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Higgs Bosons In Left-Right Symmetric Models*

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Abstract

In the context of minimal left-right symmetric extensions of the Standard $SU(2)_L \times U(1)$ Model, four possible scenarios emerge depending upon the number of neutral Higgs fields that acquire non-zero vacuum expectation values. We examine these four scenarios in the case of a simple form for the scalar potential of the model. We find that by combining the minimization conditions appropriate to each of these scenarios with constraints coming from $K_L - K_S$ mixing and absence of flavor changing neutral currents, two of the scenarios are ruled out. For the allowed scenarios rather definitive consequences for the Higgs sector can be obtained. For instance, several of the scalar bosons are forced to be very heavy by flavor changing neutral current constraints. Aside from the Higgs boson which plays the role of the SM Higgs, the three left-handed triplet members are most likely to be light. Indeed, in one of the vacuum expectation value scenarios they are forced to be light (the neutral member being massless at tree level). We catalogue the constraints from low-energy experiments on doubly charged triplet Higgs and on a very light neutral triplet Higgs, when these couple to two leptons (as in the left-right model). Many of the constraints we obtain are new. A new lower bound on the W_R mass, related to the strength of these lepton-lepton couplings is derived. The cross sections for these bosons are large out to masses of order several TeV and signatures for their decay are striking and essentially background free.

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I. Introduction

Left-right symmetric (LR) theories provide an attractive extension of the Standard Model (SM). In these models, parity is an exact symmetry of the Lagrangian, and is only broken spontaneously due to the form of the scalar field potential. They also incorporate full quark-lepton symmetry of the weak interactions and give the $U(1)$ generator of the electroweak symmetry group a definite meaning in terms of the $B - L$ quantum number. Finally, for an appropriately chosen Higgs sector they lead to a natural explanation of the smallness of neutrino masses, by relating it to the observed suppression of $V + A$ currents. The theories contain two W bosons, W_L and W_R, and two neutral gauge bosons, Z_1 and Z_2. The W_L and Z_1 are those already discovered. In the fermion sector, LR models contain the usual quarks and charged leptons, as well as three light neutrino mass eigenstates ν_k $(k = 1,2,3)$ and three heavy neutrino eigenstates N_k $(k = 1,2,3)$. The former couple predominantly to the Standard-Model-like W_L, while the latter couple mainly to W_R.

Regarding the Higgs sector of the LR theories, there are two distinct alternatives. All models contain a bi-doublet field ϕ; the masses of the standard W_L and Z_1 derive primarily from the vacuum expectation values, κ_1 and κ_2, of the two neutral members of this doublet. Since experimental constraints from K_L-K_S mixing force the W_R to be very heavy[1] ($\gtrsim 1.6\ TeV$), an additional Higgs representation, with large vacuum expectation value (v_R, with $v_R \gg \kappa_1, \kappa_2$) for its neutral member, is required that couples primarily to the W_R. To preserve LR symmetry, there must be a corresponding Higgs representation coupling to the W_L, but the vev (v_L) of its neutral member must be small ($v_L \ll max(\kappa_1, \kappa_2)$) in order to preserve the SM relation between the W_L and Z_1 masses. If the additional Higgs fields are members of doublets, then the above criteria can be met, but the theory then fails to incorporate a natural explanation of the smallness of neutrino masses. In contrast, if the extra neutral Higgs are members of triplets, then the vev v_R can induce a large Majorana mass term for the N, in addition to the Dirac mass terms induced by the vev's of the neutral bi-doublet fields that mix the N and ν. Since the ratio of Majorana to Dirac mass terms is of order $v_R/max(\kappa_1, \kappa_2)$, we are naturally led to the standard "see-saw" mechanism yielding a very small Majorana mass for the left-handed neutrinos. Because of the attractiveness of this latter alternative, we choose to investigate models containing extra triplet Higgs fields, Δ_R and Δ_L. The resulting Higgs sector has many exotic features, and our ability to experimentally probe these features is an important issue.

The principle source of uncertainty in dealing with the Higgs sector of these LR models is the exact form of the scalar potential, V. An important question regarding a candidate form for V is whether the phenomenologically required hierarchy for the vacuum expectation values,

$$v_R \gg max(\kappa_1, \kappa_2) \gg v_L, \qquad (I.1)$$

is natural. The most general form of this potential was given in ref. 2 and contains only quadratic or quartic terms. There it was demonstrated that the inclusion of quartic terms in the potential containing the product of one left-handed triplet, one right-handed triplet and two bi-doublet fields yields potential minimization conditions which imply either $v_L = v_R$ or

$$(\rho - \rho')v_L v_R = \gamma\kappa_1\kappa_2 + \beta(\kappa_1^2 + \kappa_2^2), \tag{I.2}$$

where ρ, ρ', γ, β are certain combinations of the potential parameters (γ and β derive from the above mentioned quartic terms). We may choose potential parameters so that $v_L = v_R$ is a potential maximum and eq. (I.2) corresponds to a minimum. Then, unless $\rho - \rho'$ is unnaturally small, eq. (I.2) implies that eq. (I.1) is satisfied so long as $v_R \gg max(\kappa_1, \kappa_2)$. Of course, under these same circumstances we could guarantee that $v_L = 0$ (when $v_R \neq 0$) by simply eliminating the critical quartic terms. This can be accomplished by the imposition of an appropriate discrete symmetry,[3,4] in addition to the LR symmetry requirement. It is the Higgs sector of the resulting potential that we choose to investigate here. Aside from the natural case with $\rho \neq \rho'$ and $v_L = 0$, the $\rho = \rho'$ case of this latter potential will also turn out to be very interesting since, then, the members of the left-handed triplet Higgs representation are actually forced to be light.

Whether or not a significant number of the Higgs bosons of a LR model can be sufficiently light to be detectable is, in fact, a serious issue. In the simple one bi-doublet, one left-handed triplet, one right-handed triplet model described above, all the scalar bosons other than one SM-like Higgs boson naturally have mass of order v_R. In principle, one could complicate the Higgs sector in such a way as to disconnect the W_R mass from the mass scale of these other bosons by introducing an additional triplet scalar field. In this case, all the non-SM-like scalar bosons have their mass scales set by some new vacuum expectation value, v'_R.[5] The problem is that among the physical scalar bosons, emerging after spontaneous symmetry breaking, there is always one that is a direct source of flavor-changing neutral currents (FCNC). It was shown in ref. 6 that in the context of three families the experimental limits on FCNC require that this neutral boson be heavier than $5-10\ TeV$. Thus, v_R and v'_R *must* in fact be similar in size, and there is no point in complicating the Higgs sector. Rather, the simple minimal ϕ-Δ_L-Δ_R Higgs sector is sufficient; the FCNC scalar boson will have mass proportional to v_R and will automatically be heavy provided a particular combination of Higgs potential parameters is not small. But then *all* the other non-SM-like scalar bosons of the minimal LR model will tend to also have masses set by the scale v_R, and are possibly too heavy to be experimentally accessible. This is certainly the case for the singly charged h^+ which is degenerate with the FCNC neutral boson. However, the masses of the other physical scalars involve quite different combinations of the Higgs potential parameters, and it is thus of interest to investigate the circumstances under which they might be sufficiently light to be detectable.

The simple form of the scalar potential that we investigate here is appealing in that it allows a straightforward investigation of these issues, and even allows for one vev scenario in which the left-handed triplet members are forced to be light. (More generally, it is those scalar bosons that are predominantly left-handed triplet members that are most likely to have mass below the TeV scale.) In short, it encompasses a variety of possibilities and specific experimental predictions are possible for a given vacuum expectation value scenario. Our phenomenological survey of these experimental possibilities includes an extensive summary of low-energy experimental constraints on the doubly-charged and neutral members of the left-handed triplet; a number of these constraints have not been previously discussed. We also give a new lower bound on the W_R mass, as a function of the lepton-lepton couplings which are limited by the just-mentioned low-energy constraints. This bound is generally competitive with the 1.6 TeV lower bound coming from $K_L - K_S$ mixing, and could become stronger as improved low-energy limits on the lepton-lepton couplings of the left-handed triplet Higgs bosons emerge from new experiments. We also explore possibilities for production and detection, and conclude that many striking signatures with significant event rate are predicted.

II. The Higgs Sector

The Higgs fields of the minimal model are

$$\phi(1/2, 1/2^*, 0), \quad \Delta_L(1,0,2), \quad \Delta_R(0,1,2), \tag{II.1}$$

where the $SU(2)_L$, $SU(2)_R$ and $B-L$ quantum numbers are indicated in parentheses. In the case of the Δ_R, the $B-L$ has been chosen so as to realize the "see-saw" mechanism for explaining small left-handed neutrino masses. $B-L$ for the Δ_L must be the same by L-R symmetry. A convenient representation of the fields is given by the 2×2 matrices:

$$\phi \equiv \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{II.2}$$

$$\Delta_L \equiv \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \tag{II.3}$$

$$\Delta_R \equiv \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}. \tag{II.4}$$

In our convention, a neutral field ϕ^0 is written in terms of correctly normalized real and imaginary components as $\phi^0 = (1/\sqrt{2})(\phi^{0\,r} + i\phi^{0\,i})$.

Let us now discuss the form of the scalar field potential. Left-right symmetry requires that the potential be invariant under

$$\Delta_R \leftrightarrow \Delta_L, \quad \phi \leftrightarrow \phi^\dagger. \tag{II.5}$$

Further, the most general scalar field potential cannot have any tri-linear terms. Because of the non-zero $B-L$ quantum numbers of the Δ_L and Δ_R triplets, these must always appear in the quadratic combinations $\Delta_L^\dagger\Delta_L$, $\Delta_R^\dagger\Delta_R$, $\Delta_L^\dagger\Delta_R$, or $\Delta_R^\dagger\Delta_L$. These combinations can never be combined with a single bi-doublet ϕ in such a way as to form a $SU(2)_L$ and $SU(2)_R$ singlet. Nor can three bi-doublets be combined so as to yield a singlet. However, quartic combinations of the form $\mathrm{Tr}(\Delta_L^\dagger\phi\Delta_R\phi^\dagger)$ are in general allowed by the L-R symmetry. As we have argued in the introduction, it is desirable to eliminate such terms in order that the natural minima of the potential have $v_L = 0$. To accomplish this, we shall impose invariance under the additional discrete symmetry

$$\Delta_L \rightarrow \Delta_L, \quad \Delta_R \rightarrow -\Delta_R. \tag{II.6}$$

In addition, we shall shortly argue that it is important to eliminate terms in the potential of the form:

$$-\mu_3^2[\mathrm{Tr}(\tilde{\phi}^\dagger\phi) + \mathrm{Tr}(\phi^\dagger\tilde{\phi})] \tag{II.7}$$

(where $\tilde{\phi} \equiv \tau_2\phi^*\tau_2$) in order that the natural potential minima avoid FCNC problems. Such terms are not allowed if we require that the potential be invariant under the additional discrete symmetry

$$\phi \rightarrow i\phi. \tag{II.8}$$

The most general form of V is then

$$\begin{aligned} V = &- \mu_1^2\,\mathrm{Tr}\,\phi^\dagger\phi + \lambda_1(\mathrm{Tr}\,\phi^\dagger\phi)^2 + \lambda_2\,\mathrm{Tr}\,\phi^\dagger\phi\phi^\dagger\phi + \frac{1}{2}\lambda_3(\mathrm{Tr}\,\phi^\dagger\tilde{\phi} + \mathrm{Tr}\,\tilde{\phi}^\dagger\phi)^2 \\ &+ \frac{1}{2}\lambda_4(\mathrm{Tr}\,\phi^\dagger\tilde{\phi} - \mathrm{Tr}\,\tilde{\phi}^\dagger\phi)^2 + \lambda_5\,\mathrm{Tr}\,\phi^\dagger\phi\tilde{\phi}^\dagger\tilde{\phi} + \frac{1}{2}\lambda_6(\mathrm{Tr}\,\phi^\dagger\tilde{\phi}\phi^\dagger\tilde{\phi} + \mathrm{Tr}\,\tilde{\phi}^\dagger\phi\tilde{\phi}^\dagger\phi) \\ &- \mu_2^2(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L + \mathrm{Tr}\,\Delta_R^\dagger\Delta_R) + \rho_1\left[(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L)^2 + (\mathrm{Tr}\,\Delta_R^\dagger\Delta_R)^2\right] \\ &+ \rho_2(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L + \mathrm{Tr}\,\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R) + \rho_3(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L)(\mathrm{Tr}\,\Delta_R^\dagger\Delta_R) \\ &+ \alpha_1\,\mathrm{Tr}\,\phi^\dagger\phi(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L + \mathrm{Tr}\,\Delta_R^\dagger\Delta_R) + \alpha_2(\mathrm{Tr}\,\Delta_R^\dagger\phi^\dagger\phi\Delta_R + \mathrm{Tr}\,\Delta_L^\dagger\phi\phi^\dagger\Delta_L) \\ &+ \alpha_2'(\mathrm{Tr}\,\Delta_R^\dagger\tilde{\phi}^\dagger\tilde{\phi}\Delta_R + \mathrm{Tr}\,\Delta_L^\dagger\tilde{\phi}\tilde{\phi}^\dagger\Delta_L). \end{aligned} \tag{II.9}$$

