Preprint typeset in JHEP style. - PAPER VERSION

CERN{TH/2001-044

Higgs production in hadron collisions: soft and virtual QCD corrections at NNLO

Stefano Catani[(a)] [y], Daniel de Florian[(b)] [z] and Massimiliano Grazzini[(c,d)]

[(a)] Theory Division, CERN, CH-1211 Geneva 23, Switzerland

[(b)] Theoretical Physics, ETH-Hongggerberg, CH-8093 Zurich, Switzerland

[(c)] Dipartimento di Fisica, Universita di Firenze, I-50125 Florence, Italy

[(d)] INFN, Sezione di Firenze, I-50125 Florence, Italy

Abstract: We consider QCD corrections to Higgs boson production through gluon{gluon fusion in hadron collisions. Using the recently evaluated [14] two-loop amplitude for this process and the corresponding factorization formulae [15, 16, 17, 18] describing soft-gluon bremsstrahlung at $O(\alpha_S^2)$, we compute the soft and virtual contributions to the next-to-next-to-leading order cross section. We also discuss soft-gluon resummation at next-to-next-to-leading logarithmic accuracy. Numerical results for Higgs boson production at the LHC are presented.

Keywords: QCD, Higgs production, NNLO Calculations.

This work was supported in part by the EU Fourth Framework Programme \Training and Mobility of Researchers", Network \Quantum Chromodynamics and the Deep Structure of Elementary Particles", contract FMRX{CT98{0194 (DG 12 { MIHT).

[y] On leave of absence from INFN, Sezione di Firenze, Florence, Italy.

[z] Partially supported by Fundacion Antorchas

1. Introduction

The Higgs boson [1] is a fundamental ingredient of the Standard Model (SM), but it has not yet been observed.

Direct searches at LEP imply a lower limit of $M_H > 112.3$ GeV (at 95% CL) [2] on the mass M_H of the SM Higgs boson. Global SM fits to electroweak precision measurements favour a light Higgs ($M_H \lesssim 200$ GeV) [3]. The combination of the preliminary Higgs boson search results of the four LEP experiments [4, 5] shows an excess of candidates, which may indicate the production of a SM Higgs boson with a mass near 115 GeV. The final analysis of the LEP data is expected soon, but it is unlikely that it can substantially change these results.

After the end of the LEP physics programme, the search for the Higgs boson will be carried out at hadron colliders. Depending on the luminosity delivered to the CDF and D0 detectors during the forthcoming Run II, the Tevatron experiments can yield evidence for a Higgs boson with $M_H < 180$ GeV and may be able to discover (at the 5σ level) a Higgs boson with $M_H \lesssim 130$ GeV [6]. At the LHC, the SM Higgs boson can be discovered over the full mass range up to $M_H \sim 1$ TeV after a few years of running [7].

The dominant mechanism for SM Higgs boson production at hadron colliders is gluon–gluon fusion through a heavy-quark (top-quark) loop [8]. At the Tevatron, this production mechanism leads to about 65% of the total cross section for producing a Higgs boson in the mass range $M_H = 100$–200 GeV [6]. At the LHC [9], gg fusion exceeds all the other production channels by a factor decreasing from ~ 8 to ~ 5 when M_H increases from 100 to 200 GeV. When M_H approaches 1 TeV, gg fusion still provides about 50% of the total production cross section.

QCD radiative corrections at next-to-leading order (NLO) to gg-fusion were computed and found to be large [10, 11, 12]. Since approximate evaluations [13] of higher-order terms suggest that their effect can still be sizeable, the evaluation of the next-to-next-to-leading order (NNLO) corrections is highly desirable.

In this paper, we perform a first step towards the complete NNLO calculation. We use the recently evaluated [14] two-loop amplitude for the process $gg \to H$ and the soft-gluon factorization formulae [15, 16, 17, 18] for the bremsstrahlung subprocesses $gg \to Hg$ and $gg \to Hgg, Hq\bar{q}$, and we compute the soft and virtual contributions to the NNLO partonic cross section. We also discuss all-order resummation of soft-gluon contributions to next-to-next-to-leading logarithmic (NNLL) accuracy.

We use the approximation $M_t \gg M_H$, where M_t is the mass of the top quark. The results of the NLO calculation in Ref. [12] show that this is a good numerical approximation [13] of the full NLO correction, provided the exact dependence on M_H/M_t is included in the leading-order (LO) term. We can thus assume that the limit $M_t \gg M_H$ continues to be a good numerical approximation at NNLO.

The hadronic cross section for Higgs boson production is obtained by convoluting the perturbative partonic cross sections with the parton distributions of the colliding hadrons. Besides the partonic cross sections, the other key ingredients of the NNLO calculation are the NNLO parton distributions. Even though their NNLO evolution kernels are not fully available, some of their Mellin moments have been computed [19] and, from these, approximated kernels have been constructed [20]. Recently, the new MRST [21] sets of distributions became available[†], including the (approximated) NNLO densities, which allows an evaluation of the hadronic cross section to (almost full) NNLO accuracy.

We use our NNLO result for the partonic cross sections and the MRST parton distributions at NNLO to compute the Higgs boson production cross section at the LHC. In this paper, we do not present numerical results for Run II at the Tevatron. Inclusive production of Higgs boson through gluon–gluon fusion is phenomenologically less relevant at the Tevatron: it is not regarded as a main discovery channel, because of the large QCD background [6].

The paper is organized as follows. In Sect. 2 we define the soft-virtual approximation for the cross section and present our result for the corresponding NNLO coefficient. In Sect. 3 we discuss soft-gluon resummation for Higgs production at NNLL accuracy, and we also consider the dominant contributions of collinear origin. In Sect. 4 we present the quantitative effect of the computed NNLO corrections for SM Higgs boson production at the LHC. Finally, in Sect. 5 we present our conclusions and we comment on Higgs boson production beyond the SM.

2. QCD cross section at NNLO

We consider the collision of two hadrons h_1 and h_2 with centre-of-mass energy $\sqrt{s}$. The inclusive cross section for the production of the SM Higgs boson can be written as

$$\sigma(s, M_H^2) = \sum_{a,b} \int_0^1 dx_1\, dx_1\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2) \int_0^1 dz\, \delta\!\left(z - \frac{\tau}{x_1 x_2}\right)$$

$$\cdot\, \sigma_0\, z\, G_{ab}(z; \alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2)\,, \tag{2.1}$$

where $\tau = M_H^2/s$, and μ_F and μ_R are factorization and renormalization scales, respectively. The parton densities of the colliding hadrons are denoted by $f_{a/h}(x, \mu_F^2)$ and the subscript a labels the type of massless partons ($a = g, q_f, \bar{q}_f$, with N_f different flavours of light quarks). We use parton densities as defined in the $\overline{MS}$ factorization scheme.

