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Higgs mass prediction in the MSSM at three-loop level in a pure $\overline{\text{DR}}$ context

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Abstract The impact of the three-loop effects of order $\alpha_t\alpha_s^2$ on the mass of the light CP-even Higgs boson in the MSSM is studied in a pure $\overline{\text{DR}}$ context. For this purpose, we implement the results of Kant et al. (JHEP 08:104, 2010) into the C++ module Himalaya and link it to FlexibleSUSY, a Mathematica and C++ package to create spectrum generators for BSM models. The three-loop result is compared to the fixed-order two-loop calculations of the original FlexibleSUSY and of FeynHiggs, as well as to the result based on an EFT approach. Aside from the expected reduction of the renormalization scale dependence with respect to the lower-order results, we find that the three-loop contributions significantly reduce the difference from the EFT prediction in the TeV-region of the SUSY scale M_S. Himalaya can be linked also to other two-loop $\overline{\text{DR}}$ codes, thus allowing for the elevation of these codes to the three-loop level.

1 Introduction

The measurement of the Higgs boson mass at the Large Hadron Collider (LHC) represents a significant constraint on the viability of supersymmetric (SUSY) models. Given a particular SUSY model, the mass of the Standard Model-like Higgs boson is a prediction, which must be in agreement with the measured value of $(125.09 \pm 0.21 \pm 0.11)$ GeV [2]. Noteworthy, the experimental uncertainty on the measured Higgs mass has already reached the per-mille level. Theory predictions in SUSY models, however, struggle to reach the same level of accuracy. The reason is that the Higgs mass receives large higher-order corrections, dominated by the top Yukawa and the strong gauge coupling [3–5]. Both of these two couplings are comparatively large, leading to a relatively slow convergence of the perturbative series. Furthermore, the scalar nature of the Higgs implies corrections proportional to the square of the top-quark mass, on top of the top-mass dependence due to the Yukawa coupling, which enters the loop corrections quadratically. On the other hand, corrections from SUSY particles are only logarithmic in the SUSY particle masses due to the assumption of only soft SUSY-breaking terms. If the SUSY particles are not too far above the TeV scale [6,7], the SUSY Higgs mass can be obtained from a fixed-order calculation of the relevant one- and two-point functions with external Higgs fields. In this case, higher-order corrections up to the three-loop level are known in the Minimal Supersymmetric Standard Model (MSSM) [1,5,8–23].

There are plenty of publicly available computer codes which calculate the Higgs pole mass(es) in the MSSM at higher orders: CPsuperH [24–26], FeynHiggs [9,27–31], FlexibleSUSY [32,33], H3m [1,20], ISASUSY [34], MhEFT [35], SARAH/SPheno [36–42], SOFTSUSY [43,44], SuSpect [45] and SusyHD [46]. FeynHiggs adopts the on-shell scheme for the renormalization of the particle masses, while all other codes express their results in terms of $\overline{\text{MS}}/\overline{\text{DR}}$ parameters. All these schemes are formally equivalent up to higher orders in perturbation theory, of course. The numerical difference between the schemes is one of the sources of theoretical uncertainty on the Higgs mass prediction, however. All of these programs take into account one-loop corrections, most of them also leading two-loop corrections. H3m is the only one which includes three-loop corrections of order $\alpha_t\alpha_s^2$, where α_t is the squared top Yukawa and α_s is the strong coupling. It combines these terms with the on-shell two-loop result of FeynHiggs after transforming the $\mathcal{O}(\alpha_t)$ and $\mathcal{O}(\alpha_t\alpha_s)$ terms from there to the $\overline{\text{DR}}$ scheme.

Here we present an alternative implementation of the $\mathcal{O}(\alpha_t\alpha_s^2)$ contributions of Refs. [1,20] for the light CP-even Higgs mass in the MSSM into the framework of FlexibleSUSY [32], referring to the combination as FlexibleSUSY+Himalaya in what follows. This allows us to study the effect of the three-loop contributions in a pure

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$\overline{\text{DR}}$ environment, i.e. without the trouble of combining the corrections with an on-shell calculation. The three-loop terms are provided in the form of a separate C++ package, named Himalaya, which one should be able to include in any other $\overline{\text{DR}}$ code without much effort. The Himalaya package and the dedicated version of FlexibleSUSY, which incorporates the three-loop contributions from Himalaya, can be downloaded from Refs. [47,48], respectively. In this way, we hope to contribute to the on-going effort of improving the precision of the Higgs mass prediction in the MSSM.

In the present paper we study the impact of the three-loop corrections for low and high SUSY scales and compare our results to the two-loop calculations of the public spectrum generators of FlexibleSUSY and FeynHiggs. By quantifying the size of the three-loop corrections, we also provide a measure for the theoretical uncertainty of the $\overline{\text{DR}}$ fixed-order calculation.

As will be shown below, the implementation of the $\alpha_t\alpha_s^2$ corrections also applies to the terms of order $\alpha_b\alpha_s^2$, where α_b is the bottom Yukawa coupling. Therefore, Himalaya will take such terms into account, and we will refer to the sum of top- and bottom-Yukawa induced supersymmetric QCD (SQCD) corrections as $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ in what follows. However, it should be kept in mind that this does not include effects of order $\alpha_s^2\sqrt{\alpha_t\alpha_b}$, which arise from three-loop Higgs self energies involving both a top/stop and a bottom/sbottom triangle. The results of Himalaya are thus unreliable in the (rather exotic) case where α_t and α_b are comparable in magnitude.

The remainder of this paper is structured as follows. Section 2 describes the form in which the three-loop contributions of order $(\alpha_t + \alpha_b)\alpha_s^2$ are implemented in Himalaya. Its input parameters are to be provided in the $\overline{\text{DR}}$ scheme at the appropriate perturbative order. Section 3 details how this input is prepared in the framework of FlexibleSUSY. It also summarises all the contributions that enter the final Higgs mass prediction in FlexibleSUSY+Himalaya. Section 4 analyses the impact of various three-loop contributions on this prediction as well as the residual renormalization scale dependence, and it compares the results obtained with FlexibleSUSY+Himalaya to existing fixed-order and resummed results for the light Higgs mass. In particular, this includes a comparison to the original implementation of the three-loop effects in H3m. Our conclusions are presented in Sect. 5. Technical details of Himalaya, its link to FlexibleSUSY, and run options are collected in the appendix.

2 Higgs mass prediction at the three-loop level in the MSSM

The results for the three-loop $\alpha_t\alpha_s^2$ corrections to the Higgs mass in the MSSM have been obtained in Refs. [1,20] by a Feynman diagrammatic calculation of the relevant one- and two-point functions with external Higgs fields in the limit of vanishing external momenta. The dependence of these terms on the squark and gluino masses was approximated through asymptotic expansions, assuming various hierarchies among the masses of the SUSY particles. For details of the calculation we refer to Refs. [1,20].

2.1 Selection of the hierarchy

A particular set of parameters typically matches several of the hierarchies mentioned above. In order to select the most suitable one, Ref. [1] suggested a pragmatic approach, namely the comparison of the various asymptotic expansions to the exact expression at two-loop level. Himalaya also adopts this approach, but introduces a few refinements in order to further stabilise the hierarchy selection (see also Ref. [49]).

In a first step the Higgs pole mass M_h is calculated at the two-loop level at order $\alpha_t\alpha_s$ using the result of Ref. [12] in the form of the associated FORTRAN code provided by the authors. We refer to this quantity as M_h^{DSZ} in what follows. Subsequently, for all hierarchies i which fit the given mass spectrum, M_h is calculated again using the expanded expressions of Ref. [1] at the two-loop level, resulting in $M_{h,i}$. In the original approach of Ref. [1], the hierarchy is selected as the value of i for which the difference

$$\delta_i^{2\text{L}} = \left| M_h^{\text{DSZ}} - M_{h,i} \right| \tag{1}$$

is minimal. However, we found that this criterion alone causes instabilities in the hierarchy selection in regions where several hierarchies lead to similar values of $\delta_i^{2\text{L}}$. We therefore refine the selection criterion by taking into account the quality of the convergence in the respective hierarchies, quantified by

$$\delta_i^{\text{conv}} = \sqrt{\sum_{j=1}^{n} \left(M_{h,i} - M_{h,i}^{(j)} \right)^2}. \tag{2}$$

While $M_{h,i}$ includes all available terms of the expansion in mass (and mass difference) ratios, in $M_h^{(j)}$ the highest terms of the expansion for the mass (and mass difference) ratio j are dropped. We then define the "best" hierarchy to be the one which minimises the quadratic mean of Eqs. (1) and (2),

$$\delta_i = \sqrt{\left(\delta_i^{2\text{L}}\right)^2 + \left(\delta_i^{\text{conv}}\right)^2}. \tag{3}$$

The relevant analytical expressions for the three-loop terms of order $\alpha_t\alpha_s^2$ to the CP-even Higgs mass matrix in the various mass hierarchies are quite lengthy. However, they are accessible in Mathematica format in the framework of the publicly available program H3m. We have transformed these formulas into C++ format and implemented them into Himalaya.

The hierarchies defined in H3m equally apply to the top and the bottom sector of the MSSM, so that the results of Ref. [1] can also be used to evaluate the corrections of order $\alpha_b\alpha_s^2$ to the Higgs mass matrix. Indeed, Himalaya takes these corrections into account. However, as already pointed out in Sect. 1, a complete account of the top- *and* bottom-Yukawa effects to order α_s^2 would require one to include the contribution of diagrams which involve both top/stop and bottom/sbottom loops at the same time. These were not considered in Ref. [1], and thus the Himalaya result should only be used in cases where such mixed $\sqrt{\alpha_t\alpha_b}$ terms can be neglected.

2.2 Modified $\overline{\text{DR}}$ scheme

By default, all the parameters of the calculation are renormalised in the $\overline{\text{DR}}$ scheme. However, in this scheme, one finds artificial "non-decoupling" effects [12], meaning that the two- and three-loop result for the Higgs mass depends quadratically on a SUSY particle mass if this mass gets much larger than the others. Such terms are avoided by transforming the stop masses to a non-minimal scheme, named $\overline{\text{MDR}}$ (modified $\overline{\text{DR}}$) in Ref. [1], which mimics the virtue of the on-shell scheme of automatically decoupling the heavy particles.

