Higgs and superparticle mass predictions from the landscape

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ABSTRACT: Predictions for the scale of SUSY breaking from the string landscape go back at least a decade to the work of Denef and Douglas on the statistics of flux vacua. The assumption that an assortment of SUSY breaking F and D terms are present in the hidden sector, and their values are uniformly distributed in the landscape of $D = 4$, $N = 1$ effective supergravity models, leads to the expectation that the landscape pulls towards large values of soft terms favored by a power law behavior $P(m_{\rm soft}) \sim m_{\rm soft}^n$. On the other hand, similar to Weinberg's prediction of the cosmological constant, one can assume an anthropic selection of weak scales not too far from the measured value characterized by $m_{W,Z,h} \sim 100\,{\rm GeV}$. Working within a fertile patch of gravity-mediated low energy effective theories where the superpotential μ term is $\ll m_{3/2}$, as occurs in models such as radiative breaking of Peccei-Quinn symmetry, this biases statistical distributions on the landscape by a cutoff on the parameter $\Delta_{\rm EW}$, which measures fine-tuning in the m_Z-μ mass relation. The combined effect of statistical and anthropic pulls turns out to favor low energy phenomenology that is more or less agnostic to UV physics. While a uniform selection $n = 0$ of soft terms produces too low a value for m_h, taking $n = 1$ and 2 produce most probabilistically $m_h \sim 125\,{\rm GeV}$ for negative trilinear terms. For $n \geq 1$, there is a pull towards split generations with $m_{\tilde{q},\tilde{\ell}}(1,2) \sim 10 - 30\,{\rm TeV}$ whilst $m_{\tilde{t}_1} \sim 1 - 2\,{\rm TeV}$. The most probable gluino mass comes in at $\sim 3 - 4\,{\rm TeV}$ — apparently beyond the reach of HL-LHC (although the required quasi-degenerate higgsinos should still be within reach). We comment on consequences for SUSY collider and dark matter searches.

Contents

1 Introduction

One of the great mysteries of fundamental physics is the origin of the vastly different energy scales which appear in nature. Paramount among these is the cosmological constant problem: why is the measured value of $\Lambda \simeq 10^{-47}\,\mathrm{GeV}^4$ so much smaller than the (reduced) Planck scale $M_P^4 \simeq 3.3 \times 10^{73}\,\mathrm{GeV}^4$? Weinberg proposed an anthropic explanation [1, 2]: in a vast set of possible universes each with different (uniformly distributed) possibilities for Λ, if Λ were too much larger than its measured value, then the universe would expand too rapidly for galaxies to condense, and the latter constraint seems necessary for the appearance of life as we know it. Using such reasoning, Weinberg was able to predict the value of Λ to within a factor of a few of its measured value at a time when many physicists expected its value to be zero. The expectation of a vast set of possible universes (the multiverse) found strong support in string theory where stabilization of moduli via flux compactifications [3, 4] led to the emergence of the string theory landscape [5].

Perhaps as intriguing as the cosmological constant problem is the presence of the gauge hierarchy enigma: why is the weak scale as typified by $m_{W,Z,h} \sim 100\,\mathrm{GeV}$ so much smaller

than the scale of grand unification $m_{GUT} \simeq 2 \times 10^{16}$ GeV when it is well known that fundamental scalar masses are intrinsically unstable under quantum corrections [6]? In this case, the expansion of the set of spacetime symmetries in the Standard Model (SM) to include supersymmetry (SUSY) results in cancellation of quadratic divergences to all orders [7, 8]. The remaining log divergences are relatively mild and at least allow for a stable value of the weak scale with the prospect of no funetuning. And indeed from this point of view it is possible to view the presence of spacetime SUSY with weak scale soft breaking as a necessary feature in an anthropic vacuum.

An expansion of the SM to the Minimal Supersymmetric Standard Model (MSSM) is actually supported by three disparate data sets: 1. the measured values of the gauge couplings are exactly what is needed for grand unification at a scale $m_{GUT} \simeq 2 \times 10^{16}$ GeV, 2. the measured value of the top quark mass falls in the range needed to radiatively break electroweak symmetry in the MSSM and 3. the measured value of the Higgs boson mass $m_h \simeq 125$ GeV falls squarely within the predicted narrow MSSM window where $m_h \lesssim 135$ GeV is required [9, 10]. In spite of these successes, so far no signal for superparticles has yet emerged from dedicated searches by LHC experiments using ~ 100 fb^{-1} of data from pp collisions at $\sqrt{s} = 13$ TeV. The lack of superpartners at LHC has called into question whether weak scale SUSY is indeed nature's solution to the naturalness puzzle, and whether the emergence of a *Little Hierarchy* between the weak scale and the superpartner scale is indicative of the collapse of the SUSY paradigm [11].

Early calculations of upper bounds on SUSY particles seemed to require charginos with mass $m_{\widetilde{W}_1} \lesssim 100$ GeV and gluinos with $m_{\tilde{g}} \lesssim 350$ GeV [12]. Recently, these calculations have been challenged [13, 14] in that they compute using a log derivative measure [12, 15] $\Delta_{BG} \equiv max_i |\frac{\partial \log m_Z^2}{\partial \log p_i}|$ in terms of multiple soft terms p_i (assumed independent) whereas in more fundamental theories the soft terms are all *dependent* in that they are computable in terms of more fundamental parameters (such as the gravitino mass $m_{3/2}$ in gravity-mediated SUSY breaking). By combining the dependent soft terms, then large cancellations can occur leading to much less fine-tuning. Other evaluations of fine-tuning required not-too-large logarithmic corrections to the Higgs mass squared, thus seemingly requiring three third generation squarks with mass bounded by 500 GeV [16–18]. These calculations ignore various dependent contributions to the renormalization group equation (RGE) of the up-Higgs soft term $m_{H_u}^2$ which allow for *radiatively-driven naturalness* wherein large seemingly unnatural high scale soft terms such as $m_{H_u}^2$ can be driven by radiative corrections to natural values $\sim m_Z^2$ at the weak scale.

An improved naturalness measure Δ_{EW} has been proposed [19, 20] which just requires that *weak scale* contributions to m_Z^2 should be comparable to or less than m_Z^2. From the minimization conditions for the MSSM Higgs potential [21] one finds

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2. \tag{1.1}$$

The radiative corrections Σ_u^u and Σ_d^d include contributions from various particles and sparticles with sizeable Yukawa and/or gauge couplings to the Higgs sector. Usually the most

important of these are

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right] \tag{1.2}$$

where f_t is the top-quark Yukawa coupling, $\Delta_t = (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)/2 + M_Z^2 \cos 2\beta(\frac{1}{4} - \frac{2}{3}x_W)$, $x_W \equiv \sin^2\theta_W$, $F(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right)$ and the optimized scale choice for evaluation of these corrections is $Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$. In the denominator of eq. (1.2), the tree level expressions of $m_{\tilde{t}_{1,2}}^2$ should be used. Expressions for the remaining Σ_u^u and Σ_d^d terms are given in the appendix of ref. [20].

The naturalness measure Δ_{EW} compares the largest contribution on the right-hand-side of eq. (1.1) to the value of $m_Z^2/2$. If they are comparable ($\Delta_{\text{EW}} \lesssim 30$), then no unnatural fine-tunings are required to generate $m_Z = 91.2\,\text{GeV}$. The main requirement for low fine-tuning is then that

- $|\mu| \sim 100 - 300\,\text{GeV}$ [22–24] (the lighter the more natural with $\mu \gtrsim 100\,\text{GeV}$ to accommodate LEP2 limits from chargino pair production searches).

- Also, $m_{H_u}^2$ is driven radiatively to small ($\sim -(100 - 300)^2\,\text{GeV}^2$, and not large, negative values [19, 20].

- The top squark contributions to the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized for TeV-scale highly mixed top squarks [19]. This latter condition also lifts the Higgs mass to $m_h \sim 125\,\text{GeV}$.

- First and second generation squark and slepton masses may range as high as 10-30 TeV with little cost to naturalness [20, 25]. Such a high mass range offers a decoupling solution to the SUSY flavor, CP and gravitino problems [26–29].

The question then arises: why should the soft SUSY breaking terms and the superpotential μ term adopt the specific range of values needed to satisfy the naturalness condition?

In the case of the μ term, it has been commonly assumed that μ takes a value comparable to the SUSY breaking scale as suggested in the Giudice-Masiero mechanism [30]. If that were so — and with soft terms now required to lie in the multi-TeV regime by LHC constraints — then one would have to accept a multi-TeV value of μ and the MSSM would necessarily be fine-tuned with $|\mu| \gg m(W, Z, h)$. However, in the Kim-Nilles (KN) μ term solution [31], which is a supersymmetrized version of the DFSZ axion model [32, 33], the expectation can be very different. In KN, the Higgs superfields carry a common PQ charge so that the μ term is initially forbidden by PQ symmetry. Upon spontaneous PQ symmetry breaking, an axion is generated to solve the strong CP problem but also a μ parameter is generated with value $\mu \sim \lambda_\mu v_{PQ}^2/m_P$.[1] This may be compared to the SUSY breaking scale $m_{\text{soft}} \sim m_{\text{hidden}}^2/m_P$ where m_{hidden} is some intermediate mass scale associated with the hidden sector. Then $\mu \ll m_{\text{soft}}$ is just a consequence of $v_{PQ} < m_{\text{hidden}}$.

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[1]In addition, an intermediate scale Majorana neutrino mass m_N is also generated.

Indeed, in models of radiative PQ breaking [34–36], wherein PQ breaking is derived as a consequence of SUSY breaking, then typically $\mu \ll m_{\text{soft}}$ is expected [37]. We note here that $\mu \ll m_{\text{soft}}$ arises in other well-motivated cases, such as certain classes of string models with flux compactifications [38].

Regarding natural values for the soft SUSY breaking terms, one possibility is that, with the right correlations amongst soft terms and a small superpotential μ term $\sim 100 - 300\,\text{GeV}$, then a generalized focus point mechanism [39, 40] can exist such that $m_{H_u}^2$ runs to small negative values at the weak scale roughly independently of its high scale value [41]. Another possibility arises from the string theory landscape. If — within a "fertile patch" of the landscape of string theory vacua (such that the low energy effective theory is the MSSM or related variants) — there is

1. a statistical selection towards large soft terms [42–45] and

2. an anthropic selection towards a weak scale value $m(W, Z, h)$ not too far from $\sim 100\,\text{GeV}$ [46, 47] and

3. a mechanism such as radiative PQ breaking which generates $\mu \sim m(weak)$ rather than $\mu \sim m_{\text{soft}}$,

then the soft terms are pulled towards those values which generate natural SUSY in accord with eq. (1.1) and a light Higgs mass $m_h \simeq 125\,\text{GeV}$ [48].[2] The combined draw — 1. towards large soft terms and 2. towards an anthropic weak scale — pulls the high scale value of $m_{H_u}^2$ to such large values that electroweak symmetry is "barely broken" [49–51]. This is the same as the naturalness condition that $m_{H_u}^2$ be driven to small negative values at the weak scale.