We shall assume that the potential is CP conserving and take all parameters to be real. It will be convenient to define certain combinations of the parameters

appearing in the above potential:

$$\begin{aligned} \rho_{dif} &\equiv \rho_3 - 2(\rho_2+\rho_1) & \lambda_\Sigma &\equiv \lambda_1 + \lambda_2 \\ \Delta\alpha &\equiv (\alpha_2 - \alpha_2')/2 & \alpha_\Sigma &\equiv \alpha_1 + \alpha_2 \\ \Sigma\lambda &\equiv \lambda_2 - (4\lambda_3 + \lambda_5 + \lambda_6) & \alpha_\Sigma' &\equiv \alpha_1 + \alpha_2' \\ \Sigma'\lambda &\equiv \lambda_5 - \lambda_2 - 4\lambda_4 - \lambda_6 & \rho_\Sigma &\equiv \rho_1 + \rho_2. \end{aligned} \tag{II.10}$$

The vacuum expectation values of the neutral Higgs fields can be chosen to be real and non-negative. Thus, δ_R^{0r}, δ_L^{0r}, ϕ_1^{0r} and ϕ_2^{0r} can potentially acquire vacuum expectation values, v_R, v_L, κ_1, and κ_2, respectively. More explicitly,

$$\begin{aligned} &<\delta_R^0> = \frac{1}{\sqrt{2}} < \delta_R^{0r} > = \frac{v_R}{\sqrt{2}} & &<\delta_L^0> = \frac{1}{\sqrt{2}} < \delta_L^{0r} > = \frac{v_L}{\sqrt{2}} \\ &<\phi_1^0> = \frac{1}{\sqrt{2}} < \phi_1^{0r} > = \frac{\kappa_1}{\sqrt{2}} & &<\phi_2^0> = \frac{1}{\sqrt{2}} < \phi_2^{0r} > = \frac{\kappa_2}{\sqrt{2}}. \end{aligned} \tag{II.11}$$

Of the twenty real degrees of freedom, six are absorbed in giving mass to the left and right handed gauge bosons, $W_L^\pm$, $W_R^\pm$, Z_1, and Z_2.

For the potential to be at a minimum when all the neutral fields are evaluated at their respective vev's, we must require that

$$\partial V/\partial v_L = \partial V/\partial v_R = \partial V/\partial \kappa_1 = \partial V/\partial \kappa_2 = 0. \tag{II.12}$$

In addition, at a true local minimum all the physical Higgs bosons must have positive squared-masses for a solution of eq. (II.12). The forms of the derivatives appearing in eq. (II.12) are such that $\partial V/\partial v_i = v_i f_i(v_i)$ for $v_i = v_L, v_R, \kappa_1, \kappa_2$. Thus the minimization condition for each v_i can be satisfied either by $v_i = 0$ or $f_i(v_i) = 0$. We have already learned that we must have $v_R \neq 0$ and, adopting the non-restrictive convention $\kappa_1 \geq \kappa_2$, $\kappa_1 \neq 0$. Since we will be particularly concerned with the neutral scalar bosons of the theory we present the general form of the mass-squared matrix for this sector in Appendix A, after substituting the non-trivial minimization conditions for v_R and κ_1 (also given in Appendix A). Considering the remaining two vev's, v_L and κ_2, we are left with four possible vacuum expectation value scenarios corresponding to whether each is zero or not.

In order to assess the viability of the four alternative vev scenarios, it is first necessary to review a few basic features of the couplings of the Higgs fields to gauge bosons and fermions. In terms of the Larangian level fields (which are not necessarily the physical mass eigenstates) the important tri-linear $WWHiggs$ couplings derive from the bi-doublet covariant kinetic energy term, $\mathrm{Tr}[(D_\mu\phi)^\dagger(D_\mu\phi)]$, and take the form

$$\frac{1}{2}g^2\Big[(\kappa_1\phi_1^{0\,r}+\kappa_2\phi_2^{0\,r})(W_L^+W_L^-+W_R^+W_R^-)$$
$$-(\kappa_2\phi_1^{0\,r}+\kappa_1\phi_2^{0\,r})(W_L^+W_R^-+W_L^-W_R^+)\Big]. \qquad (II.13)$$

The Lagrangian describing the interaction of the bi-doublet Higgs field with the quark fields takes the general form

$$\mathcal{L}_Y=\overline{\mathbf{Q}}_i^L(\mathbf{F}_{ij}\phi+\mathbf{G}_{ij}\widetilde{\phi})\mathbf{Q}_j^R+\text{h.c.}, \qquad (II.14)$$

where $\mathbf{Q}$ denotes the quark field doublet and the indices i,j denote different families. We may solve for $\mathbf{F}$ and $\mathbf{G}$ in terms of the up- and down-quark mass matrices, and then follow the usual procedure of diagonalizing and extracting Yukawa couplings for the down-type quark mass eigenstates, denoted by $\mathbf{d}_i$ where i is the mass eigenstate family index. In family matrix notation the resulting Yukawa couplings to the neutral members of the bi-doublet take the form:

$$\frac{\sqrt{2}}{\kappa_1^2-\kappa_2^2}\overline{\mathrm{d}}^L\Big[(\kappa_1\phi_1^{0*}-\kappa_2\phi_2^0)\mathcal{M}_d+(\kappa_1\phi_2^0-\kappa_2\phi_1^{0*})V_{CKM}^L{}^\dagger\mathcal{M}_uV_{CKM}^R\Big]\mathrm{d}^R+\text{h.c.}, \qquad (II.15)$$

where the $\mathcal{M}$ matrices are the diagonal mass matrices for down and up quarks, and the $V_{CKM}^{L,R}$ are the left and right handed Cabbibo, Kobayashi, Maskawa matrices. (Note that in the limit of $\kappa_1\to\kappa_2$ eq. (II.14) implies that $\mathcal{M}_u=\mathcal{M}_d$; thus the vanishing of the denominator of eq. (II.15) in this limit is compensated by a zero of the numerator. Since we know experimentally that $\mathcal{M}_u\neq\mathcal{M}_d$, we presume $\kappa_1\neq\kappa_2$.) It was shown in ref. 6 that it is not possible in the context of three families to avoid FCNC interactions coming from the term in eq. (II.15) involving the V_{CKM} matrices. Thus, physical eigenstates that have large ϕ_1^0 or ϕ_2^0 components are in danger of violating FCNC constraints unless they have mass in the multi-TeV range.

This observation allows us to give a strong argument in favor of requiring the $\phi\to i\phi$ symmetry for the scalar potential. If this symmetry is not required, then quadratic terms of the form (II.7) are generally present. When such a term is allowed, the minimization conditions coming from $\partial V/\partial\kappa_1=0$ and $\partial V/\partial\kappa_2=0$ (see eq. (II.12)) take the form:

$$-2\mu_1^2\kappa_1-2\mu_3^2\kappa_2+\mathcal{F}=0$$
$$-2\mu_1^2\kappa_2-2\mu_3^2\kappa_1+\mathcal{G}=0, \qquad (II.16)$$

where $\mathcal{F}$ and $\mathcal{G}$ are functions of the couplings and vacuum expectation values κ_1, κ_2, v_L, and v_R. These equations do not impose any unnatural or fine tuned

relationships between the couplings when both κ_1 and κ_2 are non-zero—values for μ_1^2 and μ_3^2 obtained by renormalization group evolution from GUT scale boundary conditions simply lead to a determination of κ_1 and κ_2 for definite choices of the Higgs coupling constants.

The problem with this scenario becomes apparent when we note that non-zero values for both κ_1 and κ_2 will almost inevitably lead to the impossibility of a relatively light Higgs boson (giving masses to the quarks and curing WW unitarity) without large FCNC. This is because for κ_1, $\kappa_2\neq 0$, the FCNC violating combination $\phi_{FCNC}^0=\kappa_1\phi_2^0-\kappa_2\phi_1^0{}^*$ in eq. (II.15) is extremely unlikely to be a mass eigenstate. (We shall see examples of this later.) Generally, when both κ_1 and κ_2 are non-zero, all mass eigenstates will overlap significantly with the ϕ_{FCNC}^0 combination and must all be heavy to avoid large FCNC.

But, by imposing the $\phi\to i\phi$ symmetry, we eliminate the μ_3^2 terms in eq. (II.16). If both κ_1 and κ_2 are non-zero, we may eliminate μ_1^2 between the two conditions of eq. (II.16), and obtain a requirement on the coupling constant/vev combinations $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F}/\kappa_1=\mathcal{G}/\kappa_2$. This condition is seen to be unnatural when one notes that, together with the requirement $v_R\gg\kappa_1$, it forces some couplings to be very small. Thus, a random choice of coupling constants will not allow *both* κ_1 and κ_2 to be non-zero when the $\phi\to i\phi$ symmetry is imposed. We will find that when $\kappa_2=0$, ϕ_1^0 and ϕ_2^0 are generally components of *orthogonal* mass eigenstates. In this case, the one(s) containing ϕ_2^0 can have large mass, thereby avoiding FCNC, while one or more of those containing ϕ_1^0 can be light and play the role of the SM-like Higgs boson(s), with large WW couplings and flavor diagonal fermion couplings.

So, we now turn to our restricted potential form and consider in detail the phenomenological viability of the four different vacuum expectation value scenarios for κ_2 and v_L. This will illustrate, in detail, the points discussed above. In particular, we shall immediately eliminate any scenario in which the following two conditions cannot be simultaneously met.

1. All physical scalar boson masses must be positive.
2. It must be possible for the neutral scalar boson having the largest coupling to W_LW_L and Z_1Z_1 to be lighter than the unitarity bound in these channels, of about 1.5 TeV, without it or some still lighter neutral scalar contributing significantly to FCNC interactions through the couplings of eq. (II.15).

Vacuum Expectation Value Scenarios

We first consider scenarios with both κ_1 and κ_2 non-zero.

(*a*) $v_L,\kappa_2,\kappa_1,v_R\neq 0$: The parameter constraints emerging from eq. (II.12) are very powerful in this case. In particular, combining the v_R and v_L derivative equations implies

$$\rho_{dif}=0, \qquad (II.17)$$

(as given by eq. (I.2) for $\gamma,\beta=0$ when we note that $\rho_{dif}=\rho-\rho'$) while combining the κ_1 and κ_2 derivative equations leads to the requirement

$$\Sigma\lambda(\kappa_1^2 - \kappa_2^2) = \Delta\alpha(v_R^2 + v_L^2). \qquad \text{(II.18)}$$

As we have noted, requirements of this type in which certain coupling constants must have highly correlated values should perhaps be considered "unnatural". However, our approach will be to investigate each vev scenario in turn, regardless of its degree of naturalness. It is also worth recalling that when $\rho_{dif} = 0$ there is no constraint requiring $v_L \ll max(\kappa_1, \kappa_2)$ when $v_R \gg max(\kappa_1, \kappa_2)$. This simply puts v_L and v_R on the same footing. Potential parameters always have to be chosen to give large v_R, and now we must make additional choices to guarantee small v_L. We cannot assess the naturalness of such choices without a specific grand unification scenario.

Turning to the neutral scalar mass matrix, we note that condition (II.18) implies that Δ_{22} (see Appendix A) is 0, and that there are various other simplifications. We have numerically diagonalized the resulting analytic form for the mass matrix of the neutral real Higgs sector over a wide range of parameters. We find that there are no symmetry breaking solutions that are acceptable in that they satisfy the criteria (1) and (2) given above. A typical example of what goes wrong can be outlined as follows. First we know that $v_L \ll \sqrt{\kappa_1^2 + \kappa_2^2}$ is required in order to prevent a large anomaly in the standard electro-weak rho parameter. Suppose $\kappa_2 \ll \kappa_1$. Then, labelling the physical eigenstates in order of increasing mass ($m_i > m_j$ for $i > j$), scalar boson number 3 turns out to be mainly $\phi_1^{0\,r}$ and has large $W_L W_L$ couplings (see eq. (II.13)), while the next lighter scalar boson number 2 is mainly $\phi_2^{0\,r}$ and will contribute to FCNC interactions (see eq. (II.15)). Thus, scalar boson number 3 plays the role of the SM-like Higgs and must have mass below 1.5 TeV. Consequently, we see that the mass scale for the flavor changing neutral current Higgs must also lie below 1.5 TeV, in contradiction to the 5 TeV bound from FCNC.

