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Higgs as (Pseudo-) Goldstone Particles

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Higgs scalars may be (pseudo-) Goldstone particles related to a spontaneous breakdown of a global symmetry which contains grand unified gauge group as a subgroup. This hypothesis gives severe constraint on possible form of Higgs mass terms in a framework of supersymmetric grand unified theories coupled to $N=1$ supergravity.

Remarkably, their tree level mass terms, both supersymmetric and soft-breaking, are definitely given by gravitino mass $m_{3/2}$ and do not depend on a detailed structure of a superpotential of GUT sector. These mass terms are best desirable for the relatively light top quark ($m_t \lesssim 50$ GeV) suggested at CERN to be active in the radiative $SU(2)\times U(1)$ breaking scenario.

§ 1.

Higgs scalars, in spite of their indispensable roles, have been mysterious and even troublesome existence throughout every stage of the progress of unified gauge theories. Although supersymmetry shed important light on them[1] and successive attempts gave some valuable hints,[2),3)] we are still far from the position to clarify their nature.

In the last couple of years some attempts have been made to assign leptons and quarks (and Higgs scalars) to (pseudo-) Goldstone multiplets in the supersymmetric non-linear σ-model in a context of composite models to get natural explanation of their lightness.[4] Even if we do not rush into composite models, this idea is appealing and suggestive enough within the standard scenario of grand unified theories (GUTs) when we recall the well-known problem of doublet-triplet Higgs mass splitting and related mess. Surely it will be most comfortable if we can attribute lightness of Higgs doublets to the Goldstone theorem supplemented by non-renormalization theorem. That is, Higgs scalars are light because they are (p-) Goldstone bosons due to spontaneous breakdown of some global symmetry G which is large enough to contain GUT gauge group $G_{\rm GUT}$ as a subgroup.

Consider, for example, a theory possessing a global symmetry $SU(6)$. The subgroup $SU(5)$ is gauged. Suppose the vacuum of the theory breaks $SU(6)$ down to $SU(4)\times SU(2)\times U(1)$ and at the same time $SU(5)$ to $SU(3)\times SU(2)\times U(1)$ ($SU(3)$ is embedded in $SU(4)$).[4] The (p-) Goldstone modes due to the spontaneous breakdown of $SU(6)$ are

$$(\mathbf{4},\mathbf{2})+(\mathbf{4},\mathbf{2})^*$$

of $SU(4)\times SU(2)$. Their $SU(3)\times SU(2)$ decomposition is

$$(\mathbf{3},\mathbf{2})+(\mathbf{3},\mathbf{2})^*+(\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2})^*\,.$$

Apparently the first two multiplets are (p-) Goldstone modes due to spontaneous breakdown of $SU(5)$ and are absorbed to massive vector multiplets. The remaining two multiplets have quantum numbers which precisely coincide with those of Higgs doublets H_1 and H_2 which are indispensable building blocks of supersymmetric (SUSY) $SU(3)$

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$\times SU(2)\times U(1)$ theory.

Though the global symmetry G is explicitly broken by gauge interaction of G_{GUT}, H_1 and H_2 remain massless to all orders in perturbation theory, as far as supersymmetry is exact, due to the non-renormalization theorem. In order to make the model realistic, we must introduce soft SUSY-breaking terms. The best suitable way will be to couple the model to spontaneously broken $N=1$ supergravity theory in the standard manner.[5] Then H_1 and H_2 receive mass terms which are expected to be of order of the gravitino mass $m_{3/2}$.

In this paper we clarify some consequences of this scenario within the context of $N=1$ SUSY GUTs coupled to $N=1$ supergravity, paying special attention to mass terms of these Higgs multiplets. In order to avoid unwanted complexities, we neglect D-term gauge contribution to the bosonic potential by simply assuming that we are always working in the vacuum with $\langle D\rangle=0$.

§ 2.

In order to see how our hypothesis is restrictive, we first look into the consequence of the "standard" SUSY GUT model where the lightness of Higgs scalars is realized by the fine tuning. The simplest version of the model has a superpotential

$$W=\alpha\left(\frac{1}{3}\Sigma^3-\frac{1}{2}M\Sigma^2\right)+\beta H_1H_2(\Sigma-M')+W_R\,, \tag{1}$$

where for simplicity superfields (or their scalar components) Σ, H_1 and H_2 are assumed to be single component and W_R contains rest of the terms which are irrelevant for the following discussions and is disregarded hereafter. According to the standard scenario we concentrate on the vacuum specified by vacuum expectation values (VEVs)

$$\langle\Sigma\rangle_0=M\,,\qquad \langle H_1\rangle_0=\langle H_2\rangle_0=0\,, \tag{2}$$

which are a solution of supersymmetric vacuum equation

$$\frac{\partial W}{\partial z_i}=0\,.\qquad (z_i=\Sigma,\ H_1,\ H_2)$$

The supersymmetric mass of Higgs scalars H_1 and H_2, which is defined by that of their fermionic partners, is given in general as

$$m(H-\text{SUSY})=\left\langle\frac{\partial^2W}{\partial H_1\partial H_2}\right\rangle=\langle\beta(\Sigma-M')\rangle\,. \tag{3}$$

Thus in order to get light H's, we have to fine-tune parameters as follows:

$$M=M'+\frac{m_S}{\beta}\,, \tag{4}$$

where M and M' are of order of unification mass M_X and m_S of order of the weak mass scale m_W.