[†] We thank J. Stirling for providing us with the new set of distributions.

From Eq. (2.1) the cross section $\hat\sigma_{ab}$ for the partonic subprocess $ab \to H + X$ at the centre-of-mass energy $\hat s = x_1 x_2 s = M_H^2/z$ is

$$\hat\sigma_{ab}(\hat s; M_H^2) = \frac{1}{\hat s}\,\sigma_0 M_H^2\, G_{ab}(z) = \sigma_0\, z\, G_{ab}(z)\,, \tag{2.2}$$

where the term $1/\hat s$ corresponds to the flux factor and leads to an overall z factor. The Born-level cross section σ_0 and the hard coefficient function G_{ab} arise from the phase-space integral of the matrix elements squared.

The incoming partons a, b couple to the Higgs boson through heavy-quark loops and, therefore, σ_0 and G_{ab} also depend on the masses M_Q of the heavy quarks. The Born-level contribution σ_0 is [8]

$$\sigma_0 = \frac{G_F}{288\pi\sqrt{2}} \left| \sum_Q A_Q\left(\frac{4M_Q^2}{M_H^2}\right)\right|^2 \,, \tag{2.3}$$

where $G_F = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant, and the amplitude A_Q is given by

$$A_Q(x) = \frac{3}{2}\, x \left[1 + (1-x) f(x) \right] ,$$

$$f(x) = \begin{cases} \arcsin^2 \dfrac{1}{\sqrt{x}}\,, & x \geq 1 \\ -\dfrac{1}{4}\left[\ln \dfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi \right]^2, & x < 1 \end{cases} . \tag{2.4}$$

In the following we limit ourselves to considering the case of a single heavy quark, the top quark, and $N_f = 5$ light-quark flavours. We always use M_t ($M_t = 176$ GeV) to denote the on-shell pole mass of the top quark.

The coefficient function G_{ab} in Eq. (2.1) is computable in QCD perturbation theory according to the expansion

$$\begin{aligned} G_{ab}(z; \alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2) &= \alpha_S^2(\mu_R^2) \sum_{n=0}^{+\infty} \left(\frac{\alpha_S(\mu_R^2)}{\pi}\right)^n G_{ab}^{(n)}(z; M_H^2/\mu_R^2; M_H^2/\mu_F^2) \\ &= \alpha_S^2(\mu_R^2)\, G_{ab}^{(0)}(z) + \frac{\alpha_S^3(\mu_R^2)}{\pi}\, G_{ab}^{(1)}\left(z; \frac{M_H^2}{\mu_R^2}; \frac{M_H^2}{\mu_F^2}\right) \\ &+ \frac{\alpha_S^4(\mu_R^2)}{\pi^2}\, G_{ab}^{(2)}\left(z; \frac{M_H^2}{\mu_R^2}; \frac{M_H^2}{\mu_F^2}\right) + O(\alpha_S^5)\,, \end{aligned} \tag{2.5}$$

where the (scale-independent) LO contribution is

$$G_{ab}^{(0)}(z) = \delta_{ag}\,\delta_{bg}\,\delta(1-z)\,. \tag{2.6}$$

The NLO coefficients $G_{ab}^{(1)}$ are known. Their calculation with the exact dependence on M_t was performed in Ref. [12]. In the large-M_t limit (i.e. neglecting corrections that vanish when $M_H/M_t \to 0$) the result is [10, 11]

$$G^{(1)}_{gg}(z;M_H^2/\mu_R^2;M_H^2/\mu_F^2) = \delta(1-z)\left(\frac{11}{2}+6\zeta(2)+\frac{33-2N_f}{6}\ln\frac{\mu_R^2}{\mu_F^2}\right) + 12D_1$$
$$+ 6D_0\ln\frac{M_H^2}{\mu_F^2} + P^{reg}_{gg}(z)\ln\frac{(1-z)^2M_H^2}{z\,\mu_F^2} - 6\frac{\ln z}{1-z} - \frac{11}{2}\frac{(1-z)^3}{z} ; \tag{2.7}$$

$$G^{(1)}_{gq}(z;M_H^2/\mu_R^2;M_H^2/\mu_F^2) = \frac{1}{2}P_{gq}(z)\ln\frac{(1-z)^2M_H^2}{z\,\mu_F^2} + \frac{2}{3}z - \frac{(1-z)^2}{z} ; \tag{2.8}$$

$$G^{(1)}_{q\bar q}(z;M_H^2/\mu_R^2;M_H^2/\mu_F^2) = \frac{32}{27}\frac{(1-z)^3}{z} ; \quad G^{(1)}_{qq}(z;M_H^2/\mu_R^2;M_H^2/\mu_F^2) = 0 ; \tag{2.9}$$

where $\zeta(n)$ is the Riemann zeta-function ($\zeta(2) = \pi^2/6 = 1.645\ldots$, $\zeta(3) = 1.202\ldots$), and we have defined

$$D_i(z) \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+ . \tag{2.10}$$

The kernels $P_{ab}(z)$ are the LO Altarelli–Parisi splitting functions for real emission,

$$P^{reg}_{gg}(z) = 6\left[\frac{1}{z} - 2 + z(1-z)\right] ; \quad P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z} ; \tag{2.11}$$

and, more precisely, $P^{reg}_{gg}(z)$ is the regular part (i.e. after subtracting the $1/(1-z)$ soft singularity) of $P_{gg}(z)$.

In Eqs. (2.7)–(2.9) we can identify three kinds of contributions:

- Soft and virtual corrections, which involve only the gg channel and give rise to the $\delta(1-z)$ and D_i terms in Eq. (2.7). These are the most singular terms when $z \to 1$.

- Purely-collinear logarithmic contributions, which are controlled by the regular part of the Altarelli–Parisi splitting kernels (see Eqs. (2.7), (2.8)). The argument of the collinear logarithm corresponds to the maximum value ($q^2_{T\,max} \sim (1-z)^2M_H^2/z$) of the transverse momentum q_T of the Higgs boson. These contributions give the next-to-dominant singular terms when $z \to 1$.

- Hard contributions, which are present in all partonic channels and lead to finite corrections in the limit $z \to 1$.