(*b*) $v_L = 0,\ \kappa_1, \kappa_2, v_R \neq 0$: This case amounts to a simplified version of the previous case (a), and allows some intuitive understanding of the previous result. The constraint eq. (II.17) does not apply, but eq. (II.18) can be used to simplify somewhat the mass-squared matrix for the four real neutral scalars (see Appendix A). The form of this matrix is such that $v_L = 0$ leaves $\delta_L^{0\,r}$ as an isolated eigenstate with mass of order v_R. The mass matrix is especially easy to analyze in the limit where $\kappa_2 \ll \kappa_1$. In this case $\phi_2^{0\,r}$ is also very nearly a massless eigenstate, and to zeroeth order in κ_2/κ_1 we are left with diagonalizing the mass-squared matrix

$$\begin{pmatrix} 2\kappa_1^2\lambda_\Sigma & v_R\kappa_1\alpha'_\Sigma \\ v_R\kappa_1\alpha'_\Sigma & 2v_R^2\rho_\Sigma \end{pmatrix} \qquad \text{(II.19)}$$

in $\phi_1^{0\,r}$-$\delta_R^{0\,r}$ space. Since $\kappa_1 \ll v_R$, one eigenstate is approximately $\phi_1^{0\,r}$. This

state can have a moderate mass of order κ_1 and it plays the role of the SM Higgs because it couples to $W_L W_L$ and gives masses to the fermions through its Yukawa couplings. However, $\phi_2^{0\,r}$ then plays the role of the FCNC Higgs, but is approximately massless. This clearly violates the phenomenological constraints. For more general values of the κ_1 and κ_2 we must use numerical techniques, as in case (*a*). We have explored a wide range of κ_1, κ_2, v_R, and coupling constant values (consistent with eq. (II.18)) and found no solution satisfying conditions (1) and (2).

Thus, we see that both cases (*a*) and (*b*) confirm the general conclusion reached earlier: namely when both κ_1 and κ_2 are non-zero, we have been unable to find a potential minimum with a light Higgs boson that does not have a FCNC component. However, the condition of eq. (II.18) that is required for $\kappa_2 \neq 0$ is unnatural and not likely to hold in any case. Indeed, the restricted form of the potential that we employ, based on requiring $\phi \to i\phi$ symmetry, will instead lead to the likelihood that κ_2 is zero and to the possibility that FCNC can be avoided. We turn now to these more natural $\kappa_2 = 0$ scenarios.

(*c*) $\kappa_2 = 0,\ v_L, \kappa_1, v_R \neq 0$: The minimization conditions for this case enforce only one "unnatural" relation among couplings constants, eq. (II.17). However, the phenomenology to be outlined below is otherwise entirely acceptable, so long as the potential parameters are chosen so that eq. (I.1) holds.

(*d*) $v_L = \kappa_2 = 0,\ \kappa_1, v_R \neq 0$: This scenario is the most "natural" in the sense that highly correlated values among the coupling constants are not imposed by the minimization conditions for κ_1 and v_R. Nonetheless, we shall see that the phenomenological interest of this model is greatly enhanced if the difference between couplings constants denoted above by ρ_{dif} happens to be small.

To summarize, we have seen that the minimal potential of eq. (II.9) implies that $\kappa_2 \neq 0$ is unnatural and, in addition, would lead to an unacceptable potential minimum. This has two additional positive implications. When $\kappa_2 = 0$, there is automatically no mixing between the W_L and W_R gauge bosons. Generally, this mixing is given by the angle ξ where $\tan 2\xi \sim 2\kappa_1\kappa_2/v_R^2$. Such mixing might lead to phenomenological difficulties. For instance, for $m_{W_R} = 1.6\ TeV$ we find $v_R \sim 3.3\ TeV$; maximal mixing occurs for $\kappa_1 = \kappa_2(\sim 170\ GeV)$, implying $\tan 2\xi \sim 0.005$. This comes close to violating the present experimental constraint of $\xi < 0.0055$.[7] Substantial improvements in this constraint would force us to require that κ_2 be significantly smaller than κ_1. An additional point is that it is desirable to have κ_1 significantly different from κ_2 in order to easily generate a large mass ratio for m_t/m_b.

In the following sections we give a broad outline of the phenomenological consequences of vacuum expectation value scenarios (c) and (d). In so doing, it will be useful to have a precise statement of the constraint on the relative size of v_L/κ_1 coming from existing experimental measurements. For $\kappa_2 = 0$ we find

$$m_{W_L}^2 \simeq \frac{1}{4}g^2(\kappa_1^2 + 2v_L^2) \qquad \text{(II.20)}$$

and

$$m^2_{Z_1} \simeq \frac{1}{4}\frac{g^2(g^2+2g'^2)}{g^2+g'^2}(\kappa_1^2+4v_L^2). \tag{II.21}$$

The appropriate definition of the Weinberg angle for the left right symmetric model is such that

$$\cos^2\theta_W = \frac{g^2+g'^2}{g^2+2g'^2}, \tag{II.22}$$

so that we have

$$\rho_{EW} \equiv \frac{m^2_{W_L}}{\cos^2\theta_W m^2_{Z_1}} = \frac{\kappa_1^2+2v_L^2}{\kappa_1^2+4v_L^2}. \tag{II.23}$$

We know experimentally that $|1-\rho_{EW}| \lesssim 0.01$, implying that

$$v_L \lesssim 0.07\kappa_1. \tag{II.24}$$

In particular, it is clearly always safe to neglect effects of order v_L/v_R, since $v_R \gg \kappa_1$. We also note at this time the formulae for the W_R and Z_2 masses:

$$m^2_{W_R} \simeq \frac{1}{4}g^2(\kappa_1^2+2v_R^2) \tag{II.25}$$

and

$$m^2_{Z_2} \simeq v_R^2(g^2+g'^2). \tag{II.26}$$

III. Higgs Boson Eigenstates

The results for vacuum expectation value scenarios (c) and (d) can be presented in a common notation so long as we adopt the approximation of neglecting terms of order v_L/v_R; terms of relative order v_L/κ_1 and κ_1/v_R are retained. The formulas appearing below can then be applied to the two different cases by simply recalling that (d) corresponds to $v_L=0$ and ρ_{dif} adjustable, while (c) corresponds to $v_L \neq 0$ and $\rho_{dif}=0$.

The mass eigenstates may be obtained from the mass matrices presented for $\kappa_2=0$, $\kappa_1, v_R \neq 0$ in Appendix A. (Note: the mass eigenvalue and eigenstate formulae below are not solutions of these matrices except in the two specific cases (c) and (d).)

1. *Doubly Charged Sector:* δ_R^{++} and δ_L^{++} are unmixed mass eigenstates with masses

$$\begin{aligned} m^2_{\delta_R^{++}} &= -\rho_2 v_R^2 + \Delta\alpha\kappa_1^2 \\ m^2_{\delta_L^{++}} &= \frac{1}{2}\rho_{dif}v_R^2 - \rho_2 v_L^2 + \Delta\alpha\kappa_1^2. \end{aligned} \tag{III.1}$$

Note that for vev scenario (c) δ_L^{++} does not have a mass term of order v_R, whereas in scenario (d) it can only be light if ρ_{dif} is small.

2. *Singly Charged Sector:* The states remaining after absorption of the two Goldstone boson states by the W_R^+ and W_L^+ are:

$$\begin{aligned} h^+ &= \frac{\phi_1^+ + \frac{\kappa_1}{\sqrt{2}v_R}\delta_R^+}{\sqrt{1+\frac{\kappa_1^2}{2v_R^2}}} \\ \tilde{\delta}_L^+ &= \frac{\delta_L^+ + \frac{\sqrt{2}v_L}{\kappa_1}\phi_2^+}{\sqrt{1+\frac{2v_L^2}{\kappa_1^2}}}, \end{aligned} \tag{III.2}$$

with masses

$$\begin{aligned} m^2_{h^+} &= \Delta\alpha(v_R^2 + \frac{1}{2}\kappa_1^2) \\ m^2_{\tilde{\delta}_L^+} &= \frac{1}{2}\rho_{dif}v_R^2 + \frac{1}{2}\Delta\alpha(\kappa_1^2+2v_L^2). \end{aligned} \tag{III.3}$$

3. *Neutral Imaginary Sector:* Neglecting terms of order v_L/v_R the two surviving states are pure $\phi_2^{0\,i}$ and $\delta_L^{0\,i}$ with masses

$$\begin{aligned} m^2_{\phi_2^{0\,i}} &= \Delta\alpha v_R^2 + \kappa_1^2\Sigma'\lambda \\ m^2_{\delta_L^{0\,i}} &= \frac{1}{2}\rho_{dif}v_R^2. \end{aligned} \tag{III.4}$$

4. *Neutral Real Sector:* Finally, again neglecting v_L/v_R, the four eigenstates of this sector may be divided into two sets of two. In the first set we have the pure eigenstates $\delta_L^{0\,r}$ and $\phi_2^{0\,r}$ with masses

$$\begin{aligned} m^2_{\delta_L^{0\,r}} &= \frac{1}{2}\rho_{dif}v_R^2 \\ m^2_{\phi_2^{0\,r}} &= \Delta\alpha v_R^2 - \kappa_1^2\Sigma\lambda. \end{aligned} \tag{III.5}$$

The second set of eigenstates is that arising from diagonalizing a 2×2 sub-matrix; the appropriate matrix appears in eq. (II.19). Diagonalization of this matrix yields the eigenstates

$$h^0 = \cos\alpha\phi_1^{0\,r} - \sin\alpha\delta_R^{0\,r}$$
$$H^0 = \cos\alpha\delta_R^{0\,r} + \sin\alpha\phi_1^{0\,r}, \tag{III.6}$$

where

$$\tan 2\alpha = \frac{v_R\kappa_1\alpha'_\Sigma}{v_R^2\rho_\Sigma - \kappa_1^2\lambda_\Sigma}. \tag{III.7}$$

In the limit where $v_R\rho_\Sigma \gg \kappa_1\alpha'_\Sigma$, the masses of these two states are approximately given by

$$m_{H^0}^2 = 2v_R^2\rho_\Sigma - 2\kappa_1^2\lambda_\Sigma + \kappa_1^2\frac{\alpha_\Sigma'^2}{\rho_\Sigma}$$
$$m_{h^0}^2 = \kappa_1^2\left(2\lambda_\Sigma - \frac{\alpha_\Sigma'^2}{2\rho_\Sigma}\right), \tag{III.8}$$

and the mixing between $\phi_1^{0\,r}$ and $\delta_R^{0\,r}$, characterized by the angle α, is small.

It is now apparent that the two vacuum expectation value cases (c) and (d) can avoid flavor changing neutral currents problems in a natural way. First, referring to eqs. (II.13) and (II.15), we see that when $\kappa_2 = 0$ it is only $\phi_1^{0\,r}$ that couples to W_LW_L (and also Z_1Z_1), while it is only $\phi_2^{0\,r}$ and $\phi_2^{0\,i}$ that are responsible for FCNC interactions. These latter will be sufficiently heavy so long as

$$\Delta\alpha v_R^2 > (5\,TeV)^2. \tag{III.9}$$

(For the minimum value of v_R ($\sim 3.3\ TeV$) coming from $m_{W_R} = 1.6\ TeV$, this constraint requires $\Delta\alpha \gtrsim 2.3$.) Note from eq. (III.3) that the h^+ is then also forced to be heavy and approximately degenerate with $\phi_2^{0\,r}$ and $\phi_2^{0\,i}$. Thus, these three bosons will not be experimentally accessible. (However, the constraint of eq. (III.9) does not necessarily imply that the remaining scalar bosons are also very heavy.) Meanwhile, it can be verified from eqs. (III.6)-(III.8) that parameters are easily chosen so that the h^0 and H^0 masses and couplings to W_LW_L and Z_1Z_1 (through their $\phi_1^{0\,r}$ components) are such that no unitarity problems arise. For instance, if $\kappa_1\alpha'_\Sigma \ll v_R\rho_\Sigma$ (implying small mixing angle α) then h^0 is nearly pure $\phi_1^{0\,r}$ and plays the role of the SM-like Higgs boson. It dominates the W_LW_L couplings and has mass that is naturally of order κ_1, although there is still the usual freedom of the magnitude of the quartic coupling strengths in setting the exact size of the mass, see eq. (III.8).