Now let us couple this model to $N=1$ supergravity theory with spontaneous SUSY breaking in a hidden sector. According to the standard procedure, the bosonic potential in the "flat" limit is given in general as[5]

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$$V(z)=\left|\frac{\partial W(z)}{\partial z_i}+m_1 z_i^*\right|^2+m_2^2 z_i z_i^*+[m_3 W(z)+\text{h.c.}]\,, \tag{5}$$

where m_1, m_2 and m_3 are all of order m_W and parametrize the effective soft SUSY breaking, and also summation over $z_i=\Sigma$, H_1, H_2 is understood. Due to the existence of soft breaking terms, the VEV of Σ which minimize this potential shifts from its SUSY limit value Eq.(2). The amount of this shift in the leading order $O(m)$ is determined by minimizing the first term in Eq.(5) because only this term contains leading order $(O(m^2M^2))$ nontrivial terms under variation of Σ around M. Thus we get $\langle\Sigma\rangle=M-m_1/\alpha+O(m^2/M)$ and this determines the supersymmetric Higgs mass as

$$m(H-\text{SUSY})=m_S-\frac{\beta}{\alpha}m_1\,. \tag{6}$$

In order to get SUSY breaking Higgs mass terms to the order $O(m^2)$, we must examine the VEV of Σ to the order $O(m^2/M)$ retaining second and third terms in Eq.(5). The result is

$$\langle\Sigma\rangle=M-\frac{m_1}{\alpha}+\frac{1}{\alpha^2 M}(m_1m_3-m_2^2)+O(m^3/M^2)\,. \tag{7}$$

By using this result we obtain the full Higgs mass terms:

$$\left\langle\frac{\partial^2 V}{\partial H_1\partial H_1^*}\right\rangle=\left\langle\frac{\partial^2 V}{\partial H_2\partial H_2^*}\right\rangle=|m(H-\text{SUSY})|^2+|m_1|^2+m_2^2\,, \tag{8}$$

$$\left\langle\frac{\partial^2 V}{\partial H_1\partial H_2}\right\rangle=m(H-\text{SUSY})(2m_1^*+m_3)+\frac{\beta}{\alpha}(m_1m_3-m_2^2)\,. \tag{9}$$

From this result we see that the Higgs mass terms are not only dependent on supergravity coupling parameters m_1, m_2 and m_3, but also deeply dependent on GUT parameters α, β and m_S. They are essentially free parameters even after we fix the values of m_1, m_2 and m_3.

Now imagine what happens if H_1 and H_2 are (p-) Goldstone particles. Because they must be massless at global SUSY limit (m_1, m_2, $m_3\to 0$), m_S must vanish or equivalently $M=M'$ in Eq. (1). This is plausible because it seems to invoke some global symmetry which transforms Σ and H's into each other. Now it is not so hard to imagine that this symmetry also requires definite relation between cubic coupling constants α and β in Eq. (1). This relation can be obtained by realizing that the mass terms of H_1 and H_2

$$\begin{aligned}V(H\text{-mass})=&\left\{\left(1+\left|\frac{\beta}{\alpha}\right|^2\right)|m_1|^2+m_2^2\right\}(H_1^*H_1+H_2^*H_2)\\&-\left\{\frac{\beta}{\alpha}(2|m_1|^2+m_2^2)H_1H_2+\text{h.c.}\right\}\end{aligned} \tag{10}$$

should contain zero-mode irrespective of m_1 and m_2 corresponding to a true Goldstone mode because the supergravity coupling effect in Eq.(5) does not disturb the global symmetry. This requires

$$|\beta/\alpha|=1\,.$$

Thus we arrive at the conclusion that in a suitable phase convention the supersymmetric

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Higgs mass is given by the superpotential

$$W(H-\text{mass})=m_1H_1H_2 \tag{11}$$

and the total Higgs mass terms are given as

$$V(H\text{-mass})=(2|m_1|^2+m_2^2)(H_1^*H_1+H_2^*H_2+H_1H_2+H_1^*H_2^*)\,. \tag{12}$$

Especially in the "minimal" supergravity coupling model, m_1, m_2 and m_3 are given as[5]

$$m_1=m_{3/2}\,,\qquad m_2=0\,,\qquad m_3=(A-3)m_{3/2}\,.$$

Notice that our results Eqs. (11) and (12) are independent of the parameter A. Though the results Eqs. (11) and (12) are derived from the simplest model Eq. (1) by using intuitive arguments, we will see in the following that they hold quite generally for wide class of models.