If the user wishes to use this scheme rather than pure $\overline{\text{DR}}$, Himalaya writes the Higgs mass matrix as

$$\begin{aligned}\hat{\mathsf{M}}(\hat{m}_{\tilde{t}}) &= \hat{\mathsf{M}}^{\text{tree}} + \hat{\mathsf{M}}^{(\alpha_t)}(\hat{m}_{\tilde{t}}) + \hat{\mathsf{M}}^{(\alpha_t\alpha_s)}(\hat{m}_{\tilde{t}}) \\ &\quad + \hat{\mathsf{M}}^{(\alpha_t\alpha_s^2)}(\hat{m}_{\tilde{t}}) + \cdots \\ &= \mathsf{M}^{\text{tree}} + \mathsf{M}^{(\alpha_t)}(m_{\tilde{t}}) + \mathsf{M}^{(\alpha_t\alpha_s)}(m_{\tilde{t}}) \\ &\quad + \delta\mathsf{M}(m_{\tilde{t}}, \hat{m}_{\tilde{t}}) + \hat{\mathsf{M}}^{(\alpha_t\alpha_s^2)}(\hat{m}_{\tilde{t}}) + \cdots, \end{aligned} \tag{4}$$

where M and $\hat{\mathsf{M}}$ are the Higgs mass matrices in the $\overline{\text{DR}}$ and the $\overline{\text{MDR}}$ scheme, respectively, $\mathsf{M}^{\text{tree}} = \hat{\mathsf{M}}^{\text{tree}}$ is the tree-level expression, and the superscript $^{(x)}$ denotes the term of order $x \in \{\alpha_t, \alpha_s, \alpha_t\alpha_s, \ldots\}$. The ellipsis in Eq. (4) symbolises any terms that involve coupling constants other than α_t or α_s, or higher orders of the latter. For brevity we suppress the stop mass indices "1" and "2" here. Himalaya provides the numerical results for $\hat{\mathsf{M}}^{(\alpha_t\alpha_s^2)}(\hat{m}_{\tilde{t}})$ as well as

$$\begin{aligned}\delta\mathsf{M}(m_{\tilde{t}}, \hat{m}_{\tilde{t}}) &\equiv \left(\hat{\mathsf{M}}^{(\alpha_t)}(\hat{m}_{\tilde{t}}) + \hat{\mathsf{M}}^{(\alpha_t\alpha_s)}(\hat{m}_{\tilde{t}})\right) \\ &\quad - \left(\mathsf{M}^{(\alpha_t)}(m_{\tilde{t}}) + \mathsf{M}^{(\alpha_t\alpha_s)}(m_{\tilde{t}})\right), \end{aligned} \tag{5}$$

where the $\overline{\text{MDR}}$ stop mass $\hat{m}_{\tilde{t}}$ is calculated from its $\overline{\text{DR}}$ value $m_{\tilde{t}}$ by the conversion formulas through $\mathcal{O}(\alpha_s^2)$, provided in Ref. [1]. Note that these conversion formulas depend on the underlying hierarchy, and may be different for $m_{\tilde{t},1}$ and $m_{\tilde{t},2}$.

Even if the result is requested in the $\overline{\text{MDR}}$ scheme, the output of Himalaya can thus be directly combined with pure $\overline{\text{DR}}$ results through $\mathcal{O}(\alpha_t\alpha_s)$ according to Eq. (4) in order to arrive at the mass matrix at order $\alpha_t\alpha_s^2$. Of course, one may also request the plain $\overline{\text{DR}}$ result from Himalaya, in which case it will simply return the numerical value for $\mathsf{M}^{(\alpha_t\alpha_s^2)}(m_{\tilde{t}})$, which can be directly added to any two-loop $\overline{\text{DR}}$ result.

In any case, the difference between the $\overline{\text{DR}}$ and $\overline{\text{MDR}}$ result is expected to be quite small unless the mass splitting between one of the stop masses and other, heavier, strongly interacting SUSY particles becomes very large. As a practical example, in Fig. 1 we show the difference of the lightest Higgs mass at the three-loop level calculated in the $\overline{\text{DR}}$ and $\overline{\text{MDR}}$ scheme. All $\overline{\text{DR}}$ soft-breaking mass parameters, the μ parameter of the MSSM super-potential, and the running CP-odd Higgs mass are set equal to M_S here. The running trilinear couplings, except A_t, are chosen such that the sfermions do not mix. The $\overline{\text{DR}}$ stop mixing parameter $X_t = A_t - \mu/\tan\beta$ is left as a free parameter. For this scenario we find that the difference between the $\overline{\text{DR}}$ and $\overline{\text{MDR}}$ scheme is below 100 MeV for different values of the stop mixing parameter.

Note that, for all terms in the Higgs mass matrix except α_t, $\alpha_t\alpha_s$, and $\alpha_t\alpha_s^2$, it is perturbatively equivalent to use either the $\overline{\text{DR}}$ or the $\overline{\text{MDR}}$ stop mass as defined above. Predominantly, this concerns the electroweak contributions as well as the terms of order α_t^2. In this paper, we use the $\overline{\text{DR}}$ stop mass for these contributions.

3 Implementation into **FlexibleSUSY**

3.1 Determination of the MSSM $\overline{\text{DR}}$ parameters

FlexibleSUSY determines the running $\overline{\text{DR}}$ gauge and Yukawa couplings as well as the running vacuum expectation value of the MSSM along the lines of Ref. [50] by setting the scale to the Z-boson pole mass M_Z. In this approach, the following Standard Model (SM) input parameters are used:

$$\begin{aligned}&\alpha_{\text{em}}^{\text{SM}(5)}(M_Z),\, \alpha_s^{\text{SM}(5)}(M_Z),\, G_F,\, M_Z, \\ &M_e,\, M_\mu,\, M_\tau,\, m_{u,d,s}(2\,\text{GeV}),\, m_c^{\text{SM}(4),\overline{\text{MS}}}(m_c), \\ &\quad m_b^{\text{SM}(5),\overline{\text{MS}}}(m_b),\, M_t,\end{aligned} \tag{6}$$

where $\alpha_{\text{em}}^{\text{SM}(5)}(M_Z)$ and $\alpha_s^{\text{SM}(5)}(M_Z)$ denote the electromagnetic and strong coupling constants in the $\overline{\text{MS}}$ scheme in the Standard Model with five active quark flavours, and G_F is the Fermi constant. M_e, M_μ, M_τ, and M_t denote the pole masses of the electron, muon, tau lepton, and top quark, respectively. The input masses of the up, down and strange quark are

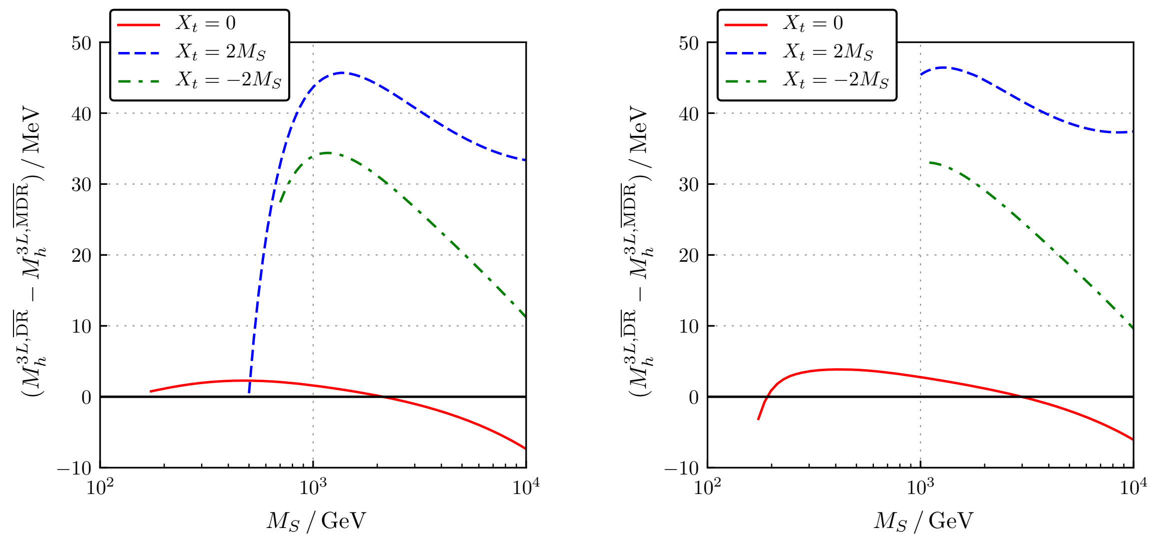

Fig. 1 Difference between the lightest Higgs pole mass calculated in the $\overline{\text{DR}}$ scheme and the $\overline{\text{MDR}}$ scheme as a function of the SUSY scale M_S for $\tan\beta = 5$. In the left panel the soft-breaking stop and gluino mass parameters are set equal to M_S. In the right panel, we use $m_{\tilde{g}} = 2M_S$. We have cut off curves with non-zero X_t around or below the TeV scale, where the $\overline{\text{DR}}$ CP-even Higgs mass becomes tachyonic at the electroweak scale

defined in the $\overline{\text{MS}}$ scheme at the scale 2 GeV. The charm and bottom quark masses are defined in the $\overline{\text{MS}}$ scheme at their scale in the Standard Model with four and five active quark flavours, respectively.

The MSSM $\overline{\text{DR}}$ gauge couplings g_1, g_2 and g_3 are given in terms of the $\overline{\text{DR}}$ parameters $\alpha_{\text{em}}^{\text{MSSM}}(M_Z)$ and $\alpha_s^{\text{MSSM}}(M_Z)$ in the MSSM as

$$g_1(M_Z) = \sqrt{\frac{5}{3}}\frac{\sqrt{4\pi\alpha_{\text{em}}^{\text{MSSM}}(M_Z)}}{\cos\theta_w(M_Z)}, \tag{7}$$

$$g_2(M_Z) = \frac{\sqrt{4\pi\alpha_{\text{em}}^{\text{MSSM}}(M_Z)}}{\sin\theta_w(M_Z)}, \tag{8}$$

$$g_3(M_Z) = \sqrt{4\pi\alpha_s^{\text{MSSM}}(M_Z)}. \tag{9}$$

The couplings $\alpha_{\text{em}}^{\text{MSSM}}(M_Z)$ and $\alpha_s^{\text{MSSM}}(M_Z)$ are calculated from the corresponding input parameters as

$$\alpha_{\text{em}}^{\text{MSSM}}(M_Z) = \frac{\alpha_{\text{em}}^{\text{SM(5)}}(M_Z)}{1 - \Delta\alpha_{\text{em}}(M_Z)}, \tag{10}$$

$$\alpha_s^{\text{MSSM}}(M_Z) = \frac{\alpha_s^{\text{SM(5)}}(M_Z)}{1 - \Delta\alpha_s(M_Z)}, \tag{11}$$

where the threshold corrections $\Delta\alpha_i(M_Z)$ have the form

$$\begin{aligned}\Delta\alpha_{\text{em}}(M_Z) = \frac{\alpha_{\text{em}}}{2\pi}\Bigg(&\frac{1}{3} - \frac{16}{9}\log\frac{m_t}{M_Z} \\ &- \frac{4}{9}\sum_{i=1}^{6}\log\frac{m_{\tilde{u}_i}}{M_Z} - \frac{1}{9}\sum_{i=1}^{6}\log\frac{m_{\tilde{d}_i}}{M_Z} \\ &- \frac{4}{3}\sum_{i=1}^{2}\log\frac{m_{\tilde{\chi}_i^+}}{M_Z} - \frac{1}{3}\sum_{i=1}^{6}\log\frac{m_{\tilde{e}_i}}{M_Z} \\ &- \frac{1}{3}\log\frac{m_{H^+}}{M_Z}\Bigg),\end{aligned} \tag{12}$$

$$\begin{aligned}\Delta\alpha_s(M_Z) = \frac{\alpha_s}{2\pi}\Bigg[&\frac{1}{2} - 2\log\frac{m_{\tilde{g}}}{M_Z} - \frac{2}{3}\log\frac{m_t}{M_Z} \\ &- \frac{1}{6}\sum_{i=1}^{6}\left(\log\frac{m_{\tilde{u}_i}}{M_Z} + \log\frac{m_{\tilde{d}_i}}{M_Z}\right)\Bigg].\end{aligned} \tag{13}$$