While ref. [48] provided a qualitative picture for understanding why the soft terms adopt values required for naturalness, in the present work we attempt to place this approach on a more quantitative footing. In section 2, we review some ideas mainly originating from Douglas and Denef regarding the draw of the string theory landscape towards large soft SUSY breaking terms as described by a power law selection $f_{SUSY}(m_{\text{soft}}) \sim m_{\text{soft}}^n$. A mild pull towards large soft SUSY breaking terms comes from values of $n \sim 1$ or 2 which arises from rather simple hidden sectors where SUSY breaking arises from just one or two fields gaining a SUSY breaking vev. In contrast, larger values of $n \geq 3$ emerge from more complicated hidden sectors where several or more fields gain comparable SUSY breaking vevs and thus exert a stronger pull towards large values of soft breaking terms. We combine this with an anthropic draw towards the measured value of the weak scale. The combination of both allows us to calculate probability distributions for expected Higgs

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[2]Condition #1, as argued by Denef and Douglas, seems generic in string theory. Condition #2 may [46, 47] or may not be generic in string theory vacua. Condition #3 emerges from the assumed solution to the SUSY μ problem. Weak scale naturalness prefers $\mu \sim m_{W,Z,h}$ while LHC results prefer the SUSY breaking scale $m_{3/2}$ in the multi-TeV regime. Since the MSSM μ term is supersymmetric and not SUSY breaking, a solution to the SUSY μ problem, such as Kim-Nilles [31] where μ can be $\ll m_{3/2}$ (while solving the strong CP problem and generating intermediate scale right-hand Majorana neutrino masses) seeeems preferred to us over other mechanisms which generate $\mu \sim m_{3/2}$. For further discussion, see e.g. ref. [37].

boson and superparticle masses. In section 3, we implement this methodology with its power law selection for large soft terms which are then passed on to the SUSY spectrum generator contained in Isajet 7.87 [52]. By assuming a μ parameter not too far from m_{weak}, then we are able to *invert* the normal useage of eq. (1.1) to calculate the value of m_Z which is in general *not* equal to its measured value. If m_Z is too large, then also the weak scale is too large, thus suppressing rates for weak interactions and increasing particle masses which arise from electroweak symmetry breaking. Requiring that the weak scale not deviate by more than a factor of a few from its measured value (in accord with calculations from Agrawal et al. [46, 47]), then we are able to present our results as probability distributions versus various observable masses. Some confidence in this approach is gained in that the probability distribution for the light Higgs mass peaks rather sharply at $m_h \sim 125\,\text{GeV}$. It is intriguing that this already occurs for the simplest case of SUSY breaking which is dominated by a single F-term field which yields $n = 1$. We then also find $m_{\tilde{g}} \sim 3 - 4\,\text{TeV}$ and $m_{\tilde{t}_1} \sim 1 - 2\,\text{TeV}$. First/second generation scalar masses are pulled into the 10-30 TeV range leading to an amelioration of the SUSY flavor and CP problems. Higher values of $n \geq 3$ tend to pull the soft terms to such large values that one is placed into charge or color breaking (CCB) electroweak vacua or else vacua where electroweak symmetry doesn't even break. In section 4 we discuss some inplications of our results for collider searches for SUSY and for dark matter searches for WIMPs and axions. In section 5 we discuss some aspects of the cosmological moduli problem and in section 6 we present a summary and conclusions.

2 String vacuum statistics and the SUSY breaking scale

In this section, we assume a vast ensemble of string vacua states which give rise to a $D = 4$, $N = 1$ supergravity effective field theory at high energies. Furthermore, the theory consists of a visible sector containing the MSSM along with a perhaps large assortment of fields that comprise the hidden sector. The scalar potential is given by the usual supergravity form [53]

$$V = e^{K/m_P^2} \left(g^{i\bar{j}} D_i W D_{\bar{j}} W^* - \frac{3}{m_P^2} |W|^2 \right) + \frac{1}{2} \sum_\alpha D_\alpha^2 \tag{2.1}$$

$$= e^{K/m_P^2} \left(\sum_i |F_i|^2 - 3\frac{|W|^2}{m_P^2} \right) + \frac{1}{2} \sum_\alpha D_\alpha^2 \tag{2.2}$$

where W is the holomorphic superpotential, K is the real Kähler potential[3] and $F_i = D_i W = DW/D\phi^i \equiv \partial W/\partial\phi^i + (1/m_P^2)(\partial K/\partial\phi^i)W$ are the F-terms and $D_\alpha \sim \sum \phi^\dagger g t_\alpha \phi$ are the D-terms and the ϕ^i are chiral superfields. Supergravity is assumed to be broken spontaneously via the super-Higgs mechanism either via F-type breaking or D-type breaking or in general a combination of both leading to a gravitino mass $m_{3/2} = e^{K/2m_P^2} |W|/m_P^2$. The (metastable) minima of the scalar potential can be found by requiring $\partial V/\partial\phi^i = 0$ with $\partial^2 V/\partial\phi^i\partial\phi^j > 0$ to ensure a local minimum. The cosmological constant is given by

$$\Lambda_{cc} = m_{\text{hidden}}^4 - 3e^{K/m_P^2} |W|^2/m_P^2 \tag{2.3}$$

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[3]Not to be confused with the (dimensionless) Kähler function $G = K/m_P^2 + \log|W/m_P^3|^2$.

where $m_{\text{hidden}}^4 = \sum_i |F_i|^2 + \frac{1}{2} \sum_\alpha D_\alpha^2$ is a mass scale associated with the hidden sector (and usually in SUGRA-mediated models it is assumed $m_{\text{hidden}} \sim 10^{12}\,$GeV such that the gravitino gets a mass $m_{3/2} \sim m_{\text{hidden}}^2/m_P$).

A key observation of Susskind [54] and Denef and Douglas [42, 43] (DD) was that W at the minima is distributed uniformly as a complex variable, and the distribution of $e^{K/m_P^2}|W|^2/m_P^2$ is not correlated with the distributions of F_i and D_α. Setting the cosmological constant to nearly zero, then, has no effect on the distribution of supersymmetry breaking scales. Physically, this can be understood by the fact that the superpotential receives contributions from many sectors of the theory, supersymmetric as well as non-supersymmetric.

Next, we would like to estimate the number of flux vacua containing spontaneously broken supergravity with a SUSY breaking scale m_{hidden}^2, $dN_{\text{vac}}[m_{\text{hidden}}^2, m_{\text{weak}}, \Lambda]/dm_{\text{hidden}}^2$. According to DD [43, 55–57], this distribution is likely to be the product of three factors: $f_{SUSY}(m_{\text{hidden}}^2)$, f_{EWFT} and f_{cc}.

$$dN_{\text{vac}}[m_{\text{hidden}}^2, m_{\text{weak}}, \Lambda] = f_{SUSY}(m_{\text{hidden}}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{\text{hidden}}^2 \qquad (2.4)$$

which contain $\Lambda \sim 0$ but with $m_{\text{weak}} \simeq m_{W,Z,h} \sim 100\,$GeV. The cosmological fine-tuning penalty is $f_{cc} \sim \Lambda/m^4$ where the above discussion leads to $m^4 \sim m_{string}^4$ rather than $m^4 \sim m_{\text{hidden}}^4$, rendering this term inconsequential for determining the number of vacua with a given SUSY breaking scale. Another key observation from examining flux vacua in IIB string theory is that the SUSY breaking F_i and D_α terms are likely to be uniformly distributed — in the former case as complex numbers while in the latter case as real numbers. In this case, one then obtains the following distribution of supersymmetry breaking scales

$$f_{SUSY}(m_{\text{hidden}}^2) \sim (m_{\text{hidden}}^2)^{2n_F+n_D-1} \qquad (2.5)$$

where n_F is the number of F-breaking fields and n_D is the number of D-breaking fields in the hidden sector [43]. The case of $n_F = 1$ is displayed in figure 1. We label the visible sector soft term mass scale as m_{soft} where in SUGRA breaking models we typically have $m_{\text{soft}} \sim m_{\text{hidden}}^2/m_P \sim m_{3/2}$. Thus, the case of $n_F = 1$ $n_D = 0$ would give a *linearly increasing* probability distribution for generic soft breaking terms simply because the area of annuli within the complex plane increases linearly. We will denote the collective exponent in eq. (2.5) as $n \equiv 2n_F + n_D - 1$ so that the case $n_F = 1$, $n_D = 0$ leads to $n = 1$ with $f_{SUSY}(m_{\text{soft}}) \sim m_{\text{soft}}^1$. The case $n_F = 0$ with $n_D = 1$ would lead to a uniform distribution in soft terms $f_{SUSY}(m_{\text{soft}}) \sim m_{\text{soft}}^0$. For the more general case with an assortment of F and D terms contributing comparably to SUSY breaking, then high scale SUSY breaking models would be increasingly favored.[4]

The third factor in the SUSY breaking distribution $f_{EWFT}(m_{\text{soft}})$ arises from anthropics and places a penalty on the calculated value of the weak scale deviating too much from its measured value $m_{\text{weak}} \sim 100\,$GeV. Following [60], DD advocated the form [55]

$$f_{EWFT} \sim m_{\text{weak}}^2/m_{\text{soft}}^2 \qquad (2.6)$$

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[4]The authors of ref. [58] argue that low scale SUSY breaking is preferred by the cosmological constant [59] but then possible formation of cosmological domain walls via R-symmetry breaking provides a lower bound on the scale of SUSY breaking and hence upon $m_{3/2}$.
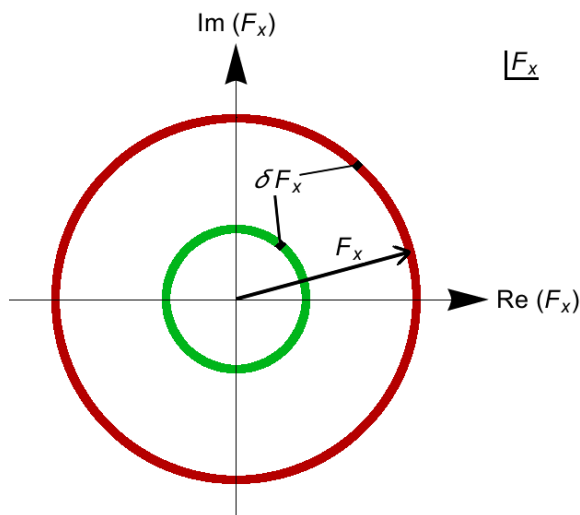
Figure 1. Annuli of the complex F_X plane giving rise to linearly increasing selection of soft SUSY breaking terms.

n_F	n_D	n
0	1	0
1	0	1
0	2	1
1	1	2
0	3	2
2	0	3
2	1	4

Table 1. Some choices of n_f and n_D leading to different n values.

so that the more the soft terms increase beyond the weak scale, the greater is the penalty. This factor must be interpreted with some care. At first glance, one would expect that the larger the value of m_{soft} becomes, then the larger is the calculated value of the weak scale. However, this does not hold true for a variety of cases.