§ 3.

Our basic assumption is that the theory has a global symmetry G under which chiral multiplets z_i are transformed as

$$\delta z_i=i\varepsilon^A(T^A)_i{}^jz_j\,, \tag{13}$$

where $(T^A)_i{}^j$ is a hermitian representation matrix of a generator of G and ε^A is a real transformation parameter. The invariance of the superpotential $W(z)$ under this transformation gives identities

$$\frac{\partial W}{\partial z_i}(T^A)_i{}^jz_j=0\,, \tag{14·a}$$

$$\frac{\partial^2 W}{\partial z_i\partial z_j}(T^A)_j{}^kz_k+\frac{\partial W}{\partial z_j}(T^A)_j{}^i=0\,. \tag{14·b}$$

The vacuum and particle spectra of the theory are characterized by the following VEVs:

$$Z_i\equiv\langle z_i\rangle,\qquad F_i^*\equiv\left\langle\frac{\partial W}{\partial z_i}\right\rangle\,,\qquad M^{ij}\equiv\left\langle\frac{\partial^2 W}{\partial z_i\partial z_j}\right\rangle\,. \tag{15}$$

These VEVs break the global symmetry G down to H, and there appear massless Goldstone bosons associated with broken generators of G/H. In the SUSY limit $(F^i\to 0)$, these Goldstone bosons lie definitely in chiral multiplets.[4),6)] These Goldstone chiral multiplets z_a are characterized by the matrix

$$\mathcal{T}^A{}_i\equiv(T^A)_i{}^jZ_j \tag{16}$$

and are given as

$$z_a=U^a{}_i{}^*(z_i-Z_i) \tag{17}$$

with

$$U^a{}_i=\mathcal{T}^A{}_iN^a{}_A\,, \tag{18}$$

where $N^a{}_A$ are defined so as to orthonormalize $U^a{}_i$:

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$$U^a{}_i U^b{}_i{}^* = \delta^a{}_b .$$

They are given explicitly as

$$N^a{}_A = \frac{1}{\sqrt{\pi^a}} V^a{}_A , \tag{19}$$

where π^a and $V^a{}_A$ are non-zero eigenvalues and associated normalized eigenvectors of the hermitian positive semi-definite matrix

$$\pi^{AB} = \mathcal{T}_i{}^{A*} \mathcal{T}_i{}^B = Z^\dagger T^A T^B Z . \tag{20}$$

Thus we see that there is one to one correspondence between these eigenvectors $V^a{}_A$ and Goldstone chiral multiplets z_a. On the other hand, broken generators, which have one to one correspondence to massless Goldstone bosons, are determined by eigenvectors (associated with non-zero eigenvalues) of a real symmetric matrix

$$S^{AB} = \frac{1}{2} Z^\dagger \{T^A T^B\} Z . \tag{21}$$

If these two matrices π^{AB} and S^{AB} coincide, that is,

$$S^{AB} = \pi^{AB} \tag{22}$$

then we get one to one correspondence between Goldstone bosons and Goldstone chiral multiplets. Thus one Goldstone chiral multiplet contains only one Goldstone boson and its scalar partner is pseudo-Goldstone boson. This case is called total doubling case.[4),6),7)] The necessary and sufficient condition for Eq. (22) is

$$D^A = Z^\dagger T^A Z = 0 \tag{23}$$

for all generators T^A of G. Remarkably if the unbroken symmetry H contains Cartan subgroup of G, this condition is automatically satisfied and the total doubling is inevitable.[7)] If D^A is not identically zero, the correspondence between Goldstone bosons and Goldstone chiral multiplets becomes incomplete, and in the extreme case where the rank of π^{AB} is smaller than that of S^{AB}, it becomes possible that both of the spinless components of a Goldstone chiral multiplet are occupied by two Goldstone bosons.

In the following we work only on models where Eq. (23) is satisfied for all generators T^A of G. This condition for generators T^α of gauged group G_{GUT} is the usual hypothesis of the standard SUSY GUTs scenario to prevent huge SUSY breaking in the observable sector. Furthermore, since we neglect D-term gauge contribution to the potential, consistency of the following analysis also requires $D^\alpha = 0$ at least for those which belong to gauge group G_{GUT}. Also it should be noticed that any field configuration with non-vanishing D^A can always be transformed to that with vanishing D^A by a suitable complexified global transformation (Eq. (13) with complex transformation parameters ε^A) which is the symmetry transformation of the superpotential $W(z)$.[8)] The proof of this statement is given in the Appendix. Since the soft SUSY breaking terms due to $N=1$ SUGRA coupling are not invariant under this complexified transformation, there is, in general, a possibility that the D-terms get non-vanishing values due to this SUSY breaking effect and may give physically interesting effects. But these are out of our present investigation. We will see that this condition gives great simplification to our analysis.