It is also useful to make a few remarks regarding δ_R^{++}. First, we note from eq. (III.1) that ρ_2 must be negative or very small to avoid having a negative mass-squared for δ_R^{++}. In the former case, it is probably natural for δ_R^{++} to be quite heavy. However, $\rho_\Sigma = \rho_1 + \rho_2$ must be positive in order that both $m_{H^0}^2$ and $m_{h^0}^2$ be positive (for $\lambda_\Sigma > 0$). Clearly, substantial cancellations are quite likely and ρ_Σ could be much smaller than either ρ_1 or ρ_2. This would imply that the H^0 could be lighter than one would at first anticipate from eq. (III.8). More generally, there is clearly substantial uncertainty associated with both the δ_R^{++} and H^0 masses, and we will not consider their phenomenology in detail in this paper.

We are now in a position to consider in more detail the left-right model scalar bosons in the two viable vacuum expectation value scenarios, (c) and (d).

IV. Higgs Boson Phenomenology

We will focus primarily on the scalar bosons that are associated with the left-handed triplet Higgs field. This is because their phenomenology is highly constrained and amenable to systematic study. In addition, they have striking experimental signatures. For instance, because they have $B - L = 2$, the doubly-charged triplet members can decay to two like-sign leptons. Other unique properties will emerge as we continue. It is useful at this point to review a few general features of their couplings to leptons and gauge bosons. The fermion couplings of the triplet fields are given by the Lagrangian

$$\mathcal{L}_Y = ih_{ij}^M\left[\psi_{i\,L}^T C\tau_2\Delta_L\psi_{j\,L} + \psi_{i\,R}^T C\tau_2\Delta_R\psi_{j\,R}\right] + \text{h.c.}, \tag{IV.1}$$

where i, j are generation indices, C is the Dirac charge-conjugation matrix, and the lepton fields $\psi_L(1/2, 0, -1)$ and $\psi_R(0, 1/2, -1)$ have the decomposition

$$\psi_L = \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} \quad \psi_R = \begin{pmatrix}\nu_R\\ e_R\end{pmatrix}. \tag{IV.2}$$

A discussion of the magnitude and role of the Majorana Yukawa coupling h^M can be found in refs. 2 and 5. Here, we briefly discuss constraints on a doubly-charged Higgs δ^{++} (where δ^{++} can be either δ_L^{++} or δ_R^{++}) arising from Bhabha scattering, $(g-2)$ of the muon, and muon decay. When there is a doubly charged Higgs present, the Bhabha scattering process receives an extra contribution from a diagram in which the δ^{++} is exchanged in the u-channel of $e^-e^+ \to e^-e^+$. From eq. (IV.1), the coupling of δ^{++} to e^+e^+ is given by $-h_{ee}^M P_L$, where $P_L \equiv (1-\gamma_5)/2$. The interference between the above-mentioned u-channel graph and the normal photon exchange graphs results in an extra contribution to the differential cross section given by

$$\frac{d\sigma}{dz} = \frac{(h^M_{ee})^2}{32\pi s} \frac{(1+z)^3}{(1-z)(1+z+2m^2_{\delta^{++}}/s)}, \tag{IV.3}$$

where $z = \cos\theta_{CM}$. Thus, requiring that there be no observable effect on the experimentally measured Bhabha cross section yields a constraint on $(h^M_{ee})^2$. The constraint obtained by requiring that the extra contribution of eq. (IV.3) lie within the experimental error bars of the data of ref. 8 is presented graphically in fig. 1. A similar constraint on $(h^M_{\mu\mu})^2$ derives from extra δ^{++}-exchange triangle-graph contributions to $(g-2)_\mu$ (in which the photon attaches either to the δ^{++} or to the μ). The total contribution from these two graphs is given by

$$\tfrac{1}{2}(g-2)_\mu = \frac{(h^M_{\mu\mu})^2}{8\pi^2}\left[\frac{\eta}{3} + \eta\ln\eta + \mathcal{O}(\eta^2)\right], \tag{IV.4}$$

where $\eta \equiv (m_\mu/m_{\delta^{++}})^2$. The restriction imposed by demanding that this contribution not exceed 2×10^{-8} is also presented in fig. 1. Finally, a δ^{++} could contribute to $\mu^+ \to e^+e^-e^+$ if $h^M_{\mu e}$ and h^M_{ee} are both non-zero.[9] We have estimated the limit from this source and find

$$h^M_{\mu e}h^M_{ee} < 1.5\times10^{-6}\left(\frac{gm_{\delta^{++}}}{m_{W_L}}\right)^2 \sim 1\times10^{-6}\left(\frac{m_{\delta^{++}}}{100\ GeV}\right)^2. \tag{IV.5}$$

If $h^M_{\mu e} = h^M_{ee}$, then this constraint is generally stronger than those of fig. 1. Alternatively, one can view this constraint as giving a limit on the μe flavor-non-diagonal component of h^M when the ee diagonal component is of significant size.

Limits on h^M_{ee} potentially lead to a significant lower bound on m_{W_R}. The argument is as follows. First, we note from eq. (IV.1) that h^M_{ee} is the same for the left and right sectors. Thus, it specifies the magnitude of the δ^0_R coupling to N_eN_e. After symmetry breaking, this coupling leads to a Majorana mass of size $\sqrt{2}h^M_{ee}v_R$ for N_e, and when eq. (I.1) holds, this mass is approximately equal to the physical N_e mass:

$$M_{N_e} = \sqrt{2}h^M_{ee}v_R. \tag{IV.6}$$

Combining this relation with eq. (II.25) (neglecting κ_1/v_R), we have:

$$M_{N_e} = 2\frac{h^M_{ee}}{g}m_{W_R}. \tag{IV.7}$$

Now, the experimental limits on neutrinoless double beta decay bound the contribution to this decay from a diagram in which two nucleons in the parent nucleus

Constraints on δ^{++} from Bhabha Scattering and $(g-2)_\mu$

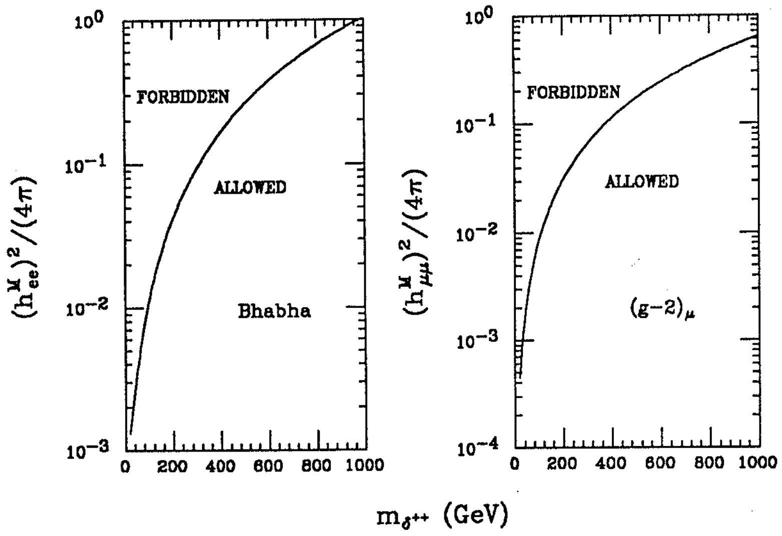

Figure 1: We plot the constraint on h^M_{ee} deriving from Bhabha scattering, and the constraint on $h^M_{\mu\mu}$ coming from $(g-2)_\mu$, as a function of the δ^{++} mass.

each emit a W_R, and then $W_RW_R \to ee$ via virtual N_e exchange. This contribution is proportional to $1/(m^4_{W_R}M_{N_e})$. For the constant of proportionality, c, we take the value implied by Haxton and Stephenson.[10] (The N_e exchange is a very short range interaction between nucleons, and the analysis of these authors takes into account the consequent suppression of the exchange due to the hard-core repulsion between nucleons, and the mitigation of this suppression due to the finite size of nucleons.) From the most recent bound of $8\times10^{23}\ yr$ on the neutrinoless double beta decay half-life of ^{76}Ge,[11] and the value of c, we find that

$$M_{N_e} > 63\ GeV\left(\frac{1.6\ TeV}{m_{W_R}}\right)^4. \tag{IV.8}$$

We may combine this result with eq. (IV.7) to obtain

$$m_{W_R} > 1.05\ TeV\left(\frac{0.1}{h^M_{ee}}\right)^{1/5}. \tag{IV.9}$$

We see that a strong upper bound on h^M_{ee} would force m_{W_R} above the lower limit coming from the $K_L - K_S$ mass difference. Furthermore, since the numerical coefficient in eq. (IV.9) only depends on the one-fifth power of c, it is rather insensitive to the theoretical uncertainties involved in obtaining a value for c.

Tri-linear couplings of the Δ_L triplet Higgs bosons to W_L and Z_1 are also potentially of phenomenological significance. First, we consider those involving one Δ_L member and two gauge bosons. The Feynman vertices are all proportional to v_L and appear below (we remove an overall factor of i from all our rules):

$$\begin{array}{llll} \delta_L^{++}W_L^-W_L^- & -\sqrt{2}g^2v_L & \delta_L^{0\,r}W_L^+W_L^- : & g^2v_L \\ \tilde{\delta}_L^+W_L^-Z_1 : & -\dfrac{g^2v_L}{\sqrt{2}c_{\theta_W}} & \delta_L^{0\,r}Z_1Z_1 : & \dfrac{2g^2v_L}{c_{\theta_W}^2}, \end{array} \qquad \text{(IV.10)}$$

where we have defined $c_{\theta_W} \equiv \cos\theta_W$. We also note that the potential couplings involving the photon are absent at tree level:

$$\tilde{\delta}_L^+W_L^-\gamma = 0 \qquad \delta_L^{0\,r}Z_1\gamma = 0. \qquad \text{(IV.11)}$$

These latter results are, as we have seen, a common feature of extended Higgs sectors. The second class of tri-linear couplings of potential importance is that describing interactions of one Δ_L member with one other scalar boson and either W_L or Z_1. They are listed below in the convention where all momenta are incoming and each rule is to be multiplied by $i(p_1-p_2)\cdot\epsilon$, where ϵ is the polarization of the W_L or Z_1, p_1 is the momentum of the first scalar boson listed, and p_2 is that of the second:

$$\begin{array}{llll} \delta_L^{--}\delta_L^{++}Z_1 : & \dfrac{gc_{2\theta_W}}{c_{\theta_W}} & \delta_L^{++}\delta_L^-W_L^- : & g \\ \delta_L^-\delta_L^+Z_1 : & -\dfrac{gs_{\theta_W}^2}{c_{\theta_W}} & \delta_L^{0\,r}\delta_L^+W_L^- : & \dfrac{g}{\sqrt{2}} \\ \delta_L^{0\,r}\delta_L^{0\,i}Z_1 : & -i\dfrac{g}{c_{\theta_W}} & \delta_L^{0\,i}\delta_L^+W_L^- : & -i\dfrac{g}{\sqrt{2}} \\ \delta_L^{0\,r,i}h^0Z_1 : & 0 & \delta_L^+h^0W_L^- : & 0 \\ \delta_L^{0\,r,i}H^0Z_1 : & 0 & \delta_L^+H^0W_L^- : & 0\,. \end{array} \qquad \text{(IV.12)}$$

We have defined $c_{2\theta_W} \equiv \cos 2\theta_W$ and $s_{\theta_W} \equiv \sin\theta_W$.

Next, we remark that there are trilinear couplings involving three Higgs bosons. The $\delta_L^{++}\delta_L^-\delta_L^-$ coupling is proportional to $v_L(\rho_1+\rho_2)$ and will vanish when $v_L = 0$, but is generally significant for $v_L \neq 0$. Because of the very large mass for the h^+, we need not concern ourselves with couplings of the δ_L's to it in considering their decays. Finally, couplings of the δ_L^0 to $\delta_L^+\delta_L^-$ or $\delta_L^{++}\delta_L^{--}$ are not relevant for δ_L^0 decays since the charged states are always more massive than the δ_L^0. This leaves us with couplings of the type $\delta_L^{0\,r,i}h^0h^0$, $\delta_L^{0\,r,i}h^0H^0$, *etc.* which are all proportional

to v_L, and thus are suppressed or, when $v_L = 0$ as in scenario (d), zero. In fact, when $v_L \neq 0$, δ_L^0 is massless since ρ_{dif} is zero, see eq. (III.5), and decays to h^0h^0 *etc.* could not occur.

