

## Higgs inflation from Standard Model criticality

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### Abstract

The observed Higgs mass  $M_H = 125.9 \pm 0.4$  GeV leads to the criticality of the Standard Model, that is, the Higgs potential becomes flat around the scale  $10^{17-18}$  GeV for the top mass 171.3 GeV. Earlier we have proposed a Higgs inflation scenario in which this criticality plays a crucial role. In this paper, we investigate the detailed cosmological predictions of this scenario in light of the latest Planck and BICEP2 results. We also consider the Higgs portal scalar dark matter model, and compute the Higgs one-loop effective potential with the two-loop renormalization group improvement. We find a constraint on the coupling between Higgs and dark matter which depends on the inflationary parameters.

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## I. INTRODUCTION

The observed value of the Higgs mass [1]<sup>1</sup>

$$M_H = 125.9 \pm 0.4 \text{ GeV} \quad (1)$$

indicates that the Standard Model (SM) Higgs potential becomes small and flat at the scale around  $10^{17-18}$  GeV for the top mass 171.3 GeV; see e.g. [4–16] for latest analyses.<sup>2</sup> This fact suggests [34] that the Higgs field beyond the ultraviolet (UV) cutoff of the SM, at the criticality [35], may play the role of the slowly rolling inflaton in the early universe; see Ref. [36] for the original proposal to use the Higgs field for the cosmological inflation and also Refs. [37–40] for the idea to use the false vacuum of the SM at criticality. Especially, under the presence of the large non-minimal coupling  $\xi \sim 10^4$  between the Higgs field and the Ricci curvature, there arises a plateau in the SM effective potential above the field value  $\varphi \sim M_P/\sqrt{\xi}$ , and enough number of  $e$ -foldings is achieved without introducing any other field beyond the SM [36, 41–46]. In Ref. [47], it has been shown by numerical analysis that smaller values of  $\xi \sim 400$  and 90 are possible in the prescriptions I and II, respectively; see Sec. III for what “prescription” means.

In Ref. [48], we have proposed to push the idea of Ref. [34] to use the criticality of the SM for the Higgs inflation scenario in order to accommodate a lower value of  $\xi = 7\text{--}100$ , as well as a wider range of the tensor-to-scalar ratio  $r \lesssim 0.2$ ; see also Refs. [49, 50].<sup>3</sup> Similar attempts have been done in some extensions of the SM [48, 51–55]. There have also been different directions of the extension of the Higgs inflation involving higher dimensional operators [56–63]. See also Refs. [64–92].

In this paper, we give detailed analyses of the Higgs inflation scenario proposed in Ref. [48] that utilizes the saddle point, at which both the first and second derivatives of the potential become very small. The scale dependence of the effective quartic coupling  $\lambda_{\text{eff}}$  is very important to determine the effective potential, whose behavior around the saddle point is characterized by the minimum value  $\lambda_{\text{min}}$  of the effective coupling  $\lambda_{\text{eff}}$ , the corresponding scale  $\mu_{\text{min}}$ , and the second derivative  $\beta_2$  of  $\lambda_{\text{eff}}$  around  $\mu_{\text{min}}$ , in addition to  $\xi$ . We examine the predictions of this model on spectral index  $n_s$ , tensor to scalar ratio  $r$ , and the running of spectral index  $dn_s/d \ln k$ .

<sup>1</sup> The latest values of the Higgs mass are  $125.03^{+0.26}_{-0.27}(\text{stat})^{+0.13}_{-0.15}(\text{syst})$  GeV (CMS)[2] and  $125.36 \pm 0.37(\text{stat}) \pm 0.18(\text{syst})$  GeV (ATLAS)[3], which are consistent with each other and also with the PDG value we are using here.

<sup>2</sup> It is an intriguing fact that the bare Higgs mass also becomes small at the same scale [9, 17–19]; see also Refs. [20–24]. The running Higgs mass after the subtraction of the quadratic divergence is considered e.g. in Ref. [25]; see also Refs. [26–33].

<sup>3</sup> See Sec. V A 3 for an explanation for the apparent discrepancy between the results from Refs. [48, 50] and those from Ref. [47, 49].

We also estimate how small the higher dimensional Planck-suppressed operators must be in order to maintain the observed values of the cosmic microwave background (CMB). For that purpose, we pick up the six-dimensional operator  $\varphi^6/M_P^2$  in the Jordan-frame potential as a concrete example, and compute the CMB spectral indices.

We also evaluate the relation between the high-scale parameters  $\mu_{\min}$ ,  $\beta_2$  and the low energy parameters in the SM, as well as in the Higgs portal scalar dark matter (DM) model, using one-loop effective potential and the two-loop renormalization group equation (RGE).

This paper is organized as follows. In Section 2, we review the criticality, namely the flatness and smallness, of the SM Higgs potential around the scale  $10^{17-18}$  GeV. In Section 3, we review the Higgs inflation scenario in a wider perspective. In Section 4, we investigate the predictions of this model in detail. In Section 5, we consider the extension with the Higgs portal scalar DM. We summarize our results in the last section.

## II. STANDARD MODEL HIGGS POTENTIAL

In the SM on the flat spacetime background, the one-loop effective potential calculated in the  $\overline{\text{MS}}$  scheme in the Landau gauge is

$$V = V_{\text{tree}} + \Delta V_{\text{1-loop}}, \quad (2)$$

with

$$V_{\text{tree}} = e^{4\Gamma(\varphi)} \frac{\lambda(\mu)}{4} \varphi^4, \quad (3)$$

$$\begin{aligned} \Delta V_{\text{1-loop}} = e^{4\Gamma(\varphi)} \left\{ -\frac{3m_t(\varphi)^4}{16\pi^2} \left( \ln \frac{m_t(\varphi)^2}{\mu^2} - \frac{3}{2} + 2\Gamma(\varphi) \right) \right. \\ \left. + \frac{6m_W(\varphi)^4}{64\pi^2} \left( \ln \frac{m_W(\varphi)^2}{\mu^2} - \frac{5}{6} + 2\Gamma(\varphi) \right) + \frac{3m_Z(\varphi)^4}{64\pi^2} \left( \ln \frac{m_Z(\varphi)^2}{\mu^2} - \frac{5}{6} + 2\Gamma(\varphi) \right) \right\}, \end{aligned} \quad (4)$$

$$\Gamma(\varphi) = \int_{M_t}^{\varphi} \gamma \, d \ln \mu, \quad (5)$$

$$\gamma = \frac{1}{(4\pi)^2} \left( \frac{9}{4} g_2^2 + \frac{3}{4} g_Y^2 - 3y_t^2 \right), \quad (6)$$

where  $m_W(\varphi) = g_2 \varphi / 2$ ,  $m_Z(\varphi) = \sqrt{g_Y^2 + g_2^2} \varphi / 2$ , and  $m_t(\varphi) = y_t \varphi / \sqrt{2}$ . We have neglected the effects from the loops of the Higgs and would-be Nambu-Goldstone bosons since we are interested in the scale where  $\lambda$  becomes small. We also neglect the quadratic term; the bare Higgs mass is canceled by the loop effect at low energies; see e.g. Appendix B in Ref. [34].

We define the effective quartic coupling as [6]

$$V(\varphi) = \frac{\lambda_{\text{eff}}(\varphi, \mu)}{4} \varphi^4. \quad (7)$$

At the one-loop level,

$$\begin{aligned} \lambda_{\text{eff}}(\varphi, \mu) = e^{4\Gamma(\varphi)} \lambda(\mu) + e^{4\Gamma(\varphi)} \frac{1}{16\pi^2} \left[ -3y_t^4 \left( \ln \frac{y_t^2 \varphi^2}{2\mu^2} - \frac{3}{2} + 2\Gamma(\varphi) \right) \right. \\ \left. + \frac{3g_2^4}{8} \left( \ln \frac{g_2^2 \varphi^2}{4\mu^2} - \frac{5}{6} + 2\Gamma(\varphi) \right) + \frac{3(g_Y^2 + g_2^2)^2}{16} \left( \ln \frac{(g_Y^2 + g_2^2) \varphi^2}{4\mu^2} - \frac{5}{6} + 2\Gamma(\varphi) \right) \right], \end{aligned} \quad (8)$$

where we have made the scale dependence explicit in the right hand side while omitting it in  $y_t$ ,  $g_2$ , and  $g_Y$ , which corresponds to the two-loop corrections.

In the SM on the flat spacetime background,  $\Delta V_{1\text{-loop}}$  is minimized by

$$\begin{aligned} \ln \frac{\varphi^2}{\mu^2} &= \frac{C_t^2 (-\ln C_t + \frac{3}{2} - 2\Gamma) - 2C_W^2 (-\ln C_W + \frac{5}{6} - 2\Gamma) - C_Z^2 (-\ln C_Z + \frac{5}{6} - 2\Gamma)}{C_t^2 - 2C_W^2 - C_Z^2} \\ &= \frac{(-\ln C_t + \frac{3}{2} - 2\Gamma) - 2\frac{C_W^2}{C_t^2} (-\ln C_W + \frac{5}{6} - 2\Gamma) - \frac{C_Z^2}{C_t^2} (-\ln C_Z + \frac{5}{6} - 2\Gamma)}{1 - 2\frac{C_W^2}{C_t^2} - \frac{C_Z^2}{C_t^2}}, \end{aligned} \quad (9)$$

where  $C_W = g_2^2/4$ ,  $C_Z = (g_Y^2 + g_2^2)/4$ , and  $C_t = y_t^2/2$ . Around  $\mu_{\text{min}} \sim 10^{17-18}\text{GeV}$ , Eq. (9) leads to  $\mu \simeq 0.23\varphi$ . However, because the difference of the numerical values of the one-loop effective potential for  $\mu = \varphi$  and  $\mu = 0.23\varphi$  is negligibly small, we use  $\mu = \varphi$  hereafter in this section.