The $\overline{\text{DR}}$ weak mixing angle in the MSSM, θ_w, is determined at the scale M_Z from the Fermi constant G_F and the Z pole mass via the relation

$$\sin^2\theta_w\cos^2\theta_w = \frac{\pi\,\alpha_{\text{em}}^{\text{MSSM}}}{\sqrt{2}M_Z^2 G_F(1-\delta_r)}, \tag{14}$$

where

$$\delta_r = \hat{\rho}\frac{\text{Re}\,\Sigma_{W,T}(0)}{M_W^2} - \frac{\text{Re}\,\Sigma_{Z,T}(M_Z^2)}{M_Z^2} + \delta_{\text{VB}} + \delta_r^{(2)}, \tag{15}$$

$$\hat{\rho} = \frac{1}{1-\Delta\hat{\rho}},$$

$$\Delta\hat{\rho} = \text{Re}\left[\frac{\Sigma_{Z,T}(M_Z^2)}{\hat{\rho}\,M_Z^2} - \frac{\Sigma_{W,T}(M_W^2)}{M_W^2}\right] + \Delta\hat{\rho}^{(2)}. \tag{16}$$

Here, $\Sigma_{V,T}(p^2)$ denotes the transverse part of the $\overline{\text{DR}}$-renormalised one-loop self-energy of the vector boson V in the MSSM. The vertex and box contributions δ_{VB} as well as the two-loop contributions $\delta_r^{(2)}$ are taken from Ref. [50]. The $\overline{\mathrm{DR}}$ vacuum expectation values of the up- and down-type Higgs doublets are calculated by

$$v_u(M_Z) = \frac{2m_Z(M_Z)\sin\beta(M_Z)}{\sqrt{3/5 g_1^2(M_Z) + g_2^2(M_Z)}}, \tag{17}$$

$$v_d(M_Z) = \frac{2m_Z(M_Z)\cos\beta(M_Z)}{\sqrt{3/5 g_1^2(M_Z) + g_2^2(M_Z)}}, \tag{18}$$

where $\tan\beta(M_Z)$ is an input parameter and $m_Z(M_Z)$ is the Z boson $\overline{\mathrm{DR}}$ mass in the MSSM, which is calculated from the Z pole mass at the one-loop level as

$$m_Z^2(M_Z) = M_Z^2 + \mathrm{Re}\,\Sigma_{Z,T}(M_Z^2). \tag{19}$$

In order to calculate the Higgs pole mass in the $\overline{\mathrm{DR}}$ scheme at the three-loop level $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$, the $\overline{\mathrm{DR}}$ top and bottom Yukawa couplings must be extracted from the input parameters M_t and $m_b^{\mathrm{SM}(5),\overline{\mathrm{MS}}}(m_b)$ at the two-loop level at $\mathcal{O}(\alpha_s^2)$. In order to achieve that, we make use of the known two-loop SQCD contributions to the top and bottom Yukawa couplings of Refs. [51–54], as described in the following: We calculate the $\overline{\mathrm{DR}}$ Yukawa couplings y_t at the scale M_Z from the $\overline{\mathrm{DR}}$ top mass m_t and the $\overline{\mathrm{DR}}$ up-type VEV v_u as

$$y_t(M_Z) = \sqrt{2}\frac{m_t(M_Z)}{v_u(M_Z)}. \tag{20}$$

In our approach, we relate the $\overline{\mathrm{DR}}$ top mass to the top pole mass M_t at the scale M_Z as

$$\begin{aligned} m_t(M_Z) = {} & M_t + \mathrm{Re}\,\Sigma_t^S(M_t^2, M_Z) \\ & + M_t\Big[\mathrm{Re}\,\Sigma_t^L(M_t^2, M_Z) + \mathrm{Re}\,\Sigma_t^R(M_t^2, M_Z) \\ & + \Delta m_t^{(1),\mathrm{SQCD}}(M_Z) + \Delta m_t^{(2),\mathrm{SQCD}}(M_Z)\Big], \end{aligned} \tag{21}$$

where the $\Sigma_t^{S,L,R}(p^2, Q)$ denote the scalar (superscript S), and the left- and right-handed parts (L, R) of the $\overline{\mathrm{DR}}$ renormalised one-loop top self-energy without the gluon, stop, and gluino contributions, and $\Delta m_t^{(1),\mathrm{SQCD}}$ and $\Delta m_t^{(2),\mathrm{SQCD}}$ are the full one- and two-loop SQCD corrections taken from Refs. [51,52],

$$\begin{aligned} \Delta m_t^{(1),\mathrm{SQCD}} = {} & -\frac{\alpha_s}{4\pi}C_F\Bigg[\Bigg(\frac{m_g m_{\tilde{t}_1}^2 s_{2\theta_t}}{m_t\left(m_{\tilde{t}_1}^2 - m_g^2\right)} - \frac{m_g m_{\tilde{t}_2}^2 s_{2\theta_t}}{m_t\left(m_{\tilde{t}_2}^2 - m_g^2\right)} + \frac{m_{\tilde{t}_1}^4}{2\left(m_{\tilde{t}_1}^2 - m_g^2\right)^2} \\ & - \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_1}^2 - m_g^2} + \frac{m_{\tilde{t}_2}^4}{2\left(m_{\tilde{t}_2}^2 - m_g^2\right)^2} - \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_2}^2 - m_g^2} + 1\Bigg)\log\frac{m_g^2}{Q^2} \\ & + \Bigg(-\frac{m_g m_{\tilde{t}_1}^2 s_{2\theta_t}}{m_t\left(m_{\tilde{t}_1}^2 - m_g^2\right)} - \frac{m_{\tilde{t}_1}^4}{2\left(m_{\tilde{t}_1}^2 - m_g^2\right)^2} + \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_1}^2 - m_g^2}\Bigg)\log\frac{m_{\tilde{t}_1}^2}{Q^2} \\ & + \Bigg(\frac{m_g m_{\tilde{t}_2}^2 s_{2\theta_t}}{m_t\left(m_{\tilde{t}_2}^2 - m_g^2\right)} - \frac{m_{\tilde{t}_2}^4}{2\left(m_{\tilde{t}_2}^2 - m_g^2\right)^2} + \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_2}^2 - m_g^2}\Bigg)\log\frac{m_{\tilde{t}_2}^2}{Q^2} \\ & + \frac{m_{\tilde{t}_1}^2}{2\left(m_{\tilde{t}_1}^2 - m_g^2\right)} + \frac{m_{\tilde{t}_2}^2}{2\left(m_{\tilde{t}_2}^2 - m_g^2\right)} - 3\log\frac{m_t^2}{Q^2} + \frac{7}{2}\Bigg], \end{aligned} \tag{22}$$

$$\Delta m_t^{(2),\mathrm{SQCD}} = \left(\Delta m_t^{(1),\mathrm{SQCD}}\right)^2 - \Delta m_t^{(2),\mathrm{dec}}. \tag{23}$$

In Eq. (22), it is $C_F = 4/3$ and $s_{2\theta_t} = \sin 2\theta_t$, with θ_t the stop mixing angle. The two-loop term $\Delta m_t^{(2),\mathrm{dec}}$ is given in Ref. [51] for general stop, sbottom, and gluino masses.

The MSSM $\overline{\mathrm{DR}}$ bottom-quark Yukawa coupling y_b is calculated from the $\overline{\mathrm{DR}}$ bottom-quark mass m_b and the down-type VEV at the scale M_Z as

$$y_b(M_Z) = \sqrt{2}\frac{m_b(M_Z)}{v_d(M_Z)}. \tag{24}$$

We obtain $m_b(M_Z)$ from the input $\overline{\mathrm{MS}}$ mass $m_b^{\mathrm{SM}(5),\overline{\mathrm{MS}}}(m_b)$ in the Standard Model with five active quark flavours by first evolving $m_b^{\text{SM(5)},\overline{\text{MS}}}(m_b)$ to the scale M_Z, using the one-loop QED and three-loop QCD renormalization group equations (RGEs). Afterwards, $m_b^{\text{SM(5)},\overline{\text{MS}}}(M_Z)$ is converted to the $\overline{\text{DR}}$ mass $m_b^{\text{SM(5)},\overline{\text{DR}}}(M_Z)$ by the relation

$$m_b^{\text{SM(5)},\overline{\text{DR}}}(M_Z) = m_b^{\text{SM(5)},\overline{\text{MS}}}(M_Z) \times \left(1 - \frac{\alpha_s}{3\pi} + \frac{3g_2^2}{128\pi^2} + \frac{13g_Y^2}{1152\pi^2}\right). \tag{25}$$

Finally, the MSSM $\overline{\text{DR}}$ bottom mass $m_b(M_Z)$ is obtained from $m_b^{\text{SM(5)},\overline{\text{DR}}}(M_Z)$ via

$$m_b(M_Z) = \frac{m_b^{\text{SM(5)},\overline{\text{DR}}}(M_Z)}{1 + \Delta m_b^{(1)} + \Delta m_b^{(2)}}, \tag{26}$$

$$\Delta m_b^{(1)} = -\,\text{Re}\,\Sigma_b^S\left(\left(m_b^{\text{SM(5)},\overline{\text{MS}}}\right)^2, M_Z\right)\Big/ m_b - \text{Re}\,\Sigma_b^L\left(\left(m_b^{\text{SM(5)},\overline{\text{MS}}}\right)^2, M_Z\right) - \text{Re}\,\Sigma_b^R\left(\left(m_b^{\text{SM(5)},\overline{\text{MS}}}\right)^2, M_Z\right), \tag{27}$$

$$\Delta m_b^{(2)} = \Delta m_b^{(2),\text{dec}} - \frac{\alpha_s}{3\pi}\Delta m_b^{(1)}, \tag{28}$$

where $\Sigma_b^{S,L,R}(p^2, Q)$ are the scalar, left- and right-handed parts of the $\overline{\text{DR}}$ renormalised one-loop bottom quark self-energy in the MSSM, in which all Standard Model particles, except the bottom quark, the top quark and the W, Z, and Higgs bosons, are omitted. In Eq. (28) $\Delta m_b^{(2),\text{dec}}$ denotes the two-loop decoupling relation of order $\mathcal{O}(\alpha_s^2)$ between the $\overline{\text{MS}}$ bottom mass $m_b^{\text{SM(5)},\overline{\text{MS}}}$ and the $\overline{\text{DR}}$ bottom mass in the MSSM calculated in Refs. [53,54].