- In one case, as trilinear soft terms increase, then the visible sector scalar potential develops charge and/or color breaking (CCB) minima (see figure 1 of [48]), leading to a universe not as we know it, and likely not conducive to observers. Another possibility is that as soft terms such as $m_{H_u}^2$ increase relative to other soft terms, then its value is too large to be driven radiatively to negative values so that electroweak symmetry doesn't even break. Such string vacua — even within the context of spontaneously broken SUGRA in the MSSM+hidden sector paradigm — must be vetoed by our selection rules.

- Even in the case where EW symmetry is properly broken, it is not always the case that increasing soft terms lead to larger values of the calculated weak scale. One case consists of the soft term $m^2_{H_u}$: the larger its high scale value becomes, then the larger is its cancelling correction from radiative corrections/RG running [61]. For too small values of $m^2_{H_u}$, then it runs deeply negative at the weak scale leading to some required fine-tuning by adopting a large value of μ to compensate and keep m_Z or m_h at its measured value. But for larger values of $m^2_{H_u}(m_{GUT})$, then $m^2_{H_u}$ runs to small weak scale values, thus barely breaking EW symmetry [48–51]. For yet higher values of $m^2_{H_u}(m_{GUT})$, then $m^2_{H_u}$ doesn't even run negative at the weak scale, and EW symmetry remains unbroken.

Another case consists of the trilinear soft term A_t. For small values of A_t, then there is little mixing in the stop sector. Not only is it difficult to raise m_h up to its measured value [62], but the radiative corrections $\Sigma^u_u(\tilde{t}_{1,2})$ in eq. (1.1) become large, leading to either large fine-tuning, or in the case where μ is fixed and m_{weak} floats, to a too large value of m_{weak}. As the weak scale value of A_t increases, then large cancellations occur in both $\Sigma^u_u(\tilde{t}_1)$ and $\Sigma^u_u(\tilde{t}_2)$ leading to greater naturalness and an increased $m_h \sim 125\,\text{GeV}$ [19].

2.1 f_{EWFT}: case A

To ameliorate this situation, we advocate two different replacements of eq. (2.6).

$$case\ \mathbf{A}:\quad f_{EWFT} \to \Theta(30 - \Delta_{\text{EW}}), \tag{2.7}$$

where $\Theta(x)$ is the usual Heaviside unit step function $\Theta(x) = 0\ (1)$ for $x \leq 0\ (x > 0)$. In our methodology, we assume μ is generated to small values not too far from m_{weak} but then we invert the usual useage of eq. (1.1) to let m_Z float so that large values of $\sqrt{|m^2_{H_u}(weak)|}$ or Σ^u_u generate large values of the weak scale $m_{\text{weak}} \gg 100\,\text{GeV}$. The value of $\Delta_{\text{EW}} < 30$ then corresponds to calculated anthropic requirements from Agrawal et al. that the weak scale not deviate by more than a factor of several from its measured value [46, 47]. In this case, $\Delta_{\text{EW}} = 30$ corresponds to a Z mass nearly four times its measured value.

2.2 f_{EWFT}: case B

We also examine

$$case\ \mathbf{B}:\quad f_{EWFT} \to \Delta^{-1}_{\text{EW}} \tag{2.8}$$

which is more closely tied to the DD prescription in that

$$\Delta^{-1}_{\text{EW}} \sim (m^2_Z/2)/max\left[|m^2_{H_u}(weak)|\ or\ \mu^2\ \ or\ \ |\Sigma^u_u(i)|\right]. \tag{2.9}$$

Instead of placing a generic m^2_{soft} in the denominator of eq. (2.9), we place the maximal weak scale contribution to the magnitude of the weak scale. Rather than placing a sharp cutoff on the calculated magnitude of the weak scale as in Case **A**, case **B** places an increasing rejection penalty the more the calculated value of the weak scale strays from its measured value. However, calculated values which differ by large factors from the measured weak scale are nonetheless sometimes allowed.

2.3 Some general comments

The purpose of the present work is to explore several questions that emerge in this framework. On the one hand, the pull towards large supersymmetry breaking scales is evident from eq. (2.5), especially for a large number n_F and/or n_D of SUSY breaking fields. Already at $n_F = 2$, a distribution $f_{SUSY} \sim m_{\text{soft}}^3$ emerges that is heavily biased towards high scale supersymmetry breaking. This leads to the question of whether one should expect to see any signatures of supersymmetry at low energies, since, naively, the soft terms in the infrared (IR or weak scale) should similarly be pulled to larger and larger values. On the other hand, one could also ask how predictive low-energy phenomenology is for a given scale of SUSY breaking m_{hidden}. A given scale m_{hidden} can accommodate various statistical distributions corresponding to the different powers n_F or n_D in eq. (2.5). Naively, superpartner masses in the IR should show a corresponding statistical distribution, raising the question of predictive power. For the case of the Higgs mass, which receives corrections from the supersymmetric spectrum, the question becomes even more critical — can one argue for a natural value preferred from the landscape?

While the statistical distribution f_{SUSY} clearly pulls m_{hidden} (and hence soft masses in the IR) to large values, the imposition of additional constraints can balance this effect. The most important constraint may be anthropic in nature: it is that the calculated value of the weak scale not deviate from its measured value by more than a factor of several. Calculations by Agrawal et al. maintain that anthropically the weak scale should not deviate by more than a factor 5 from its measured value: we will adopt a slightly more conservative bound

$$m_{\text{weak}} \sim m_{W,Z,h} \lesssim 350 \text{GeV} \tag{2.10}$$

corresponding to $\Delta_{\text{EW}} \lesssim 30$. This rests on the observation that rates of nuclear fusion processes and beta decays scale as $\sim 1/m_{\text{weak}}^4$, and a large value of m_{weak} would severely alter the production of heavy elements during Big Bang Nucleosynthesis and in stars. A higher weak scale, with all other constants remaining the same, would also result in heavier particles which receive mass from EWSB. Susskind suggests that the increased masses would speed up numerous astrophysical processes [54] (for more details on astrophysical constraints on a too-large weak scale, see Ref's [46, 47, 63–65]). A caveat that should be kept in mind is that this conclusion is true if the weak scale is the only parameter that is varied; for example, if one is also allowed to sample other technically natural parameters of the Standard Model, perfectly habitable vacua where the Higgs mass resides near the Planck scale may be obtained (the so-called "Weakless Universe" models [63–65]). Nevertheless, small fermion masses are more likely to be obtained in a chiral rather than vector theory.

In the context of supersymmetry, the requirement of an anthropic weak scale can be expressed as a concrete requirement on superpartner masses, namely, that the naturalness parameter should satisfy $\Delta_{\text{EW}} < 30$. A natural Universe where supersymmetry resolves the hierarchy problem, then, would be one in which $\Delta_{\text{EW}} \leq \mathcal{O}(10)$, not only in our vacuum, but also in vacua like ours. This would ensure that all terms in eq. (1.1) are not too far above the measured value of the weak scale. The distribution of vacua in eq. (2.4) can then

be usefully written as

$$dN_{\text{vac}} \sim \Theta(30 - \Delta_{\text{EW}}) \times (m_{\text{hidden}}^2)^n d(m_{\text{hidden}}^2) \qquad (2.11)$$

where $n = 2n_F + n_D - 1$. This is a mathematical statement of the strongest sense in which supersymmetry can be taken as a solution to the gauge hierarchy problem while not also generating a Little Hierarchy where $\Delta_{\text{EW}} \gg 30$.

We note that our philosophy with regard to the landscape is similar to the one pursued by Douglas [55], with the difference being what we consider to be the correct measure of naturalness. In Douglas's 2012 paper [55], the measure adopted was simply $f_{EWFT} = m_{\text{weak}}^2/m_{\text{soft}}^2$. Naturalness quantified in this manner is clearly in tension with the findings of the LHC so far, since mass limits on gluinos (top squarks) exceed $2\,\text{TeV}$ ($1\,\text{TeV}$).

Adopting, instead, the more robust measure Δ_{EW}, we see that the expected low-energy mass spectrum is the one described in the Introduction. The question then arises: how robust is the expected natural spectrum against different values of n in eq. (2.11)? This isn't an entirely trivial question. There are two tendencies in eq. (2.11) — the first is the pull towards heavier scalars as increasing n pulls the distribution towards larger m_{soft}. In fact, there is no reason to expect that only one field dominates supersymmetry breaking in the hidden sector. On the other hand, however, increasing n tends to increase contributions to the radiative corrections Σ_u^u and Σ_d^d on the right hand side of eq. (1.1) which pulls the calculated value of m_{weak} beyond its measured value. The step function in eq. (2.11) then rejects these vacua through the anthropic weak scale. In fact, it is not only the low m_{weak} requirement that rejects these vacua — many of them are unacceptable because they fall into color-breaking minima or do not break electroweak symmetry at all. It is thus clear that some distribution of soft masses, centered around a presumably natural set of values, is expected as one increases n. We now go on to show that this is indeed the case.