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The VEVs equation (15) are determined to minimize the potential

$$V(z)=\left|\frac{\partial W}{\partial z_i}+m_1 z_i^*\right|^2+m_2^2 z_i^* z_i+[m_3 W(z)+\text{h.c.}]\,. \tag{24}$$

Thus they satisfy the vacuum equation

$$(F_k+m_1^* Z_k)M^{ki}+(F_i^*+m_1 Z_i^*)m_1^*+m_2^2 Z_i^*+m_3 F_i^*=0\,. \tag{25}$$

Now we make some physically plausible assumptions on the solution of this equation. First we demand, according to the standard SUSY GUTs scenario, that VEVs equation (15) have smooth limit with respect to (m_1, m_2, $m_3 \to 0$), and the vacuum in this limit expresses the exact SUSY world, that is, $F_i|_{m_1,m_2,m_3=0}=0$. Next we demand that those chiral multiplets which are neither Goldstone chiral multiplets nor those whose mass terms are forbidden by unbroken symmetry H, have masses of order M_X. This is the straightforward extension of our hypothesis "Higgs as (pseudo-) Goldstone particles."

Now we look at the vacuum equation, Eq. (25). Here we only need to pay attention to H-singlet fields because only they have nonvanishing VEVs. At a glance we see that the first term of Eq. (25) must vanish in leading order because only this term contains $O(mM^2)$ quantities.[9] Therefore if M^{ij} is an $O(M)$ regular matrix, we obtain

$$F_k+m_1^* Z_k=O(m^2)\,. \tag{26}$$

Unfortunately, M^{ij} contains in general Goldstone modes whose masses are expected to be much smaller than M_X, that is, it is not an $O(M)$ regular matrix. From identities Eq. (14), however, we get

$$(F_k^*+m_1 Z_k^*)\mathcal{T}^A{}_k=m_1 D^A\,. \tag{27}$$

Therefore in the present case where $D^A=0$, the vector $F_k^*+m_1 Z_k^*$ is orthogonal to the Goldstone modes, therefore Goldstone modes in M^{ij} completely decouple in Eq. (25), and the naive expectation Eq.(26) holds precisely.

The SUSY mass terms of Goldstone chiral multiplets are given in general as

$$M^{ab}=M^{ij}U_i^a U_j^b=-\frac{1}{\sqrt{\pi^a\pi^b}}V_A{}^a V_B{}^b F_i^*(T^A T^B)_i{}^j Z_j\,, \tag{28}$$

where the identity Eq.(14·b) has been used in the second equality. The previous result Eq. (26) then gives

$$M^{ab}=\frac{m_1}{\sqrt{\pi^a\pi^b}}V_A{}^a V_B{}^b \pi^{AB}=\frac{m_1}{\sqrt{\pi^a\pi^b}}\pi^b V_A{}^a V_A{}^b\,.$$

Furthermore, $D^A=0$ implies $\pi^{AB}=S^{AB}$ is a real symmetric matrix and so eigenvectors $V_A{}^a$ are essentially real vectors. Therefore in this phase conventions,

$$V_A{}^a V_A{}^b=\delta^{ab}$$

and we get

$$M^{ab}=m_1\delta^{ab}\,. \tag{29}$$

Also it is straightforward to show that the mixing mass matrix elements between Goldstone modes and massive ($O(M)$ modes are suppressed as $O(m^2/M)$ and completely

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negligible.

The mass matrices of (p-) Goldstone bosons

$$\left\langle \frac{\partial^2 V}{\partial z_a{}^* \partial z_b} \right\rangle = U_i{}^{a*} U_j{}^b \left\langle \frac{\partial^2 V}{\partial z_i{}^* \partial z_j} \right\rangle , \tag{30·a}$$

$$\left\langle \frac{\partial^2 V}{\partial z_a \partial z_b} \right\rangle = U_i{}^a U_j{}^b \left\langle \frac{\partial^2 V}{\partial z_i \partial z_j} \right\rangle \tag{30·b}$$

are also related to the SUSY mass matrix M^{ab} through the relation

$$\left\langle \frac{\partial^2 V}{\partial z_i{}^* \partial z_j} \right\rangle = M^{ki*} M^{kj} + (|m_1|^2 + m_2{}^2) \delta_i{}^j \tag{31}$$

and the supplemented relation

$$\left\langle \frac{\partial^2 V}{\partial z_i \partial z_j} \right\rangle \mathcal{T}_j{}^A - \left\langle \frac{\partial^2 V}{\partial z_i \partial z_j{}^*} \right\rangle \mathcal{T}_j{}^{A*} = 0 \tag{32}$$

which comes from the invariance of the potential V under the transformation of G, Eq. (13).