Note that the matching of the SM to the MSSM leads to large logarithmic contributions in the MSSM $\overline{\text{DR}}$ parameters in the case of a heavy SUSY particle spectrum. These contributions can be resummed in a so-called EFT approach [31, 33,46,55,56].

3.2 Calculation of the CP-even Higgs pole masses

FlexibleSUSY calculates the two CP-even Higgs pole masses M_h and M_H by diagonalising the loop-corrected mass matrix[1]

$$\mathsf{M} = \mathsf{M}^{\text{tree}} + \mathsf{M}^{1L}(p^2) + \mathsf{M}^{2L} + \mathsf{M}^{3L} \tag{29}$$

at the momenta $p^2 = M_h^2$ and $p^2 = M_H^2$, respectively (M^{2L} and M^{3L} are evaluated at $p^2 = 0$). The one-loop correction $\mathsf{M}^{1L}(p^2)$ contains the full one-loop MSSM Higgs self-energy and tadpole contributions, including electroweak corrections and the momentum dependence. The two-loop correction M^{2L} contains the known corrections of order $\mathcal{O}\big(\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2\big)$ [12–16]. The three-loop correction M^{3L} incorporates the terms of order $\mathcal{O}\big(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2\big)$ from the Himalaya package, as described in Sect. 2. In Eq. (29) all contributions are defined in the $\overline{\text{DR}}$ scheme by default.[2] The renormalization scale is chosen to be $Q = \sqrt{m_{\tilde{t},1} m_{\tilde{t},2}}$ and the $\overline{\text{DR}}$ parameters which enter Eq. (29) are evolved to that scale by using the three-loop RGEs of the MSSM [57,58]. Since the two CP-even Higgs pole masses are the output of the diagonalization of M but at the same time must be inserted into $\mathsf{M}^{1L}(p^2)$, an iteration over the momentum is performed for each mass eigenvalue until a fixed point for the Higgs masses is reached with sufficient precision.

4 Results

4.1 Size of three-loop contributions from different sources

In the $\overline{\text{DR}}$ calculation within FlexibleSUSY+Himalaya, there are three sources of contributions which affect the Higgs pole mass at order $\mathcal{O}\big(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2\big)$: The one-loop threshold correction $\mathcal{O}(\alpha_s)$ to the strong coupling constant, the two-loop threshold correction $\mathcal{O}\big(\alpha_s^2\big)$ to the top and bottom Yukawa couplings, and the genuine three-loop contribution to the Higgs mass matrix. In Fig. 2, the impact of these three sources on the Higgs pole mass is shown relative to the two-loop calculation without these three corrections. The left panel shows the impact as a function of the SUSY scale M_S, and the right panel as a function of the relative stop mixing parameter X_t/M_S for the scenario defined in Sect. 2.2.

First, we observe that the inclusion of the one-loop threshold correction to α_s, Eq. (13), (blue dashed line) leads to a significant positive shift of the Higgs pole mass of around +2.5 GeV for $M_S \approx 1$ TeV. For larger SUSY scales the shift increases logarithmically as is to be expected from the logarithmic terms on the r.h.s. of Eq. (13). The inclusion of the full two-loop SQCD corrections to y_t (green dash-dotted line) leads to a shift of similar magnitude, but in the opposite direction (the effect due to y_b is negligible). Thus, there is a significant cancellation between the three-loop contributions from the one-loop threshold correction to α_s and the two-loop SQCD corrections to y_t. The genuine three-loop contribution

[1] We do not distinguish between $\overline{\text{DR}}$ and $\overline{\text{MDR}}$ parameters here, and drop the hat over $\hat{\mathsf{M}}$ introduced in Eq. (4) for simplicity.

[2] FlexibleSUSY+Himalaya provides a flag to calculate the corrections of order $\mathcal{O}(\alpha_t(1 + \alpha_s + \alpha_s^2) + \alpha_b(1 + \alpha_s + \alpha_s^2))$ in the $\overline{\text{MDR}}$ scheme, as described in Sect. 2.2. See "Appendix C" for more details.

to the Higgs pole mass (black dotted line) is again positive and around +2 GeV for $M_S \approx 1$ GeV. This is consistent with the findings of Ref. [1], of course. As a result, the sum of these three three-loop effects (red solid line) leads to a net positive shift of the Higgs mass relative to the two-loop result without all these corrections.

The size of the individual three-loop contributions depends on the stop mixing parameter X_t/M_S, as can be seen from the r.h.s. of Fig. 2: between minimal ($X_t/M_S = 0$) and maximal stop mixing ($X_t/M_S \approx \sqrt{6}$) the size of the individual three-loop contributions changes by 1–2 GeV. For maximal (minimal) mixing, their impact is maximal (minimal). The direction of the shift is independent of X_t/M_S.

Note that the nominal two-loop result of the original `FlexibleSUSY` (i.e., without `Himalaya`) includes by default the one-loop threshold correction to α_s and the SM QCD two-loop contributions to the top Yukawa coupling [32, 33]. This means that the two-loop Higgs mass as evaluated by the original `FlexibleSUSY` already incorporates partial three-loop contributions. As a result, the two-loop result of the original `FlexibleSUSY` does not correspond to the zero-line in Fig. 2, but it is rather close to the blue dashed line. This implies that, compared to the two-loop result of the original `FlexibleSUSY`, the effect of the remaining $\alpha_t\alpha_s^2$ contributions in the Higgs mass prediction is *negative*.

4.2 Scale dependence of the three-loop Higgs pole mass

To estimate the size of the missing higher-order corrections, Fig. 3 shows the renormalization scale dependence of the one-, two- and three-loop Higgs pole mass for the scenario defined in Sect. 2.2 with $\tan\beta = 5$ and $X_t = 0$. The one- and two-loop calculations correspond to the original `FlexibleSUSY`. In the one-loop calculation the threshold corrections to α_s and y_t are set to zero, and in the two-loop calculation the one-loop threshold corrections to α_s and the two-loop QCD corrections to y_t are taken into account. The three-loop result of `FlexibleSUSY+Himalaya` includes all three-loop contributions at $(\alpha_t + \alpha_b)\alpha_s^2$ discussed above, i.e. the one-loop threshold correction to α_s, the full two-loop SQCD corrections to $y_{t,b}$, and the genuine three-loop correction to the Higgs pole mass from `Himalaya`. In addition, the Higgs mass predicted at the two-loop level in the pure EFT calculation of `HSSUSY` is shown as the black dotted line, see Sect. 4.3. The bands show the corresponding variation of the Higgs pole mass when the renormalization scale is varied using the three-loop renormalization group equations [57–63] for all parameters except for the vacuum expectation values, where the β-functions are known only up to the two-loop level [64,65]. In `FlexibleSUSY` and `FlexibleSUSY+Himalaya`, the renormalizaion scale is varied in the full MSSM within the interval $[M_S/2, 2M_S]$, while in `HSSUSY` it is varied in the Standard Model within the interval $[M_t/2, 2M_t]$, keeping the matching scale fixed at M_S. The plot shows that the successive inclusion of higher-order corrections reduces the scale dependence, as expected. In particular, the three-loop corrections to the Higgs mass reduce the scale dependence by around a factor two, compared to the two-loop calculation. The scale dependence of `HSSUSY` is almost independent of M_S, because scale variation is done within the SM after integrating out all SUSY particles at M_S. Note that the variation of the renormalization scale only serves as an indicator of the theoretical uncertainty due to missing higher-order effects.

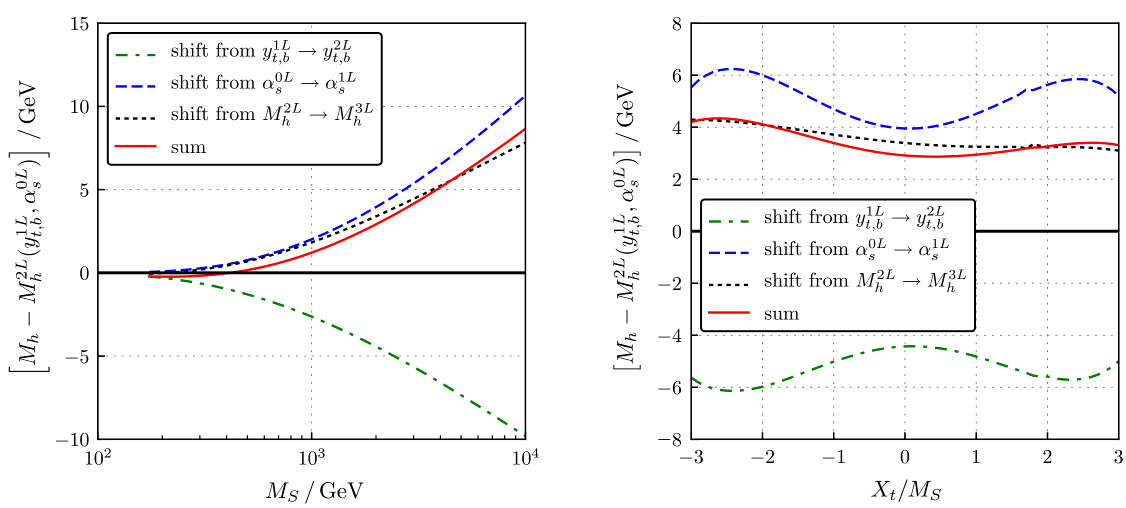

Fig. 2 Influence of different three-loop contributions to the Higgs pole mass. In the left panel we show the shift in the Higgs pole mass with respect to $M_h^{2L}(y_{t,b}^{1L}, \alpha_s^{0L})$ for $\tan\beta = 5$ and $X_t = 0$ as a function of the SUSY scale. In the right panel we fix $\tan\beta = 5$ and $M_S = 2$ TeV and vary the relative stop mixing parameter X_t/M_S