3 Numerical results

A quantitative investigation of these questions will require us to work within a particular mediation scheme with suitable boundary conditions at the GUT scale. We choose gravity mediation and a selection of soft terms following the NUHM3 (three-extra-parameter non-universal Higgs) model [66–71] although our broad conclusions are independent of specific UV boundary conditions for the soft terms. The NUHM3 model is convenient in that it allows for μ as an independent input parameter, and since we require μ not too far from $m_{W,Z,h} \sim 100\,\text{GeV}$. The NUHM3 model is inspired by previous work on mini-landscape investigations of heterotic string theory compactified on a $Z_6 - II$ orbifold [72–74]. In these models, sparticle masses are dictated by the geography of their wavefunctions within the compactified manifold. These models exhibit *localized* grand unification [75, 76] wherein the first/second generation matter superfields lie near fixed points (the twisted sector) and thus lie in **16**-dimensional spinor reps of SO(10). Meanwhile, third generations fields and Higgs and vector boson multiplets lie more in the bulk and thus occur in split multiplets (solving the doublet-triplet splitting problem) and receive smaller soft masses [77]. Such a set-up motivates the NUHM3 model with the following parameters

$m_0(1, 2)$, $m_0(3)$, $m_{1/2}$, A_0, $\tan\beta$, m_{H_u}, m_{H_d} where all mass parameters are taken as GUT scale values. The soft Higgs masses can be traded for weak scale values of μ and m_A. Thus, the final parameter space is taken as

$$m_0(1, 2),\ m_0(3),\ m_{1/2},\ A_0,\ \tan\beta,\ \mu,\ m_A \quad \text{(NUHM3)} \tag{3.1}$$

With the gravitino mass $m_{3/2} \sim m_{\text{hidden}}^2/m_P$, then we will adopt

$$\begin{aligned}
m_0(1, 2) &= c_{1,2} \times m_{3/2} \\
m_0(3) &= c_3 \times m_{3/2} \\
m_{1/2} &= c_{1/2} \times m_{3/2} \\
A_0 &= -c_{A_0} \times m_{3/2} \\
m_A &= c_A \times m_{3/2}
\end{aligned} \tag{3.2}$$

i.e. each of these mass terms will scan as m_{soft}^n.

We scan according to m_{soft}^n over:

- $m_0(1, 2):\ 0.1 - 60\,\text{TeV}$,

- $m_0(3):\ 0.1 - 20\,\text{TeV}$,

- $m_{1/2}:\ 0.5 - 10\,\text{TeV}$,

- $A_0:\ -50 -\ 0\,\text{TeV}$,

- $m_A:\ 0.3 - 10\,\text{TeV}$,

with $\mu = 150\,\text{GeV}$ while $\tan\beta : 3 - 60$ scanned uniformly. The goal here is to choose upper limits to our scan parameters which will lie beyond the upper limits imposed by the anthropic selection from f_{EWFT}. Lower limits are motivated by current LHC search limits. Our final results will hardly depend on the chosen value of μ so long as μ is with an factor of a few of $m_{W,Z,h} \sim 100\,\text{GeV}$.

3.1 Case A

While μ is fixed to be small, nonetheless large values of Δ_{EW} can still be generated. This often occurs due to large contributions to Δ_{EW} from $m_A/\tan\beta$ or large contributions to $\Sigma_u^u(\tilde{t}_{1,2})$. Usually, in such cases the value of $m_{H_u}^2(weak)$ is adjusted/fine-tuned to guarantee that m_Z lies at its measured value. Then $m_{H_u}^2$ is run back up to $Q = m_{GUT}$ to whatever value is consistent with its weak scale value. Alternatively, if we do not fine-tune $m_{H_u}^2(weak)$, then the weak scale will attain a value

$$m(weak) \simeq \sqrt{\Delta_{\text{EW}} \cdot m_Z^2/2}. \tag{3.3}$$

The procedure followed in case **A** is to not tune $m_{H_u}^2(weak)$ and then reject solutions with $\Delta_{\text{EW}} > 30$ which would generate a weak scale $m_Z \gtrsim 350\,\text{GeV}$, nearly four times the measured value of the Z mass.
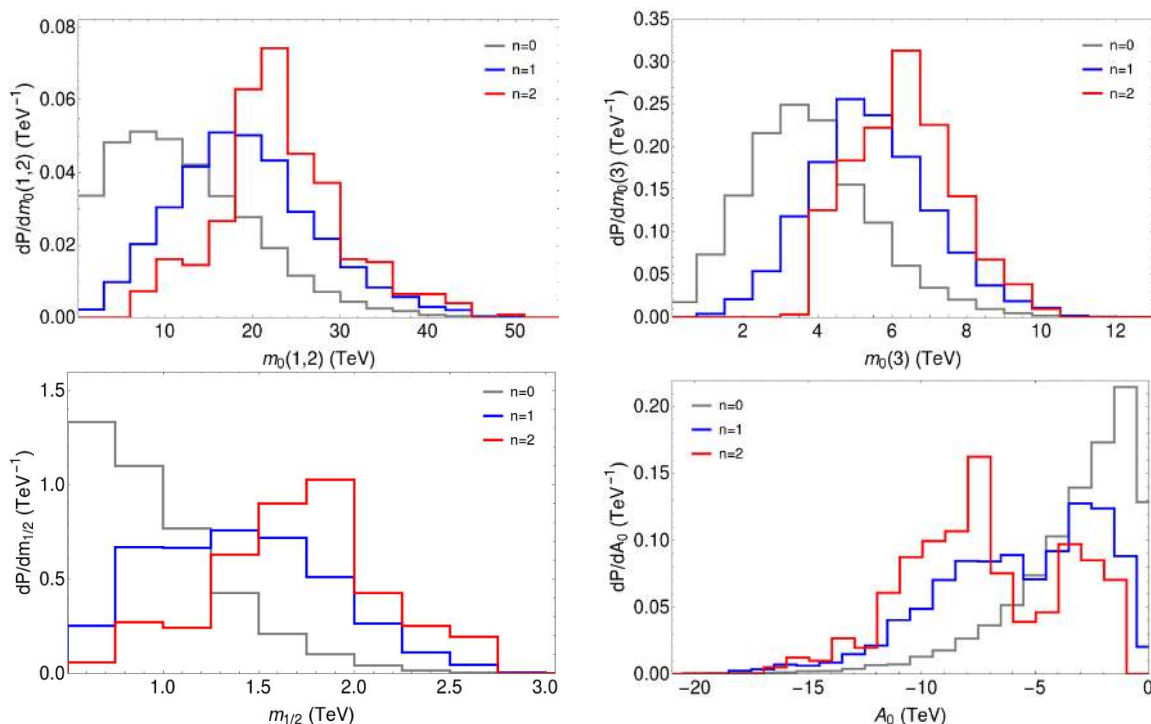
Figure 2. Case **A**: $f_{EWFT} \to \Theta(30 - \Delta_{EW})$ — Upper panels: probability distributions in $m_0(1,2)$ (left) and $m_0(3)$ (right). Lower panels: probability distributions in $m_{1/2}$ (left) and A_0 (right). All distributions are shown following eq. (2.11) and in addition rejecting non-standard scalar potential minima. Results for different values of $n = 2n_F + n_D - 1$ are displayed for each plot.

In figure 2, we plot the probability distributions from our statistical scan over soft terms versus first/second generation scalar mass $m_0(1,2)$ and third generation soft mass $m_0(3)$ in the top panels. For the generation 1,2 soft SUSY breaking matter scalar masses, we immediately see from frame a) that for the cases $n = 1$ and 2 that the probability distributions peak in the vicinity of $m_0(1,2) \sim 20 \, \text{TeV}$ with tails extending out to $30 \, \text{TeV}$. Such large scalar masses occur because of the linear ($n = 1$) and quadratic ($n = 2$) pull on these soft terms with only minimal suppression which sets in at $m_0(1,2) \gtrsim 20 \, \text{TeV}$. One avenue for suppression arises from electroweak D-term contributions to the $\Sigma_{u,d}^{u,d}$ terms which depend on weak isospin and electric charge assignments. For nearly degenerate scalars of each generation, these nearly cancel out [78]. Another avenue for suppression comes from two loop terms in the MSSM RGEs [79]: if scalar masses enter the multi-TeV range, then these terms can become large and help drive third generation scalar masses tachyonic leading to CCB minima in the scalar potential [80]. Both these rather mild suppressions are insufficient to prevent first/second generation scalar masses from rising to the 20-30 TeV range. Such heavy scalars go a long way to suppressing possible FCNC and CP violating SUSY processes [26–29]. For the $n = 0$ case, $dP/dm_0(1,2)$ peaks around 5-10 TeV before suffering a drop-off.

In contrast, in frame b) we plot the distribution of third generation scalar masses $m_0(3)$. In this case, for $n = 1, 2$ the distribution peaks around 5-6 TeV while dropping to near zero around 10 TeV for $n = 1$ and 12 TeV for $n = 2$. Large values of $m_0(3)$ generate

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large stop masses which result in $\Sigma_u^u(\tilde{t}_{1,2})$ exceeding ~ 30 i.e. generating a weak scale typically in excess of $m(weak) \sim 400\,\text{GeV}$. For $n = 0$, the distribution peaks around $3\,\text{TeV}$.

In frame c), we plot the distribution in $m_{1/2}$. In this case, the $n = 1$ distribution peaks around $1.5\,\text{TeV}$ whilst $n = 2$ peaks slightly higher. If the (unified) gaugino masses become too big, then the large gluino mass also lifts the top squarks to higher masses thus causing the $\Sigma_u^u(\tilde{t}_{1,2})$ to again become too large. The distributions fall to near zero by $m_{1/2} \sim 3\,\text{TeV}$ leading to upper limits on gaugino masses. The $n = 0$ distribution actually peaks at its lowest allowed values followed by a steady decline.

In frame d), we show the distribution versus A_0. Here we only show the more lucrative negative A_0 case which leads to higher Higgs masses m_h [62]. The $n = 0$ distribution peaks at $A_0 \sim 0$ with a steady fall-off at large negative A_0 values. In this case, the typically small mixing in the stop sector leads to values of m_h below its measured result. In contrast, for $n = 1, 2$ the distributions increase (according to the statistical pull) to peak values around $A_0 \sim -(5 - 10)\,\text{TeV}$. Such large A_0 values lead to large mixing in the top-squark sector which can enhance m_h whilst decreasing the $\Sigma_u^u(\tilde{t}_{1,2})$ values [19]. The $n = 1$ curve actually features a double bump structure: we have traced the lower peak to the presence of large $m_A \sim m_{H_d} \sim 5 - 10\,\text{TeV}$ values which increase the S term in the third generation matter scalar RGEs. This term (along with large two-loop effects from first/second generation matter scalars) acts to suppress $m_{U_3}^2$ leading to lighter $\tilde{t}_1$ states even without large mixing. For even larger negative A_0 values, the distributions rapidly fall to zero since they start generating CCB minima in the MSSM scalar potential.