Thus we arrive at a conclusion that in the case $D^A = 0$, SUSY masses of Goldstone chiral multiplets are given by the superpotential

$$W(\text{mass}) = \frac{1}{2} m_1 z_a z_a \tag{33}$$

and total mass terms of Goldstone and p-Goldstone bosons are given as

$$V(\text{mass}) = (2|m_1|^2 + m_2{}^2) z_a{}^* z_a + (2|m_1|^2 + m_2{}^2) \frac{1}{2} (z_a z_a + \text{h.c.}) . \tag{34}$$

In the case where there are only two Goldstone chiral multiplets z_1 and z_2, we define

$$H_1 = \frac{1}{\sqrt{2}} (z_1 + i z_2) , \qquad H_2 = \frac{1}{\sqrt{2}} (z_1 - i z_2)$$

and we obtain

$$W(\text{mass}) = m_1 H_1 H_2 ,$$

$$V(\text{mass}) = (2|m_1|^2 + m_2{}^2)(H_1{}^* H_1 + H_2{}^* H_2) + (2|m_1|^2 + m_2{}^2)(H_1 H_2 + \text{h.c.})$$

which precisely coincide with Eqs.(11) and (12).

§ 4.

Now we briefly discuss the phenomenological implication of the scheme. As was stressed earlier, once we assume that Higgs bosons are (pseudo-) Goldstone particles, their masses, both supersymmetric and soft breaking ones, are completely fixed by parameters m_1 and m_2 irrespective of the detailed structure of the GUT superpotential. If the low energy theory is a minimal supersymmetric $SU(3) \times SU(2) \times U(1)$ theory which contains two Higgs doublet H_1 and H_2, their supersymmetric mass term is given by the superpotential

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$$W(H-\text{mass})=m_1H_1H_2 \tag{35}$$

and total mass terms in the potential are given as

$$V(H-\text{mass})=(2|m_1|^2+m_2^2)(H_1^\dagger H_1+H_2^\dagger H_2+H_1H_2+H_1^*H_2^*)\,, \tag{36}$$

where a contraction of $SU(2)$ indices should be properly understood. In the usual "minimal" $N=1$ supergravity coupling model, m_2 vanishes and m_1 is nothing but a gravitino mass $m_{3/2}$. Of course we must take into account the radiative corrections to these mass terms. Strictly speaking, there is a correction due to renormalization effect between energy range $M_X \lesssim \mu < M_{\text{Planck}}$ governed by GUT Lagrangian. However, this effect is expected to be order $\alpha \ln M_X/M_{\text{Planck}}$ and here we optimistically neglect it in

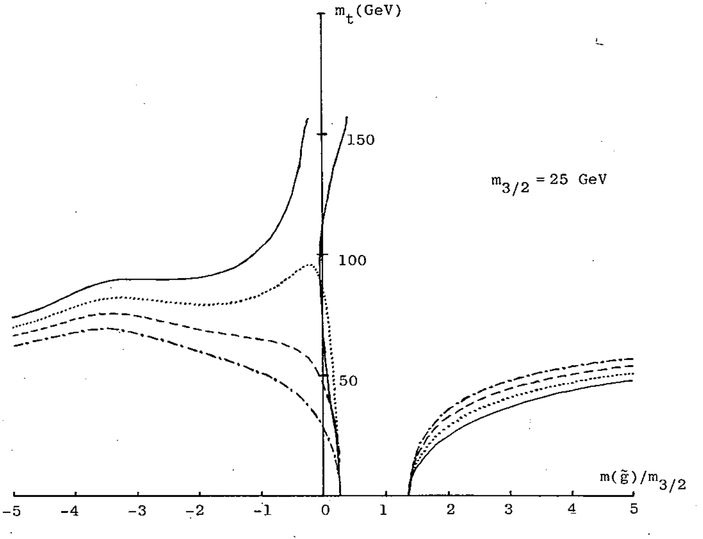
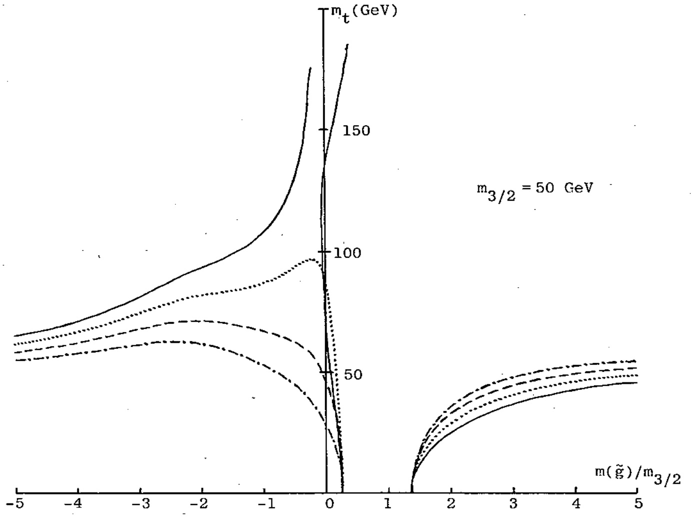


Fig. 1.(a) Top quark mass as a function of $m(\tilde{g})/m_{3/2}$ in the case of $m_{3/2}=25\,\text{GeV}$. The solid line corresponds to the case $A=3$, the dotted line to $A=1$, the dashed line to $A=-1$ and the dot-dashed line to $A=-3$.