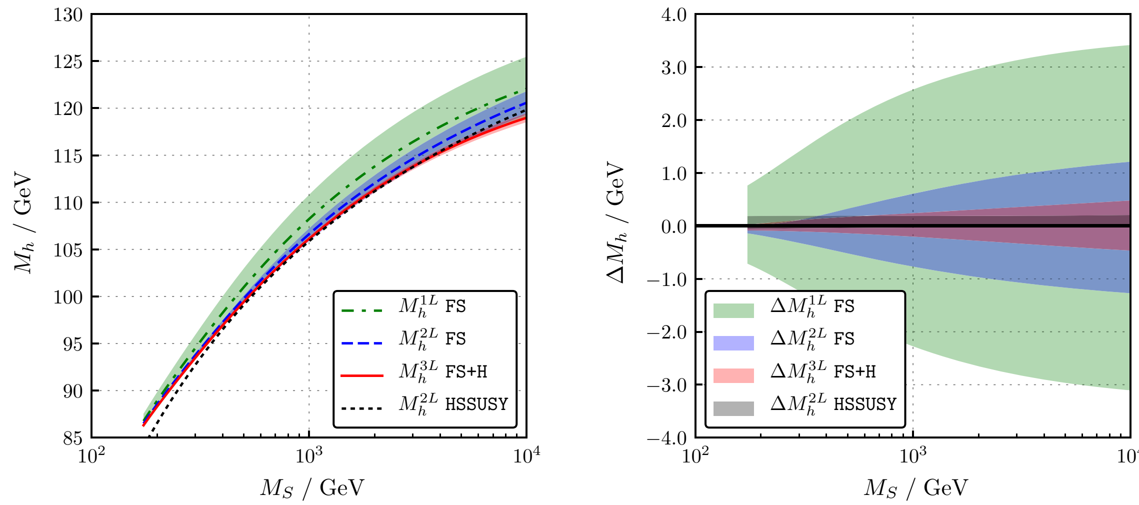

Fig. 3 Variation of the Higgs pole mass when the renormalization scale is varied by a factor two at which the Higgs pole mass is calculated, for $\tan\beta = 5$ and $X_t = 0$

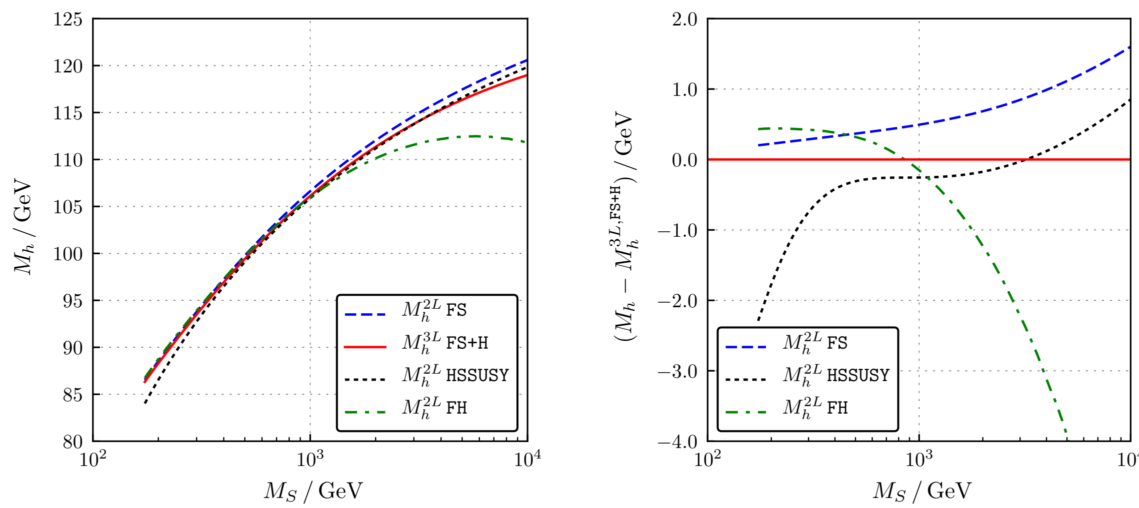

Fig. 4 Comparison of Higgs mass predictions between two- and three-loop fixed-order programs and a two-loop EFT calculation as a function of the SUSY scale for $\tan\beta = 5$ and $X_t = 0$. In the left panel the absolute Higgs pole mass and in the right panel the difference w.r.t. the three-loop calculation is shown (FS = FlexibleSUSY, FS+H = FlexibleSUSY+Himalaya, FH = FeynHiggs)

4.3 Comparison with lower-order and EFT results

In Figs. 4, 5, we compare the three-loop calculation of FlexibleSUSY+Himalaya (red) with other MSSM spectrum generators. As input we use $M_t = 173.34\,\text{GeV}$, $\alpha_{\text{em}}^{\text{SM(5)}}(M_Z) = 1/127.95$, $\alpha_s^{\text{SM(5)}}(M_Z) = 0.1184$ and $G_F = 1.1663787 \cdot 10^{-5}\,\text{GeV}^{-2}$. All $\overline{\text{DR}}$ soft-breaking mass parameters as well as the μ parameter of the super-potential in the MSSM, and the running CP-odd Higgs mass are set equal to M_S. The running trilinear couplings, except for A_t, are chosen such that there is no sfermion mixing. The stop mixing parameter $X_t = A_t - \mu/\tan\beta$ is defined in the $\overline{\text{DR}}$ scheme and left as a free parameter. The lightest CP-even Higgs pole mass is calculated at the scale $Q = \sqrt{m_{\tilde{t},1} m_{\tilde{t},2}}$.

FlexibleSUSY 1.7.4 The blue dashed line shows the original two-loop calculation with FlexibleSUSY 1.7.4 [32]. Note that, by construction of FlexibleSUSY, this result coincides exactly with the one of SOFTSUSY 3.5.1. As described above, it includes the one-loop threshold corrections to α_s and the two-loop QCD contributions to y_t, and it uses the three-loop RGEs of the MSSM [57,58]. FlexibleSUSY 1.7.4 (and SOFTSUSY) use the explicit two-loop Higgs pole mass contribution of order $\mathcal{O}\big(\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2\big)$ of Refs. [12–16].

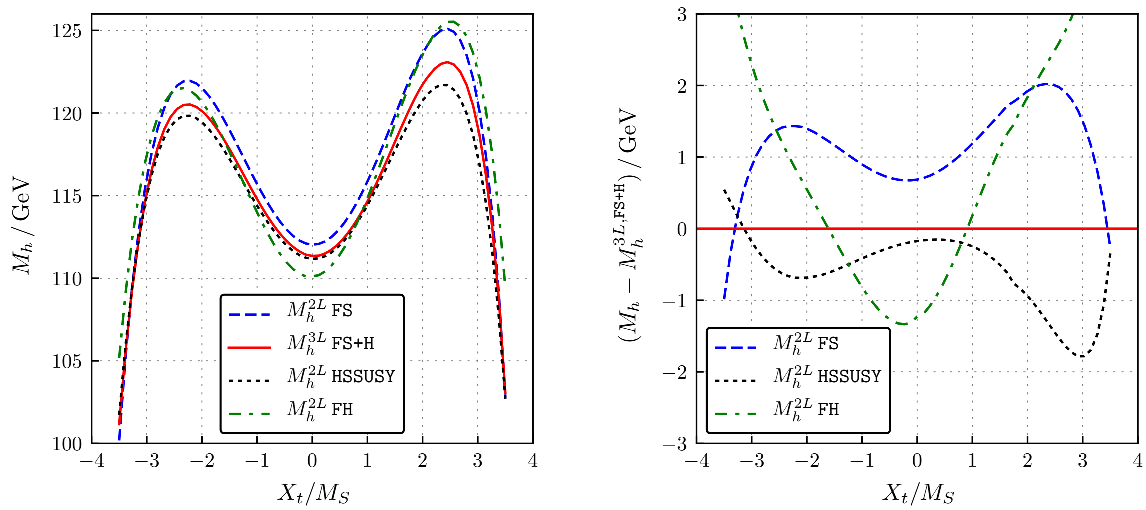

Fig. 5 Comparison of Higgs mass predictions between two- and three-loop fixed-order programs and a two-loop EFT calculation as a function of the relative stop mixing parameter X_t/M_S for $\tan\beta = 5$ and $M_S = 2\,\text{TeV}$. In the left panel the absolute Higgs pole mass and in the right panel the difference w.r.t. the three-loop calculation is shown

HSSUSY 1.7.4 The black dotted line has been obtained using the pure two-loop effective field theory (EFT) calculation of `HSSUSY` [48]. `HSSUSY` is a spectrum generator from the `FlexibleSUSY` suite, which implements the two-loop threshold correction for the quartic Higgs coupling of the Standard Model at $\mathcal{O}(\alpha_t(\alpha_t + \alpha_s))$ when integrating out the SUSY particles at a common SUSY scale [46,55]. Renormalization group running is performed down to the top mass scale using the three-loop RGEs of the Standard Model [59–63] and, finally, the Higgs mass is calculated at the two-loop level in the Standard Model at order $\mathcal{O}(\alpha_t(\alpha_t + \alpha_s))$. In terms of the implemented corrections, `HSSUSY` is equivalent to `SusyHD` [46], and resums large logarithms up to NNLL level while neglecting terms of order $v^2/{M_S}^2$. The $\mathcal{O}(v^2/{M_S}^2)$ corrections calculated in Ref. [66] have not been taken into account here.

FeynHiggs 2.13.0-beta The green dash-dotted line shows the Higgs mass prediction using `FeynHiggs` 2.13.0-beta without large log resummation [9,27–31].[3] `FeynHiggs` 2.13.0-beta includes the two-loop contributions of order $\mathcal{O}(\alpha_t\alpha_s + \alpha_b\alpha_s + \alpha_t^2 + \alpha_t\alpha_b)$.

Consider first Fig. 4. The left panel shows the Higgs mass prediction as a function of M_S according to three codes discussed above, together with the `FlexibleSUSY+Himalaya` result (solid red). The stop mixing parameter X_t is set to zero. The right panel shows the difference of these curves to the latter. Note that the resummed result of `HSSUSY` neglects terms of order $v^2/{M_S}^2$, and thus forfeits reliability towards lower values of M_S. The deviation from the fixed-order curves below $M_S \approx 400\,\text{GeV}$ clearly underlines this. In contrast, the fixed-order results start to suffer from large logarithmic contributions toward large M_S, which on the other hand are properly resummed in the `HSSUSY` approach. From Fig. 4, we conclude that the fixed-order $\overline{\text{DR}}$ result loses its applicability once M_S is larger than a few TeV, while the deviation between the non-resummed on-shell result of `FeynHiggs` and `HSSUSY` increases more rapidly above $M_S \approx 1\,\text{TeV}$. Note that the good agreement of `FlexibleSUSY` with `HSSUSY` above the few-TeV region is accidental, as shown in Ref. [33].

