex in the similarity graph G'[l]) remains a root at higher levels, this is not true in gray scale images while the similarity graph continues to evolve. That is, the root may disappear at subsequent levels, its receptive field being fused into a larger region.
The decimation process can be modified to be biased toward cells with high informational value. Jolion and Montanvert [1989] proposed an adaptive pyramid in which cells belonging to the most homogeneous regions have priority to become survivors. Such an approach, however, is not successful for labeled pictures in which many cells carry identical descriptions.

V. CONCLUSION

In this paper we have presented an image analysis technique in which a separate hierarchy is built over every compact object of the input. The approach is made possible by a stochastic decimation algorithm which adapts the structure of the hierarchy to the analyzed image. For labeled images the final description is unique. For gray level images the classes are defined by converging local processes and slight differences may appear. At the apex every root can recover information about the represented object in logarithmic number of processing steps, and thus the adjacency graph can become the foundation for a relational model of the scene.

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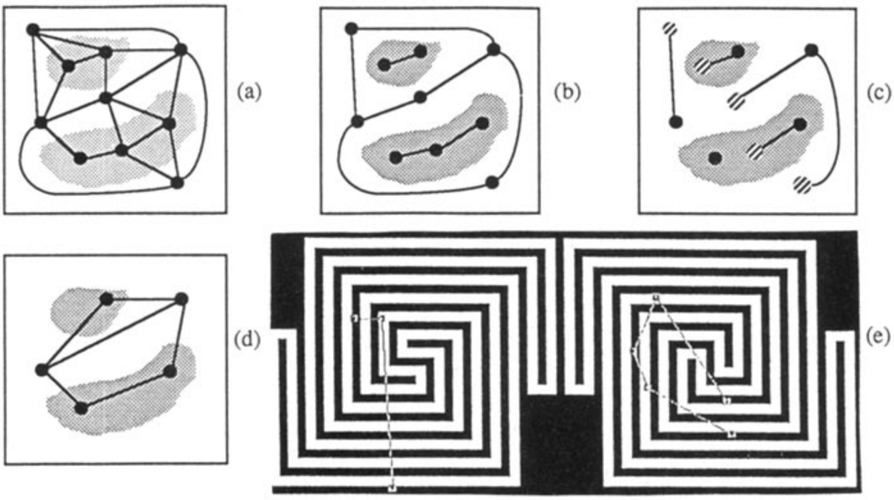
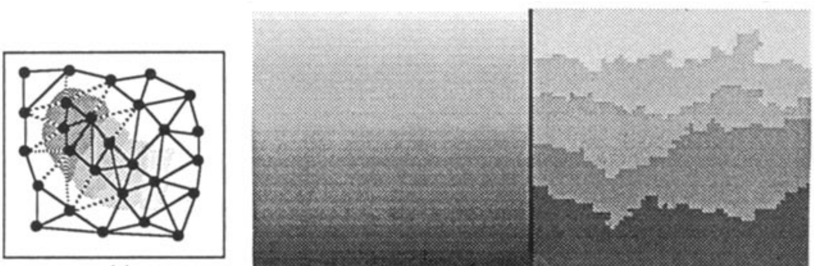
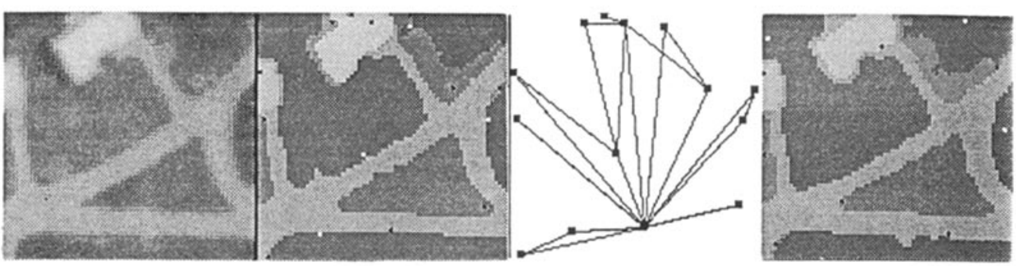


Figure 1. Labeled images. (a) G[l] : adjacency graph (b) G'[l] : similarity graph
(c) Allocation of non survivors (hashed circles) at level l (d) G[l+1]
(e) The root level projected on the bottom for the puzzle of Minsky and Papert



(a) (b) (c)

Figure 2. Symmetrical criterion on gray scale images
(a) Dot lines show the differences between G[l] and G'[l]
(b) The original image : a uniform gray level slope
(c) The result provides at the root level shows the artefacts provide by the use of a symmetrical criterion



(a) (b) (c) (d)

Figure 3. Non symmetrical criterion on gray scale images
(a) The original picture (b) The result with the root locations superposed
(c) The adjacency graph of the root level (d) The result for another outcome of the random variable