(b) The same as (a) for $m_{3/2}=50\,\text{GeV}$.

(continued)

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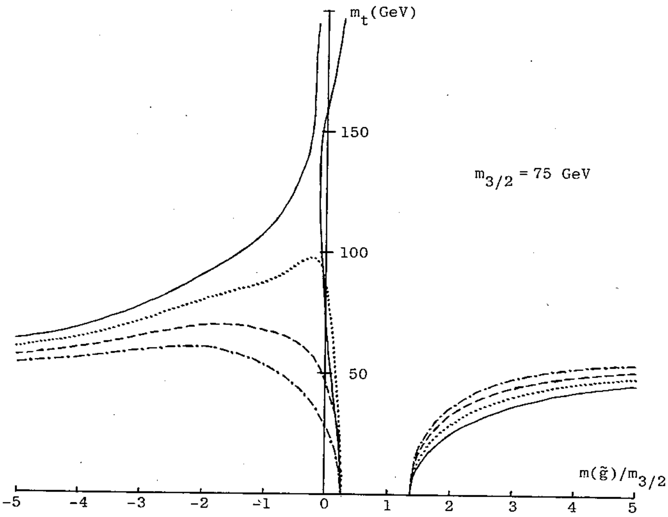
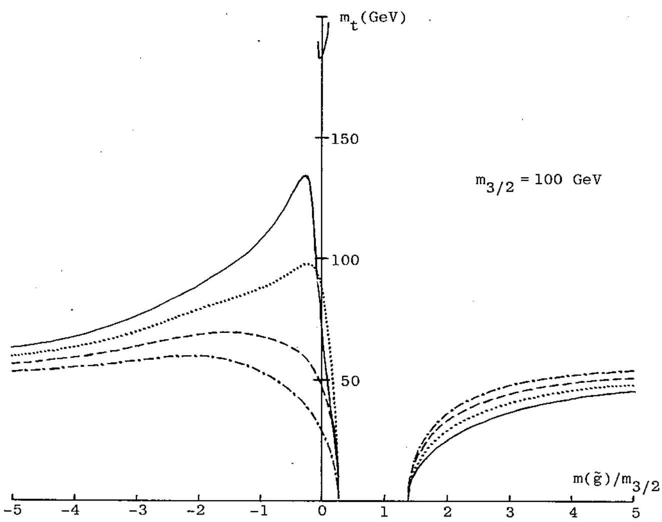


(c) The same as (a) for $m_{3/2}$=75 GeV.



(d) The same as (a) for $m_{3/2}$=100 GeV.

comparison with the radiative correction of order $\alpha \ln M_X/m_W$ due to the low energy effective theory. Thus we regard Eqs.(35) and (36) as a boundary condition for each operator in the low energy effective theory renormalized at $\mu = M_X$.

Now it is customary to work in minimal low energy theory with three generations of leptons and quarks where the top quark Yukawa coupling triggers the radiative $SU(2) \times U(1)$ breaking. Here we notice a remarkable feature of Eq. (36). This potential is flat in the direction $H_1 \sim H_2^*$. This is an automatic consequence of our scheme. It is nothing but a Goldstone mode. Therefore even a tiny radiative effect of top quark Yukawa coupling can occasionally pull down the potential to a negative value, that is, it admits a relatively light top quark ($m_t < 60$ GeV) which is suggested by the CERN SPS

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Table I. Masses of sleptons ($\tilde{\nu}$, $\tilde{e}$), squarks ($\tilde{u}$, $\tilde{d}$, $\tilde{t}$, $\tilde{b}$), gluino ($\tilde{g}$), winos ($\tilde{W}^{\pm}$), neutralinos ($\tilde{N}^0$), charged Higgs scalar ($H^{\pm}$) and neutral Higgs scalars (H^0) for $m_{3/2}$=25, 50 and 75 GeV, in the case $A=1$ and $A=3$. The suffices $i(=1, 2, 3)$ and $j(=1, 2)$ are generation indices. For the mixing effects of scalars, only the top quark Yukawa coupling is taken into account, which is fixed to give the top quark mass $m_t=42$ GeV.