The terms proportional to the distributions $D_i(z)$ and $\delta(1-z)$ can be used to define what we call the soft-virtual (SV) approximation. In this approximation only the gg channel contributes and we have

$$G^{(1)SV}_{ab}(z; M_H^2/\mu_R^2; M_H^2/\mu_F^2) = \delta_{ag}\,\delta_{bg} \left[\delta(1-z) \left(\frac{11}{2} + 6\zeta(2) + \frac{33-2N_f}{6} \ln\frac{\mu_R^2}{\mu_F^2} \right) + 6 D_0 \ln\frac{M_H^2}{\mu_F^2} + 12 D_1 \right] . \tag{2.12}$$

The SV terms are certainly the dominant contributions to the cross section in the kinematic region near threshold ($\tau = M_H^2/s \to 1$). At fixed s, this means that the SV terms certainly dominate in the case of heavy Higgs bosons. However, these terms can give the dominant effect even long before the threshold region in the hadronic cross section is actually approached. This is a consequence of the fact that the partonic cross section $\hat\sigma(\hat s, M_H^2)$ has to be convoluted with the parton densities, and the QCD evolution of the latter sizeably reduces the energy that is available in the partonic hard-scattering subprocess. Thus, the partonic cross section $\hat\sigma(\hat s, M_H^2)$ (or the coefficient function $G(z)$) in the factorization formula (2.1) is typically evaluated much closer to threshold than the hadronic cross section. In other words, the parton densities are large at small x and are strongly suppressed at large x (typically, when $x \to 1$, $f(x;\mu^2) \sim (1-x)^\lambda$ with $\lambda > 3$ and $\lambda > 6$ for valence quarks and sea-quarks or gluons, respectively); after integration over them, the dominant value of the square of the partonic centre-of-mass energy $\langle \hat s \rangle = \langle x_1 x_2 \rangle s$ is therefore substantially smaller than the corresponding hadronic value s. Note, also, that this effect is enhanced, in gluon-dominated processes, by the stronger suppression of the gluon density at large x. In the case of Higgs boson production at the LHC, these features were emphasized in Ref. [13], where the authors pointed out that the SV approximation gives a good numerical approximation (see also Sect. 4) of the complete NLO corrections down to low values ($M_H \sim 100$ GeV) of the Higgs boson mass.

The NNLO coefficients $G^{(2)}_{ab}$ are not yet known. Their computation, including their exact dependence on M_t, is certainly very difficult, since it requires the evaluation of three-loop Feynman diagrams.

The computation is certainly more feasible in the large-M_t limit, where one can exploit the effective-lagrangian approach introduced in Ref. [22] and developed up to $O(\alpha_S^4)$ in Refs. [23, 13]. Using this approach, the contribution of the heavy-quark loop is embodied by an effective vertex, thus reducing by one the number of loop integrals to be explicitly carried out.

Within the effective-lagrangian formalism, an important step has recently been performed by Harlander [14], who has evaluated the two-loop amplitude for the process $gg \to H$ by using dimensional regularization in $d = 4 - 2\epsilon$ space-time dimensions. The two-loop amplitude has poles of the type $1/\epsilon^n$ with $n = 4, 3, 2, 1$. The coefficients of the poles of order $n = 4;3;2$ had been predicted in Ref. [24]. The agreement [25] with this prediction is a non-trivial check of Harlander's result.

To compute the NNLO cross section, the two-loop amplitude for the process $gg \to H$ has to be combined with the phase-space integrals of the squares of the one-loop matrix element for the process $gg \to Hg$ and of the tree-level matrix elements for the processes $gg \to Hgg$ and $gg \to Hq\bar{q}$. We have computed these matrix elements in the limit where the final-state partons are soft, by using the one-loop and tree-level factorization formulae derived in Refs. [15,16] and Refs. [17,18], respectively. Then, we have carried out the phase-space integrals by using the technique of Ref. [26]. The result contains ϵ-poles and finite terms. The ϵ-poles (including the single pole $1/\epsilon$) exactly cancel those in the two-loop amplitude [14], thus providing a non-trivial cross-check of our and Harlander's results. The remaining finite terms give the complete soft and virtual contributions to the NNLO cross section.

Details of our calculation will be presented elsewhere [27]. In this paper we limit ourselves to presenting the final result. We obtain the following soft and virtual contributions to the NNLO coefficient function $G^{(2)}_{gg}$:

$$
\begin{aligned}
G_{gg}^{(2)SV}(z;M_H^2/\mu_R^2;M_H^2/\mu_F^2) = {}& \delta(1-z)\Bigg[\frac{11399}{144}+\frac{133}{2}\zeta(2)-\frac{9}{20}\zeta(2)^2-\frac{165}{4}\zeta(3)\\
&+\left(\frac{19}{8}+\frac{2}{3}N_f\right)\ln\frac{M_H^2}{M_t^2}+N_f\left(-\frac{1189}{144}-\frac{5}{3}\zeta(2)+\frac{5}{6}\zeta(3)\right)\\
&+\frac{(33-2N_f)^2}{48}\ln^2\frac{\mu_F^2}{\mu_R^2}-18\,\zeta(2)\ln^2\frac{M_H^2}{\mu_F^2}\\
&+\left(\frac{169}{4}+\frac{171}{2}\zeta(3)-\frac{19}{6}N_f+(33-2N_f)\zeta(2)\right)\ln\frac{M_H^2}{\mu_F^2}\\
&+\left(-\frac{465}{8}+\frac{13}{3}N_f-\frac{3}{2}(33-2N_f)\zeta(2)\right)\ln\frac{M_H^2}{\mu_R^2}\Bigg]\\
&+\mathcal{D}_0\Bigg[-\frac{101}{3}+33\zeta(2)+\frac{351}{2}\zeta(3)+N_f\left(\frac{14}{9}-2\zeta(2)\right)+\left(\frac{165}{4}-\frac{5}{2}N_f\right)\ln^2\frac{M_H^2}{\mu_F^2}\\
&-\frac{3}{2}(33-2N_f)\ln\frac{M_H^2}{\mu_F^2}\ln\frac{M_H^2}{\mu_R^2}+\left(\frac{133}{2}-45\zeta(2)-\frac{5}{3}N_f\right)\ln\frac{M_H^2}{\mu_F^2}\Bigg]\\
&+\mathcal{D}_1\Bigg[133-90\zeta(2)-\frac{10}{3}N_f+36\ln^2\frac{M_H^2}{\mu_F^2}+(33-2N_f)\left(2\ln\frac{M_H^2}{\mu_F^2}-3\ln\frac{M_H^2}{\mu_R^2}\right)\Bigg]\\
&+\mathcal{D}_2\left[-33+2N_f+108\ln\frac{M_H^2}{\mu_F^2}\right]\\
&+72\,\mathcal{D}_3\;. \qquad\qquad (2.13)
\end{aligned}
$$