In figure 3, we show string landscape probability predictions for quantities associated with the Higgs and electroweak-ino sector. Special attention should be paid to the Higgs mass distributions. In frame a), we show dP/dm_h vs. m_h for $n = 0$, 1 and 2. For $n = 0$, we find a broad peak ranging from $m_h \sim 119 - 125\,\text{GeV}$. This may be expected for the $n = 0$ case since we have a uniform scan in soft terms and low Δ_{EW} can be found for $A_0 \sim 0$ which leads to little mixing in the stop sector and hence too light values of m_h. Taking $n = 1$, instead we now see that the distribution in m_h peaks at $\sim 125\,\text{GeV}$ with the bulk of probability between $123\,\text{GeV} < m_h < 127\,\text{GeV}$ — in solid agreement with the measured value of $m_h = 125.09 \pm 0.24\,\text{GeV}$ [81].[5] This may not be surprising since the landscape is pulling the various soft terms towards large values including large mixing in the Higgs sector which lifts up m_h into the $125\,\text{GeV}$ range. By requiring the $\Sigma_u^u(\tilde{t}_{1,2}) \lesssim 30$ (which would otherwise yield a weak scale in excess of $350\,\text{GeV}$) then too large of Higgs masses are vetoed. For the $n = 2$ case with a stronger draw towards large soft terms, the m_h distribution hardens with a peak at $m_h \sim 126\,\text{GeV}$.

In figure 3b), we show the distribution in pseudoscalar mass m_A. Here, for $m_A \gg m_h$, then $m_A \sim m_{H_d}$ (at the weak scale) and we have a statistical draw to large m_A values which is tempered by the presence of $m_{H_d}/\tan\beta$ in eq. (1.1). While the $n = 0$ uniform draw peaks at the lowest m_A values, the $n = 1$ and 2 cases yield a broad distribution peaking around $m_A \sim 3\,\text{TeV}$ which drops thereafter. In frame c), we show the distribution

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[5]Here, we rely on the Isajet 7.87 theory evaluation of m_h which includes renormalization group improved 1-loop corrections to m_h along with leading two-loop effects. Calculated values of m_h are typically within 1-2 GeV of similar calculations from latest FeynHiggs [82] and SUSYHD [83] codes.
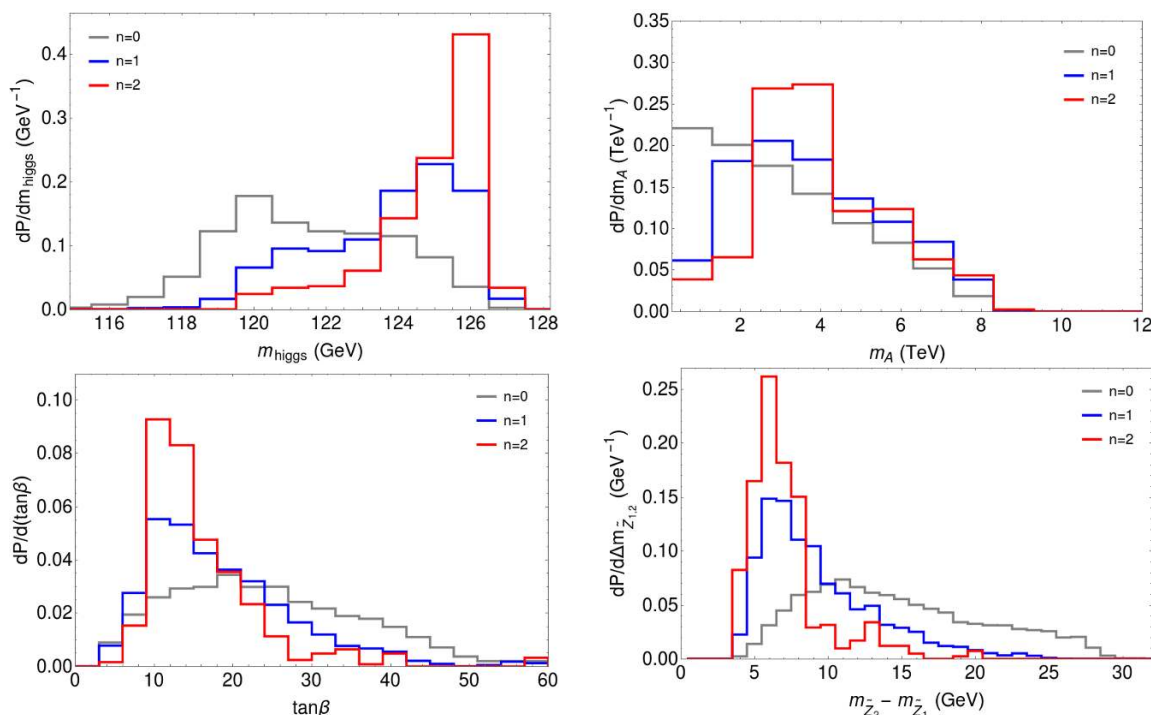
Figure 3. Case **A**: $f_{EWFT} \to \Theta(30 - \Delta_{EW})$ — Upper panels: probability distributions in m_h (left) and m_A (right). Lower panels: probability distributions in $\tan\beta$ (left) and $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ (right). All distributions are shown following eq. (2.11) and in addition rejecting CCB and noEWSB minima. Results for different values of $n = 2n_F + n_D - 1$ are displayed for each plot.

in $\tan\beta$. Here, the $n = 0$ case has a broad distribution with a peak around $\tan\beta \sim 20$ while the $n = 1$ and 2 cases have sharper distributions peaking around $\tan\beta \sim 10 - 15$. The suppression of $\tan\beta$ for large values can be understood due to the draw towards large soft terms in the sbottom sector. As $\tan\beta$ increases, the b (and τ) Yukawa couplings increase so that the $\Sigma_u^u(\tilde{b}_{1,2})$ terms become large. Then the anthropic cutoff on $\Delta_{EW} < 30$ disfavors the large $\tan\beta$ regime. In frame d), we show the $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ mass splitting. For our case with $\mu = 150\,\text{GeV}$, the light higgsinos $\widetilde{W}_1^\pm$, $\widetilde{Z}_{1,2}$ all have masses around 150 GeV. The phenomenologically important mass gap $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ becomes smaller the more gauginos are decoupled from the higgsinos. The landscape draw towards large gaugino masses thus suppressed $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ for the $n = 1$ and 2 cases so that the mass gap peaks at around $5 - 8\,\text{GeV}$. For the uniform scan with $n = 0$, then the gap is larger — typically $10 - 20\,\text{GeV}$.

In figure 4 we show string landscape probability distributions for some strongly interacting sparticles. In frame a), we show the distribution in gluino mass $m_{\tilde{g}}$. From the figure, we see that the $n = 1$ distribution rises to a peak probability around $m_{\tilde{g}} = 3.5\,\text{TeV}$. This may be compared to current LHC13 limits which require $m_{\tilde{g}} \gtrsim 2\,\text{TeV}$ [84, 85]. Thus, it appears LHC13 has *only begun* to explore the relevant string theory predicted mass values. The distribution fall steadily such that essentially no probability exists for $m_{\tilde{g}} \gtrsim 6\,\text{TeV}$. This is because such heavy gluino masses lift the top-squark sector soft terms under RG running so that $\Sigma_u^u(\tilde{t}_{1,2})$ then exceeds 30. For $n = 2$, the distribution is somewhat harder, peaking at around $m_{\tilde{g}} \sim 4.5\,\text{TeV}$. The uniform $n = 0$ distribution peaks around 2 TeV.
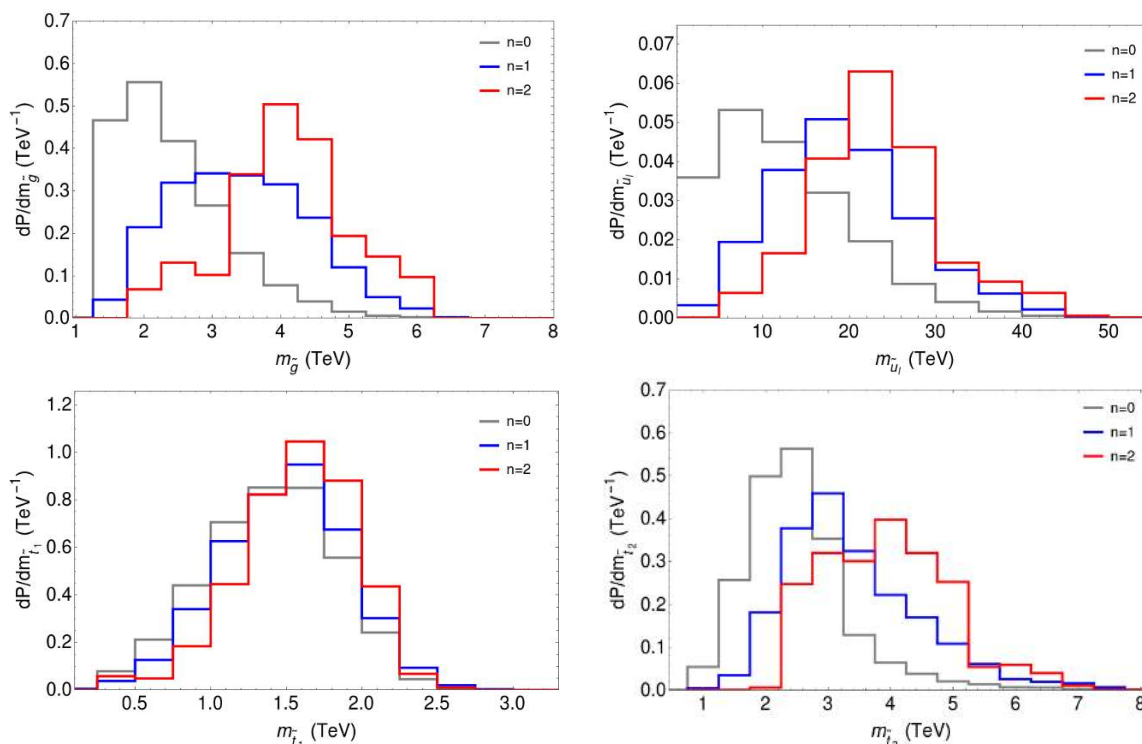
Figure 4. Case **A**: $f_{EWFT} \to \Theta(30 - \Delta_{\mathrm{EW}})$ - Upper panels: probability distributions in $m_{\tilde{g}}$ (left) and $m_{\tilde{u}_L}$ (right). Lower panels: probability distributions in $m_{\tilde{t}_1}$ (left) and $m_{\tilde{t}_2}$ (right). All distributions are shown following eq. (2.11) and in addition rejecting CCB and noEWSB minima. Results for different values of $n = 2n_F + n_D - 1$ are displayed for each plot.

