# Hierarchical Self-Organization of Perylene Bisimide-Melamine Assemblies to Fluorescent Mesoscopic Superstructures 

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## Derivation of a General Formula to Estimate the Chain Length of Supramolecular [.-AA…BB.•] ${ }_{n}$ Type Polymers

For the estimation of the chain length of a supramolecular [ $\cdot \cdot \mathrm{AA} \cdots \mathrm{BB} \cdot \cdot]_{n}$ type polymer let us consider a mixture of ditopic receptors $A A$ and $B B$ where each receptor functionality A may associate with the complementary receptor $B$ with a microscopic association constant $K_{\mu}$ and no competing association processes between A $\cdots$. or $B \cdots \cdot B$ have to be taken into account.


AA BB $\quad[\because \mathrm{AA} \cdots \cdot \mathrm{BB} \cdot \cdot]_{n}$

From the law of mass action (Eq. 1), we derive the concentration of bound receptors [AB] (Eq. 2), where $c_{A}{ }^{0}$ and $c_{B}{ }^{0}$ are the total concentrations of the respective receptor units (i.e. $c_{A}{ }^{0}=2 \cdot c_{A A}{ }^{0}$ and $c_{B}{ }^{0}=2 \cdot c_{B B}{ }^{0}$ ). Thus, the probability $p$ for a receptor to be bound is given by Eq. 3, where the factor 2 results from the fact that the complex [AB] involves two bound receptors, one of type A and one of type B.

$$
\begin{equation*}
[\mathrm{AB}]=K_{\mu}[\mathrm{A}][\mathrm{B}]=K_{\mu}\left(c_{\mathrm{A}}^{0}-[\mathrm{AB}]\right)\left(c_{\mathrm{B}}^{0}-[\mathrm{AB}]\right) \tag{1}
\end{equation*}
$$

$[\mathrm{AB}]=\frac{\left(c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}+\frac{1}{K_{\mu}}\right)-\sqrt{\left(c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}+\frac{1}{K_{\mu}}\right)^{2}-4 c_{\mathrm{A}}^{0} c_{\mathrm{B}}^{0}}}{2}$
$p=\frac{2[\mathrm{AB}]}{c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}}=1+\frac{\frac{1}{K_{\mu}}-\sqrt{\left(c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}+\frac{1}{K_{\mu}}\right)^{2}-4 c_{\mathrm{A}}^{0} c_{\mathrm{B}}^{0}}}{c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}}$

Assuming a system containing an excess of $B$, we calculate the concentration of chain-terminating free receptors $C_{F}$ according to Eq. 4:

$$
\begin{equation*}
c_{F}=2 c_{A}^{0}(1-p)+\underbrace{\left(c_{B}^{0}-c_{A}^{0}\right)}_{\text {Excess of } B} \tag{4}
\end{equation*}
$$

Using the definition of the Degree of Polymerization $\left(^{--}, D P\right)^{[21]}$ a general formula for the mean chain length of the supramolecular $[\cdot A A \cdots \cdot B B \cdot \cdot]_{n}$ polymer is derived with Eq. 5. For exact stoichiometries $c_{A}{ }^{0}=C_{B}{ }^{0}$, Eq. 5 simplifies into Eq. 6, which is the supramolecular version of the well-known Carothers' equation of condensation polymer chemistry:

$$
\begin{equation*}
\overline{D P}=\frac{\text { number of monomers }}{\text { number of assemblies }}=\frac{\left(c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}\right)^{2}}{2 c_{\mathrm{A}}^{0}\left(-\frac{1}{K_{\mu}}+\sqrt{\left(c_{\mathrm{A}}^{0}+c_{\mathrm{B}}^{0}+\frac{1}{K_{\mu}}\right)^{2}-4 c_{\mathrm{A}}^{0} c_{\mathrm{B}}^{0}}\right)+\left(c_{\mathrm{B}}^{0}\right)^{2}-\left(c_{\mathrm{A}}^{0}\right)^{2}} \tag{5}
\end{equation*}
$$

$\overline{D P}=\frac{1}{1-p}=\frac{2 c_{\mathrm{A}}^{0}}{-\frac{1}{K_{\mu}}+\sqrt{\left(2 c_{\mathrm{A}}^{0}+\frac{1}{K_{\mu}}\right)^{2}-4\left(c_{\mathrm{A}}^{0}\right)^{2}}}$

If we plot the degree of polymerization as a function of the receptor concentrations $c_{A}{ }^{0}, c_{B}{ }^{0}$ for a given binding constant $K_{\mu}$ according to Eq. 5, we obtain the threedimensional plot depicted in Figure 1.