Note that our result in Eq. (2.13) gives the complete soft contributions (all the terms proportional to the distributions $\mathcal{D}_i(z)$) to the NNLO coefficient functions

$G^{(2)}_{ab}(z)$. It also gives the complete virtual contribution (the term proportional to $\delta(1-z)$) to the gg channel. The expression in Eq. (2.13) is an approximation of the exact NNLO calculation in the sense that it differs from $G^{(2)}_{ab}(z)$ by terms that are less singular when $z \to 1$. More precisely, in the large-z limit we have (see Sect. (3))

$$G^{(2)}_{gg}(z) - G^{(2)SV}_{gg}(z) = O(\ln^3(1-z)) \; ; \tag{2.14}$$

$$G^{(2)}_{gq}(z) = O(\ln^3(1-z)) \; ; \tag{2.15}$$

$$G^{(2)}_{q\bar{q}}(z) \propto \delta(1-z) + O((1-z)\ln^2(1-z)) \; ; \quad G^{(2)}_{qq}(z) = O((1-z)\ln^2(1-z)) \; . \tag{2.16}$$

Note also that, unlike the NLO term $G^{(1)}_{ab}(z)$, the NNLO coefficient function $G^{(2)}_{ab}(z)$ is not independent of M_t in the large-M_t limit. The virtual contribution in Eq. (2.13) contains a term, proportional to $\ln M_H^2/M_t^2$, that derives from the integration of the heavy-quark degrees of freedom in the effective lagrangian [23,13].

Our result in Eq. (2.13) can be useful as a non-trivial check of a future complete calculation at NNLO. It can also be used to extend the accuracy of the soft-gluon resummation formalism to NNLL order (see Sect. 3).

As previously discussed, the SV approximation turns out to be a good numerical approximation of the full NLO correction for Higgs boson production at the LHC. Thus, the NNLO-SV result in Eq. (2.13) can also be exploited to obtain an approximate numerical estimate of the complete NNLO correction (see Sect. 4).

3. Soft-gluon resummation at NNLL accuracy

The soft (and virtual) contributions $\alpha_S^2 \alpha_S^n D_m(z)$ (with $m \le 2n-1$) to the coefficient function $G_{gg}(z)$ can be summed to all orders in QCD perturbation theory. Using the soft-gluon resummation formulae that are known at present, we can check the coefficients of some of the soft contributions presented in Eq. (2.13). The remaining coefficients can then be used to extend the accuracy of the resummation formulae to NNLL order. Both points are discussed in this section.

The formalism to systematically perform soft-gluon resummation for processes initiated by $q\bar{q}$ annihilation and gg fusion was set up in Refs. [28,29,30,31]. Soft-gluon resummation has to be carried out in the Mellin (or N-moment) space. The N-moments G_N of the coefficient function $G(z)$ are defined by

$$G_N \equiv \int_0^1 dz\, z^{N-1}\, G(z) \; . \tag{3.1}$$

In N-moment space the soft (or threshold) region $z \to 1$ corresponds to the limit $N \to \infty$, and the distributions $D_m(z)$ lead to logarithmic contributions, $D_m(z) \to \ln^{m+1} N$. The singular contributions in the large-N limit can be organized in the following all-order resummation formula:

$$G_{gg,N} = \bar{C}_{gg}(\alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2) \cdot \Delta_N^H(\alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2) + O(1/N) \; . \tag{3.2}$$

The radiative factor Δ_N^H embodies all the large contributions $\ln N$ due to soft radiation. The function $\bar{C}_{gg}(\alpha_S)$ contains all the terms that are constant in the large-N limit and has a perturbative expansion analogous to Eq. (2.5):

$$\begin{aligned}\bar{C}_{gg}(\alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2) &= \\ &= \alpha_S^2(\mu_R^2)\left[1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{\pi}\right)^n \bar{C}_{gg}^{(n)}(M_H^2/\mu_R^2; M_H^2/\mu_F^2)\right] \; .\end{aligned} \tag{3.3}$$

These constant terms are due to virtual contributions, and the perturbative coefficients $\bar{C}_{gg}^{(n)}$ are thus directly related to the coefficients of the contribution proportional to $\delta(1-z)$ in $G_{gg}^{(n)}(z)$. The term $O(1/N)$ on the right-hand side of Eq. (3.2) denotes all the contributions that are suppressed by some power of $1/N$ (modulo $\ln N$ enhancement) when $N \to \infty$.

The radiative factor Δ_N^H for Higgs boson production has the following general expression [28, 29, 32]:

$$\begin{aligned}\Delta_N^H(\alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2) = &\left[\Delta_N^g(\alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2)\right]^2 \\ &\cdot \Delta_N^{(\mathrm{int})H}(\alpha_S(\mu_R^2), M_H^2/\mu_R^2) \; .\end{aligned} \tag{3.4}$$

Each term Δ_N^g embodies the effect of soft-gluon radiation emitted collinearly to the initial-state partons and depends on both the factorization scheme and the factorization scale μ_F. In the $\overline{MS}$ factorization scheme we have the exponentiated result

$$\Delta_N^a(\alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2) = \exp\left\{\int_0^1 dz \, \frac{z^{N-1}-1}{1-z}\int_{\mu_F^2}^{(1-z)^2 M_H^2}\frac{dq^2}{q^2}A_a(\alpha_S(q^2))\right\} \; , \tag{3.5}$$

where $A_a(\alpha_S)$ is a perturbative function

$$A_a(\alpha_S) = \frac{\alpha_S}{\pi}A_a^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A_a^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A_a^{(3)} + O(\alpha_S^4) \; . \tag{3.6}$$

The factor $\Delta_N^{(\mathrm{int})}$ is independent of the factorization scale and scheme and contains the contribution of soft-gluon emission at large angles with respect to the direction of the colliding gluons. It can also be written in exponentiated form as

$$\Delta_N^{(\mathrm{int})H}(\alpha_S(\mu_R^2), M_H^2/\mu_R^2) = \exp\left\{\int_0^1 dz \, \frac{z^{N-1}-1}{1-z} D_H(\alpha_S((1-z)^2 M_H^2))\right\} \; ; \tag{3.7}$$

where the function $D_H(\alpha_S)$ for Higgs production has the following perturbative expansion:

$$D_H(\alpha_S) = \left(\frac{\alpha_S}{\pi}\right)^2 D_H^{(2)} + O(\alpha_S^3) \; . \tag{3.8}$$