Then, we obtain

$$V(\varphi) = \frac{\lambda_{\text{eff}}(\mu = \varphi)}{4} \varphi^4, \quad (10)$$

where  $\lambda_{\text{eff}}(\mu)$  is written by

$$\begin{aligned} \lambda_{\text{eff}}(\mu) = e^{4\Gamma} \lambda(\mu) + e^{4\Gamma} \frac{1}{16\pi^2} \left[ -3y_t^4 \left( \ln \frac{y_t^2}{2} - \frac{3}{2} + 2\Gamma \right) \right. \\ \left. + \frac{3g_2^4}{8} \left( \ln \frac{g_2^2}{4} - \frac{5}{6} + 2\Gamma \right) + \frac{3(g_Y^2 + g_2^2)^2}{16} \left( \ln \frac{(g_Y^2 + g_2^2)}{4} - \frac{5}{6} + 2\Gamma \right) \right], \end{aligned} \quad (11)$$

at the one-loop level.

The effective coupling  $\lambda_{\text{eff}}$  is quartically sensitive to  $y_t$  thus the top quark mass,  $M_t$ , which is scheme dependently defined. The actual value of  $M_t$  is known with large uncertainties at the level

of GeV scales depending on the measurements:

$$M_t^{\text{pole}} = \begin{cases} 171.2 \pm 2.4 \text{ GeV}, & \text{MITP [93]}, \\ 176.7_{-3.4}^{+4.0} \text{ GeV}, & \text{PDG [94]}, \end{cases} \quad (12)$$

$$M_t^{\text{Pythia}} = \begin{cases} 173.21 \pm 0.51 \pm 0.71 \text{ GeV}, & \text{direct measurement, PDG [94]}, \\ 174.98 \pm 0.76 \text{ GeV}, & \text{D0 [95]}, \\ 174.34 \pm 0.64 \text{ GeV}, & \text{D0+CDF [96]}, \\ 173.34 \pm 0.76 \text{ GeV}, & \text{ATLAS [97]}, \\ 172.38 \pm 0.10 \pm 0.65 \text{ GeV}, & \text{CMS [98]}. \end{cases} \quad (13)$$

One should note that the ‘‘directly measured value’’ in Eq. (13) obtained by Tevatron (D0 and CDF) and by LHC (ATLAS and CMS) is indeed a parameter in Monte Carlo simulation code [7, 99], so-called the Pythia mass [100], whose physical relation to the pole and  $\overline{\text{MS}}$  masses is not well established. In discussing the Higgs inflation near criticality, however, the only important fact is that the critical value for the pole mass  $M_t \simeq 171.3 \text{ GeV}$ , shown just below, is perfectly consistent with both the MITP and PDG within  $2\sigma$  confidence level. In below, we will take the MITP value as a benchmark.

In Fig. 1, we can see that  $\lambda_{\text{eff}}$  has the minimum around  $10^{17-18} \text{ GeV}$ . Interestingly, if  $M_t \simeq 171.3 \text{ GeV}$ , the minimum value of  $\lambda_{\text{eff}}$  becomes zero around the scale  $10^{17-18} \text{ GeV}$ , and the Higgs potential has a plateau around  $10^{17-18} \text{ GeV}$  as shown in Fig. 2.<sup>4</sup>

Let us expand the effective potential of the Higgs field  $V_{\text{eff}}(\varphi)$  on the flat space-time background around its minimum in terms of  $\ln \varphi$ :

$$V(\varphi) = \frac{\lambda_{\text{eff}}(\mu = \varphi)}{4} \varphi^4, \quad \lambda_{\text{eff}}(\mu) = \lambda_{\text{min}} + \sum_{n=2}^{\infty} \frac{\beta_n}{(16\pi^2)^n} \left( \ln \frac{\mu}{\mu_{\text{min}}} \right)^2, \quad (14)$$

where the overall factor  $\varphi^4$  is put to make the expansion well-behaved. In the potential analysis around the minimum, we can safely neglect the higher order terms with  $n \geq 3$ , and will omit them hereafter. By tuning the top mass for a given Higgs mass, we can obtain arbitrarily small  $\lambda_{\text{min}}$ . This fact is crucial for our inflation scenario.

We note that for the potential to be monotonically increasing [48],  $\lambda_{\text{min}}$  must be larger than a

<sup>4</sup> It has been known that such a position of plateau is unphysical and can vary by an order of magnitude depending on the gauge choice [101]. The gauge dependence of the effective potential can be absorbed by a field redefinition [102]. The eventual field equation for  $\varphi$  should not depend on such a choice, but the field value here necessarily contains this amount of uncertainty. See also Refs. [103, 104] for a further account on the gauge (in)dependence.

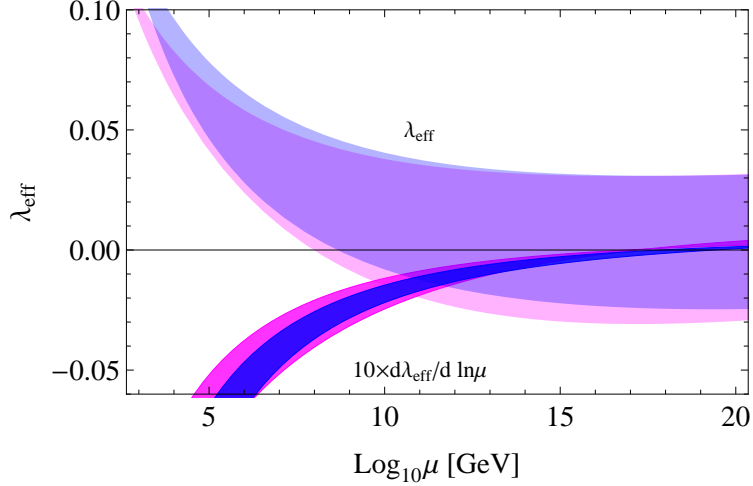


FIG. 1: The light red (lower) and blue (upper) bands are 2-loop RGE running of  $\lambda(\mu)$  and  $\lambda_{\text{eff}}(\mu)$  (11), respectively. The dark red (upper) and blue (lower) bands are the beta function times ten  $10 \times d\lambda_{\text{eff}}/d \ln \mu$  evaluated at the tree and 1-loop levels, respectively. We take  $M_H = 125.9 \text{ GeV}$  and  $\alpha_s = 0.1185$ . The band corresponds to 95% CL deviation of  $M_t$  [93]; see Eq. (12).

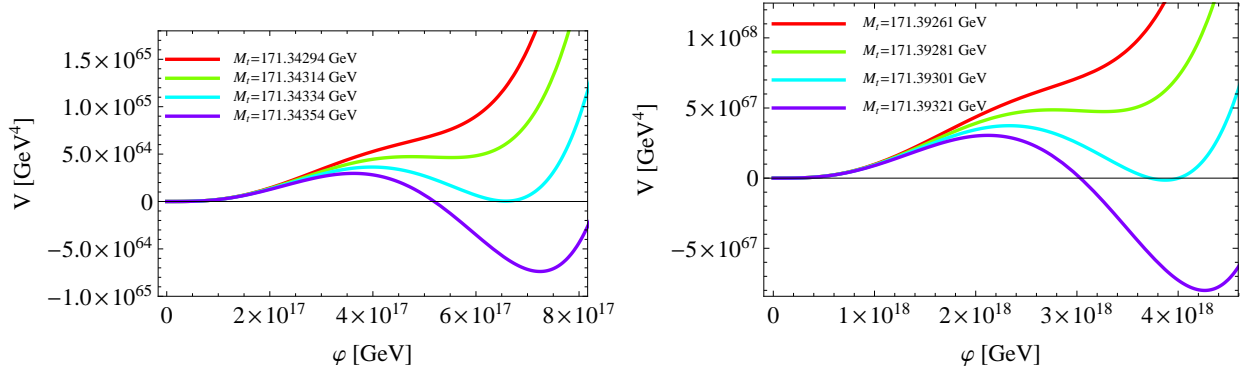


FIG. 2: Left: The tree level Higgs potential (3) as a function of Higgs field  $\varphi$ . Right: The one-loop Higgs potential (3) and (4). We take  $M_H = 125.9 \text{ GeV}$  and  $\alpha_s = 0.1185$ .

critical value  $\lambda_c$ :

$$\lambda_{\min} \geq \lambda_c := \frac{\beta_2}{(64\pi^2)^2}. \quad (15)$$

When  $\lambda_{\min}$  saturates this inequality,

$$\lambda_{\min} = \lambda_c, \quad (16)$$

there appears a true saddle point of the potential  $V_\varphi = V_{\varphi\varphi} = 0$ . We will see in Section V A that in the prescription I, this value  $\lambda_c$  also gives the true saddle point of the modified potential:

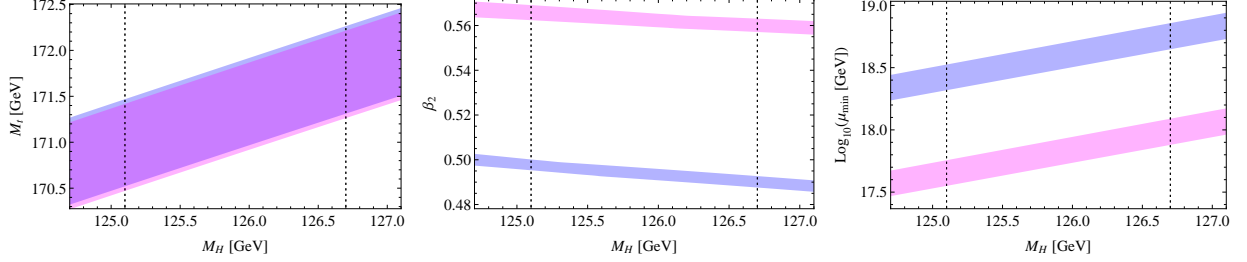


FIG. 3:  $M_t$  (left),  $\beta_2$  (center), and  $\mu_{\min}$  (right) that realize the condition  $\lambda_{\min} = \lambda_c$  are plotted as functions of  $M_H$ . We have imposed the condition  $\lambda_{\min} = \lambda_c$  using the tree-level potential (3) and the one-loop one, (3) and (4), for the red and blue bands, respectively. (The one-loop blue band is the upper one for left and right, whereas the lower for center.) The width of the bands corresponds to the 95% CL of  $\alpha_s(M_Z)$ . Dotted lines show the current 95% CL for  $M_H$ ; see Eq. (1).

$$U_\varphi = U_{\varphi\varphi} = 0.^5$$

In the left, center, and right of Fig. 3, we plot  $M_t$ ,  $\beta_2$ , and  $\mu_{\min}$ , respectively, with the critical value of  $\lambda_{\min}$  given in Eq. (16). The band corresponds to the 95% CL for the strong coupling constant measured at  $\mu = M_Z$ , where

$$\alpha_s(M_Z) = 0.1185 \pm 0.0006 \quad (17)$$

at the  $1\sigma$  level [1]. We see that  $\beta_2$  does not depend much on  $M_H$ . In the following figures except Fig. 12, we take a reference value  $\beta_2 = 0.5$ .<sup>6</sup>  $\mu_{\min}$  changes by an order of magnitude when one includes the one-loop corrections to the effective potential as shown in the right of Fig. 3. The two-loop corrections are negligible compared with the one-loop corrections; see e.g. Ref. [6]. In Fig. 3, we see that  $\beta_2$  and  $\mu_{\min}$  differ between at tree and one-loop levels, but note that  $M_t$  is almost identical at both levels.

### III. INFLATION MODEL

Let us consider the effective action of the SM-gravity system in the local potential approximation. As we are interested in the spatially constant field configuration and the case where the Hubble parameter is much smaller than the Planck scale, we restrict ourselves to the terms con-

<sup>5</sup> Numerical difference between the results from the condition  $\lambda_{\min} = 0$  and from Eq. (16) is much smaller than the deviation coming from the  $\alpha_s(M_Z)$  error. We have imposed  $\lambda_{\min} = 0$  within a precision of  $10^{-5}$  in the actual numerical computation in writing Fig. 3. Note that  $\lambda_c = 2.5 \times 10^{-6} \beta_2$ .

<sup>6</sup> We have checked that the changes of spectral index, its running, its running of running, and tensor-to-scalar ratio are hardly seeable when we vary  $\beta_2$ .

taining up to second derivative of the fields. We can write down the effective action schematically as<sup>7</sup>

$$\begin{aligned}
S = \int d^4x \sqrt{-g} & \left[ \frac{M_P^2}{2} A(\varphi) \mathcal{R} \right. \\
& - \frac{1}{2} B(\varphi) g^{\mu\nu} (\partial_\mu \varphi \partial_\nu \varphi + A_\mu A_\nu \varphi^2) - V(\varphi) \\
& - C(\varphi) \bar{\psi} \gamma^\mu D_\mu \psi - \frac{y}{\sqrt{2}} D(\varphi) (\varphi \bar{\psi} \psi + \text{h.c.}) \\
& \left. - \frac{E(\varphi)}{4g_A^2} F_{\mu\nu} F^{\mu\nu} \right], \tag{18}
\end{aligned}$$

where  $g_A$  and  $y$  are gauge and Yukawa couplings, respectively,  $M_P := 1/\sqrt{8\pi G} = 2.4 \times 10^{18}$  GeV is the reduced Planck scale,  $\varphi$  is the physical (real) Higgs field, and

$$A(\varphi) = 1 + a_2 \frac{\varphi^2}{M_P^2} + a_4 \frac{\varphi^4}{M_P^4} + \dots, \quad B(\varphi) = 1 + b_2 \frac{\varphi^2}{M_P^2} + b_4 \frac{\varphi^4}{M_P^4} + \dots, \quad \text{etc.}, \tag{19}$$

with  $a_2, \dots, b_2, \dots, \text{etc.}$  being dimensionless constants. Generically the potential  $V$  also contains higher dimensional terms

$$V(\varphi) = \frac{m^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4 + \left( \lambda_6 \frac{\varphi^6}{M_P^2} + \lambda_8 \frac{\varphi^8}{M_P^4} + \dots \right). \tag{20}$$

We can recast the Jordan frame action (18) by the field redefinition

$$g_{\mu\nu}^E = A(\varphi) g_{\mu\nu}, \tag{21}$$

to get the action in the Einstein frame

$$\begin{aligned}
S = \int d^4x \sqrt{-g_E} & \left[ \frac{M_P^2}{2} \mathcal{R}_E \right. \\
& - \frac{1}{2} \left[ \frac{B(\varphi)}{A(\varphi)} + \frac{3}{2} \frac{B(\varphi) A'(\varphi)^2}{A(\varphi)^2} \right] g_E^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \left[ \frac{B(\varphi)}{A(\varphi)} \right] g_E^{\mu\nu} A_\mu A_\nu \varphi^2 - \frac{V(\varphi)}{A(\varphi)^2} \\
& - \frac{C(\varphi)}{A(\varphi)^{3/2}} \bar{\psi} \gamma_E^\mu D_\mu \psi - \frac{y}{\sqrt{2}} \frac{D(\varphi)}{A(\varphi)^2} (\varphi \bar{\psi} \psi + \text{h.c.}) \\
& \left. - \frac{E(\varphi)}{4g_A^2} F_{\mu\nu} F_E^{\mu\nu} \right], \tag{22}
\end{aligned}$$

see e.g. Refs. [105, 106].

<sup>7</sup> In the Letter [48], we have used  $h$  instead of  $\varphi$ .



By the field redefinition:

$$\frac{d\chi}{d\varphi} = \sqrt{\frac{B(\varphi)}{A(\varphi)} + \frac{3}{2} \frac{B(\varphi)A'(\varphi)^2}{A(\varphi)^2}}, \quad \tilde{\psi} = \frac{C(\varphi)^{1/2}}{A(\varphi)^{3/4}}\psi, \quad (23)$$

we get the canonically normalized kinetic term for  $\chi$  and  $\tilde{\psi}$ .<sup>8</sup> For a given background field  $\varphi$  in the Jordan frame, the effective mass for the canonically normalized field  $\tilde{\psi}$  is

$$m_{\tilde{\psi}} = \frac{y\varphi}{\sqrt{2}} \frac{D(\varphi)}{\sqrt{A(\varphi)}C(\varphi)}. \quad (24)$$

Similarly, the effective mass for a canonically normalized gauge field is

$$m_{\tilde{A}} = g_A\varphi \sqrt{\frac{B(\varphi)}{A(\varphi)E(\varphi)}}. \quad (25)$$

For later convenience, we define the Einstein frame potential

$$U(\varphi) := \frac{V(\varphi)}{A(\varphi)^2}. \quad (26)$$

In the original version of the Higgs inflation [36, 44, 45], it is assumed that  $\xi := a_2$  happens to be large:  $\xi \sim 10^4$ , whereas the other couplings are not much larger than unity:  $\xi \gg a_2, a_4, \dots; b_2, \dots$  etc. In that limit, we can write

$$A(\varphi) = 1 + \frac{\xi\varphi^2}{M_P^2}, \quad B(\varphi) = C(\varphi) = D(\varphi) = E(\varphi) = 1, \quad \lambda_6 = \lambda_8 = \dots = 0. \quad (27)$$

As a side remark, we note that we can instead assume  $b_2 \sim 10^5$  while keeping all other coefficients, including  $a_2$ , not much larger than unity in order to realize another version of Higgs inflation [61]. It may be interesting to look for more possibilities of putting a large number in other places. In this paper, we restrict ourselves to more conventional set of the non-minimal couplings (27), and later take into account the term  $\lambda_6\varphi^6$  in the potential (20) as a next step.