The final class of couplings which must be mentioned are the quartic couplings. These do not involve any vacuum expectation values. The quartic Higgs self couplings are determined by the size of the coefficients appearing in the scalar potential. Note that they must involve even numbers of L or R triplet members. The quartic couplings of two Higgs bosons and two gauge bosons emerge from the covariant derivative in the Higgs kinetic energy terms. Specific couplings that we shall need are given below. Again, we remind the reader that since the h^+ is heavy, couplings involving it are not of interest here. Removing an overall factor of i we have:

$$\begin{array}{llll} \delta_L^{++}W_L^-W_L^+\delta_L^{--} : & g^2 & & \\ \delta_L^+W_L^-W_L^+\delta_L^- : & 2g^2 & & \\ \delta_L^0W_L^-W_L^+(\delta_L^0)^* : & g^2 & & \\ \delta_L^{++}W_L^-W_L^-(\delta_L^0)^* : & -2g^2 & \delta_L^{++}\delta_L^-\delta_L^-\delta_L^0 : & -2\rho_2 \\ \delta_L^{++}W_L^-Z_1\delta_L^- : & -\dfrac{g^2}{c_{\theta_W}}(1-3s_{\theta_W}^2) & \delta_L^{++}\delta_L^-\delta_L^-h^0 : & 0 \\ & & \delta_L^{++}\delta_L^-\delta_L^-H^0 : & 0, \\ \delta_L^+W_L^-Z_1(\delta_L^0)^* : & -\dfrac{g^2}{c_{\theta_W}}(1+s_{\theta_W}^2) & & \\ \delta_L^{++}W_L^-W_L^-h^0(H^0) : & 0 & & \\ \delta_L^+W_L^-Z_1h^0(H^0) : & 0 & & \end{array} \qquad \text{(IV.13)}$$

where the zeroes derive from the fact that h^0, H^0 do not mix with left-handed triplet states, see eq. (III.6).

With this background we can now turn to the phenomenology of the Δ_L scalar bosons in the two vacuum expectation value scenarios.

Case (c)

We recall that this case is somewhat unnatural in that $v_L \neq 0$ requires $\rho_{dif} = 0$. Thus we present only its most striking features. In case (c) we have $m_{\delta_L^{0\,r}}^2 = m_{\delta_L^{0\,i}}^2 = 0$. It is important to note that this is not an approximate statement—these two states are exactly massless at tree level. Of course, radiative corrections might lead to some non-zero mass, but we shall assume that it is still very small and that the two fields can be thought of as combining to form a single complex field δ_L^0. The other eigenstates of interest are δ_L^{++}, and in the approximation where we drop terms of order v_L/κ_1, δ_L^+. The δ_L^+ and δ_L^{++} also tend to be light — their masses do not depend on v_R:

$$m^2_{\delta_L^+} \simeq \frac{1}{2}\Delta\alpha\kappa_1^2, \quad m^2_{\delta_L^{++}} \simeq \Delta\alpha\kappa_1^2. \tag{IV.14}$$

In addition, we cannot choose $\Delta\alpha$ to be arbitrarily large. This is because these left handed triplet members couple to the W_L, and if their mass splittings are too large they will cause an experimentally unacceptable deviation in ρ_{EW}.* Defining

$$f(x,y) \equiv x^2 + y^2 - \frac{2x^2y^2}{x^2-y^2}\log\left(\frac{x^2}{y^2}\right), \tag{IV.15}$$

we have

$$\Delta\rho_{EW} = \frac{2g^2}{64\pi^2 m^2_{W_L}}\left[f(m_{\delta_L^0}, m_{\delta_L^+}) + f(m_{\delta_L^+}, m_{\delta_L^{++}})\right]. \tag{IV.16}$$

Requiring that $\Delta\rho_{EW} \lesssim 0.01$ leads to the constraint

$$m_{\delta_L^+} \lesssim 200\ GeV. \tag{IV.17}$$

Of course, smaller values of $m_{\delta_L^+}$ are allowed. Since we know the magnitude of κ_1 from the W_L mass (eq. (II.20)), we may consider $\Delta\alpha$ to be a function of $m_{\delta_L^+}$. We do not plot this dependence but note that $\Delta\alpha < 1$ for $m_{\delta_L^+} < 170\ GeV$. Given a value of $\Delta\alpha$ it is then amusing to compute the minimum value of v_R which satisfies the FCNC constraint of eq. (III.9). This value of v_R then determines the minimum W_R and Z_2 masses allowed for a given choice of $m_{\delta_L^+}$. This functional dependence is illustrated in fig. 2. Note that small values of $m_{\delta_L^+}$ are only possible if the W_R and Z_2 are very heavy.

There are several severe additional experimental constraints on this scenario. We first note that, at tree level, δ_L^0 (assuming radiative corrections to its zero tree-level mass are small) can only decay invisibly to $\nu\nu$. (The partial widths for particular final state neutrinos are determined by the h^M's.) In this case the decays $Z_1 \to \delta_L^0\delta_L^0$ are on the verge of ruling out this scenario since these decays are invisible and must be included in the neutrino-like Z_1 decay modes for which limits from astrophysics and the CERN $Sp\bar{p}S$ are available. The decay width is $\Gamma(Z_1 \to \delta_L^0\delta_L^0) = 2\Gamma(Z_1 \to \nu_e\bar{\nu}_e)$, *i.e.*, the $\delta_L^0\delta_L^0$ decay mode is equivalent to two new neutrino modes. (Actually, because of Bose statistics, Z_1 decays to $\delta_L^{0i}\delta_L^{0r}$, and we employ the coupling of eq. (IV.12).) This is close to being ruled out.[12]

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* We note that the $h^+h^0W_L$ coupling is very small, and, hence, no significant contribution to $\Delta\rho_{EW}$ arises from h^+-h^0 loops.

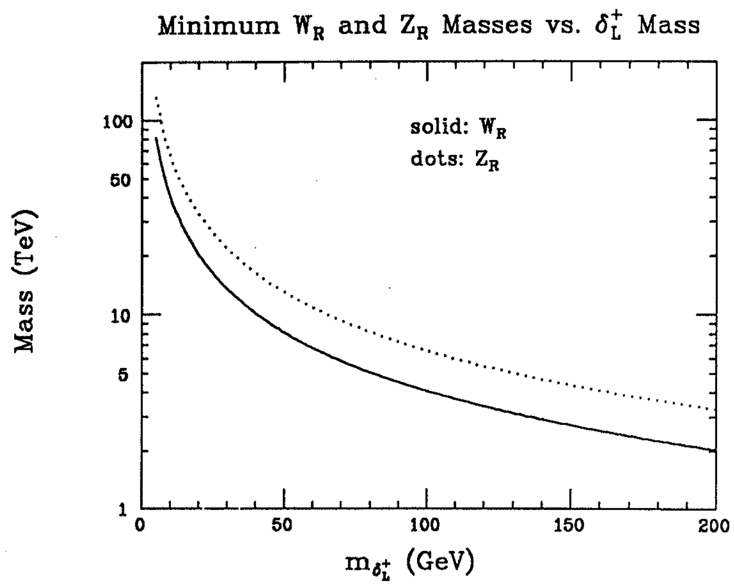


Figure 2: The minimum allowed W_R and Z_2 masses as a function of $m_{\delta_L^+}$, as determined by the requirement that the FCNC Higgs have mass larger than $5\ TeV$ in vev scenario (c).

For a nearly massless δ_L^0 there are also strong constraints on the allowed size of the h^M Yukawa couplings of eq. (IV.1). The strongest of these arises from a consideration of neutrino-less double-beta nuclear decays with "Majoron" emission, denoted $\beta\beta_{0\nu,Maj}$. Neglecting mixing, the important term in eq. (IV.1) can be written

$$\mathcal{L}_{ee} = \frac{h^M_{ee}}{\sqrt{2}}(\delta_L^{0r} + i\delta_L^{0i})\overline{(\nu_{eL})^c}\nu_{eL} + \text{h.c.}. \tag{IV.18}$$

The light neutrino mass state is $\nu_e \simeq \gamma_5(\nu_{eL} + (\nu_{eL})^c)/\sqrt{2}$ and we have

$$\mathcal{L}_{ee} = \sqrt{2}h^M_{ee}\left[\delta_L^{0r}\overline{\nu}_e\nu_e + i\delta_L^{0i}\overline{\nu}_e\gamma_5\nu_e\right]. \tag{IV.19}$$

If the δ_L^{0r} and δ_L^{0i} have very small masses, but are distinct mass eigenstates, then these couplings lead to two incoherent neutrino-less double-beta decays; one in which which the δ_L^{0r} is emitted and one in which the δ_L^{0i} is emitted. Constraints from $\beta\beta_{0\nu,Maj}$ decays may thus be reinterpreted in the present context by simply noting that in the Majoron models it is assumed that there is only one light particle, that can be emitted, whereas in our case there are two. We have taken the average value of the coupling constant limit given in Table 2 of ref. 11 coming from the lifetime limit on $\beta\beta_{0\nu,Maj}$ for ^{82}Se. After noting that g_X of that table is equivalent to $\sqrt{2}h^M_{ee}$, and after correcting for the fact that emission of two light particles is possible in our case, we obtain the limit:

$$\left|h^M_{ee}\right| < 2 \times 10^{-4}. \tag{IV.20}$$

Another bound on h^M_{ee} in the $v_L \neq 0$ case arises from the requirement that the "see-saw" mechanism operates as expected. In particular, one should have

$$h^M_{ee}\langle\delta^0_L\rangle \equiv M^{Maj}_L(\nu_e)/2 \ll M_D(\nu_e) \simeq M_e, \tag{IV.21}$$

where M_D is the Dirac mass for the ν_e and M^{Maj}_L is the Majorana type mass arising from the v_L expectation value; the see-saw mechanism requires that this latter mass be small compared to the Dirac mass, which in turn must be small compared to the Majorana mass arising from the v_R vev. Taking the upper bound on $\langle\delta^0_L\rangle = v_L/\sqrt{2}$ from eq. (II.24) we obtain

$$h^M_{ee} \ll 0.5\ MeV/(0.07\kappa_1/\sqrt{2}) \sim .5 \times 10^{-4}. \tag{IV.22}$$

Note that if v_L is smaller than the upper bound from eq. (II.24) then the experimental bound from $\beta\beta_{0\nu,Maj}$ could easily be the stronger bound of the two.

Before continuing, we note that the limits on h^M_{ee} from eqs. (IV.20) and (IV.22) combine with eq. (IV.9) to impose significantly increased lower bounds on m_{W_R}. For instance, using eq. (IV.20) yields $m_{W_R} > 3.2\ TeV$. Referring to fig. 2, we see that in the present scenario this means that only the range $m_{\delta^+_L} \lesssim 125\ GeV$ is allowed if the FCNC Higgs boson ϕ^0_2 were to have mass as low as 5 TeV. Of course, if $m_{\phi^0_2}$ is larger than this lower limit (as is likely), then no upper limit on $m_{\delta^+_L}$ can be inferred.

Strictly speaking the above limits are for h^M_{ee} only, but moderately restrictive bounds on many of the other h^M's can also be obtained in the case where δ^{0r}_L and δ^{0i}_L have very small mass. Once again, the relevant experimental constraints parallel those that emerge in Majoron models. The latter were reviewed in ref. 13. However, several details must be considered in interpreting the bounds given in ref. 13 for application to the left-right model being considered. First, as in previously discussed Majoron-like bounds, in our model both the δ^{0i}_L and δ^{0r}_L will contribute to the processes of relevance. In ref. 13 the Majoron model considered has both a light scalar S and a Majoron M. The couplings of the S and M are of the same form as the couplings of δ^{0r}_L and δ^{0i}_L, respectively, in our model, see eq. (IV.19). However, one must be careful to note that the couplings g employed in ref. 13 are equivalent to the couplings $\sqrt{2}h^M$ defined in this paper. This means the bounds on g^2's must be divided by 2 to obtain bounds on $(h^M)^2$'s. The physical processes that lead to the constraints divide into four sets. In the first set we have: (i) $\mu^+ \to \delta^{0r,i}_L\delta^{0r,i}_L e^+$; (ii) $\mu^+ \to \bar\nu_\mu\nu_e e^+$; and (iii) $\mu^+ \to \nu\nu\delta^{0r,i}_L e^+$. In process (i) the leading contribution to the rate for $\delta^{0r}_L\delta^{0i}_L$ emission is zero, while the rates for $\delta^{0r}_L\delta^{0r}_L$ and $\delta^{0i}_L\delta^{0i}_L$ are equal. In process (ii) the Standard Model expectation for the rate is modified by interference of the usual W exchange diagram with extra diagrams involving a virtual δ^{0r}_L or δ^{0i}_L in addition to the W. The δ^{0r}_L and δ^{0i}_L diagrams are equal. In process (iii), and in all those to be discussed shortly, δ^{0r}_L and δ^{0i}_L emissions are equally likely. (All these same observations apply to the calculations performed and quoted in ref. 13 for the M and S particles of the Majoron model. Thus, as stated earlier, the bounds of that reference apply directly after correcting for the different coupling constant definition.) The constraints of our second set derive from: (iv) $\pi^+ \to \mu^+X$; (v) $K^+ \to \mu^+X$; (vi) $K^+ \to e^+X$; and (vii) $\pi^+ \to e^+X$, where $X = \bar\nu\delta^{0r}_L$ or $\bar\nu\delta^{0i}_L$. Measurements of the spectrum in m_X limit the rate for the latter two final states for m_X values above a few MeV. The next two constraints derive from considering the total rates for (viii) $K^+ \to l^+L^0$ and (ix) $\pi^+ \to l^+L^0$, where $L^0 = \nu$, $\bar\nu\delta^{0r}_L$ or $\bar\nu\delta^{0i}_L$. Here, one looks for deviations of the ratio $\Gamma(M^+ \to e^+L^0)/\Gamma(M^+ \to \mu^+L^0)$ ($M = \pi$ or K) from SM expectations based on $L^0 = \nu$ only. Finally, we have the bremsstrahlung processes (x) $\nu_\mu N \to \mu^+\delta^{0r,i}_L X$ and (xi) $\nu_\mu N \to e^+\delta^{0r,i}_L X$. Here the δ^{0r}_L or δ^{0i}_L is bremsstrahlunged from the incoming ν_μ leaving behind a $\bar\nu_\mu$ or $\bar\nu_e$ which then scatters on the nuclear target as usual. Limits on the relevant couplings result from experimental limits on the cross section for producing a 'wrong'-sign final state lepton of a given type. The coupling constant limit coming from each process is given in Table 1 in terms of our h^M's. (In the table we temporarily drop the superscript M for convenience.)