Figure 1. Plot of the mean chain length of an [ $\cdot \cdot \mathrm{AA} \cdots \mathrm{BB} \cdot \cdot]_{n}$-type supramolecular polymer as a function of the concentrations of the receptors A, B for a binding constant of $13500 \mathrm{~L} \mathrm{~mol}^{-1}$ (Eq. 5). For the plot, one has to take into account that also receptor A may prevail in excess. This can be done by using the absolute value of $\left(c_{B}{ }^{0}-c_{A}{ }^{0}\right)$ in Eq. 5.

As shown in the three-dimensional plot of Eq. 5 given in Figure 1, long-chain assemblies are only predicted for an exact stoichiometry of the two receptors $A$ and $B$, whereas for unequal stoichiometry the chain length drops rapidly to one. For example, a solution of two ditopic receptors $A A$ and $B B$ at a stoichiometric concentration of $C_{A A}{ }^{0}=C_{B B}{ }^{0}=10^{-2} \mathrm{~mol} \mathrm{~L}^{-1}$ affords a ${ }^{--}, D P$ of 16.9 for a binding constant $K_{\mu}=13500 \mathrm{~L} \mathrm{~mol}^{-1}$. However, if $c_{A A}{ }^{0}$ is only $0.99 \cdot 10^{-2} \mathrm{~mol} \mathrm{~L}^{-1}$ and $c_{B B}{ }^{0}$ is $1.01 \cdot 10^{-2} \mathrm{~mol} \mathrm{~L}^{-1}$, the calculated ${ }^{--}, D P$ drops to only 10.8 .

Even for an exact stoichiometry long chains of associated molecules are only obtained at high concentrations and for high binding constants. The vertex ridge of the three-dimensional plot describes this situation of the stoichiometric case when $C_{A A}{ }^{0}=C_{B B}{ }^{0}$ leads to the longest possible supramolecular chain at a given concentration for a given binding constant. Accordingly, for $K_{\mu}=13500 \mathrm{~L} \mathrm{~mol}^{-1}$ at least concentrations $c_{A A}{ }^{0}=C_{B B}{ }^{0}>10^{-2} \mathrm{~mol} \mathrm{~L}^{-1}\left(\approx 100 \cdot K_{\mu}{ }^{-1}\right)$ are required to overcome the entropic cost of the association process and to afford polymer-like species. This curve may be also calculated by the simpler Eq. 6, which describes stoichiometric $[\cdots \mathrm{AA} \cdots \mathrm{BB} \cdot \cdot]_{n}$ (nylon-type) and $[\cdots \mathrm{AB} \cdots \mathrm{AB} \cdot \cdot]_{n}$ (perlon-type) supramolecular polymers. As with perlon these association polymers are easier to prepare because the problem of exact stoichiometries, e.g. errors in weighing of the compounds, is not given.

## Additional Figures



Figure 2. Absorption and Fluorescence spectra of perylene bisimide $5\left(c=10^{-6} \mathrm{~mol}^{-1}\right)$ in methylcyclohexane in the presence of increasing amounts of melamine 9. The fluorescence spectra are independent of the excitation wavelength. The excitation wavelength was 530 nm .


Figure 3. Changes in the UV/Vis absorption spectra of a $5 \cdot 10^{-5} \mathrm{~mol} \mathrm{~L}^{-1}$ solution of $\mathbf{3 b} \cdot \mathbf{8 c}$ in methylcyclohexane upon addition of monotopic melamine 9 (conc. up to $2.5 \cdot 10^{-4} \mathrm{~mol} \mathrm{~L}^{-1}$ ). The concentration of $\mathbf{3 b} \mathbf{8 c}$ was kept constant. Arrows indicate changes upon increasing concentrations of 9.