The coefficients $A^{(1)}$ and $A^{(2)}$ fully control soft-gluon resummation up to next-to-leading logarithmic (NLL) accuracy [28, 29, 32]. In the case of a generic incoming parton a, they are given by

$$A_a^{(1)} = C_a \; , \quad A_a^{(2)} = \frac{1}{2} C_a K \; , \tag{3.9}$$

where $C_a = C_F = 4/3$ if $a = q, \bar{q}$ and $C_a = C_A = 3$ if $a = g$, while the coefficient K is the same both for quarks and for gluons [33, 30, 34] and it is given by

$$K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \; . \tag{3.10}$$

Expanding the resummation formula (3.2) up to $O(\alpha_S^3)$ and transforming the result back to z-space, it is straightforward to check that we correctly obtain the soft contributions, $D_0(z)$ and $D_1(z)$, to $G_{ab}^{(1)SV}(z)$ in Eq. (2.12). By comparison with the virtual term in Eq. (2.12), we can also extract the coefficient $\overline{C}_{gg}^{(1)}$ in Eq. (3.3):

$$\overline{C}_{gg}^{(1)}(M_H^2/\mu_R^2; M_H^2/\mu_F^2) = \frac{11}{2} + 6\,\zeta(2) + \frac{33 - 2N_f}{6} \ln \frac{\mu_R^2}{\mu_F^2} \; . \tag{3.11}$$

Then, we can expand the resummation formula (3.2) up to $O(\alpha_S^4)$, and we can compare the result with our NNLO soft-virtual calculation in Eq. (2.13). It is straightforward to check that the knowledge of $A_g^{(1)}, A_g^{(2)}$ and $\overline{C}_{gg}^{(1)}$ predicts the coefficients of $D_3(z), D_2(z)$ and $D_1(z)$ in $G_{gg}^{(2)SV}(z)$, and that the prediction fully agrees with our result in Eq. (2.13).

The comparison[‡] at $O(\alpha_S^4)$ and our calculation of the D_0-term in Eq. (2.13) also allows us to extract the (so far unknown) coefficient $D_H^{(2)}$ that controls soft-gluon resummation at NNLL order. We obtain

$$D_H^{(2)} = C_A^2 \left(-\frac{101}{27} + \frac{11}{3}\,\zeta(2) + \frac{7}{2}\,\zeta(3) \right) + C_A N_f \left(\frac{14}{27} - \frac{2}{3}\,\zeta(2) \right) \; . \tag{3.12}$$

Note that the corresponding NNLL coefficient for the Drell–Yan process [35] differs from $D_H^{(2)}$ by the simple replacement of colour factors $C_F \to C_A$. This could have straightforwardly been predicted from the general structure of the soft-factorization formulae at $O(\alpha_S^2)$ (see Sect. 5 of Ref. [16] and the Appendix of Ref. [18]). The exact expression of the remaining NNLL coefficient $A_g^{(3)}$ is still unknown, but an approximate numerical estimate can be found in Ref. [35].

‡ We can also extract the virtual coefficient $\overline{C}_{gg}^{(2)}$ in Eq. (3.3).

The integrals over z and q^2 in Eqs. (3.5) and (3.7) can be carried out to any required logarithmic accuracy (see Refs. [32, 35]) and used for phenomenological analyses. Quantitative studies of soft-gluon resummation effects for Higgs boson production are left to future investigations.

3.1 Collinear-improved resummation

In Ref. [13] Kramer, Laenen and Spira (KLS) exploited the resummation formalism to obtain approximate expressions for the NNLO corrections to Higgs boson production. Their resummation formula is a simplified version of Eq. (3.2) that includes only the first-order coefficients (the coefficients $A^{(1)}, \overline{C}^{(1)}$ and the first-order coefficient β_0 in the expression of the running coupling $\alpha_S(q^2)$). Therefore, the NNLO expressions obtained in Ref. [13] correctly predict only the coefficients of the contributions D_3 and D_2 to the soft and virtual coefficient function $G^{(2)SV}_{gg}$ in Eq. (2.13).

KLS also pointed out [13] that the resummation formalism can be extended to include subdominant contributions in the large-z limit. These contributions are the terms proportional to powers of $\ln(1-z)$ that appear in $G_{gg}(z)$ (see, e.g., Eq. (2.7)). In N-moment space, they lead to contributions of the type $\frac{1}{N}\ln^k N$, which are usually (and consistently) neglected within the soft and virtual approximation (i.e. in the limit $N \to \infty$).

We agree with KLS that the highest power[x] of $\ln(1-z)$ at the n-th perturbative order, namely, $\ln^{2n-1}(1-z)$ in $G^{(n)}_{gg}(z)$ (or, equivalently, the term $\frac{1}{N}\ln^{2n-1}N$ in $G^{(n)}_{gg,N}$), can correctly and consistently be implemented in the all-order resummation formula (3.2). The key observation [13] is that these terms have a collinear origin. They arise from the transverse-momentum evolution of initial-state collinear radiation up to the maximum value of q_T permitted by kinematics. In the large-z limit, the maximum value is $q^2_{T\,max} \sim (1-z)^2 M_H^2$, which is very different from the typical hard scale M_H^2 of the process. The large transverse-momentum region $(1-z)^2 M_H^2 < q_T^2 < M_H^2$ is thus responsible for the leading $\ln(1-z)$-enhancement. The resummation formalism correctly embodies the transverse-momentum evolution of soft radiation up to the kinematical limit $(1-z)^2 M_H^2$ (see Eq. (3.5)). Therefore, the leading collinear enhancement can be taken into account by supplementing the integrand in Eq. (3.5) with the regular (i.e. non-soft) part of the Altarelli–Parisi splitting function (see Eq. (2.11)). Both for the $q\bar{q}$ annihilation (Drell–Yan process) and gg fusion (Higgs production) channels, we can simply perform the following replacement on the right-hand side of Eq. (3.5):

$$\frac{z^{N-1}-1}{1-z} A_a^{(1)} \to \frac{z^{N-1}-1}{1-z} A_a^{(1)} + z^{N-1}\frac{1}{2}P^{reg}_{aa}(z) =$$

$$= \frac{z^{N-1}-1}{1-z} - z^{N-1} A_a^{(1)} + O(1/N^2) \; . \qquad (3.13)$$

---

[x] As for lower powers, KLS acknowledge [13] that their result is not complete.

Having performed the replacement of Eq. (3.13) in Δ_N^a, we can insert its ensuing collinear-improved expression in Eq. (3.2). The resummed expression for the N moments of the coefficient function $G_{gg,N}$ can then be expanded in powers of α_S in the large-N limit by consistently computing and keeping all the terms of the type $\alpha_S^2\,\alpha_S^{n}\frac{1}{N}\ln^{2n-1}N$. Transforming the result back to z-space, this procedure gives the soft and virtual contributions to $G_{gg}^{(n)}(z)$ plus its leading subdominant correction (the contribution proportional to $\ln^{2n-1}(1-z)$) when $z \to 1$.