	$A=3$				(GeV)		$A=1$				(GeV)	
	$m_{3/2}=25$		$m_{3/2}=50$		$m_{3/2}=75$		$m_{3/2}=25$		$m_{3/2}=50$		$m_{3/2}=75$	
	I	II	I	II	I	II	I	II	I	II	I	II
$\tilde{\nu}_L{}^i$	23	62		145	75	229	24	55	50	123	75	193
$\tilde{e}_L{}^i$	26	79		157	76	238	26	69	51	134	76	201
$\tilde{e}_R{}^i$	26	52		96	76	143	26	46	51	85	76	126
$\tilde{u}_L{}^j$	25	292		609	78	942	28	251	57	508	85	778
$\tilde{d}_L{}^j$	28	296		612	80	944	29	255	58	511	87	780
$\tilde{u}_R{}^j$	26	286		596	78	921	28	246	57	497	85	761
$\tilde{d}_R{}^i$	27	287		595	79	919	29	247	57	497	85	759
$\tilde{t}_L-\tilde{t}_R$	65	321		631	112	957	54	278	74	529	98	794
	16	249		556	17	878	46	214	62	462	85	724
$\tilde{b}_L$	26	293		606	74	935	29	252	56	506	84	773
$\tilde{g}$	8.8	316		657	26	1020	15	272	30	548	45	837
$\tilde{W}_h{}^{\pm}$	100	122		186	148	263	99	113	122	162	147	222
$\tilde{W}_l{}^{\pm}$	67	72		89	49	118	68	77	59	94	53	122
$\tilde{N}_1{}^0$	111	125		187	158	263	110	117	132	163	157	223
$\tilde{N}_2{}^0$	78	89		104	106	129	78	92	71	108	105	133
$\tilde{N}_3{}^0$	35	44		86	58	128	35	38	68	74	61	111
$\tilde{N}_4{}^0$	1.3	33		64	3.7	98	2	34	4.3	64	6.4	96
$H^{\pm}$	100	119		201	193	296	101	116	145	462	198	272
$H_1{}^0$	58	86		184	175	284	60	82	120	188	180	259
$H_2{}^0$	109	124		201	198	294	110	123	152	170	202	272
$H_3{}^0$	1.3	24		45	5.8	52	0.6	17	1.7	190	2.4	39

experiment.[10] The detailed method of the analysis is available in the literature.[2),11)] In Fig. 1 we give a value of top quark mass which is required to reproduce right W and Z masses in radiative $SU(2)\times U(1)$ breaking scenario as a function of $m(\tilde{g})/m_{3/2}$ where $m(\tilde{g})$ is a common gaugino mass at $\mu\simeq M_X$ for $m_{3/2}$=25, 50, 75, 100 GeV and $A=3, 1, -1, -3$. All these graphs have the common structure. Especially there are two solutions which reproduce experimentally suggested top quark mass $m_t\sim40$ GeV. One solution (type II) requires $m(\tilde{g})/m_{3/2}\gtrsim2$, that is, relatively heavy gauginos, on the other hand the alternative solution (type I) admits light gauginos $|m(\tilde{g})/m_{3/2}|\ll1$. In general gaugino mass $m(\tilde{g})$ feeds masses to squarks and sleptons through radiative correction.[2)] Therefore in the former case (heavy gaugino) most of sparticles become considerably massive and it is phenomenologically less appealing than the latter case (light gauginos). In Table I we give numerical result for particle masses for each value of parameters A and $m_{3/2}$ which reproduce a top quark mass $m_t=42$ GeV. The blank in type I solutions are due to the negative mass square of the stop, which makes the usual $SU(3)_c\times U(1)_{em}$ vacuum unstable.

Finally we mention about a serious difficulty of our scheme which makes it almost impossible to construct a realistic model within the standard GUT scenario. As is well

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known, the Goldstone-matter coupling is severely constrained by a symmetry principle. Taking a derivative of Eq.(14·b), we get

$$\frac{\partial^3 W}{\partial z_i \partial z_j \partial z_k}(T^A)_k{}^l z_e + \frac{\partial^2 W}{\partial z_i \partial z_k}(T^A)_k{}^j + \frac{\partial^2 W}{\partial z_j \partial z_k}(T^A)_k{}^i = 0 .$$

This equation implies that the Goldstone-matter coupling is proportional to matter masses (m_i, m_j):

$$\Gamma_{ijA} \sim \frac{m_i}{M} + \frac{m_j}{M} .$$

On the other hand, leptons and quarks are massless at the unification level. Therefore Yukawa couplings of Higgs multiplets to leptons and quarks are "kinematically" forbidden. One may expect that radiative correction may improve this situation since G_{GUT} gauge interactions explicitly break the global symmetry G. Due to the non-renormalization theorem, however, these cannot give any nontrivial effect as far as we are working on perturbation theories. Therefore only possible way to escape this difficulty will be to invoke non-perturbative effects of gauge interactions. Encouragingly enough, recent progress in this branch clearly shows that the non-perturbative effects of supersymmetric gauge interactions surely generate a superpotential against the non-renormalization theorem.[12)]

It will be probable that the theory which governs high energy world $(M_X \lesssim \mu \lesssim M_{\mathrm{Planck}})$ contains a large number of particles, and therefore asymptotic freedom is badly violated. If this is a case, we have a chance that the gauge interactions become strong at $\mu \cong M_{\mathrm{Planck}}$ and instantons with a scale of Planck length induce superpotential which seems to be almost a local operator in the low energy effective theory.[13)] Since this superpotential is generated through gauge interactions, it does not respect global symmetry G, so it may escape the difficulty. Of course this time we need some additional mechanism, such as the R-invariance,[*)] in model building which forbids a huge mass correction to Higgs scalars from this non-perturbative effect.