For  $\varphi \gg M_P/\sqrt{\xi}$ , we have  $d\chi/d\varphi \simeq \sqrt{6}M_P/\varphi$ , which leads to  $\varphi \simeq \frac{M_P}{\sqrt{\xi}} \exp(\chi/\sqrt{6}M_P)$ , and the potential becomes [36]

$$U(\chi) = \frac{V}{\left(1 + e^{2\chi/\sqrt{6}M_P}\right)^2}. \quad (28)$$

<sup>8</sup> There appear extra derivative terms from the kinetic term of the fermion. We neglect such terms, since we are interested in the expression of the fermion mass for a constant background field  $\varphi$  and for the Hubble parameter much smaller than the Planck scale.

The analysis of this model without taking into account the running of  $\lambda$  gives following predictions [36]

$$\begin{aligned} n_s &= 1 - 6\epsilon_V + 2\eta_V \simeq 0.967, \\ r &= 16\epsilon_V \simeq 3 \times 10^{-3}, \\ \frac{dn_s}{d \ln k} &= 16\epsilon_V \eta_V - 24\epsilon_V^2 - 2\zeta_V^2 \simeq -5.4 \times 10^{-4}, \end{aligned} \quad (29)$$

where

$$\epsilon_V = \frac{M_P^2}{2} \left( \frac{dU/d\chi}{U} \right)^2, \quad (30)$$

$$\eta_V = M_P^2 \frac{d^2 U/d\chi^2}{U}, \quad (31)$$

$$\zeta_V^2 = M_P^4 \frac{(d^3 U/d\chi^3)(dU/d\chi)}{U^2}. \quad (32)$$

As is seen in Eq. (8), the loop corrections to the effective potential contain large logarithms. They can be written as  $\ln(M(\varphi)/\mu)$ , where  $\mu$  is the renormalization scale and  $M(\varphi)$  stands for the field-dependent mass of the particle running in the loop, namely the top quark and the gauge bosons. The problem is that there are two possibilities in defining the field dependent mass [44]. In the so-called prescription I, we use the field dependent mass in the Einstein frame, as in Eqs. (24) and (25), whereas in the prescription II, we use the ones in the Jordan frame, namely  $m_\psi = y\varphi/\sqrt{2}$  and  $m_A = g_A\varphi$ . We leave the possibilities open for future research and present our results for both prescriptions.

As we have done below Eq. (9), in either prescriptions I or II, we can drop the gauge and Yukawa couplings in the field dependent mass. For the prescription I, assuming the minimal set of coefficients (27), we put

$$\mu = \frac{\varphi}{\sqrt{1 + \xi\varphi^2/M_P^2}} \quad (33)$$

and for the prescription II,

$$\mu = \varphi. \quad (34)$$

Therefore, the effective potential is

$$V = \frac{\lambda_{\text{eff}}(\mu)}{4} \varphi^4, \quad (35)$$

with the scale (33) for the prescription I and scale (34) for the prescription II, where  $\lambda_{\text{eff}}(\mu)$  in the SM is given by Eq. (11).

#### IV. COSMOLOGICAL CONSTRAINTS

The overall normalization of the CMB fluctuation fixes [107]

$$A_s = \frac{V}{24\pi^2\epsilon_V M_P^4} = (2.196_{-0.058}^{+0.053}) \times 10^{-9}, \quad (36)$$

within  $1\sigma$  CL. Current Planck+WMAP bounds on the spectral index, its running, its running of running, and the tensor-to-scalar ratio are [107]

$$\begin{aligned} n_s &= 0.9514_{-0.0090}^{+0.0087}, & \frac{dn_s}{d \ln k} &= 0.001_{-0.014}^{+0.016}, & \frac{d^2 n_s}{d \ln k^2} &= 0.020_{-0.015}^{+0.016}, & r &= 0 \quad (\text{assumed}), \\ n_s &= 0.9583 \pm 0.0081, & \frac{dn_s}{d \ln k} &= -0.021 \pm 0.012, & \frac{d^2 n_s}{d \ln k^2} &= 0 \quad (\text{assumed}), & r &< 0.25 \quad (2\sigma \text{ CL}), \end{aligned} \quad (37)$$

at the pivot scale  $k_* = 0.05 \text{ Mpc}^{-1}$ , within  $1\sigma$  CL unless otherwise stated. The BICEP2 experiment has reported an observation of  $r$  [108]:

$$r = 0.20_{-0.05}^{+0.07}, \quad (38)$$

within  $1\sigma$  CL.

It has been pointed out that the BICEP2 result may become consistent with  $r = 0$  because the foreground effect can be sizable [109, 110]. We also note that by including isocurvature perturbation, the 95% CL bound on  $n_s$  is roughly loosened to [111]

$$0.93 \lesssim n_s \lesssim 0.99 \quad (39)$$

and that by including sterile neutrinos, the allowed range is shifted to [112]

$$0.95 \lesssim n_s \lesssim 1.02. \quad (40)$$

Given above, we will plot our results within wider ranges than those in Eqs. (37) and (38):

$$0.93 \leq n_s \leq 1.02, \quad (41)$$

$$-0.05 \leq \frac{dn_s}{d \ln k} \leq 0.05, \quad (42)$$

$$0 \leq r \leq 0.3. \quad (43)$$

#### V. HIGGS INFLATION FROM STANDARD MODEL CRITICALITY

In this section, we start from the minimal set of coefficients (27), and later include the term  $\lambda_6 \varphi^6$ .

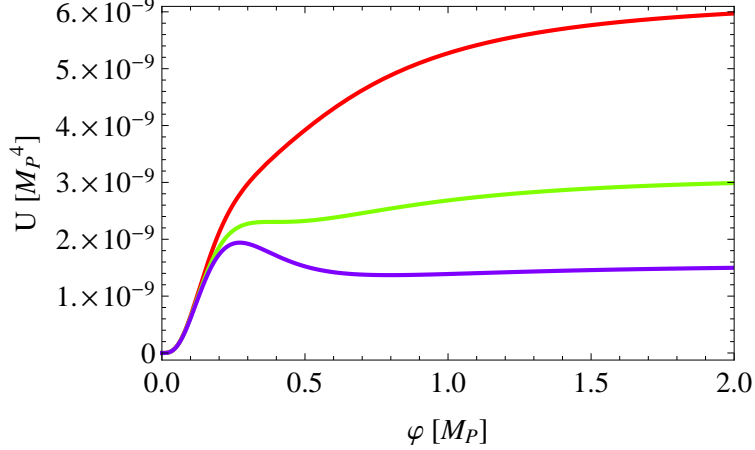


FIG. 4: SM Higgs potential in the prescription I with  $\xi = 10$  and  $c = 1$ , corresponding to  $\mu_{\min} = 7.6 \times 10^{17}$  GeV, and with  $\beta_2 = 0.5$ . The red (upper), green (center) and purple (lower) lines are drawn with  $\lambda_{\min} = 2\lambda_c$ ,  $\lambda_c$ , and  $\lambda_c/2$ , respectively. The values of  $\lambda_{\min} = 2\lambda_c$  and  $\lambda_c/2$  are chosen just for illustration. Each line roughly corresponds to the one with the same color in Fig. 2.

We expand the effective potential of the Higgs field  $V_{\text{eff}}$  on the flat space-time background around its minimum as in Eq. (14):

$$V = \frac{\lambda_{\text{eff}}(\mu)}{4} \varphi^4, \quad (44)$$

$$\lambda_{\text{eff}}(\mu) = \lambda_{\min} + \sum_{n=2}^{\infty} \frac{\beta_n}{(16\pi^2)^n} \left( \ln \frac{\mu}{\mu_{\min}} \right)^2. \quad (45)$$

The choice of scale (33) and (34) corresponds to the prescription I and II, respectively. As in Section II, we can safely neglect the higher order terms with  $n \geq 3$ , and we continue to omit them.

## A. Prescription I

### 1. Analysis in prescription I

In the prescription I, the Higgs potential is given by Eqs. (26) and (44) with the scale (33). Concretely,

$$U(\varphi) = \frac{\varphi^4}{4(1 + \xi\varphi^2/M_P^2)^2} \left\{ \lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \left[ \ln \left( \frac{1}{c} \sqrt{\frac{\xi\varphi^2/M_P^2}{1 + \xi\varphi^2/M_P^2}} \right) \right]^2 \right\}$$

$$U'(\varphi) = \frac{\varphi^3 M_P^6}{(M_P^2 + \xi\varphi^2)^3} \left\{ \lambda_{\min} + \frac{\beta_2}{2(16\pi^2)^2} \left[ 1 + 2 \ln \left( \frac{1}{c} \sqrt{\frac{\xi\varphi^2}{M_P^2 + \xi\varphi^2}} \right) \right] \ln \left( \frac{1}{c} \sqrt{\frac{\xi\varphi^2}{M_P^2 + \xi\varphi^2}} \right) \right\}, \quad (46)$$

where we define  $c$  by

$$\mu_{\min} = c \frac{M_P}{\sqrt{\xi}}. \quad (47)$$

Note that we have defined  $\mu_{\min}$  to give the minimum of the effective coupling  $\lambda_{\text{eff}}(\mu_{\min}) = \lambda_{\min}$  on the flat spacetime background Eq. (45). The stationary points  $U'(\varphi_1) = 0$  are given by

$$\varphi_1 = \frac{cM_P}{\sqrt{\xi}} \frac{1}{\left( e^{\frac{1}{2} \left[ 1 \pm \sqrt{1 - \frac{\lambda_{\min}}{\lambda_c}} \right]} - c^2 \right)^{1/2}}. \quad (48)$$

We can see the following:

- When  $\lambda_{\min} > \lambda_c$ , the potential is a monotonically increasing function of  $\varphi$ . This case corresponds to the red (upper) line in Fig. 4.
- When  $\lambda_{\min} = \lambda_c$ :
  - For  $c \geq e^{1/4}$ , the potential is monotonically increasing.
  - For  $c < e^{1/4}$ , the potential has a stationary point at

$$\varphi_c = \frac{cM_P}{\sqrt{\xi}} \frac{1}{(\sqrt{e} - c^2)^{1/2}}. \quad (49)$$

In this case,  $\varphi_c$  becomes a saddle point:  $U'(\varphi_c) = U''(\varphi_c) = 0$ . This case corresponds to the green (center) line in Fig. 4.