We name soft-virtual-collinear (SVC) approximation this improved version of the SV expressions in Eqs. (2.12) and (2.13). We find

$$G_{gg}^{(1)SVC}(z; M_H^2/\mu_R^2; M_H^2/\mu_F^2) = G_{gg}^{(1)SV}(z; M_H^2/\mu_R^2; M_H^2/\mu_F^2) - 12 \ln(1-z) \; , \tag{3.14}$$

$$G_{gg}^{(2)SVC}(z; M_H^2/\mu_R^2; M_H^2/\mu_F^2) = G_{gg}^{(2)SV}(z; M_H^2/\mu_R^2; M_H^2/\mu_F^2) - 72 \ln^3(1-z) \; . \tag{3.15}$$

The coefficient of $\ln(1-z)$ in Eq. (3.14) correctly reproduces that obtained by the exact NLO expression in Eq. (2.7). The coefficient of $\ln^3(1-z)$ in Eq. (3.15) agrees with that computed in Ref. [13].

The numerical study of Ref. [13] shows that the effect of the contribution $\ln(1-z)$ at NLO is not small (see also Sect. 4), in particular at low values of the Higgs boson mass. Therefore, in our estimate (Sect. 4) of the NNLO corrections to Higgs boson production at the LHC, we consider both the SV approximation in Eq. (2.13) and the SVC approximation in Eq. (3.15). In the gg partonic channel we thus neglect NNLO of the type

$$G_{gg}^{(2)}(z) - G_{gg}^{(2)SVC}(z) = O(\ln^2(1-z)) \; , \tag{3.16}$$

Note, however, that the coefficient function of the gq channel still contains contributions proportional to $\ln(1-z)$ at NLO (see Eq. (2.8)) and to $\ln^3(1-z)$ at NNLO (see Eq. (2.15)). We do not consider the latter. At low values of the Higgs boson mass their effect is small, because the parton density luminosity of the gq channel is smaller than that of the gg channel. The effect increases by increasing the Higgs boson mass.

4. Numerical results at the LHC

In this section we study the phenomenological impact of the higher-order QCD corrections on the production of the SM Higgs boson at the LHC, i.e. proton–proton collisions at $\sqrt{s} = 14$ TeV. We recall that we include the exact dependence on M_t in the Born-level cross section σ_0 (see Eq. (2.3)), while the coefficient function $G_{ab}(z)$ is evaluated in the large-M_t approximation. At NLO [12, 13] this is a very good numerical approximation when $M_H \lesssim 2M_t$, and it is still accurate to better than 10% when $M_H \lesssim 1$ TeV.

Unless otherwise stated, cross sections are computed using the new MRST2000 [21] sets of parton distributions, with densities and coupling constant evaluated at each corresponding order, i.e. using LO distributions and 1-loop α_S for the LO cross section, and so forth. The corresponding values of $\Lambda^{(5)}_{QCD}$ ($\alpha_S(M_Z)$) are 0.132 (0.1253), 0.22 (0.1175) and 0.187 GeV (0.1161), at 1-loop, 2-loop and 3-loop order, respectively. In the NNLO case we use the 'central' set of MRST2000, obtained from a global fit of data (deep inelastic scattering, Drell–Yan production and jet E_T distribution) by using the approximate NNLO evolution kernels presented in Ref. [20]. The result we refer to as NNLO-SV (SVC) corresponds to the sum of the LO and exact NLO (including the qg and $q\bar{q}$ channels) contributions plus the SV (SVC) corrections at NNLO, given in Eq. (2.13) (Eq. (3.15)). The LO and NLO results obtained by using the CTEQ5 distributions [36] are very similar to the ones computed with the MRST2000 sets (the differences are smaller than the uncertainties arising, for instance, from scale dependence). Therefore, we will not show those results{.

The comparison between different sets of parton distributions, however, cannot be regarded as a way to quantitatively estimate the uncertainty on the parton distributions. The theoretical and experimental errors that affect present determinations of the parton distributions are typically larger [38] than the differences between the parton distribution sets provided by different groups [21,36,37]. In the case of Higgs boson production at the LHC, the study of the CTEQ Collaboration [39] recommends an uncertainty of about $\pm 10\%$ on the corresponding gluon–gluon and quark–gluon parton luminosities.

We begin the presentation of our results by showing in Fig. 1 the scale dependence of the cross section for the production of a Higgs boson with $M_H = 115$ GeV. The scale dependence is analysed by varying the factorization and renormalization scales by a factor of 4 up and down from the default value M_H. The plot on the left corresponds to the simultaneous variation of both scales, $\mu_F = \mu_R = \chi M_H$, whereas the plots in the centre and on the right correspond, respectively, to the results of the independent variation of the factorization or renormalization scale, keeping the other scale fixed at the default value.

As expected from the QCD running of α_S, the cross sections typically decrease when μ_R increases around the characteristic hard scale M_H. In the case of variations of μ_F, we observe the opposite behaviour. In fact, the cross sections are mainly sensitive to partons with momentum fraction $x \sim 10^{-2}$, and in this x-range scaling violation of the parton densities is (moderately) positive. As a result, the scale dependence is mostly driven by the renormalization scale, because the lowest-order contribution to the process is proportional to α_S^2, a (relatively) high power of α_S.

{Larger deviations (for instance, the NLO cross section increases by $\sim 10\%$ for $M_H = 100 - 200$ GeV) appear when comparing to the GRV98 distributions [37], where both the gluon distribution and the value of $\alpha_S(M_Z)$ are different from those of MRST2000 and CTEQ5.