Appendix

In this appendix we show that any field configuration with non-vanishing D^A can always be transformed to that with vanishing D^A by a suitable complexified global transformations, Eq.(13) with complex transformation parameter ε^A. The essential ingredient for the proof has been already given in the textbook by Wess and Bagger.[8)] But their proof seems to be insufficient. Therefore we give it here for definiteness.

Let us start from a general configuration of vacuum expectation values a_i which gives non-vanishing D-term

$$D^A = a^\dagger T^A a .$$

Now we prove that by a suitable complex transformation

$$a' = e^{\theta \cdot T} a ,$$

$$\theta \cdot T \equiv \theta^A T^A ; \quad \theta^A = \text{real} , \quad T^A : \text{hermitian}$$

*) We thank Dr. T. Yanagida for pointing out this possibility.

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all D-terms

$$D^A(\theta)=a'^{\dagger}T^A a'=a^{\dagger}e^{\theta\cdot T}T^A e^{\theta\cdot T}a$$

can be vanished, that is, there exists a set of parameters $\{\theta^A\}$ which solves

$$D^A(\theta)=0\,.$$

Consider the following positive semidefinite quantity

$$X(\theta)\equiv a^{\dagger}e^{2\theta\cdot T}a=|e^{\theta\cdot T}a|^2\geqq 0\,.$$

$X(\theta)$ is a real function of d_G variables θ^A, where d_G is a dimension of group G. The equality of this equation requires $e^{\theta\cdot T}a=0$ and this occurs, if possible, only in the limit $|\theta|\to\infty$ ($|\theta|\equiv(\sum_A\theta^A\theta^A)^{1/2}$). If this is the case, all $D^A(\theta)$ trivially vanish in this limit. The non-trivial case is that there is no direction which makes $X(\theta)$ vanish in the limit $|\theta|\to\infty$. In this case $X(\theta)\to\infty$ in the limit $|\theta|\to\infty$ unless $\theta\cdot T$ is entirely contained in the algebra of unbroken subgroup H of G where $X(\theta)\equiv a^{\dagger}a$ identically. It will be easy to convince that in this nontrivial case there is a stationary point of $X(\theta)$ in the range of finite $|\theta|$. That is, there necessarily exists a solution for equation,

$$\frac{\partial X(\theta)}{\partial\theta^A}=0\,.$$

In the following we will work on this stationary point of $X(\theta)$.

The straightforward calculation gives

$$\begin{aligned}\frac{\partial X(\theta)}{\partial\theta^A}&=a^{\dagger}\left(\frac{\partial}{\partial\theta^A}e^{2\theta\cdot T}\right)a\\&=2a^{\dagger}\int_0^1 dx e^{2\theta\cdot T(1-x)}T^A e^{2\theta\cdot Tx}a\\&\equiv 2a^{\dagger}e^{2\theta\cdot T}T^A(\theta)\,a\,,\end{aligned}$$

where

$$\begin{aligned}T^A(\theta)&\equiv\int_0^1 dx e^{-2\theta\cdot Tx}T^A e^{2\theta\cdot Tx}\\&=\left(\frac{e^{-2\theta\cdot F}-1}{-2\theta\cdot F}\right)^{AB}T^B\end{aligned}$$

and $(F^A)^{BC}$ is a generator in the adjoint representation. Then for all generators T^A, we get

$$\begin{aligned}a^{\dagger}e^{2\theta\cdot T}T^A a&=\left(\frac{-2\theta\cdot F}{e^{-2\theta\cdot F}-1}\right)^{AB}a^{\dagger}e^{2\theta\cdot T}T^B(\theta)\,a\\&=0\,,\end{aligned}$$

because the matrix $-2\theta\cdot F/(e^{-2\theta\cdot F}-1)$ is finite and regular for $|\theta|<\infty$. Thus

$$\begin{aligned}D^A(\theta)=a^{\dagger}e^{\theta\cdot T}T^A e^{\theta\cdot T}a&=(e^{-\theta\cdot F})^{AB}a^{\dagger}e^{2\theta\cdot T}T^B a\\&=0\,.\end{aligned}$$

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Therefore the stationary point of $X(\theta)$ gives a solution of $D^A(\theta)=0$. Q.E.D.

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