In frame b), we show the distribution versus one of the first generation squark masses $m_{\tilde{u}_L}$. Here, it is found for $n = 1$, 2 that the distribution peaks around $m_{\tilde{q}} \sim 20 - 25\,\mathrm{TeV}$ — well beyond LHC sensitivity, but in the range to provide at least a partial decoupling solution to the SUSY flavor and CP problems. It would also seem to reflect a rather heavy gravitino mass $m_{3/2} \sim 10 - 30\,\mathrm{TeV}$ in accord with a decoupling solution to the cosmological gravitino problem [86]. The $n = 0$ distribution peaks around $m_{\tilde{q}} \sim 8\,\mathrm{TeV}$ and drops steadily to the vicinity of $40\,\mathrm{TeV}$. For much heavier squark masses, then two-loop RGE terms tend to drive the stop sector tachyonic resulting in CCB minima.

In frame c), we show the probability distribution versus $m_{\tilde{t}_1}$. In this case, all three n values lead to a peak around $m_{\tilde{t}_1} \sim 1.5\,\mathrm{TeV}$. While this may seem surprising at first, in the case of $n = 1$, 2 we gain large A_t trilinear terms which lead to large mixing and a diminution of the eigenvalue $m_{\tilde{t}_1}$ [19] even though the soft terms entering the stop mass matrix may be increasing. There is not so much probability below $m_{\tilde{t}_1} = 1\,\mathrm{TeV}$ which corresponds to recent LHC13 mass limits [87, 88]. Thus, again, LHC13 has only begun to explore the predicted string theory parameter space. The distributions taper off such that hardly any probability is left beyond $m_{\tilde{t}_1} \sim 2.5\,\mathrm{TeV}$. This upper limit is apparently within reach of high-energy LHC operating with $\sqrt{s} \sim 27\,\mathrm{TeV}$ where the reach in $m_{\tilde{t}_1}$ extends to about $2.5 - 3\,\mathrm{TeV}$ [89]. In frame d), we show the distribution in $m_{\tilde{t}_2}$. In this case, the

suppression of $m_{\tilde{t}_2}$ from large mixing A_t is far less and so the $n = 1$, 2 distributions peak at higher values $m_{\tilde{t}_2} \sim 3 - 5\,\text{TeV}$ as compared to the uniform $n = 0$ scan where $m_{\tilde{t}_2}$ peaks around $2\,\text{TeV}$. The distributions fall steadily so that hardly any probability exists beyond $m_{\tilde{t}_2} \gtrsim 6\,\text{TeV}$ because the $\Sigma_u^u(\tilde{t}_2)$ values become too large.

Let us summarize our main conclusions from this section. We find that the anthropic requirement of a weak scale not too removed (by a factor 4) from its measured value (which is imposed by requiring $\Delta_{\text{EW}} \leq 30$) centers the low-energy supersymmetric spectrum around central values that are relatively agnostic about the precise distribution of supersymmetry breaking scales in the UV so long as $n \geq 1$. There is some shift in the predicted supersymmetric spectrum as $n = 2n_F + n_D - 1$ is varied, but the shift is relatively minor.

The $n = 0$ case we regard as rather implausible compared to $n = 1$, 2 in that it typically generates $m_h < 123\,\text{GeV}$ (allowing for a couple GeV theory error in our m_h calculation). It is intriguing that the best prediction for $m_h \sim 125\,\text{GeV}$ is obtained with $n = 1$ which corresponds to SUSY breaking dominated by a single auxiliary field F, a situation that is rather common in the literature.

3.1.1 Cases with $n \geq 4$

We have also tried a case with $n = 4$. In that case, the soft term generation became extremely inefficient since almost always one is placed into either CCB or no EWSB vacua or else $\Delta_{\text{EW}} \gg 30$. This may be understood from examining figure 1 of ref. [48]. If the A_0 parameter is generated at too large values compared to $m_0(3)$, then the $m_{\tilde{t}_R}^2$ soft term gets driven to negative values at the weak scale resulting in CCB minima for the scalar potential. If $m_{H_u}^2$ is generated at too large values, then it isn't even driven negative so that electroweak symmetry isn't properly broken.

The situation is illustrated in figure 5 where we plot the locus of $n = 4$ scan points using the scan limits below eq. (3.2). We show for clarity just 100K points although we have generated 1M. The large value of n selects almost always huge values of soft terms which then either lead to invalid scalar potential minima or else, if EW symmetry is properly broken, a huge value for the weak scale due to huge values of $\Sigma_u^u(i)$ or $-m_{H_u}^2(weak)$. The large n scenario only gets worse if we increase the (artificial) scan upper limits from below eq. (3.2). This may be an important result for string model builders in that $n \geq 4$ is difficult to accommodate phenomenologically: realistic vacua with the weak scale $m_{W,Z,h} \sim 100\,\text{GeV}$ seem to prefer $n \sim 1 - 2$.

3.1.2 Varying the Δ_{EW} cutoff

What happens if we vary the cutoff for Δ_{EW}? In figure 6 we show the probability distribution for the Higgs mass m_h for $n = 1$ but for three choices of cutoff $\Delta_{\text{EW}} < 20$, 30 and 40. From the distributions, we see that the m_h distributions slightly hardens with an increasing cutoff but overall $m_h \sim 124 - 126\,\text{GeV}$ is still predicted. In the next subsection we explore what happens using instead the case **B** prescription for f_{EWFT}.
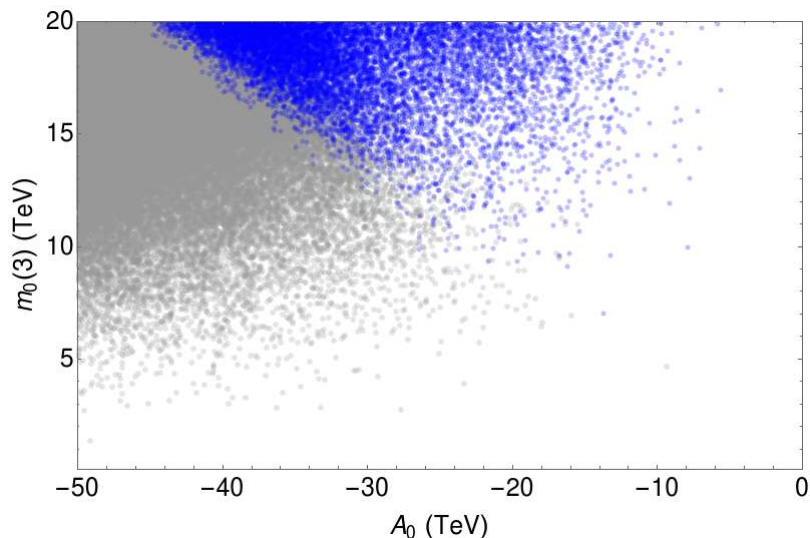
Figure 5. Locus of 100K scan points from a scan with $n = 4$ and scan range as below eq. (3.2). The gray points have either CCB scalar potential minima or no EWSB. The blue points admit EWSB but all have $\Delta_{EW} > 240$ corresponding to a weak scale greater than $\sim 1\,\mathrm{TeV}$.
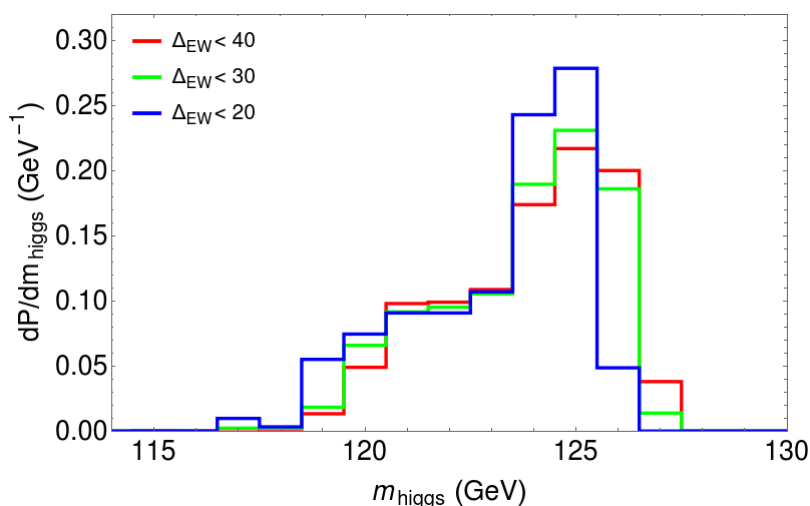

Figure 6. Probability distribution for Higgs mass m_h for the case of $n = 1$ but with varying cutoff $\Delta_{EW} < 20$, 30 and 40.

3.1.3 Conclusions for case A

It would thus appear that when statistical questions of distributions in the landscape are tempered with anthropic requirements, more or less solid predictions about the IR spectrum are obtained. We also note our other main conclusion — the mass of the Higgs comes out close to its observed value — is robust against variations in $n = 1$ or 2 and also against variations in the cutoff value of Δ_{EW}.
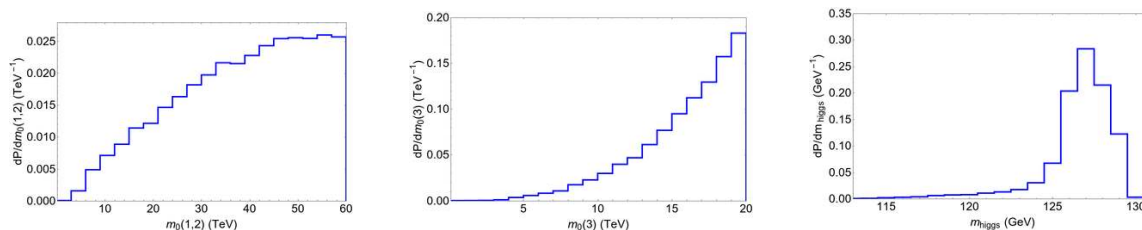
Figure 7. Case **B**: $f_{EWFT} \to \Delta_{\rm EW}^{-1}$: distributions in a) $m_0(1,2)$, b) $m_0(3)$ and c) m_h for $n = 1$.
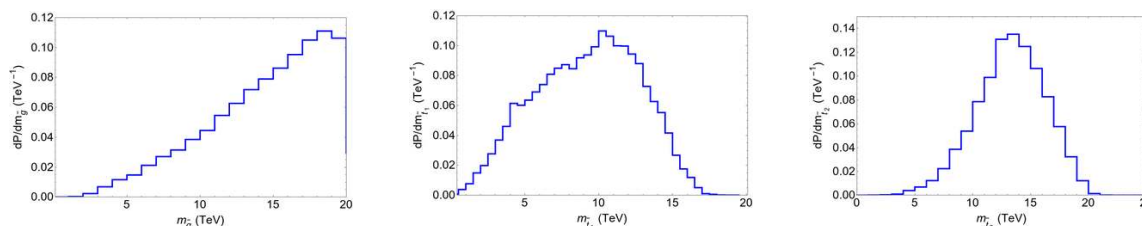

Figure 8. Case **B**: $f_{EWFT} \to \Delta_{\rm EW}^{-1}$: distributions in a) $m_{\tilde{g}}$, b) $m_{\tilde{t}_1}$ and c) $m_{\tilde{t}_2}$ for $n = 1$.