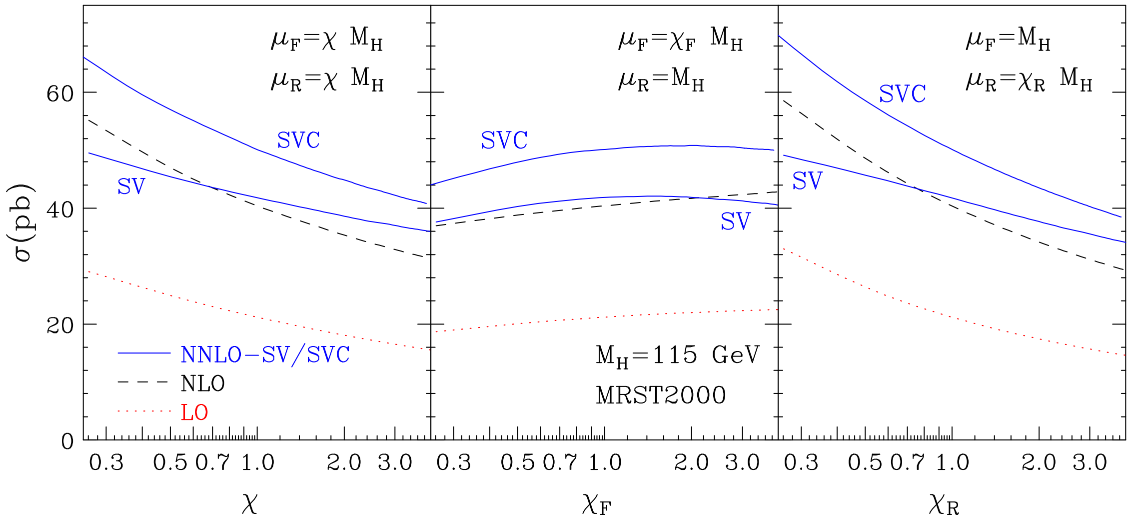

Figure 1: Scale dependence of the Higgs production cross section for $M_H = 115$ GeV at LO, NLO, NNLO-SV and NNLO-SVC.

Figure 1 shows that the scale dependence is reduced when higher-order corrections are included and, in the case of the factorization-scale dependence, a maximum appears at NNLO-SV and NNLO-SVC, showing the improved stability of the result. Also note that there is an increase in the scale dependence when going from NNLO-SV to NNLO-SVC. This is due to the fact that the dominant collinear terms included in the SVC approximation give a sizeable contribution and are scale-independent (see Eqs. (3.14) and (3.15)), so their effect cannot be compensated by scale variations. Similar results are obtained for higher masses, with a reduction in the scale dependence when approaching high mass values.

The impact of higher-order corrections is usually studied by computing K-factors, defined as the ratio of the cross section evaluated at each corresponding order over the LO result. The K-factors are shown in Fig. 2 where the bands account for the 'theoretical uncertainty' due to the scale dependence, quantified by using the minimum and maximum values of the cross sections when the scales μ_R and μ_F are varied (simultaneously and independently, as in Fig. 1) in the range $0.5 \leq \chi_R, \chi_F \leq 2$. The LO result that normalizes the K-factors is computed at the default scale M_H in all cases.

The plot on the left-hand side of Fig. 2 shows the uncertainty at LO and compares the exact NLO result with the NLO-SV and NLO-SVC approximations. In the case of light Higgs production, the NLO-SV approximation tends to underestimate the exact result by about 15 to 20%, whereas the NLO-SVC approximation only slightly overestimates it, showing the numerical importance of the term $\ln(1-z)$ added in the SVC approximation. Nevertheless, all the results agree within the theoretical bands: this confirms the validity of the large-z approximation to estimate higher-

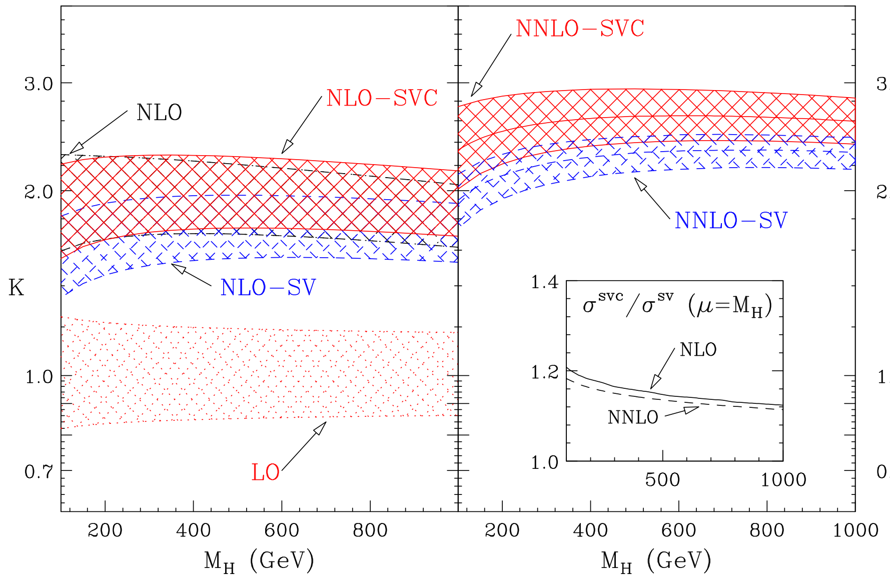

Figure 2: K-factors for Higgs production for the full NLO result and the NLO-SV, NLO-SVC, NNLO-SV and NNLO-SVC approximations.

order corrections, and, in particular, allows us to assume that a similar situation occurs at NNLO. As expected, the agreement between the three results improves for larger masses.

The right-hand side of Fig. 2 shows the SV and SVC results at NNLO. Again, the SVC band sits higher than the SV one, the ratio of the corresponding cross sections being almost the same as the one at NLO, as shown in the inset plot. The contribution from non-leading terms $\ln^k(1-z)$, with $k < 3$ (which are not under control within the SVC approximation), is not included, but it is expected to be numerically less important[k].

As is well known, the customary procedure (that we also are using) of varying the scales to estimate the theoretical uncertainty can only give a lower limit on the 'true' uncertainty. This is well demonstrated by Fig. 2, which shows no overlap between the LO and NLO bands. However, the NLO and NNLO bands do overlap, thus suggesting that the perturbative expansion begins to converge from NNLO. Note also that the size of the NNLO bands is smaller than that of the NLO bands: the scale dependence at NNLO is smaller than at NLO.

Considering the results obtained at NLO, it is reasonable to expect the full NNLO K-factor to lie inside the SV and SVC bands, and most probably, closer to the SVC one. In particular, for a light Higgs boson ($M_H < 200$ GeV), this expectation would correspond to an increase of 15 to 25% with respect to the full NLO result, i.e. a

[k] We have tried to add a term $\ln^2(1-z)$ with a coefficient as large as that of the term $\ln^3(1-z)$, finding only a small (about 5%) modification.

factor of about 2.2 to 2.4 with respect to the LO result. Taking into account that the NLO result increases the LO cross section by about 90% our result anticipates a good convergence of the perturbative series.