The above limits are important in considering the decays of the various Δ_L scalar bosons. If all the h^M's are as small as the limit (IV.20), then widths for $\delta^{++}_L \to e^+e^+$ and $\delta^+_L \to e^+\nu_e$ and so forth are very small, and any other open channel would dominate over these spectacular leptonic signatures. (Nonetheless, we have checked that the lifetimes are short enough that these decays would be contained in typical detectors.) Let us carefully consider these alternative possibilities for this $v_L \neq 0$ scenario. Let us first focus on the δ^{++}_L. We must consider whether the decays

$$\begin{array}{ll} 1)\ \delta^{++}_L \to W^+_L W^+_L, & 4)\ \delta^{++}_L \to W^+_L W^+_L \delta^0_L, \\ 2)\ \delta^{++}_L \to \delta^+_L W^+_L, & 5)\ \delta^{++}_L \to \delta^+_L\delta^+_L\delta^0_L, \\ 3)\ \delta^{++}_L \to \delta^+_L\delta^+_L, & \end{array} \tag{IV.23}$$

are kinematically allowed, and if so which will dominate. It is obvious that the third

Table 1

Limits on Couplings of a Massless δ_L^{0i}

Coupling Combination	Upper Limit	Process
$h_{\mu e}^* h_{ee} + h_{\mu\mu}^* h_{\mu e} + h_{\mu\tau}^* h_{\tau e}$	9×10^{-3}	i
$\lvert h_{\mu e}\rvert^2$	3.3×10^{-3}	ii
$\sum_{i=\mu,e}(\lvert h_{ei}\rvert^2 + \lvert h_{\mu i}\rvert^2 + \lvert h_{\tau i}\rvert^2)$	1.6×10^{-3}	iii
$\lvert h_{e\mu}\rvert^2 + \lvert h_{\mu\mu}\rvert^2 + \lvert h_{\tau\mu}\rvert^2$	1.1×10^{-2}	iv
$\lvert h_{e\mu}\rvert^2 + \lvert h_{\mu\mu}\rvert^2 + \lvert h_{\tau\mu}\rvert^2$	1.2×10^{-4}	v
$\lvert h_{ee}\rvert^2 + \lvert h_{\mu e}\rvert^2 + \lvert h_{\tau e}\rvert^2$	9×10^{-5}	vi
$\lvert h_{ee}\rvert^2 + \lvert h_{\mu e}\rvert^2 + \lvert h_{\tau e}\rvert^2$	1.4×10^{-4}	vii
$\lvert h_{ee}\rvert^2 + \lvert h_{\mu e}\rvert^2 + \lvert h_{\tau e}\rvert^2$	2.2×10^{-5}	viii
$\lvert h_{ee}\rvert^2 + \lvert h_{\mu e}\rvert^2 + \lvert h_{\tau e}\rvert^2$	1.6×10^{-4}	ix
$\lvert h_{\mu\mu}\rvert^2$	1.3×10^{-2}	x
$\lvert h_{\mu e}\rvert^2$	1×10^{-2}	xi

and fifth are not allowed by the mass relations of eq. (IV.14). Whether or not the first, second and fourth modes are allowed depends upon the value chosen for $m_{\delta_L^+}$. In the range $m_{\delta_L^+} < 200\ GeV$ allowed by $\Delta\rho_{EW}$ limits, $m_{\delta_L^{++}} - m_{\delta_L^+} \lesssim 80\ GeV$, and the second decay is also not possible. However, the first and fourth decays are allowed once $m_{\delta_L^{++}} = \sqrt{2} m_{\delta_L^+}$ is larger than $2m_{W_L}$. For the $\delta_L^{++} \to W_L^+ W_L^+$ mode, since the relevant coupling from eq. (IV.10) is of order $g^2 v_L$, we see that unless v_L is very much smaller than the maximum allowed by eq. (II.24) the $W_L^+ W_L^+$ mode will dominate the e^+e^+ decay mode. The decay $\delta_L^{++} \to W_L^+ W_L^+ \delta_L^0$ will be relatively suppressed compared to the two-body mode because of the three-body phase space, but could be important if v_L is small. Nonetheless, its signature will be quite similar to that of the two-body mode when δ_L^0 is approximately massless and invisible. The signature for either mode will still be fairly spectacular, with some events containing two like-sign di-leptons plus missing energy, although with some branching ratio penalty. Note, though, that for the $W_L^+ W_L^+$ mode the leptons need not be of the same family, whereas, to the extent that h^M is fairly diagonal in family space, the directly produced leptons would tend to be from the same family.

In the case of the δ_L^+ we must consider the competing modes

$$\begin{aligned} \delta_L^+ &\to Z_1 W_L^+ \\ \delta_L^+ &\to Z_1 W_L^+ \delta_L^0 \\ \delta_L^+ &\to \delta_L^0 W_L^+ , \end{aligned} \tag{IV.24}$$

where we remind the reader that there is no interaction capable of yielding $\delta_L^+ \to h^0 W_L^+$ decays, and $\delta_L^+ \to h^+ X$ is impossible because of the large h^+ mass. We may ignore the first mode of eq. (IV.24), since the third has a much larger coupling, the same cubic dependence on $m_{\delta_L^+}$, and is always allowed when the first is. The second mode will be suppressed compared to the third by three-body phase space. Of course, the $\delta_L^+ \to \delta_L^0 W_L^+$ mode can produce a very similar final state to that coming from direct $\delta_L^+ \to \nu l^+$ decays—the δ_L^0 decays invisibly and the W_L^+ can decay leptonically.

Regarding production mechanisms, we limit ourselves to a relatively few remarks. First, in the present vev scenario where all the Δ_L members are likely to be light, eq. (IV.12) indicates that on-shell Z_1 and W_L decays could be a copious source of Higgs pair production. We have already noted the importance of the $Z_1 \to \delta_L^0 \delta_L^0$ decay, but here we quote all the widths relative to that for $Z_1 \to \nu\bar{\nu}$:

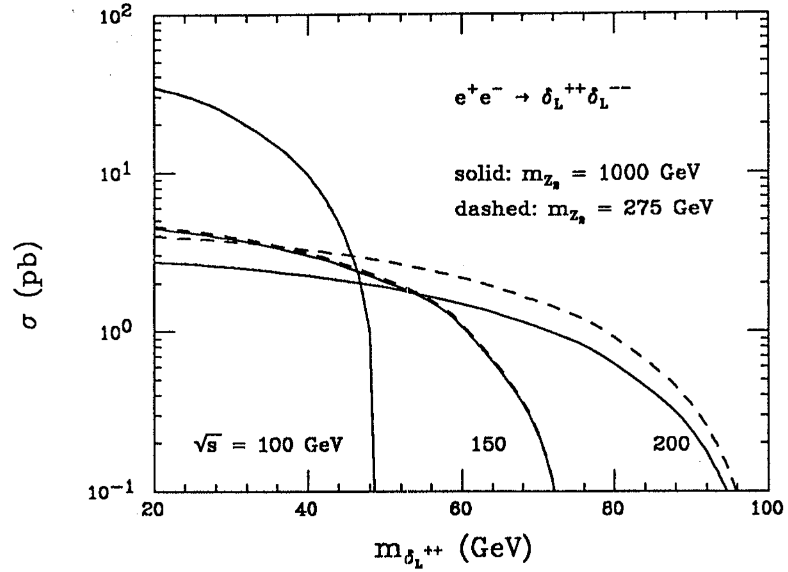
$$\nu\bar{\nu} : \delta_L^{0r}\delta_L^{0i} : \delta_L^+\delta_L^- : \delta_L^{++}\delta_L^{--} = 1 : 2 : 2s_{\theta_W}^4 : 2c_{2\theta_W}^2 . \tag{IV.25}$$

We see that the $\delta_L^+\delta_L^-$ mode is suppressed compared to $\delta_L^{0r}\delta_L^{0i}$, but that $Z_1 \to \delta_L^{++}\delta_L^{--}$ could be similar in size if phase space allowed, with a typical branching ratio of order several per cent. For the W_L^+ decays we normalize relative to the $e^+\nu$ channel and find:

$$e^+\nu : \delta_L^+\delta_L^{0r} : \delta_L^+\delta_L^{0i} : \delta_L^{++}\delta_L^- = 1 : \tfrac{1}{2} : \tfrac{1}{2} : 1. \tag{IV.26}$$

(In both eqs. (IV.25) and (IV.26) phase space corrections have been ignored.) Any copious source of real Z_1's or W_L's, such as a e^+e^- collider or the SSC, could be used to search for such decays. Presumably, processes like $Z_1 \to e^+e^+\mu^-\mu^-$, with the e^+e^+ and $\mu^-\mu^-$ having the same unique mass, would not be easily mimicked by backgrounds.

For higher δ_L^+ and δ_L^{++} masses, a TeV scale linear collider or the SSC would be appropriate machines. Production would be via virtual γ^*, Z_1^*, Z_2^* annihilation channel graphs. The cross sections would be substantial at both machines, and would clearly provide an abundance of spectacular signatures. The cross sections from Drell-Yan annihilation at an e^+e^- machine and at the SSC for production of $\delta_L^{++}\delta_L^{--}$ were computed in ref. 14. For the reader's convenience we reproduce the two relevant graphs in fig. 3.



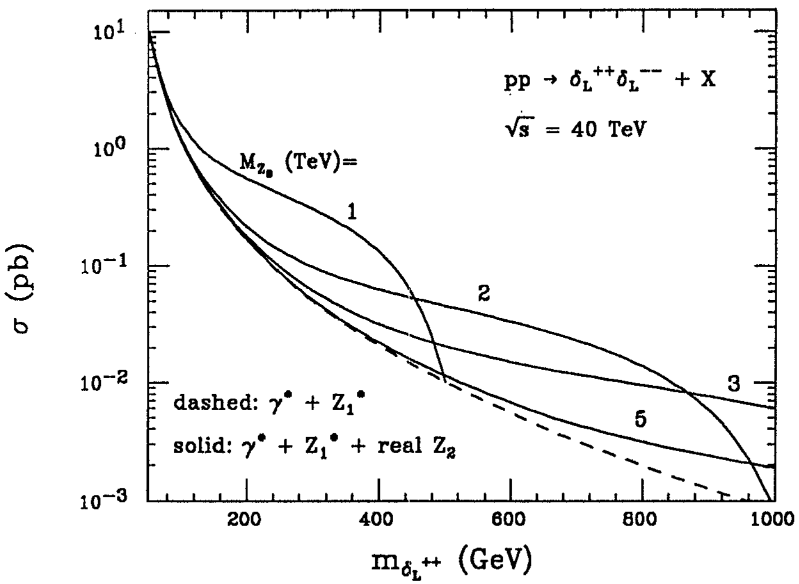


Figure 3: The cross sections for $\delta_L^{++}\delta_L^{--}$ production via Drell-Yan annihilation processes in e^+e^- and pp collisions, from ref. 14.