The effect of the three-loop $\alpha_t\alpha_s^2$ terms on the fixed-order result is negative, as discussed in Sect. 4.1, and amounts to a few hundred MeV in the region where the fixed-order approach is appropriate. They significantly improve the agreement between the fixed-order and the resummed prediction for M_h in the intermediate region of M_S, where both approaches are expected to be reliable. Between M_S of about 500 GeV and 5 TeV, our three-loop curve from `FlexibleSUSY+Himalaya` deviates from the `HSSUSY` result by less than 300 MeV. This corroborates the compatibility of the two approaches in the intermediate region. Considering the current estimate of the theoretical uncertainty in the Higgs mass prediction [28,33,46,55,67], our observation even legitimates a naive switching between the fixed-order and the resummed approach at $M_S \approx 1\,\text{TeV}$, instead of a more sophisticated matching procedure along the lines of Refs. [31,56]. Nevertheless, the latter is clearly desirable through order $\alpha_t\alpha_s^2$, in particular in the light of the observa-

[3] We use the SLHA input interface of `FeynHiggs`, which performs a conversion of the $\overline{\text{DR}}$ input parameters to the on-shell scheme. Resummation is disabled, as it would lead to an inconsistent result in combination with the $\overline{\text{DR}}$ to on-shell conversion of `FeynHiggs` [56]. We call `FeynHiggs` with the flags 4002020110.

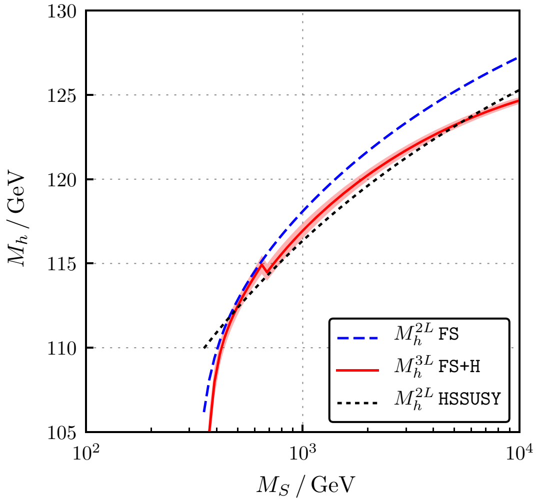

Fig. 6 Comparison of the lightest Higgs pole mass calculated at the two- and three-loop level with `FlexibleSUSY`, `FlexibleSUSY+Himalaya` and `HSSUSY` as a function of the SUSY scale M_S for $\tan\beta = 5$ and $X_t = -\sqrt{6}M_S$. The red band shows the size of the hierarchy selection criterion δ_i. In the fixed-order calculations of `FlexibleSUSY` and `FlexibleSUSY+Himalaya` the Higgs mass becomes tachyonic for $M_S \lesssim 350$ GeV

tions for non-zero stop mixing to be discussed below, but has to be deferred to future work at this point.

Figure 5 shows the three-loop effects as a function of X_t, where the value of $M_S = 2$ TeV is chosen to be inside the intermediate region. The figure shows that, for $|X_t| \lesssim 3M_S$, the qualitative features of the discussion above are largely independent of the mixing parameter, whereupon the quantitative differences between the fixed-order and the resummed results are typically larger for non-zero stop mixing. Figure 6 underlines this by setting $X_t = -\sqrt{6}M_S$ and varying M_S. The kink in the three-loop curve originates from a change of the optimal hierarchy chosen by `Himalaya`. The red band shows the uncertainty δ_i as defined in Eq. (3), which is used to select the best fitting hierarchy. We find that δ_i is comparable to the size of the kink, which indicates a reliable treatment of the hierarchy selection criterion.

4.4 Comparison with other three-loop results

The three-loop $\mathcal{O}(\alpha_t\alpha_s^2)$ corrections to the light MSSM Higgs mass discussed in this paper were originally implemented in the `Mathematica` code `H3m`. We checked that the implementation of the α_t and $\alpha_t\alpha_s$ terms in `Himalaya` leads to the same numerical results as in `H3m`, if the same set of $\overline{\text{DR}}$ parameters is used as input. Since the $\alpha_t\alpha_s^2$ terms of `Himalaya` are derived from their implementation in `H3m`, it is not surprising that they also result in the same numerical value if the same set of input parameters is given *and* the same mass hierarchy is selected. But since `Himalaya` has a slightly more sophisticated way of choosing this hierarchy (see Sect. 2.1), its numerical $\alpha_t\alpha_s^2$ contribution does occasionally differ slightly from the one of `H3m`.

In Fig. 7 we compare our results to the three-loop calculation presented in Ref. [68], assuming the input parameters for the "heavy sfermions" scenario defined in detail in the `example` folder of Ref. [69]. In the left panel the blue circles show the `H3m` result, including only the terms of $\mathcal{O}(\alpha_t + \alpha_t\alpha_s + \alpha_t\alpha_s^2)$, where the MSSM $\overline{\text{DR}}$ top mass is calculated using the "running and decoupling" procedure described in Ref. [68]. The black crosses show the same result, except that the $\overline{\text{DR}}$ top mass

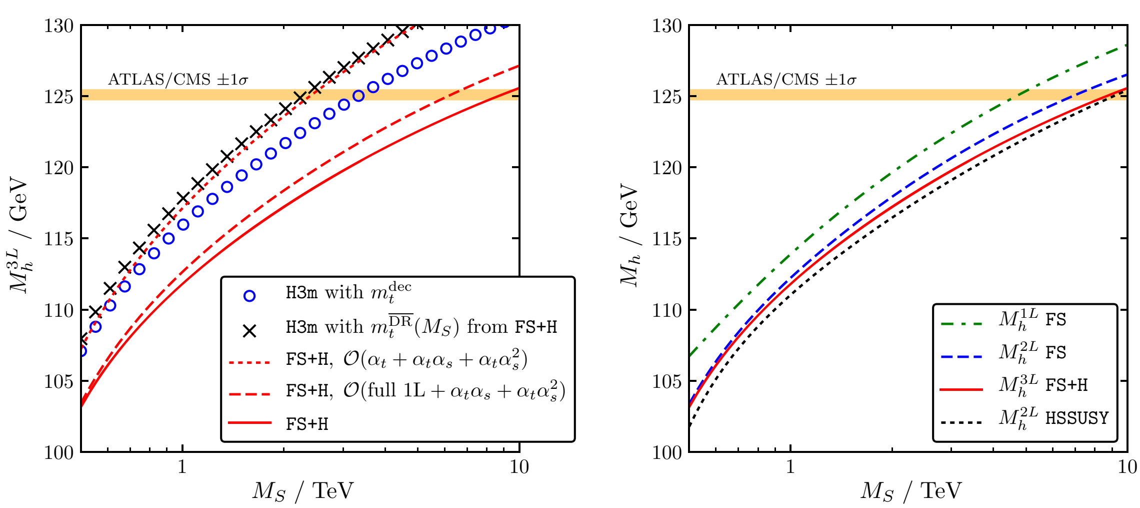

Fig. 7 Comparison of the lightest Higgs pole mass calculated at the one-, two- and three-loop level with `FlexibleSUSY`, `FlexibleSUSY+Himalaya`, `H3m` and `HSSUSY` as a function of the SUSY scale for the "heavy sfermions" scenario of Ref. [68]. The horizontal orange band shows the measured Higgs mass $M_h = (125.09 \pm 0.32)$ GeV including its experimental uncertainty

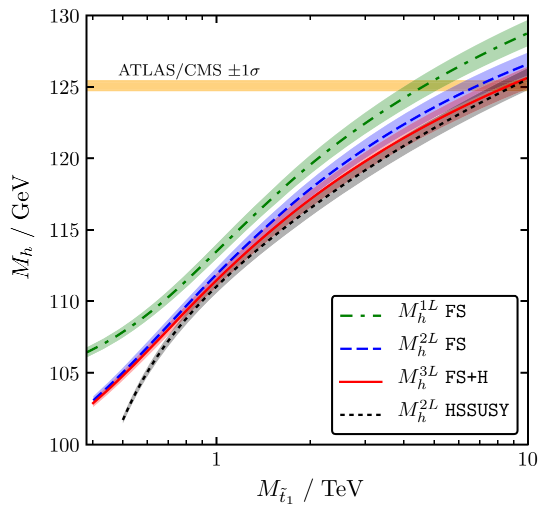

Fig. 8 Comparison of the lightest Higgs pole mass calculated at the one-, two- and three-loop level with FlexibleSUSY, FlexibleSUSY+Himalaya and HSSUSY as a function of the lightest stop pole mass for the benchmark point of Fig. 1 of Ref. [70]. The horizontal orange band shows the measured Higgs mass $M_h = (125.09 \pm 0.32)$ GeV including its experimental uncertainty. The bands around the calculated Higgs mass values show the parametric uncertainty from $M_t = (173.34 \pm 0.98)$ GeV and $\alpha_s^{\text{SM}(5)}(M_Z) = 0.1184 \pm 0.0006$

at the SUSY scale is taken from the spectrum generator FlexibleSUSY+Himalaya. We can reproduce the latter result with FlexibleSUSY+Himalaya if we take the same terms into account, i.e., $\mathcal{O}\big(\alpha_t + \alpha_t\alpha_s + \alpha_t\alpha_s^2\big)$; see the dotted red line in Fig. 7. The small differences between the two results are due to the fact that H3m works with on-shell electroweak parameters, while FlexibleSUSY+Himalaya uses $\overline{\text{DR}}$ parameters. The inclusion of all one-loop contributions to M_h and the momentum iteration reduces the Higgs mass by 4–6 GeV, as shown by the red dashed line. Including all two- and three-loop corrections which are available in FlexibleSUSY+Himalaya, i.e., $\mathcal{O}\big((\alpha_t + \alpha_b)\alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2 + (\alpha_t + \alpha_b)\alpha_s^2\big)$, further reduces the Higgs mass by up to 2 GeV, as shown by the red solid line.[4] The right panel of Fig. 7 shows again our one-, two-, and three-loop predictions obtained with FlexibleSUSY, FlexibleSUSY+Himalaya, as well as the EFT result of HSSUSY. Similar to Fig. 4, we observe that the higher-order terms lower the predicted Higgs mass and render it closer to the resummed result. A detailed comparison of FlexibleSUSY+Himalaya to a result where H3m is combined with the lower-order results of FeynHiggs is beyond the scope of this paper and left to a future publication.

Figure 8 shows the lightest MSSM Higgs mass as obtained by FlexibleSUSY at one- and two-loop level, the FlexibleSUSY+Himalaya result, as well as the EFT prediction obtained with HSSUSY. The MSSM parameters are defined in the $\overline{\text{DR}}$ scheme and are chosen in the style of Ref. [70]:[5] The soft-breaking mass parameters of the left- and right-handed stops are set equal at the SUSY scale $M_S = \sqrt{m_{\tilde{t},1} m_{\tilde{t},2}}$, i.e. $m_{\tilde{t}_L}(M_S) = m_{\tilde{t}_R}(M_S)$. All other soft-breaking sfermion mass parameters are set to $m_{\tilde{f}}(M_S) = m_{\tilde{t}_{L,R}}(M_S) + 1\,\text{TeV}$. Stop mixing is disabled, $X_t(M_S) = 0$, and the remaining trilinear couplings are set to zero at the scale M_S. The gaugino mass parameters, the super-potential μ parameter and the CP-odd $\overline{\text{DR}}$ Higgs mass are set to $M_1(M_S) = M_2(M_S) = M_3(M_S) = 1.5\,\text{TeV}$, $\mu(M_S) = 200\,\text{GeV}$ and $m_A(M_S) = M_S$, respectively, and we fix $\tan\beta(M_Z) = 20$. As opposed to the results shown in Fig. 1 of Ref. [70],[6] we observe a reduction of M_h towards higher loop orders, thus leading to the opposite conclusion of a heavy SUSY spectrum in this scenario, given the current experimental value for the Higgs mass. Reassuringly, the higher-order corrections move the fixed-order result closer to the resummed result, leading to agreement between the two at the level of about 1 GeV even at comparatively large SUSY scales.