3.2 Case B

In this subsection, we examine the results of our numerical scans using $f_{SUSY} \sim m_{\rm soft}^n$ but now with $f_{EWFT} = \Delta_{\rm EW}^{-1}$. In this case we can veto parameter space points statistically according to a $\Delta_{\rm EW}(min)/\Delta_{\rm EW}$ algorithm or else bin surviving events with a variable weight given by the same factor. In either case, the surviving weights will be penalized by a factor $\Delta_{\rm EW}^{-1}$. Although such a factor penalizes events with a large computed weak scale, it does nonetheless allow many to survive. The question is: is the penalty sufficient to offset the $f_{SUSY} \sim m_{\rm soft}^n$ draw towards large soft terms for $n \geq 1$.

In figure 7, we show our first results from Case **B**. We scan over the same soft parameter ranges as in case **A**. In frame a) (b)), we see the probability distribution of vacua versus first/second generation matter scalar soft masses $m_0(1,2)$ (third generation soft masses $m_0(3)$). For these cases, the $\Delta_{\rm EW}^{-1}$ penalty is insufficient to create an upper bound on matter scalar masses and hence the upper bounds come merely from our scan limits above. For this case, in frame c) we show the vacua probability versus m_h. Here, the value of $m_h \sim 126 - 129 \, {\rm GeV}$ which is a reflection of the rather high values of the soft terms which are allowed. For case **B**, the penalty $\Delta_{\rm EW}^{-1}$ allows for events with far higher values of $m_{\rm weak}$ in the TeV range, well beyond the $\sim 100 \, {\rm GeV}$ value.

In figure 8, we show distributions in a) $m_{\tilde{g}}$, b) $m_{\tilde{t}_1}$ and c) $m_{\tilde{t}_2}$ from the case **B** scan. We see that much higher mass scales are favored due to allowing much higher values of $m_{\rm weak}$. In particular, here values of $m_{\tilde{g}} \sim 20 \, {\rm TeV}$, $m_{\tilde{t}_1} \sim 10 \, {\rm TeV}$ and $m_{\tilde{t}_2} \sim 14 \, {\rm Tev}$ are favored. In this case, the spectra has clearly entered the unnatural region and so we do not pursue the case **B** avenue any further.

4 Implications for collider and dark matter searches

4.1 Colliders

Here we will focus on our case **A** results with $n = 1$ or 2 since these results predict a Higgs boson mass very close to or at its measured value. In this case, we may wish to take the remaining sparticle mass predictions seriously as well. As far as LHC searches go, we have found from figure 4 that there is only a tiny probability that $m_{\tilde{g}}$ lies below the $m_{\tilde{g}} > 2\,\text{TeV}$ mass bound. This means LHC has only begun to explore the string theory parameter space. Recently, the reach of HL-LHC (high luminosity LHC with $\sqrt{s} = 14\,\text{TeV}$ and $\sim 3\,\text{ab}^{-1}$ of integrated luminosity) has been estimated for gluinos [90] and for top squarks [91, 92]: it extends at 5σ level to $m_{\tilde{g}} \sim 2.8\,\text{TeV}$ and $m_{\tilde{t}_1} \sim 1.4\,\text{TeV}$. Thus, from figure 4 we see that there is a large probability that SUSY would escape HL-LHC searches in the gluino pair or top-squark pair production channels. However, the HE-LHC (high energy LHC with $\sqrt{s} = 27\,\text{TeV}$ and $10 - 15\,\text{ab}^{-1}$) has a reach extending to $m_{\tilde{g}} \sim 5.5\,\text{TeV}$ [93] and $m_{\tilde{t}_1} \sim 3\,\text{TeV}$ [89]. This should be enough to cover the probability distributions in figure 4.

Of relevance for HL-LHC searches is the same sign diboson SUSY discovery channel arising from charged/neutral wino pair production in models with light higgsinos [94–96]: $pp \to \widetilde{W}_2 \widetilde{Z}_4 \to (W^{\pm} \widetilde{Z}_{1,2}) + (\widetilde{W}_1^{\mp} W^{\pm})$ where the heavier higgsinos are quasi-visible due to their low visible energy release and the lightest higgsino $\widetilde{Z}_1$, which comprises a portion of dark matter, is completely invisible. The HL-LHC reach in this channel is to $m_{\widetilde{W}_2} \sim 1\,\text{TeV}$ corresponding roughly to $m_{1/2} \sim 1.2\,\text{TeV}$. Again, this covers only a portion of string parameter space from figure 2c).

A final LHC SUSY discovery channel [97–100] arises from direct higgsino pair production $pp \to \widetilde{Z}_1 \widetilde{Z}_2 + jet$ with $\widetilde{Z}_2 \to \widetilde{Z}_1 \ell^+ \ell^-$.[6] This challenging channel is potentially most powerful for SUSY models with light higgsinos although in our case from figure 3d) the expected $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ mass gap is expected to occur in the $4 - 8\,\text{GeV}$ range so the dilepton pair will occur with very low p_T values.[7]

Of course, a higher energy $e^+ e^-$ collider operating with $\sqrt{s} > 2m(higgsino)$ would be able to cover all parameter space and indeed would then function as a *higgsino factory* [104, 105]. In our case, with $\Delta_{\text{EW}} < 30$, this corresponds to higgsino masses below about $350\,\text{GeV}$ so a machine such as ILC with $\sqrt{s} \sim 500 - 700\,\text{GeV}$ may be needed.

4.2 Dark matter searches

For all of our discussion, we have assumed a weak scale $m_{\text{weak}} \lesssim 350\,\text{GeV}$ which corresponds to $\mu \lesssim 350\,\text{GeV}$ so that the lightest higgsino is the lightest SUSY particle and constitutes a portion of dark matter. If some mechanism such as radiative PQ breaking generates the μ parameter, as discussed in section 1, then the remainder of dark matter would be

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[6]A related channel is monojet production from $pp \to \widetilde{Z}_1 \widetilde{Z}_1 j$ production yielding a $jet + \slashed{E}_T$ signature from initial state radiation recoiling against the two WIMPs. This channel has been investigated in ref. [101] where the signal is found to occur at the 1% level compared to SM background from $pp \to Zj$ production with $Z \to \nu \bar{\nu}$ and where signal and BG have very similar $\slashed{E}_T$ and $p_T(jet)$ distributions. Thus, the monojet channel does not seem to be a viable discovery channel for SUSY.

[7]We refer to [102, 103] for some additional LHC studies conducted in this direction.
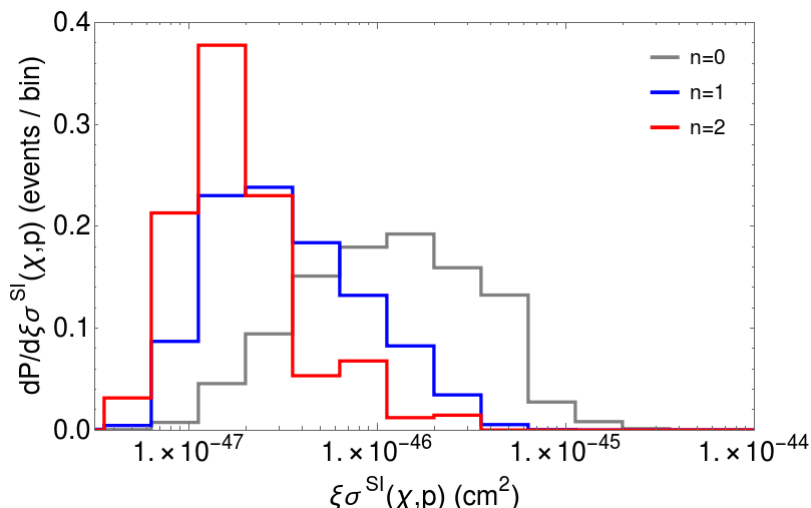
Figure 9. Plot of $dP/d\xi\sigma^{SI}(\widetilde{Z}_1 p)$ versus $\xi\sigma^{SI}(\widetilde{Z}_1 p)$ for case **A** scans with $n = 0, 1$ and 2.

a SUSY DFSZ axion [106]. Calculations of the mixed axion/higgsino dark matter relic density typically predict the bulk of DM to lie in axions (typically 80-90%) while 10-20% lies in higgsino-like WIMPs [107]. Nonetheless, prospects for WIMP detection are good at ton-scale noble liquid detectors even though the WIMP target abundance is typically well below that which is usually assumed. Detailed calculations show multi-ton WIMP detectors should cover all of parameter space [108–110].

In figure 9, we show the distribution $dP/d\xi\sigma^{SI}(\widetilde{Z}_1, p)$ versus $\xi\sigma^{SI}(\widetilde{Z}_1, p)$ for various n values. Current limits from LUX [111] and PandaX [112] require $\xi\sigma^{SI}(\widetilde{Z}_1 p) < 2 \times 10^{-46}$ cm^2 for $m_{\widetilde{Z}_1} \sim 150$ GeV. The quantity $\xi \equiv \Omega^{TP}_{\widetilde{Z}_1} h^2/0.12$ measures the minimal fraction of dark matter as composed of thermally-produced WIMPs rather than axions and is typically $0.05 - 0.1$ for mixed light higgsino/axion dark matter. While about half the parameter space seems explored by ton-scale WIMP detectors for the uniform scan with $n = 0$, the distribution skews to lower values as n increases to 1 or 2. This is because as n increases, the gaugino masses are drawn to larger values while μ remains fixed and the $\widetilde{Z}_1$ becomes more purely higgsino-like. The $\widetilde{Z}_1 - \widetilde{Z}_1 - h$ coupling is a product of gaugino times higgsino components [108–110] so typically decreases as the gaugino-higgsino mass gap increases. Only a small portion of parameter space is ruled out for $n = 1$ or 2 although future probes down to $\sim 10^{-47}$ cm^2 will cover just about all parameter space.