- When  $\lambda_{\min} < \lambda_c$ , we define  $c_{\pm} := \exp \frac{1 \pm \sqrt{1 - \frac{\lambda_{\min}}{\lambda_c}}}{4}$ :
  - For  $c \geq c_+$ , the potential is monotonically increasing.
  - For  $c_- < c < c_+$ , the potential has a stationary point given by the plus sign of Eq. (48).
  - For  $c \leq c_-$ , the potential has two stationary points given by Eq. (48). This case corresponds to the purple (lower) line in Fig. 4.

In this paper, we pursue the possibility that  $\lambda(\mu_{\min}) \simeq 0$  is realized by a principle beyond the ordinary local field theory, such as the multiple point criticality principle [35, 113, 114], classical conformality [26–31, 115–117], asymptotic safety [118], the hidden duality and symmetry [119, 120], and the maximum entropy principle [121–124].

In practice, this amounts to the following:  $\mu_{\min}$  is fixed for a given set of  $M_H$  and  $\alpha_s(M_Z)$  in the SM. For a given  $\mu_{\min}$ , we require  $\xi$  to sit in

$$\xi = c^2 \frac{M_P^2}{\mu_{\min}^2} < \sqrt{e} \frac{M_P^2}{\mu_{\min}^2}. \quad (50)$$

That is, we consider the case  $c < e^{1/4}$ . By tuning the top quark mass, we can always choose a  $\lambda_{\min}$  that is very close but larger than  $\lambda_c$  so that we realize  $U'(\varphi) \ll U(\varphi)/M_P$  and  $U''(\varphi) \ll U(\varphi)/M_P^2$  around  $\varphi \simeq \varphi_c$ .<sup>9</sup> In Fig. 4, our choice is very close but slightly above the green (middle) line.

In extensions of the SM,  $\mu_{\min}$  depends on newly-added parameters too. Anyway we require Eq. (50), and choose a  $\lambda_{\min}$  that is very close to  $\lambda_c$ , with  $\lambda_{\min} > \lambda_c$ , by the tuning of the top mass and possible other parameters.

We also need to consider the effect of the running of  $\xi$  [125–128]. However, this effect is small. More concretely, if  $\xi$  and  $\varphi$  are sufficiently large,  $\xi$  is given around  $\mu = \mu_{\min}$  by [44]

$$\xi(\mu) \simeq \xi_0 \left\{ 1 - \left( \frac{3}{2}g_Y^2 + 3g_2^2 - 6y_t^2 \right) \frac{1}{16\pi^2} \ln \frac{\mu}{\mu_{\min}} \right\} \simeq \xi_0 \left\{ 1 + 0.001 \ln \frac{\mu}{\mu_{\min}} \right\}, \quad (51)$$

We treat  $\xi$  as a constant in this paper.

## 2. Results in prescription I

The Higgs potential is determined by three parameters,  $\xi$ ,  $c$  and  $\lambda_{\min}$ . Qualitatively,  $\xi$  determines the total suppression of the potential above the scale  $\phi \gtrsim M_P/\sqrt{\xi}$ ,  $c$  determines the maximum value of  $\epsilon_V$  above the almost-saddle point, and  $\lambda_{\min}$  determines the number of  $e$ -folding. We choose  $\lambda_{\min}$  such that we can have sufficient  $e$ -folding  $N = 60$ . For a fixed  $A_s = 2.2 \times 10^{-9}$ , other cosmological parameters  $n_s, r$  and  $dn_s/d \ln k$  can be calculated as functions of  $\xi$  and  $c$ .

We show the typical predictions of this model in Fig. 5. Each solid line corresponds to a constant  $c$ . Dashed lines correspond to the values of  $\xi$  from 6 to 50 as indicated in the figure. In Fig. 5, we see that there is a minimum value of  $\xi$  that can result in  $r \lesssim 0.2$ , namely  $\xi_{\min} \sim 7$ . The model can reproduce  $r = O(10^{-3}) \sim 0.2$  and  $n_s = 0.9 \sim 1.0$ . These predictions are consistent with Planck or BICEP2 result [107, 108]. However, the value of  $dn_s/d \ln k$  is slightly large. The prediction is  $dn_s/d \ln k = O(0.01)$  for  $r \gtrsim 0.05$ . In Section V A 4, we will see that the introduction of small coefficients of higher dimensional operators may ameliorate the situation.

Finally, we discuss the field value  $\varphi_*$  that corresponds to the observed CMB fluctuation. The left panel of Fig. 6 shows  $\varphi_*$  in the case of  $c = 1$  and  $\beta_2 = 0.5$ .<sup>10</sup> We see that  $\varphi_*$  is around the Planck scale:  $\varphi_* \sim M_P$ .

<sup>9</sup> More precisely, we need  $U_\chi \ll U/M_P$  and  $U_{\chi\chi} \ll U/M_P^2$ , which are satisfied when  $U_\varphi \ll U/M_P$  and  $U_{\varphi\varphi} \ll U/M_P^2$  because we have  $d\varphi/d\chi \sim 1/\sqrt{6\xi}$  during the inflation.

<sup>10</sup> Precisely speaking, there are two  $\varphi_*$  which satisfies Eq. (36) given  $c, \xi$ . We plot the one solution which gives more desirable predictions on cosmological parameters, namely,  $n_s \lesssim 0.99$ .

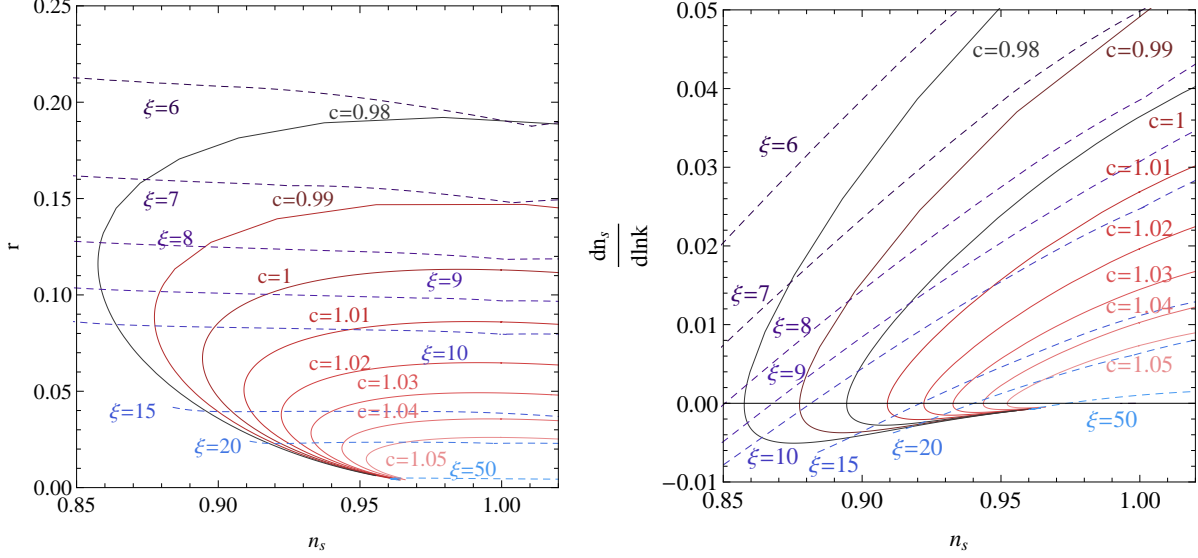


FIG. 5: Left:  $r$  vs  $n_s$ . Right:  $dn_s/d\ln k$  vs  $n_s$ . The solid and dashed contours are for fixed  $c$  and  $\xi$ , respectively. The left end of each dashed line for  $\xi = 15, 20$  and  $50$  corresponds to  $c = 0.94$ . The lower end of each solid line corresponds to  $\xi = 50$ .