5 Conclusions

We have presented the implementation Himalaya of the three-loop $\mathcal{O}\big(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2\big)$ terms of Refs. [1,20] for the light CP-even Higgs mass in the MSSM, and its combination with the $\overline{\text{DR}}$ spectrum generator framework FlexibleSUSY. These three-loop contributions have been available in the public program H3m before, where they were combined with the on-shell calculation of FeynHiggs. With the implementation into FlexibleSUSY presented here, we were able to study the size of the three-loop contributions within a pure $\overline{\text{DR}}$ environment. Despite the fact that the genuine $\mathcal{O}\big(\alpha_t\alpha_s^2\big)$ corrections are positive [1], the combination with the two-loop decoupling terms in the top Yukawa coupling lead to an overall reduction of the Higgs mass prediction relative to the "original" two-loop FlexibleSUSY result by about 2 GeV, depending on the value of the stop masses and the stop mixing. This moves the fixed-order prediction for the Higgs mass significantly closer to the result obtained from a pure EFT calculation in the region where both approaches are expected to give sensible results. Contributions of order $\mathcal{O}\big(\alpha_b\alpha_s^2\big)$ are found to be negligible in all scenarios studied here.

[4] By default all available two- and three-loop corrections are included in FlexibleSUSY+Himalaya.

[5] The scenario of Ref. [70] appears to be not fully defined; in particular, M_A and the sfermion mixing parameters other than X_t remain unspecified.

[6] Note that, in contrast to Ref. [70], we are using a logarithmic scale in Fig. 8.

To indicate the remaining theory uncertainty due to higher-order effects, we have varied the renormalization scale which enters the calculation by a factor two. The results show that the inclusion of the three-loop contributions reduces the scale uncertainty of the Higgs mass by around a factor two, compared to a calculation without the genuine three-loop effects. We conclude that our implementation leads to an improved CP-even Higgs mass prediction relative to the two-loop results. Our implementation of the three-loop terms should be useful also for other groups that aim at a high-precision determination of the Higgs mass in SUSY models.

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Appendix A: Installation of `Himalaya`

Himalaya can be downloaded as a compressed package from [47]. After the package has been extracted, Himalaya can be configured and compiled by running

```
cd $HIMALAY_PATH
mkdir build
cd build
cmake ..
make
```

where $HIMALAY_PATH is the path to the Himalaya directory. When the compilation has finished, the build directory will contain the Himalaya library libHimalaya.a. For convenience, a library named libDSZ.a is created in addition, which contains the two-loop $\mathcal{O}(\alpha_t\alpha_s)$ corrections from Ref. [12].

Appendix B: Installation of `FlexibleSUSY` with `Himalaya`

We provide a dedicated version of FlexibleSUSY 1.7.4, which uses Himalaya to calculate the Higgs pole mass at the three-loop level. This package contains three pre-generated MSSM models:

- MSSMNoFVHimalaya This model represents the MSSM without (s)fermion flavour violation, where $\tan\beta$ is fixed at the scale M_Z and the other SUSY parameters are fixed at a user-defined input scale. The parameters μ and $B\mu$ are fixed by the electroweak symmetry breaking conditions. The SUSY mass spectrum, including the Higgs pole masses, is calculated at the scale $Q = \sqrt{m_{\tilde{t},1} m_{\tilde{t},2}}$, where $m_{\tilde{t},i}$ are the two $\overline{\text{DR}}$ stop masses.
- MSSMNoFVatMGUTHimalaya This is the same model as the MSSMNoFVHimalaya, except that the input scale is the GUT scale M_X, defined to be the scale where $g_1(M_X) = g_2(M_X)$.
- NUHMSSMNoFVHimalaya This is the same model as the MSSMNoFVHimalaya, except that the soft-breaking Higgs mass parameters $m_{H_u}^2$ and $m_{H_d}^2$ are fixed by the electroweak symmetry breaking conditions.

The package FlexibleSUSY-1.7.4-Himalaya.tar.gz can be downloaded from Ref. [48]. To extract the package at the command line, run

```
tar -xf FlexibleSUSY-1.7.4-Himalaya.tar.gz
cd FlexibleSUSY-1.7.4-Himalaya/
```

After the extraction, FlexibleSUSY must be configured and compiled by running

```
./configure \
   --with-himalaya-incdir=$HIMALAY_PATH/source/include \
   --with-himalaya-libdir=$HIMALAY_PATH/build
make
```

See `./configure --help` for more options. One can use `-j<N>` to speed-up the compilation if `<N>` CPU cores are available. When the compilation has finished, the MSSM spectrum generators can be run from the command line as

```
models/MSSMNoFVHimalaya/run_MSSMNoFVHimalaya.x \
  --slha-input-file=models/MSSMNoFVHimalaya/LesHouches.in.MSSMNoFVHimalaya
  --slha-output-file=LesHouches.out.MSSMNoFVHimalaya
```

The file `LesHouches.out.MSSMNoFVHimalaya` will then contain the SUSY particle spectrum in SLHA format. Alternatively, the `Mathematica` interface of `FlexibleSUSY` can be used:

```
math -run "<< \"models/MSSMNoFVHimalaya/run_MSSMNoFVHimalaya.m\""
```

For each model an example SLHA input file and an example `Mathematica` script can be found in `models/<model>/`..

Appendix C: Configuration options to calculate the Higgs mass at three-loop level with **FlexibleSUSY**

To calculate the CP-even Higgs pole masses at order $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ at the scale $Q = M_S$, the top and bottom Yukawa couplings $y_t(M_S)$ and $y_b(M_S)$ as well as the strong coupling constant $\alpha_s(M_S)$ must be extracted from the input parameters at the appropriate loop level.

Strong coupling constant To calculate M_h at the three-loop level at $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ correctly, $\alpha_s(M_S)$ must be extracted at the one-loop level from the input value $\alpha_s^{\text{SM(5)}}(M_Z)$ as described in Sect. 3.1. To achieve that in `FlexibleSUSY`, the global threshold correction loop order (`EXTPAR[7]`) must be set to 1 (or higher) and the specific threshold correction loop order for α_s (3rd digit from the right in `EXTPAR[24]` must be set to 1 (or higher) in the SLHA input file. See the next paragraph for an example.

Top and bottom Yukawa couplings `FlexibleSUSY` by default determines $y_t(M_Z)$ from the top pole mass at the full one-loop level including two-loop Standard Model QCD corrections; see Ref. [32]. The bottom Yukawa coupling $y_b(M_Z)$ is determined at the full one-loop level from the running bottom quark mass in the Standard Model with five active quark flavours, $m_b^{\text{SM(5)},\overline{\text{MS}}}(m_b)$, where $\tan\beta$-enhanced higher-order corrections are resummed. Both calculations are not sufficient for the calculation of M_h at the three-loop level at $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$, because strong two-loop corrections from SUSY particles would be missing. For this reason, the complete two-loop strong corrections to the top and bottom Yukawa couplings of Refs. [51–54] have been implemented into `FlexibleSUSY`. They must be activated by setting the global threshold correction loop (`EXTPAR[7]`) order to 2 and by setting the threshold correction loop order for y_t and y_b (7th and 8th digit from the right in `EXTPAR[24]`) to 2 in the SLHA input file:

```
Block FlexibleSUSY
    7   2          # threshold corrections loop order
   24   122111121  # individual threshold correction loop orders
```

In the `Mathematica` interface of `FlexibleSUSY` these two settings are controlled using the `thresholdCorrectionsLoopOrder` and `thresholdCorrections` symbols:

```
handle = FS<model>OpenHandle[
    fsSettings -> {
        thresholdCorrectionsLoopOrder -> 2,
        thresholdCorrections -> 122111121
    }
    ...
];
```

Here, `<model>` is the used `FlexibleSUSY` model from above, i.e. either `MSSMNoFVHimalaya`, `MSSMNoFVatMGUTHimalaya` or `NUHMSSMNoFVHimalaya`.

Three-loop corrections to the CP-even Higgs mass To use the three-loop corrections of order $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ to the light CP-even Higgs mass in the MSSM from Refs. [1,20], the pole mass and EWSB loop orders must be set to 3 in the SLHA input file. In addition, the individual three-loop corrections should be switched on, by setting the flags 26 and 27 to 1. The user can select between the $\overline{\text{DR}}$ and $\overline{\text{MDR}}$ scheme for the three-loop corrections by setting the flag 25 to 0 or 1, respectively:

```
Block FlexibleSUSY
    4   3   # pole mass loop order
    5   3   # EWSB loop order
   25   0   # ren. scheme for Higgs 3L corrections (0 = DR, 1 = MDR)
   26   1   # Higgs 3-loop corrections O(alpha_t alpha_s^2)
   27   1   # Higgs 3-loop corrections O(alpha_b alpha_s^2)
```

In the Mathematica interface of FlexibleSUSY the pole mass and EWSB loop orders are controlled using the `poleMassLoopOrder` and `ewsbLoopOrder` symbols, respectively. The individual three-loop corrections can be switched on/off by using the `higgs3loopCorrectionAtAsAs` and `higgs3loopCorrectionAbAsAs` symbols. The renormalization scheme is controlled by `higgs3loopCorrectionRenScheme`. The above shown SLHA input settings read in FlexibleSUSY's Mathematica interface

```
handle = FS<model>OpenHandle[
    fsSettings -> {
        poleMassLoopOrder -> 3,
        ewsbLoopOrder -> 3,
        higgs3loopCorrectionRenScheme -> 0,
        higgs3loopCorrectionAtAsAs -> 1,
        higgs3loopCorrectionAbAsAs -> 1
    }
    ...
];
```

Three-loop renormalization group equations Optionally, the known three-loop renormalization group equations can be used to evolve the MSSM $\overline{\text{DR}}$ parameters from M_Z to M_S [57,58]. To activate the three-loop RGEs, the β function loop order must be set to 3 in the SLHA input file:

```
Block FlexibleSUSY
    6   3   # beta-functions loop order
```

In the Mathematica interface of FlexibleSUSY the β function loop order is controlled using the `betaFunctionLoopOrder` symbol:

```
handle = FS<model>OpenHandle[
    fsSettings -> {
        betaFunctionLoopOrder -> 3
    }
    ...
];
```