Additional production mechanisms that one might consider are the many $W_L W_L$ fusion processes that can make Higgs pairs. An example is $W_L^+ W_L^+ \to \delta_L^{++}\delta_L^0$ via a δ_L^+ exchange graph. However, these turn out to be unimportant except possibly for large δ_L^{++}, δ_L^+ or δ_L^0 masses, and will be deferred to the case (d) discussion. Another amusing mechanism is $W_L^+ W_L^+ \to \delta_L^{++}$, which proceeds only in the present $v_L \neq 0$ scenario. Roughly speaking this cross section can be estimated by comparison to the production of a SM Higgs boson of similar mass by $W_L^+ W_L^-$ fusion. Luminosity differences between $W_L^+ W_L^+$ and $W_L^+ W_L^-$ should be relatively minor and one need only compare the $W_L^+ W_L^+ \to \delta_L^{++}$ and $W_L^+ W_L^- \to h^0$ couplings. Referring to eq. (IV.10), we see that the former is suppressed by a factor of $(\sqrt{2}g^2 v_L)/(g^2\kappa_1/2)$ compared to the latter. For v_L near the upper limit of eq. (II.24) this factor is roughly 0.2 and it enters squared in the cross section. Nonetheless, for such maximal values, cross sections from this source would be substantial. Consider, for instance, $m_{\delta_L^{++}} = 100\ GeV$. At the SSC, the cross section for a SM Higgs boson of this mass from $W_L^+ W_L^-$ fusion is of order 50 pb, and the δ_L^{++} suppression factor from the square of $2\sqrt{2}v_L/\kappa_1$ is roughly 4×10^{-2}. This leaves us with $\gtrsim 10^4\ l^+l^+$ events from δ_L^{++} production and decay, for which it is hard to imagine a background. Clearly, v_L could be as small as one tenth the limit allowed by eq. (II.24) before the rate for these exotic events would begin to become too low for easy detection. Of course, once $m_{\delta_L^{++}} > 2m_W$, the $\delta_L^{++} \to W_L^+ W_L^+$ decays are likely to dominate, as we have discussed. In this case, the background from $qq \to qqW_L^+ W_L^+$, where the W's are radiated from a strong interaction quark scattering process, is expected to be quite substantial,[15] and, in view of the modest δ_L^{++} production rate, the δ_L^{++} would probably not be detectable in the $W_L^+ W_L^+$ decay mode.

Case (d)

As we have discussed, this case, specified by $v_L = 0$ and arbitrary ρ_{dif}, is the most natural scenario. However, as we easily discover from the mass formulae of eqs. (III.1)-(III.8) with $\rho_{dif} \neq 0$, all the scalar bosons other than the SM-like h^0 have mass contributions proportional to v_R. It is quite possible for all of them to be unobservably heavy. But the suppression of FCNC interactions does not *require* this; only ϕ_2^{0r} and ϕ_2^{0i} must be very heavy, implying that $\Delta\alpha$ cannot be small. The v_R terms in the masses of the H^0, δ_R^{++}, δ_L^0, δ_L^+ and δ_L^{++} are all determined by the $\rho_{1,2,3}$ potential parameters. As we have already discussed, ρ_2 must be negative while $\rho_1+\rho_2 = \rho_\Sigma$ must be positive. Thus, there is clearly potential for cancellation so that it is not inconceivable that ρ_{dif} and ρ_Σ could be modest in size. We will explore here the systematics that apply to this possibility.

Once again, the left-handed triplet members are potentially the most interesting of the physical scalars. For $v_L = 0$ they are all exact eigenstates and their masses are given by

$$
\begin{aligned}
m^2_{\delta^0_L} =& \frac{1}{2}\rho_{dif} v_R^2 \\
m^2_{\delta^+_L} =& \frac{1}{2}\rho_{dif} v_R^2 + \frac{1}{2}\Delta\alpha\kappa_1^2 \qquad \text{(IV.27)} \\
m^2_{\delta^{++}_L} =& \frac{1}{2}\rho_{dif} v_R^2 + \Delta\alpha\kappa_1^2 .
\end{aligned}
$$

Our strategy will be to choose a lower limit for the W_R mass and the mass of the FCNC scalar boson (we adopt 1.6 TeV and 5 TeV, respectively). The m_{W_R} lower limit then determines a lower limit for v_R through eq. (II.25), and the FCNC constraint then determines a lower limit on $\Delta\alpha$ via eq. (III.9). We can then use eq. (IV.27) to determine the minimum left-handed triplet masses as a function of ρ_{dif}. This dependence is illustrated in fig. 4. We note that only at extremely tiny values of ρ_{dif} does the mass splitting between the triplet members result in a $\Delta\rho_{EW}$ larger than 0.01. The most important point to note from this graph is that the left-handed triplet members remain less massive than 1 TeV for $\rho_{dif} \lesssim 0.2$, a not unreasonably small value for a difference of coupling constants.

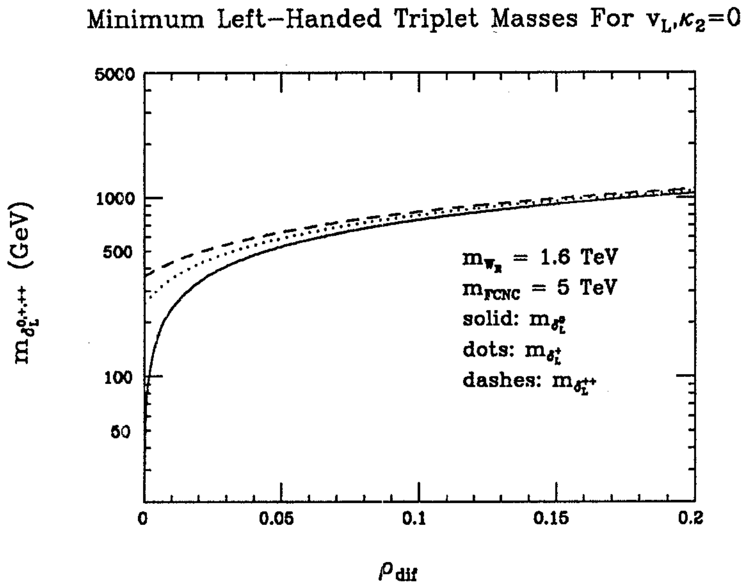


Figure 4: The minimum δ^0_L, δ^+_L and δ^{++}_L masses as a function of ρ_{dif}, for $m_{W_R} >$ 1.6 TeV and FCNC boson mass > 5 TeV.

The list of decays competing with the direct lepton decays of the δ^{++}_L and δ^+_L is shorter than before, since with $v_L = 0$ the direct couplings to two gauge bosons or

two δ_L Higgs vanish. However, three-body modes involving two gauge bosons plus the δ^0_L or two δ_L Higgs plus the δ^0_L survive. Let us focus on the δ^{++}_L for the moment. The mass systematics of fig. 4 are such that the $\delta^{++}_L \to \delta^+_L\delta^+_L\delta^0_L$ mode is forbidden for all ρ_{dif}, while $\delta^{++}_L \to \delta^+_L W^+_L$ is forbidden for $\rho_{dif} \gtrsim 0.01$ and $\delta^{++}_L \to W^+_L W^+_L \delta^0_L$ is forbidden for $\rho_{dif} \gtrsim 0.02$. Thus, the bosonic modes are allowed only for quite small values of ρ_{dif}. Of course, if $\Delta\alpha$ is made large (implying an FCNC scale much above the m_{W_R} lower limit of 1.6 TeV, see eq. (III.9)) the situation changes since the splitting between the δ^{++}_L, δ^+_L and δ^0_L masses becomes much larger. For instance, if we require the FCNC Higgs to have mass above 15 TeV, but keep $m_{W_R} = 1.6$ TeV, then these other decays will be allowed for all ρ_{dif}.

Whenever the bosonic decays are allowed, one must compare the direct lepton decay widths to the bosonic decays widths. In the case of the δ^{++}_L we have, for example:

$$\Gamma(\delta^{++}_L \to \delta^+_L W^+_L) = \frac{g^2}{2\pi}\frac{|\vec{p}_{final}|^3}{m^2_{W_L}} \qquad \Gamma(\delta^{++}_L \to l^+l^+) = \frac{|h^M_{ll}|^2}{32\pi} m_{\delta^{++}_L}. \qquad \text{(IV.28)}$$

If $\rho_{dif} \simeq 0$, $m_{W_R} = 1.6$ TeV and the FCNC-Higgs mass is 5 TeV, then from fig. 4 we find $m_{\delta^{++}_L} \simeq 360$ and $m_{\delta^+_L} \simeq 250$ GeV, and eq. (IV.28) yields $\Gamma(\delta^{++}_L \to \delta^+_L W^+_L) \simeq 2$ GeV. The only constraints in the present case on the h^M_{ll}'s derive from Bhabha scattering, $(g-2)_\mu$, and $\mu^+ \to e^+e^-e^+$. From fig. 1 we see that, for the δ^{++}_L mass in question, $\left|h^M_{ee,\mu\mu}\right|^2$ values as large as 1 are possible, yielding $\Gamma(\delta^{++}_L \to l^+l^+) \simeq 3.5$ GeV, for each l. Clearly, the direct lepton-lepton modes would be dominant despite the presence of the bosonic mode. If we consider the case of $\rho_{dif} \simeq 0$, $m_{W_R} = 1.6$ TeV and an FCNC-Higgs mass of 15 TeV, then $m_{\delta^{++}_L} = 1.07$ TeV, $m_{\delta^+_L} = 0.76$ TeV, and we find $\Gamma(\delta^{++}_L \to \delta^+_L W^+_L) \simeq 180$ GeV. From fig. 1 we see that, for this larger δ^{++}_L mass, $\left|h^M_{ee,\mu\mu}\right|^2$ values as large as 4π are possible, yielding $\Gamma(\delta^{++}_L \to l^+l^+) \simeq 130$ GeV, for each l. Clearly, the direct lepton-lepton modes would again be dominant despite the presence of the bosonic mode. Of course, if constraints on the h^M's improve, one would be pushed in the direction of dominance by the bosonic modes. Alternatively, one can adopt the attitude that h^M values larger than unity tend to force the theory into a non-perturbative domain and are not naturally incorporated into a unification framework. In this case, bosonic modes would generally be dominant when there is a large difference between the W_R mass and the FCNC-Higgs mass. However, a large separation in scales between m_{W_R} and the FCNC Higgs boson mass is not natural in the context of the theory we consider here in that it requires large $\Delta\alpha$ ($\Delta\alpha > 18$ for FCNC scale at 15 TeV). Thus, in the present scenario (d), it is most likely that bosonic decays of the δ^{++}_L will be either forbidden or phase space suppressed, and the δ^{++}_L would decay primarily to like-sign dilepton pairs if the lepton-lepton couplings are as large as allowed by current bounds.

Of course, entirely similar remarks apply to the δ_L^+. For the mass scenario of fig. 4, the bosonic decay modes are generally not allowed for the δ_L^+ decays; $m_{\delta_L^+} - m_{\delta_L^0} < m_{W_L}$ except for the region of $\rho_{dif} \lesssim 0.02$, while $\delta_L^+ \to Z_1 W_L \delta_L^0$ is kinematically allowed only for $\rho_{dif} \lesssim 0.002$.

Since $m_{\delta_L^0} \neq 0$ in this scenario, it could conceivably have some visible decay modes. However, the coupling of δ_L^0 to $W_L^+ W_L^-$ and $Z_1 Z_1$ vanishes in the present $v_L = 0$ situation; $\delta_L^0 \to h^0 h^0, h^0 H^0$, etc. couplings are also zero for $v_L = 0$. In addition, the fact that $v_L = 0$ and that δ_L^0 is a pure eigenstate guarantees that there is no tree-level coupling to give rise to $\delta_L^0 \to h^0 Z_1$ or $\delta_L^0 \to H^0 Z_1$. Thus, we conclude that the invisible $\delta_L^0 \to \nu\nu$ modes are still dominant. The only exception to this statement would arise if the relevant h^M's are so small that one-loop mediated decays become significant.

Production via Drell-Yan processes at a e^+e^- machine or at the SSC is unchanged from the discussion of case (c), and the cross sections appear in fig. 3. Our discussion of decays above implies that we would be guaranteed to have spectacular like-sign charged lepton decays in the $\delta_L^{++}\delta_L^{--}$ pair production case; for instance, one could have e^+e^+ on one side of the event, and $\mu^-\mu^-$ on the other side. Of course, in the present scenario, quite large δ_L^{++} masses are possible, and if the m_{W_R} mass were near 1.6 TeV, the Z_2 mass would be near 2.3 TeV and a substantial enhancement to the $\delta_L^{++}\delta_L^{--}$ pair cross section would arise. This is indicated also in fig. 3.

Finally, there are the $W_L^+ W_L^+ \to \delta_L^{++}(\delta_L^0)^*$, $W_L^+ W_L^- \to \delta_L^+ \delta_L^-$, $W_L^+ W_L^- \to \delta_L^{++}\delta_L^{--}$, $W_L^+ W_L^- \to \delta_L^0(\delta_L^0)^*$, *etc.* processes, alluded to earlier, to consider. (We do not consider analogous reactions with an initial Z_1 due to the much lower Z_1 luminosities at both hadron and e^+e^- colliders.) These are potentially capable of yielding substantial cross sections at large masses. However the cross sections are crucially dependent upon the mass splitting between the final state bosons and the virtual Δ_L bosons that appear in the Feynman graphs. In the present scenario this splitting is small, and the cross sections small. We illustrate this for $\delta_L^{++}\delta_L^{--}$ production in fig. 5, where we also illustrate what happens if we artificially increase the mass splitting between δ_L^{++} and δ_L^+ and hold it fixed as we change $m_{\delta_L^{++}}$. Even though these cross sections are small, it should be noted that $\delta_L^{++}(\delta_L^0)^*$ production cannot occur via Drell-Yan processes; thus, $W_L^+ W_L^+$ fusion is the primary tree-level source of such final states.