In figure 10 we show the distribution in $dP/d\xi^2\langle\sigma v\rangle$ vs. $\xi^2\langle\sigma v\rangle$ for the case **A** scans with $n = 0, 1$ and 2. Here the ξ factor is squared due to the necessity of having indirect detection of WIMP-WIMP annihilation in the cosmos. The best limits for $m_{\widetilde{Z}_1} \sim 150$ GeV come from Fermi-LAT/MAGIC combined limits [113] on observation of gamma rays from dwarf spheroidal galaxies; these require $\xi^2\langle\sigma v\rangle < 3 \times 10^{-26}$ cm^3/s. As can be seen, all predictions are well below the limits due partly to the depleted WIMP abundance squared. As n increases, the detection rates drop due to increasing sparticle masses which suppress the WIMP annihilation cross section.
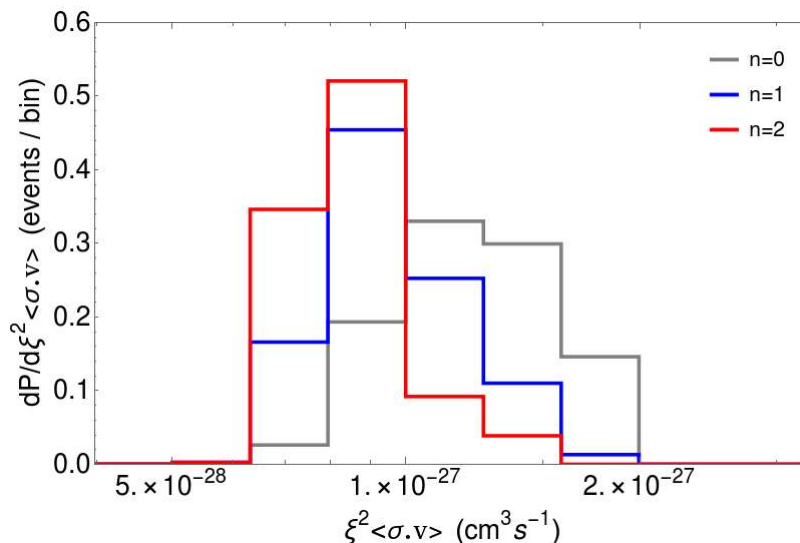
Figure 10. Plot of $dP/d\xi^2 \langle \sigma v \rangle$ versus $\xi^2 \langle \sigma v \rangle$ for case **A** scans with $n = 0$, 1 and 2.

In the case of axions, the SUSY DFSZ axion coupling to photons has been found to be severely diminished (by about an order of magnitude) compared to expectations from non-SUSY models due to the presence of light higgsinos in the axion-γ-γ triangle diagram [114]. Thus, axion detectors which probe much more deeply into small $a\gamma\gamma$ coupling strengths will be needed.

5 The cosmological moduli problem

We have seen in the previous sections that introducing anthropic constraints on the landscape had two kinds of effects on the low energy supersymmetric spectrum: (i) for the vacuum energy, the constraint did not affect the selection of our supersymmetry breaking vacuum; (ii) for the electroweak scale, the constraint had the effect of selecting natural values of the superpartner masses.

A generic issue that affects the kind of arguments we have presented here is the cosmological moduli problem (originally the Polonyi problem, dating from the earliest theories of supergravity [115–117]). The energy density of the Universe can be dominated by moduli fields, which, being gravitationally coupled to matter, can decay at late times. If the lifetime of moduli exceeds the era of Big Bang Nucleosynthesis, then late decay of moduli can disassociate the newly created nuclei and ruin the successful prediction of abundances of the light elements.

Over the last decade and a half, significant progress has been made on the issue of moduli stabilization in string theory [118]. Most moduli acquire masses near the string scale from a combination of effects — fluxes, branes, and strong coupling in the hidden sector. However, one also generally expects moduli which are parametrically lighter than the string scale, and satisfy [120]

$$m_{\text{modulus}} \sim m_{3/2}\,. \tag{5.1}$$

Such light moduli decay around $t \sim M_P^2/m_{\text{modulus}}^3 \sim 10^3$ s for $m_{\text{modulus}} \sim 1\,\text{TeV}$. This clearly interferes with the successful predictions of Big Bang Nucleosynthesis [119, 121, 122]. An equivalent way to express this is in terms of the reheat temperature

$$T_r = c^{1/2} \left(\frac{10.75}{g_*}\right)^{1/4} \left(\frac{m_{\text{modulus}}}{50\,\text{TeV}}\right)^{3/2} T_{BBN} \qquad (5.2)$$

where the decay width is given by $\Gamma = \frac{c}{2\pi} \frac{m_{\text{modulus}}^3}{M_P^2}$.

To avoid conflicts with Big Bang Nucleosynthesis, one thus typically requires

$$m_{\text{modulus}} \geq 50\,\text{TeV} . \qquad (5.3)$$

From the point of view of distributions of permissible vacua, this would introduce a biasing factor

$$dN_{\text{vac}} \sim \Theta(m_{\text{modulus}} - 50\,\text{TeV}) \times f_{EWFT} \times (m_{\text{hidden}}^2)^n d(m_{\text{hidden}}^2) \qquad (5.4)$$

Now, using the fact that $m_{moduli} \sim m_{3/2}$, and the relations between the gravitino mass and soft terms from eq. (3.2), we can recast the condition of avoiding the cosmological moduli problem as

$$dN_{\text{vac}} \sim \Theta(m_0 - c_1 * 50\,\text{TeV}) \times \Theta(m_{1/2} - c_2 * 50\,\text{TeV}) \times f_{EWFT} \times (m_{\text{hidden}}^2)^n d(m_{\text{hidden}}^2) . \qquad (5.5)$$

We then see that there are two opposing tendencies here. The pull to natural solutions, embodied by the f_{EWFT} term, is opposed to the pull for vacua where the moduli problem is avoided, which are the origin of the first two step functions in eq. (5.5). Indeed, in our scan, we specifically imposed upper limits $m_0(1,2) < 60\,\text{TeV}$, $m_0(3) < 20\,\text{TeV}$, and $m_{1/2} < 10\,\text{TeV}$. This was in anticipation of the fact that solutions beyond the imposed upper limits would be cut off by the requirement on Δ_{EW}, leading to inefficient scanning. However, these larger values of the soft terms turn out to be precisely the ones needed to solve the moduli problem.

In our opinion, this points to the fact that regions of the landscape where the coefficients c_1 and c_2 are small [120]

$$c_1 \sim c_2 \sim \mathcal{O}(1/10 - 1/100) \qquad (5.6)$$

are preferred. This would correspond, for example, to regions where the mediation scheme follows a mirage pattern [123, 124].

6 Summary and conclusions

In this paper we have implemented a statistical calculation of the SUSY breaking scale assuming a fertile patch of the string landscape where the low energy effective theory is comprised of the MSSM plus a hidden sector as described by $N = 1$ $d = 4$ supergravity with SUGRA assumed spontaneously broken via the super-Higgs mechanism. We have further assumed the existence of a vast array of scalar potential minima leading to different SUSY breaking scales. It is assumed that an assortment of SUSY breaking F and D terms are

present and that their vevs are uniformly distributed. Such an assumption leads generally to the expectation of a landscape pull towards large values of hidden sector mass scales favored by a power law behavior $f_{SUSY} \sim (m_{\text{hidden}}^2)^{2n_F + n_D - 1}$ which at first glance would seem to favor high scale SUSY breaking for $n = 2n_F + n_D - 1 \geq 1$. If such were the case — then provided electroweak symmetry even breaks properly — one would expect a value of the weak scale far beyond its measured value characterized by $m_{W,Z,h} \sim 100\,\text{GeV}$. A huge value of the weak scale would lead to far heavier particle masses and a suppression of weak interactions as we know them, and quite likely to a universe not conducive to complexity and life as we know it.

As in Weinberg's estimate of the magnitude of the cosmological constant, one may then assume an anthropic selection of weak scale values not-too-far from its measured value. Requiring in addition a not-too-large value for the weak scale, corresponding to $\Delta_{\text{EW}} \lesssim 30$ or $m_Z \lesssim 350\,\text{GeV}$ (or $f_{EWFT} = \Theta(30 - \Delta_{\text{EW}})$), we are able to compute superparticle and Higgs mass probability distributions for any assumed value of n. Remarkably, we find that for the simplest case, $n_F = 1$, $n_D = 0$ yielding a linear draw of $f_{SUSY} \sim m_{\text{soft}}^1$, that the Higgs mass m_h probability distribution is sharply peaked at $m_h \simeq 125\,\text{GeV}$. The $n = 2$ result gives $m_h \sim 126\,\text{GeV}$ while a uniform scan corresponding to $n_F = 0$ $n_D = 1$ usually yields too low a value of m_h (although $m_h \sim 125\,\text{GeV}$ is still possible — see figure 3). Values of $n \geq 3$ leads to a hard pull on soft terms that tend to place one in a situation with either CCB vacua or vacua without electroweak symmetry breaking. Thus, our results favor a rather simple hidden sector for SUSY breaking leading to $n \sim 1$ or 2. Higher $n \geq 3$ values as might be expected for instance from F-theory constructions [125, 126] will have difficulty in generating a proper breakdown of electroweak symmetry.

We also examined a different anthropic suppression factor $f_{SUSY} \sim \Delta_{\text{EW}}^{-1}$ which penalizes large values of m_{weak} but does not eliminate them. This anthropic suppression allows for much higher SUSY breaking scales and typically too large a value of m_h. A combination of the two — $f_{EWFT} \sim \Theta(30 - \Delta_{\text{EW}}) \cdot \Delta_{\text{EW}}^{-1}$ — leads back to results similar to case **A** with $m_h \sim 125\,\text{GeV}$.

From our $n = 1$, 2 results which favor a value $m_h \sim 125\,\text{GeV}$, then we also expect

- $m_{\tilde{g}} \sim 4 \pm 2\,\text{TeV}$,

- $m_{\tilde{t}_1} \sim 1.5 \pm 0.5\,\text{TeV}$,

- $m_A \sim 3 \pm 2\,\text{TeV}$,

- $\tan\beta \sim 13 \pm 7$,

- $m_{\widetilde{W}_1, \tilde{Z}_{1,2}} \sim 200 \pm 100\,\text{GeV}$ and

- $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 7 \pm 3\,\text{GeV}$ with

- $m(\tilde{q}, \tilde{\ell}) \sim 20 \pm 10\,\text{TeV}$ (for first/second generation matter scalars).

These results can provide some guidance as to SUSY searches at future colliders and also a convincing rationale for why SUSY has so far eluded discovery at LHC. They provide

a rationale for why SUSY might contain its own decoupling solution to the SUSY flavor and CP problems and the cosmological gravitino and moduli problems. They predict that precision electroweak and Higgs coupling measurements should look very SM-like until the emergence of superpartners at LHC and/or ILC. They also help explain why no WIMP signal has been seen: dark matter may be higgsino-like-WIMP plus axion admixture with far fewer WIMP targets than one might expect under a WIMP-only dark matter hypothesis. The rather large value of $m_{3/2}$ expected from these results points perhaps towards mirage mediation [123, 124] as another lucrative scenario.

Acknowledgments

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