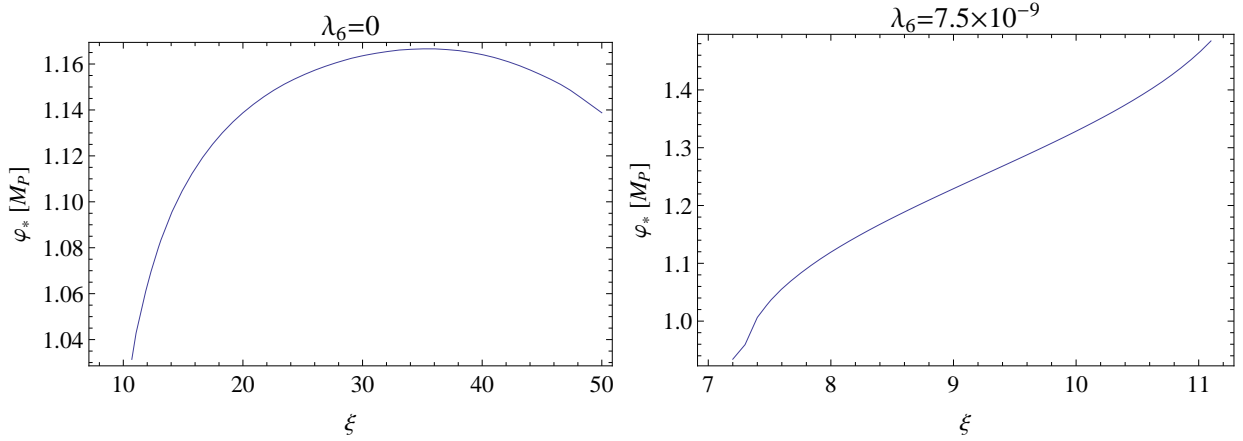


FIG. 6: Left and right:  $\varphi_*$  as a function of  $\xi$ , with  $\lambda_6 = 0$  and  $7.5 \times 10^{-9}$ , respectively. Other parameters are taken as  $c = 1$  and  $\beta_2 = 0.5$ .

### 3. Constraint on $\mu_{\min}$

The above analysis shows the existence of the lowest possible value of  $\xi$ , which is  $\xi_{\min} \sim 7$ . It is a necessary condition that  $\mu_{\min}$ , which is obtained from the parameters at low energy, satisfies  $\mu_{\min} \lesssim M_P/\sqrt{\xi_{\min}}$  for any successful Higgs inflation with  $\xi > \xi_{\min}$ . However, as we have observed in Sec. II, SM one-loop effective potential takes its minimum above  $M_P/\sqrt{\xi_{\min}}$  although the tree

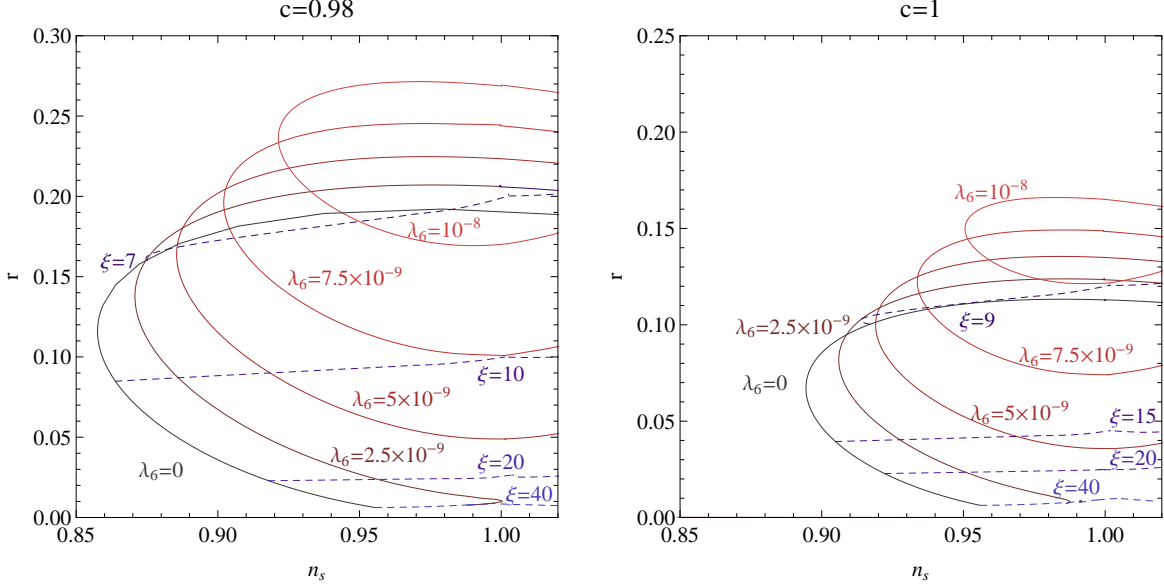


FIG. 7:  $r$  vs  $n_s$ , with  $c = 0.98$  (left) and  $1$  (right). Solid and dashed contours are for fixed  $\lambda_6$  and  $\xi$ , respectively. We put  $\beta_2 = 0.5$ .

level potential can realize  $\mu_{\min} \lesssim M_P/\sqrt{\xi_{\min}}$ . It appears that it is difficult to do our Higgs inflation in SM.

However, taking into account the ambiguity coming from non-renormalizable non-minimal coupling  $\xi$ , there still remains a possibility of realizing  $\mu_{\min} \lesssim M_P/\sqrt{\xi_{\min}}$  [50]. Around the scale  $M_P/\xi$ , we match  $\lambda$  in the SM without  $\xi$  and  $\lambda_\xi$  in the SM with  $\xi$ :

$$\lambda = \lambda_\xi + (\text{threshold corrections}), \quad (52)$$

where the threshold corrections generally contain power divergences and cannot be determined unless we specify a UV theory beyond the cutoff. One expects that the threshold corrections start from one loop order. Because they are of the same order as the difference between the tree and one-loop effective potentials, it may result in  $\mu_{\min} \lesssim M_P/\sqrt{\xi_{\min}}$ . See also the similar discussion in footnote 4 regarding the gauge dependence.

In Section VI, we will see that we can easily obtain  $\mu_{\min} \lesssim M_P/\sqrt{\xi_{\min}}$  in the Higgs portal scalar DM model without referring to such arguments. Argument of this section applies also to the prescription II shown in Sec. VB.



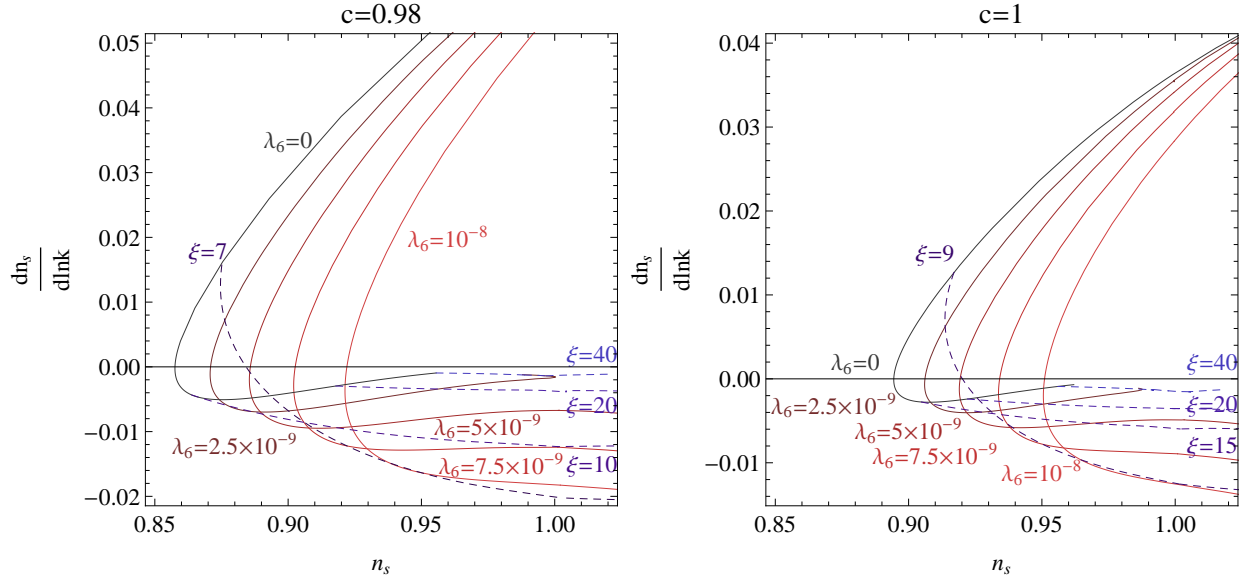


FIG. 8:  $dn_s/d \ln k$  vs  $n_s$ , with  $c = 0.98$  (left) and  $1$  (right). Solid and dashed contours are for fixed  $\lambda_6$  and  $\xi$ , respectively. We put  $\beta_2 = 0.5$ .

#### 4. Estimation of the effects of higher dimensional operators in prescription I

As we have seen in the previous sections, the extrapolation of the low energy data shows that the Higgs potential in the SM is flat around the string scale. This flatness can be broken if we introduce arbitrary strengths to the higher dimensional Planck-suppressed operators. In order to examine the effects of such operators on the cosmological data, we consider, for example, a small sixth order term in the Jordan frame

$$\Delta V = \lambda_6 \frac{\varphi^6}{M_P^2}. \quad (53)$$

Here we do not claim that Eq. (53) gives the leading contribution among the higher dimensional operators, but simply estimate the ambiguity in the value of  $dn_s/d \ln k$  discussed in the previous section. Of course, we can give any form to  $dn_s/d \ln k$  as a function of  $k$  if we introduce arbitrary strengths to the higher dimensional operators. As we will see below, the single term (53) allows the value of  $dn_s/d \ln k$  at  $k_*$  to reside in the favored region. However, this should not be taken as a prediction of the value of  $\lambda_6$  but as an estimation for the allowed magnitude of the coefficients of the higher dimensional operators.