Recommended configuration options for `FlexibleSUSY+Himalaya` We recommend to run FlexibleSUSY+Himalaya with the following SLHA configuration options:

```
Block FlexibleSUSY
    4   3           # pole mass loop order
    5   3           # EWSB loop order
    6   3           # beta-functions loop order
    7   2           # threshold corrections loop order
   24   122111121   # individual threshold correction loop orders
   25   0           # ren. scheme for 3L corrections (0 = DR, 1 = MDR)
   26   1           # Higgs 3-loop corrections O(alpha_t alpha_s^2)
   27   1           # Higgs 3-loop corrections O(alpha_b alpha_s^2)
```

At the Mathematica level we recommend to use:

```
handle = FS<model>OpenHandle[
    fsSettings -> {
        poleMassLoopOrder -> 3,
        ewsbLoopOrder -> 3,
        betaFunctionLoopOrder -> 3,
        thresholdCorrectionsLoopOrder -> 2,
        thresholdCorrections -> 122111121,
        higgs3loopCorrectionRenScheme -> 0,
        higgs3loopCorrectionAtAsAs -> 1,
        higgs3loopCorrectionAbAsAs -> 1
    }
    ...
];
```

Appendix D: Himalaya interface

Input parameters To calculate the three-loop corrections to the light CP-even Higgs pole mass at order $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ with Himalaya, the set of $\overline{\text{DR}}$ parameters is needed, which is shown in the following code snippet. The parameters are stored in the struct Parameters which contains the following members:

All these parameters are given at the scale stored in the scale variable, which is typically the SUSY scale. The input values of the stop/sbottom masses and their associated mixing angle are optional, so their default value is set to nan (std::numeric_limits<T>::quiet_NaN()). If no input is provided, the $\overline{\text{DR}}$ stop masses will be calculated by diagonalising the stop mass matrix,

$$\mathcal{M}_{\tilde{t}} = \begin{pmatrix} (m^2_{\tilde{Q}})_{33} + m_t^2 + g_t M_Z^2 c_{2\beta} & \tilde{X}_t \\ \tilde{X}_t & (m^2_{\tilde{u}})_{33} + m_t^2 + Q_t s_W^2 M_Z^2 c_{2\beta} \end{pmatrix}. \tag{30}$$

Here, $(m_{\tilde{Q}})_{33}$ is the left third generation scalar quark mass parameter, $g_t = 1/2 - Q_t s_W^2$, $\tilde{X}_t = m_t(A_t - \mu\cot\beta)$, $(m_{\tilde{u}})_{33}$ the right scalar top mass parameter, $Q_t = 2/3$, s_W the sine of the weak mixing angle and $c_{2\beta} = \cos(2\beta)$. The sbottom mass matrix is obtained by replacing $t \to b$ and $\tilde{u} \to \tilde{d}$ in (30) with $g_b = -(1/2 + Q_b s_W^2)$, $\tilde{X}_b = m_b(A_b - \mu\tan\beta)$ and $Q_b = -1/3$.

```
typedef Eigen::Matrix<double,2,1> V2;
typedef Eigen::Matrix<double,2,2> RM22;
typedef Eigen::Matrix<double,3,3> RM33;

struct Parameters {
   // DR-bar parameters
   double scale{};          // renormalization scale
   double mu{};             // mu parameter
   double g3{};             // gauge coupling g3 SU(3)
   double vd{};             // VEV of down Higgs
   double vu{};             // VEV of up Higgs
   RM33 mq2{RM33::Zero()}; // soft-breaking squared left-handed squark
                            // mass parameters
   RM33 md2{RM33::Zero()}; // soft-breaking squared right-handed
                            // down-squark mass parameters
   RM33 mu2{RM33::Zero()}; // soft-breaking squared right-handed
                            // up-squark mass parameters
   double At{};             // trilinear stop-Higgs coupling
   double Ab{};             // trilinear sbottom-Higgs coupling

   // DR-bar masses
   double MG{};             // gluino
   double MW{};             // W
   double MZ{};             // Z
   double Mt{};             // top quark
   double Mb{};             // down quark
   double MA{};             // CP-odd Higgs
   V2 MSt{nan, nan};        // stops
   V2 MSb{nan, nan};        // sbottoms

   // DR-bar mixing angles
   double s2t{nan};         // sine of 2 times the stop mixing angle
   double s2b{nan};         // sine of 2 times the sbottom mixing angle
};
```

Table 1 Description of the member functions of the `HierarchyObject` class

Function name	Returned value
`getIsAlphab()`	Returns the `bool` `isAlphab`
`getSuitableHierarchy()`	Returns the suitable hierarchy as an `int`
`getAbsDiff2L()`	Returns the `double` $\delta_{i_0}^{2L}$ for the suitable hierarchy
`getRelDiff2L()`	Returns the `double` $\delta_{i_0}^{2L}/M_h^{\mathrm{DSZ}}$ for the suitable hierarchy
`getExpUncertainty(int loops)`	Returns the uncertainty of the expansion at the given loop order (cf. Sect. 2.1)
`getDMh(int loops)`	Returns the Higgs mass matrix proportional to α_t or α_b at the given loop order. Note that at the two-loop level only corrections of order $\mathcal{O}(\alpha_t\alpha_s)$ are considered
`getDRToMDRShift()`	Returns the loop correction to the Higgs mass matrix to convert from the $\overline{\mathrm{DR}}$ to $\overline{\mathrm{MDR}}$ scheme, according to Eq. (5). The $\overline{\mathrm{MDR}}$ corrections are of order $\mathcal{O}(\alpha_s + \alpha_s^2)$ by convention
`getMDRMasses()`	Returns the vector of $\overline{\mathrm{MDR}}$ masses $\{\hat{m}_{\tilde{t},1}, \hat{m}_{\tilde{t},2}\}$ ($\{\hat{m}_{\tilde{b},1}, \hat{m}_{\tilde{b},2}\}$), if `isAlphab` is `false` (`true`)

Calculation of the three-loop corrections All the functions which are required for the calculation of the three-loop corrections are implemented as methods of the class `HierarchyCalculator`.

In the context of `Himalaya`, the procedure described in Sect. 2 is implemented by the member function

```
HierarchyObject ho = HierarchyCalculator::calculateDMh3L(bool isAlphab,
   int mdrFlag);
```

where δ_i^{2L} is defined in Eq. (1), and i_0 denotes the "optimal" hierarchy as determined by the procedure of Sect. 2.1. The latter represents a lower limit on the expected accuracy of the expansion by comparison to the exact two-loop result M_h^{DSZ}. In addition to that, the `HierarchyObject` offers a set of member functions which provide access to all intermediate results. These functions are summarised in Table 1. The

Here, the integer `mdrFlag` is optional and can be used to switch between the $\overline{\mathrm{DR}}$- (0) and the $\overline{\mathrm{MDR}}$-scheme (1). The $\overline{\mathrm{DR}}$-scheme is chosen as default. The returned object holds all information of the hierarchy selection process, such as the best fitting hierarchy, or the relative error $\delta_{i_0}^{2L}/M_h^{\mathrm{DSZ}}$, selection method described in Sect. 2 is also applied to the (s)bottom contributions by replacing $t \to b$, so that only terms of order $\mathcal{O}(\alpha_b\alpha_s)$ are considered in the comparison. By setting the Boolean parameter `isAlphab` to `false` (`true`) the `calculateDMh3L` function returns the `HierarchyObject` for the loop corrections proportional to α_t (α_b).

Example Function calls for the benchmark point SPS2:

```
#include "HierarchyCalculator.hpp"
#include "HierarchyObject.hpp"

h3m::Parameters setupSPS2()
{
   h3m::Parameters pars;

   pars.scale = 1.11090135E+03;
   pars.mu = 3.73337018E+02;
   pars.g3 = 1.06187116E+00;
   pars.vd = 2.51008404E+01;
   pars.vu = 2.41869332E+02;
   pars.mq2 << 2.36646981E+06, 0, 0,
               0, 2.36644973E+06, 0,
               0, 0, 1.63230152E+06;
   pars.md2 << 2.35612778E+06, 0, 0,
               0, 2.35610884E+06, 0,
               0, 0, 2.31917415E+06;
   pars.mu2 << 2.35685097E+06, 0, 0,
               0, 2.35682945E+06, 0,
               0, 0, 9.05923409E+05;
   pars.Ab = -784.3356416708631;
   pars.At = -527.8746242245387;

   pars.MA = 1.48446235E+03;
   pars.MG = 6.69045022E+02;
   pars.MW = 8.04001915E+01;
   pars.MZ = 8.97608307E+01;
   pars.Mt = 1.47685846E+02;
   pars.Mb = 2.38918959E+00;
   pars.MSt << 9.57566721E+02, 1.28878643E+03;
   pars.MSb << 1.27884964E+03, 1.52314587E+03;
   pars.s2t = sin(2*asin(1.13197339E-01));
   pars.s2b = sin(2*asin(-9.99883015E-01));

   return pars;
}

int main() {
   h3m::HierarchyCalculator hc(setupSPS2());

   // get the HierarchyObject with entries proportional to alpha_t
   // in the DR scheme
   auto hoTop = hc.calculateDMh3L(false);

   // get the 3-loop correction O(alpha_t * alpha_s^2)
   auto DMh_top_3L = hoTop.getDMh(3);
}
```

Estimation of the uncertainty of the expansion In addition to the relative error of the hierarchy choice $\delta_{i_0}^{2L}/M_h^{DSZ}$ (see above), we provide a member function which returns a measure for the quality of convergence of the expansion at a given loop order, given by $\delta_{i_0}^{\text{conv}}$ defined in Eq. (2), where again i_0 labels the "optimal" hierarchy. It can be called with

```
Eigen::Matrix2d HierarchyCalculator::getExpansionUncertainty(
   HierarchyObject ho, const Eigen::Matrix2d& massMatrix
   int oneLoopFlag, int twoLoopFlag, int threeLoopFlag);
```

Its arguments are a `HierarchyObject`, the Higgs mass matrix `massMatrix` up to the loop order of interest, and three flags (`oneLoopFlag`, `twoLoopFlag`, `threeLoopFlag`) to define the desired loop orders. Using the member function `calculateDMh`, the returned `HierarchyObject` provides the user with the quantity $\delta_{i0}^{\text{conv}}$ at two and three loops by default.

Example For the benchmark point SPS2 one could estimate the uncertainty by calling

```
...
// get the HierarchyObject with entries proportional to alpha_t
// in the DR scheme
auto hoTop = hc.calculateDMh3L(false);

// get the expansion uncertainty for the
// 3-loop correction O(alpha_t * alpha_s^2)
auto expansionUncertainty3LTop = hoTop.getExpUncertainty(3);

// calculate the expansion uncertainty for the
// 1-loop correction O(alpha_t)
auto expansionUncertainty1LTop = hc.getExpansionUncertainty(hoTop,
   ho.getDMh(0), 1, 0, 0);
```

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