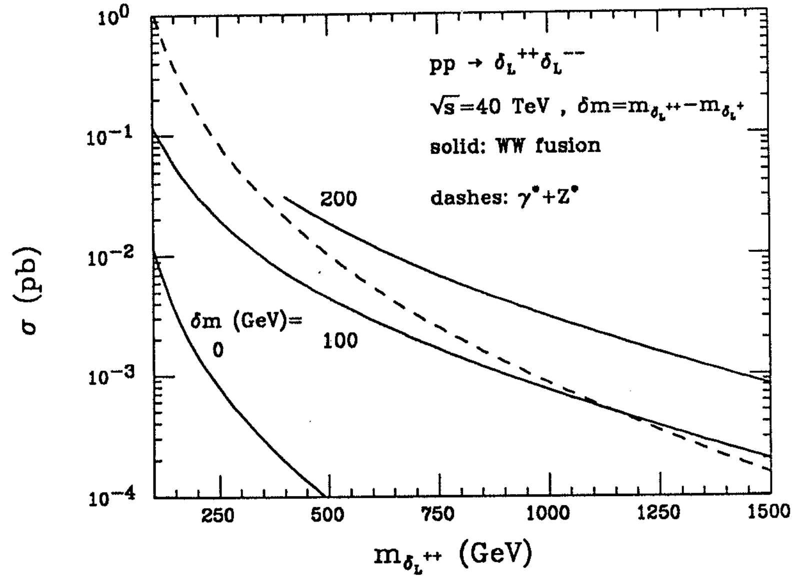


Figure 5: The cross section for $\delta_L^{++}\delta_L^{--}$ production via $W_L^+ W_L^+$ fusion in pp collisions at $\sqrt{s} = 40\ TeV$, for various values of $\delta m \equiv m_{\delta_L^{++}} - m_{\delta_L^+}$. For comparison the Drell-Yan cross section is also given (dashed curve).

V. Conclusions

We have investigated in detail the Higgs sector of the simplest possible left-right symmetric model, containing one bi-doublet Higgs field, one left-handed triplet field, and one right-handed triplet field. The basic motivations for considering this model seriously are several. The choice of extra triplet fields, as opposed to doublet fields, allows for the standard see-saw mechanism for small left-handed neutrino mass, provided the vacuum expectation value associated with the left-handed triplet field (v_L) is small compared to those for the bi-doublet (κ_1 and κ_2) which, in turn, must be small compared to that for the right-handed triplet (v_R).

Of course, generally one could have numerous triplet and bi-doublet fields. In principle, such additions to the simplest model could have advantages. For instance, appropriately chosen extra triplet fields allow for the possibility of a large W_R mass scale (as required by the lower bound of $m_{W_R} > 1.6\ TeV$), set by a new vev scale v_R', without forcing the lightest neutral gauge boson Z_1 to be heavy. This would be accomplished by taking $v_R \sim \kappa_1, \kappa_2 \ll v_R'$. However, in extensions studied to date, this scenario has severe difficulties. First, v_R' does not participate in the see-saw mechanism and small left-handed neutrino masses would require unnaturally large values for the relevant Majorana Yukawa couplings compared to the Dirac Yukawa couplings. Secondly, the masses of the physical scalar boson eigenstates associated with the triplet responsible for the Z_1 mass would all be small if $v_R \sim \kappa_1, \kappa_2$. We have reviewed the fact that this leads to a light scalar boson eigenstate which mediates flavor changing neutral currents, in violation of existing experimental constraints. Thus, the vev's associated with the usual triplet field are actually required to be large for acceptable phenomenology.

Additional bi-doublet fields have also been considered.[2] However, the original motivation for this was to allow for light neutrino masses via the see-saw mechanism even when the W_R is light. Since we now know that the W_R must be heavy, these considerations are no longer relevant, and there is no real need for extra bi-doublets.

Even within the simplest ϕ-Δ_L-Δ_R model, there is still flexibility associated with the exact form of the scalar potential. We have argued in favor of a particularly simple form appropriate when both left-right symmetry and certain discrete symmetries are imposed. By performing a complete and systematic study of the potential minima associated with various vacuum expectation value scenarios, we have shown that this simple form is particularly attractive for the following reasons.

1. Only $\kappa_2 = 0$ vacua lead to all Higgs bosons having positive mass-squared (implying a true local minimum) while being consistent with FCNC constraints and unitarity requirements on $W_L W_L$ scattering. This implies that the vacua that are consistent with the above requirements are also such that no W_L-W_R mixing occurs. This is desirable from the point of view of simplicity and since experimental constraints on such mixing could become significant in the future.
2. There is only one fully natural vev expectation value scenario, and in it $v_L = 0$ as well. This guarantees that the see-saw mechanism for small neutrino masses works properly and that corrections from this source to ρ_{EW} are absent. In addition, even though for large m_{W_R} it is natural that the FCNC violating scalar bosons also have large mass (thereby satisfying experimental limits), not all scalar bosons must be heavy. In particular, it is not unreasonable that the left-handed triplet members could be light enough to be produced at e^+e^- colliders or the SSC. The doubly charged left-handed triplet members would yield spectacular like-sign charged lepton decay signatures.
3. The only other vev scenario that could be phenomenologically acceptable is one with $v_L \neq 0$. It suffers from a degree of unnaturalness, and will be easily ruled out if limits on invisible decays of the Z_1 become stronger. On the other hand, should this scenario be nature's choice, we would again have many spectacular leptonic signatures coming from the left-handed triplet scalar bosons. For instance, the doubly charged left-handed triplet members could even be light enough to be pair-produced in on-shell Z_1 decays. Certainly they are light enough to yield copious Drell-Yan pair cross sections.

We note that, in this paper, we have explored, and included in our analysis of direct discovery possibilities, a large variety of constraints from low-energy experiments upon the left-handed triplet members. The constraints of interest are those deriving from their couplings to lepton-lepton channels. (All such couplings are related to one-another by $SU(2)_L$ Clebsch-Gordon coefficients.) Many of these constraints have not been previously discussed or applied in the left-right model.

We have found that there are significant limits on the charged-lepton—charged-lepton coupling of the doubly-charged triplet Higgs when it has a moderate mass. In addition, the neutrino-neutrino coupling of a very light neutral triplet Higgs is very strongly constrained. Thus, the size of lepton-lepton—left-handed-triplet couplings is generally quite restricted. This is particularly amusing since, as we have demonstrated, a significant upper bound on the magnitude of this lepton-lepton coupling results in a significant lower bound on the W_R mass. This lower bound is generally competitive with the 1.6 TeV bound coming from $K_L - K_S$ mixing, and could become even stronger as experimental limits on the lepton-lepton couplings of the left-handed triplet and on neutrino-less double-beta decay improve.

Overall, we find that existing and planned accelerators would have a significant chance of detecting the exotic scalar bosons emerging from the left-handed triplet of the Higgs sector in the simplest and most strongly motivated left-right symmetric model. It is these that are most likely to be experimentally accessible, simply because the large scales of right-handed symmetry breaking and FCNC experimental constraints do not necessarily force them to be extremely heavy. These exotic scalar bosons are one of the most unique consequences of gauge theories in which left-right symmetry is spontaneously broken.

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APPENDIX A

Higgs Boson Mass Matrices

In this appendix we give a variety of useful results for the mass-squared matrices of the various Higgs sectors. We begin with that for the neutral scalar Higgs bosons, which is in general a 4×4 matrix. We denote this matrix by $\mathcal{M}^2_{0r}$, where the $0r$ subscript indicates the matrix for the neutral-real fields. Since both v_R and κ_1 must be non-zero, we give the general form of this matrix after substituting the non-trivial conditions from eq. (II.12) resulting from the derivative conditions in these two variables. These two conditions are:

$$\mu_1^2 = (\lambda_\Sigma - \Sigma\lambda)\kappa_2^2 + \frac{1}{2}\alpha'_\Sigma(v_R^2 + v_L^2) + \lambda_\Sigma\kappa_1^2, \qquad (A.1)$$

for the κ_1 derivative, and

$$\mu_2^2 = \frac{1}{2}(\rho_{dif} + \rho_\Sigma)v_L^2 + \rho_\Sigma v_R^2 + \frac{1}{2}(\alpha_\Sigma\kappa_2^2 + \alpha'_\Sigma\kappa_1^2), \qquad (A.2)$$

from the v_R derivative. To simplify the notation we use the parameter combinations defined in the text. We find $\mathcal{M}^2_{0r}$ =

$$\begin{pmatrix} 2\kappa_1^2\lambda_\Sigma & 2\kappa_1\kappa_2[\lambda_\Sigma - \Sigma\lambda] & v_R\kappa_1\alpha'_\Sigma & v_L\kappa_1\alpha'_\Sigma \\ 2\kappa_1\kappa_2[\lambda_\Sigma - \Sigma\lambda] & 2\kappa_2^2\lambda_\Sigma + \Delta_{22} & v_R\kappa_2\alpha_\Sigma & v_L\kappa_2\alpha_\Sigma \\ v_R\kappa_1\alpha'_\Sigma & v_R\kappa_2\alpha_\Sigma & 2v_R^2\rho_\Sigma & v_Lv_R(2\rho_\Sigma + \rho_{dif}) \\ v_L\kappa_1\alpha'_\Sigma & v_L\kappa_2\alpha_\Sigma & v_Lv_R(2\rho_\Sigma + \rho_{dif}) & 2v_L^2\rho_\Sigma + \frac{1}{2}(v_R^2 - v_L^2)\rho_{dif} \end{pmatrix} \quad \text{(A.3)}$$

Our basis is ϕ_1^{0r}-ϕ_2^{0r}-δ_R^{0r}-δ_L^{0r} and $\Delta_{22} = \Delta\alpha(v_L^2 + v_R^2) - (\kappa_1^2 - \kappa_2^2)\Sigma\lambda$.

The other mass matrices for the neutral-pseudoscalar sector (for the imaginary components of the neutral fields), the singly charged sector, and the doubly charged sector can also be easily presented. Since we have shown that only $\kappa_2 = 0$ leads to acceptable minima for the neutral-scalar sector mass matrix above, we present the remaining mass matrices taking $\kappa_2 = 0$ in addition to employing eqs. (A.1) and (A.2). They are given below:

$$\mathcal{M}^2_{++} = \begin{pmatrix} -\rho_2 v_R^2 + \Delta\alpha\kappa_1^2 & 0 \\ 0 & -\rho_2 v_L^2 + \frac{\rho_{dif}}{2}v_R^2 + \Delta\alpha\kappa_1^2 \end{pmatrix}, \quad \text{(A.4)}$$

in the δ_R^{++}-δ_L^{++} basis;

$$\mathcal{M}^2_{+} = \begin{pmatrix} \Delta\alpha v_R^2 & 0 & \frac{\Delta\alpha}{\sqrt{2}}v_R\kappa_1 & 0 \\ 0 & \Delta\alpha v_L^2 & 0 & \frac{\Delta\alpha}{\sqrt{2}}v_L\kappa_1 \\ \frac{\Delta\alpha}{\sqrt{2}}v_R\kappa_1 & 0 & \frac{\Delta\alpha}{2}\kappa_1^2 & 0 \\ 0 & \frac{\Delta\alpha}{\sqrt{2}}v_L\kappa_1 & 0 & \frac{\rho_{dif}}{2}(v_R^2 - v_L^2) + \frac{\Delta\alpha}{2}\kappa_1^2 \end{pmatrix}, \quad \text{(A.5)}$$

in the ϕ_1^+-ϕ_2^+-δ_R^+-δ_L^+ basis; and

$$\mathcal{M}^2_{0i} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta\alpha(v_R^2 + v_L^2) + \Sigma'\lambda\kappa_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\rho_{dif}}{2}v_R^2 \end{pmatrix}, \quad \text{(A.6)}$$

in the ϕ_1^{0i}-ϕ_2^{0i}-δ_R^{0i}-δ_L^{0i} basis, and the various parameter combinations are defined in eq. (II.10). We observe that, as required in order to give the W_L, W_R, Z_1, and Z_2 mass, there are two zero mass Goldstone boson eigenstates for both $\mathcal{M}^2_+$ and $\mathcal{M}^2_{0i}$.

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