In the Einstein frame potential, the term (53) becomes,

$$\Delta U = \lambda_6 \frac{\varphi^6}{(1 + \xi \varphi^2/M_P^2)^2}. \quad (54)$$

In Figs. 7 and 8, we plot the contours for fixed  $\lambda_6 \leq 10^{-8}$  with the solid lines, in the  $r$  vs  $n_s$  plane and the  $dn_s/d \ln k$  vs  $n_s$  one, respectively. We also plot the contours for fixed  $\xi$  and  $\lambda_6$  in the dashed and solid lines, respectively. We can realize the  $r \simeq 0.1$ ,  $n_s \simeq 0.96$ , and  $dn_s/d \ln k \simeq -0.01$  simultaneously. Finally,  $\varphi_*$  has been plotted in the right panel of Fig. 6 with  $\lambda_6 = 5 \times 10^{-9}$ ,  $c = 1$ , and  $\beta_2 = 0.5$ .

The other higher dimensional operators should also have the coefficients  $\lesssim \mathcal{O}(10^{-8})$  in order to keep the flatness of the potential. Their smallness may be understood for example as a tiny explicit breaking of the asymptotic scale invariance in Jordan frame (the shift symmetry in Einstein frame) [129].

## B. Prescription II

### 1. Analysis in prescription II

In the prescription II, the Higgs potential is given by Eqs. (26) and (44) with  $\mu = \varphi$ ,

$$U(\varphi) = \frac{\lambda(\varphi)}{4} \frac{\varphi^4}{(1 + \xi\varphi^2/M_P^2)^2}, \quad (55)$$

which gives

$$U = \frac{X^4}{(1 + X^2)^2} \frac{1}{4} \left( \lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \left( \ln \frac{X}{c} \right)^2 \right) \left( \frac{M_P}{\sqrt{\xi}} \right)^4, \quad (56)$$

$$U_\varphi = \frac{X^3}{(1 + X^2)^3} \left\{ \lambda_{\min} + \frac{\beta_2}{2(16\pi^2)^2} (1 + X^2) \ln \frac{X}{c} + \frac{\beta_2}{(16\pi^2)^2} \left( \ln \frac{X}{c} \right)^2 \right\} \left( \frac{M_P}{\sqrt{\xi}} \right)^3, \quad (57)$$

where  $X = \frac{\varphi}{M_P/\sqrt{\xi}}$ . Then the slow-roll parameter (30) becomes

$$\epsilon_V = \frac{8\xi}{X^2 + (1 + 6\xi)X^4} \frac{\left( \lambda_{\min} + \frac{\beta_2}{2(16\pi^2)^2} \ln \frac{X}{c} (1 + X^2 + 2 \ln \frac{X}{c}) \right)^2}{\left( \lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \left( \ln \frac{X}{c} \right)^2 \right)^2}. \quad (58)$$

For  $\varphi \gg M_P/\sqrt{6\xi}$  and  $\xi \gg 1/6$ , we obtain

$$\epsilon_V \simeq \frac{4}{3X^4} \frac{\left( \lambda_{\min} + \frac{\beta_2}{2(16\pi^2)^2} \ln \frac{X}{c} (1 + X^2 + 2 \ln \frac{X}{c}) \right)^2}{\left( \lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \left( \ln \frac{X}{c} \right)^2 \right)^2}. \quad (59)$$

Similarly we have

$$\eta_V \simeq \frac{4}{3X^4} \left( 1 - X^2 + \frac{\beta_2}{4(16\pi^2)^2} \frac{(1 + X^2)(1 + X^2 + 6 \ln \frac{X}{c})}{\lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \left( \ln \frac{X}{c} \right)^2} \right). \quad (60)$$

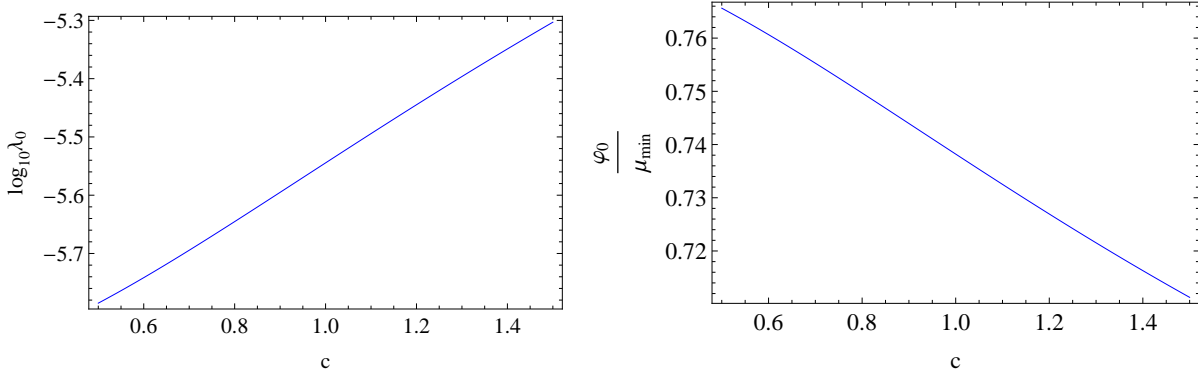


FIG. 9: Left:  $\lambda_0$ , the minimal value of  $\lambda_{\min}$  to maintain monotonicity of the potential, as a function of  $c$ . Right:  $\varphi_0$ , the position of the saddle point when we set  $\lambda_{\min} = \lambda_0$ , as a function of  $c$ .

These expressions are in agreement with those in the original Higgs inflation [36] if we take  $\beta_2 = 0$  and  $X \gg 1$ .

The  $e$ -folding  $N$  is written by

$$N = \int_{X_{\text{end}}}^{X_*} \frac{d\chi}{M_P} \frac{1}{\sqrt{2\epsilon_V}} = \int \frac{d\chi}{dX} \frac{dX}{\sqrt{2\epsilon_V}} \frac{1}{M_P}, \quad (61)$$

where

$$\frac{d\chi}{dX} = \frac{\sqrt{1 + (1 + 6\xi) X^2} M_P}{1 + X^2} \frac{1}{\sqrt{\xi}} \simeq \frac{\sqrt{6X}}{1 + X^2} M_P. \quad (62)$$

In the last step, we have used the same limit as above:  $X \gg 1/\sqrt{6}\xi$  and  $\xi \gg 1/6$ . Finally, we can write  $N$  as a function of  $X$  in that limit:

$$N \simeq \int_{X_{\text{end}}}^{X_*} dX \frac{3X^3}{1 + X^2} \frac{\lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \left(\ln \frac{X}{c}\right)^2}{2\lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2} \ln \frac{X}{c} \left(1 + X^2 + 2 \ln \frac{X}{c}\right)}. \quad (63)$$

This is also in agreement with Ref. [36] if we put  $\beta_2 = 0$  and  $X_* \gg 1$ .

## 2. Results in prescription II

Let us numerically estimate the lowest possible value of  $\lambda_{\min}$  that allows  $U(\varphi)$  to be monotonically increasing. We call this value  $\lambda_0$ . In the prescription I, such a value was  $\lambda_c$ , whereas in the prescription II,  $\lambda_0$  is a function of  $\beta_2$  and  $c$ . Note that  $\lambda_0$  is independent of  $\xi$  because the expression in the braces in Eq. (57) only depends on  $X$ , and explicit dependence on  $\xi$  drops out of it. In Fig. 9, we plot  $\lambda_0$  and the position of the saddle point  $\varphi_0$  as functions of  $c$  for a fixed  $\beta_2 = 0.5$ . We see that  $\lambda_0 \sim 10^{-5.5}$  and  $\varphi_0 \simeq 0.73\mu_{\min}$  for  $c = 1$ .

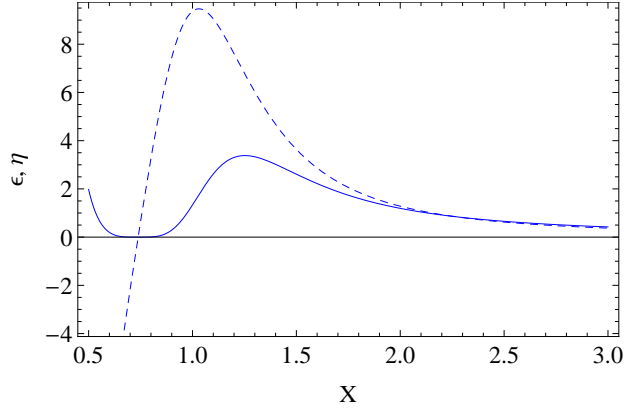


FIG. 10: Slow roll parameters  $\epsilon$  (solid) and  $\eta$  (dashed) as functions of  $X = \varphi / (M_P / \sqrt{\xi})$ . We have set  $c = 1$ ,  $\beta_2 = 0.5$ , and  $\lambda_{\min} = \lambda_0$ .

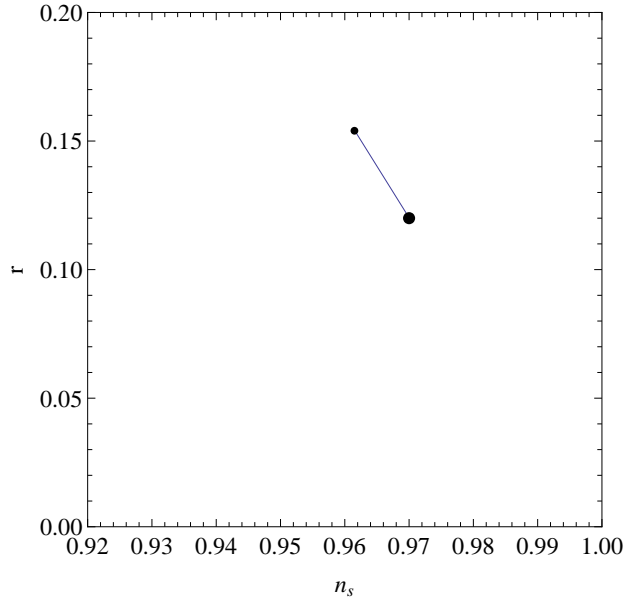


FIG. 11:  $n_s$  vs.  $r$ . The small and large dots represent  $N_* = 50$  and 65.

The potential is determined by  $\lambda_{\min}$ ,  $c$  and  $\xi$ . To be specific, we consider the  $c = 1$  case hereafter. We plot the  $\epsilon_V$  in Fig 10 with  $c = 1$ ,  $\beta_2 = 0.5$ , and  $\lambda_{\min} = \lambda_0$ . The solid and dashed lines represent  $\epsilon_V$  and  $\eta_V$ , respectively. We can see that  $\epsilon_V \simeq \eta_V \simeq 1$  around  $X \simeq 2$ . Therefore the end of inflation corresponds to  $X_{\text{end}} \simeq 2$ .

We can calculate the prediction of inflationary parameters with  $c = 1$ ,  $\beta_2 = 0.5$ , and  $\lambda_{\min} = \lambda_0$ .  $N = 50$  and 65 correspond to  $X_* \simeq 360$  and 790, respectively. We fix  $\xi$  in such a way that Planck

normalization is satisfied,

$$A_s = \frac{U}{24\pi^2 M_P^4 \epsilon_V} = 2.2 \times 10^{-9}. \quad (64)$$

By using this condition,  $\xi$  becomes 190 and 240 for  $N = 50$  and 65, respectively. The prediction of  $n_s$  and  $r$  is shown in Fig. 11.  $dn_s/d \ln k$  is small in this case,  $dn_s/d \ln k \ll O(10^{-2})$ . These predictions are just the same as the chaotic inflation, as discussed in Ref. [48].

We note that the argument in this subsection implicitly assumes that Planck scale physics does not modify the Higgs potential above the UV cutoff.

## VI. SCALAR DARK MATTER MODEL

Next we consider the model which includes Higgs portal singlet scalar DM  $S$  [130, 131]; see also Ref. [132]. The Lagrangian is [133]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 - \frac{\rho}{4!}S^4 - \frac{\kappa}{2}S^2 H^\dagger H. \quad (65)$$

We put subscript  $Z$  on the new parameters at the  $Z$  mass scale  $\mu = M_Z$ , that is,  $\kappa_Z = \kappa(\mu = M_Z)$ , and  $\rho_Z = \rho(\mu = M_Z)$ . If we require perturbativity up to the cutoff scale, these parameters should be  $\kappa_Z \lesssim 0.4$  and  $\rho_Z \lesssim 0.6$  [134]. The one-loop effective potential in this model is given by

$$V = V_{\text{tree}} + \Delta V_{1\text{-loop, DM}}, \quad (66)$$

$$V_{\text{tree}} = e^{4\Gamma(\varphi)} \frac{\lambda(\mu)}{4} \varphi^4, \\ \Delta V_{1\text{-loop, DM}} = \Delta V_{1\text{-loop}} + \frac{m_{\text{DM}}^4}{64\pi^2} \left( \ln \frac{m_{\text{DM}}(\varphi)^2}{\mu^2} - \frac{3}{2} \right), \quad (67)$$

where  $m_{\text{DM}}(\varphi) = \sqrt{\frac{\kappa\varphi^2}{2}e^{2\Gamma(\varphi)} + m_S^2}$ .  $\Delta V_{1\text{-loop}}$  and  $\Gamma$  are given by SM one-loop potential (4) and Eq. (5), respectively.

We plot  $M_t$ ,  $\beta_2$ , and  $\mu_{\text{min}}$  as functions of  $\kappa_Z$  imposing the existence of the saddle point in Fig. 12. Here we use two loop RGEs [134] and put  $M_H = 125.9 \text{ GeV}$ ,  $\alpha_s = 0.1184$ . The band width comes from the requirement of perturbativity of  $\rho$  up to string scale [134]:  $0 \leq \rho_Z \leq 0.6$ . The red (lower) and blue (upper) bands correspond to the tree and one-loop effective potentials, respectively. From this figure, we see that  $\mu_{\text{min}}$  can become smaller than  $M_P$  by adding  $\kappa$  and that  $\beta_2$  remains to be  $O(1)$ . In particular, the addition of the scalar DM does not alter the existence of the minimum of  $\lambda_{\text{eff}}(\mu)$ , which is essential in this inflation scenario with criticality.

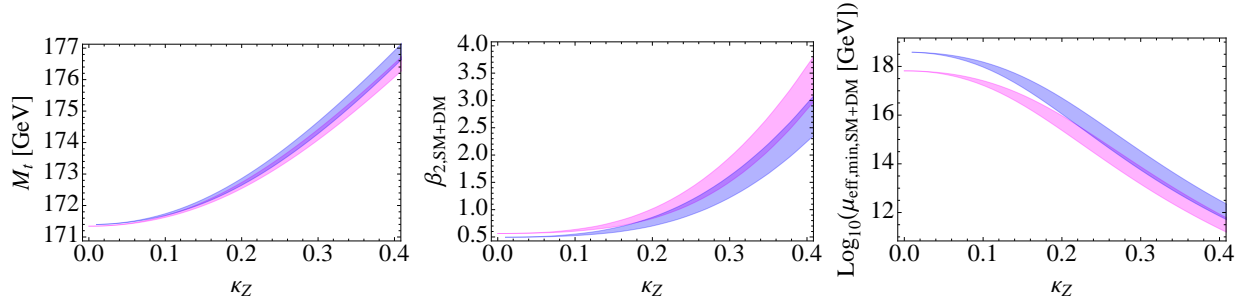


FIG. 12:  $M_t$  (left),  $\beta_2$  (center), and  $\mu_{\min}$  (right) are plotted as functions of  $\kappa_Z$ . Red (lower) and blue (upper) bands correspond to the tree and 1-loop potentials, respectively. The band width comes from the requirement of perturbativity of  $\rho$  up to the string scale [134]:  $0 \leq \rho_Z \leq 0.6$ .  $M_H$  and  $m_S$  are set to be 125.9 GeV and 0, respectively.

## VII. SUMMARY

We have considered the Higgs inflation model which contains non-minimal coupling  $\xi\varphi^2\mathcal{R}$  [36]. Conventional wisdom was that a large non-minimal coupling  $\xi \sim 10^4$  is required to fit the COBE normalization,  $\delta T/T \sim 10^{-5}$ , and cosmological predictions are  $n_s = 0.967$  and the small tensor-to-scalar ratio,  $r = 3 \times 10^{-3}$ . In the letter [48], we have reconsidered this model in light of the discovery of the Higgs boson, which indicates the criticality of the SM. That is, if the SM parameters are tuned so that the saddle point appears, it is possible to realize a Higgs inflation with moderate  $\xi$  and generate  $O(0.1)$  tensor to scalar ratio  $r$ . The value of  $\xi$  is  $O(10)$  for the prescription I and  $O(100)$  for the prescription II.

In this paper, we investigate the cosmological predictions of this Higgs inflation in greater detail. To realize this Higgs inflation scenario, it is essential that the effective Higgs quartic coupling  $\lambda_{\text{eff}}$  takes its minimum around the scale  $10^{17-18}$  GeV. The Higgs potential around the inflation scale is determined by the position  $\mu_{\min}$  of the minimum of  $\lambda_{\text{eff}}$ , the minimum value  $\lambda_{\min}$ , and the second derivative  $\beta_2$  around the minimum, in addition to the non-minimal coupling  $\xi$ . We calculate the cosmological predictions as functions of above parameters. We also estimate the effects of higher dimensional operators by considering the  $\lambda_6\varphi^6/M_P^2$  term as an example. We find that the coefficients of the higher dimensional operators should be as small as  $10^{-8}$  in order to account for the scalar and tensor perturbations consistent with the Planck and BICEP2 results. Although we have concentrated on the SM and the Higgs portal scalar DM model in this paper, one may apply our analysis to the other models beyond the SM by evaluating  $\lambda_{\min}$ ,  $\mu_{\min}$ ,  $\beta_2$  in terms of the model parameters.

Finally, we comment on the problem of the unitarity [62, 135–140]. The problem of unitarity does not threaten the consistency of the Higgs inflation by itself. Concretely speaking, the physical momentum scale during the inflation, which is given by the de Sitter temperature  $H_{\text{inf}} \simeq 10^{14} \text{ GeV} (r/0.2)^{1/2}$ , is smaller than the unitarity violation scale  $M_P/\xi$  that is evaluated on the electroweak vacuum.<sup>11</sup> In general, a new physics would appear around the unitarity violation scale. It is interesting that it is around the GUT or string scale in our model.

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<sup>11</sup> In Ref. [140], it is shown that the unitarity constraints are relieved for moderate values of non-minimal coupling,  $\xi = O(10 - 100)$ .

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