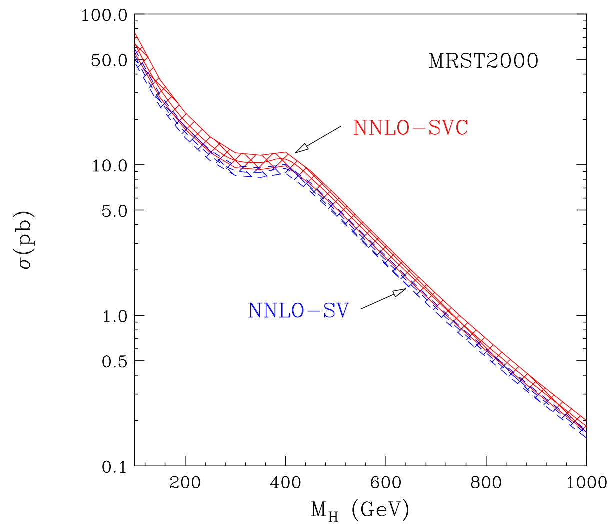

Figure 3: Cross section for Higgs boson production at the LHC in the NNLO-SV and NNLO-SVC approximations.

In Fig. 3 we present the NNLO-SV and SVC cross sections as a function of the Higgs mass and including the corresponding uncertainty bands computed as defined above. To facilitate the comparison with other calculations and more refined predictions, we report the values of the cross sections for the production of a Higgs boson with $M_H = 115$ GeV. The NNLO-SVC band corresponds to $\sigma = 43.51$–58.56 pb (50.13 pb at the default scales), the NNLO-SV to $\sigma = 37.73$–45.69 pb (41.66 pb at the default scales), whereas for the full NLO it is $\sigma = 34.14$–48.48 pb (40.37 pb at the default scales).

Finally we want to quantify the effect of the (approximated) NNLO parton distributions in the gluonic channel. In Fig. 4 we study this effect for the NNLO-SV result at $M_H = 115$ GeV, by plotting the cross section as a function of the scale. We use different combinations of NNLO and NLO parton distributions and coupling constant expressions. The inset plot shows the ratio R of the different results with respect to the one obtained by using NNLO distributions and 3-loop α_S. The use of NNLO distributions and 3-loop α_S reduces the NNLO cross section by 10% with respect to the result that would be obtained if NLO distributions and 2 loop α_S were used. Since the values of $\alpha_S(M_Z)$ from MRST2000 are very similar at 2 and 3 loops and the typical scale of the process is not far from M_Z, the effect of going from 2 to 3 loops α_S amounts to only $1/3$ of the 10% change. The biggest effect comes from the difference in the distributions, mostly due to the decrease of the NNLO gluon

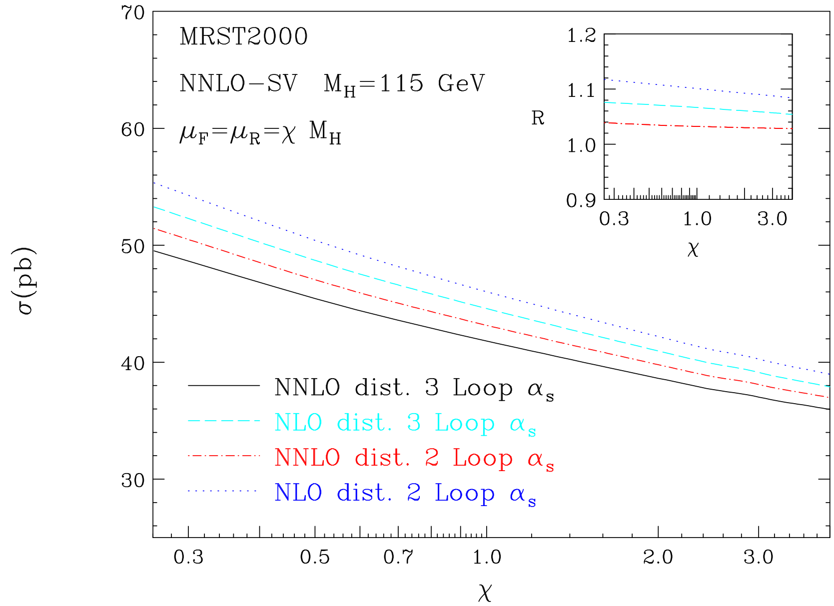

Figure 4: Cross section for Higgs production with M_H = 115 GeV computed using different NLO and NNLO parton distributions and coupling constant.

density at small x [21]. Similar results are obtained in the SVC approximation and for different masses.

5. Conclusions

In this paper we have studied the QCD corrections to Higgs boson production through gluon–gluon fusion in hadronic collisions, within the framework of the large-M_t approximation. Using a recent result for the two-loop correction to the $gg \to H$ amplitude [14] and the soft-factorization formulae for soft-gluon emission at $O(\alpha_S^2)$ [15,16,17,18] we have evaluated the soft and virtual QCD correction to this process at NNLO (SV approximation). We have also considered [13] the leading $\ln^3(1-z)$ contribution from the collinear region (SVC approximation). Our result for the coefficient $G_{gg}^{(2)SV}$ in Eq. (2.13) is consistent with the present knowledge of soft-gluon resummation at NLL accuracy; it also allows us to fix the NNLL coefficient $D_H^{(2)}$ in Eq. (3.8).

We have then studied the phenomenological impact of our results at the LHC by using the (approximate) NNLO set MRST2000 of parton distributions [21]. We have shown that the exact NLO result lies in between the NLO-SV approximation and the NLO-SVC approximation, the latter being a better numerical approximation in the case of low values of the Higgs boson mass. Comparing the results in the SV and SVC approximations at NNLO for a light Higgs ($M_H < 200$ GeV), we estimate that the NNLO correction will increase the NLO result between 15 and 25%.

The results presented here are a first consistent (though approximate) estimate of QCD corrections to Higgs boson production through gg fusion at NNLO and will eventually be a stringent check of a future full NNLO calculation.

In this paper we have only considered the production of the SM Higgs boson. The Minimal Supersymmetric extension of the Standard Model (MSSM) leads to two CP-even neutral Higgs bosons [1]. They are produced by gg fusion through loops of heavy quarks (top, bottom) and squarks. For small values ($\tan\beta \stackrel{<}{\sim} 5$) of the MSSM parameter $\tan\beta$, the NLO QCD corrections to this production mechanism are comparable (to better than 10%) [12,40,9] to those for SM Higgs boson production. Therefore, the NNLO K-factors computed in this paper could also be applicable to MSSM Higgs boson production.

Note added: The calculation of the soft and virtual NNLO corrections to Higgs boson production has independently been performed in Ref.[41]. The method used in Ref.[41] is different from ours. The analytical results fully agree.

References

[1] For a review on Higgs physics in and beyond the Standard Model, see J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, The Higgs Hunter's Guide (Addison-Wesley, Reading, Mass., 1990).

[2] T. Junk [LEP Collaborations], hep-ex/0101015, to appear in the Proc. of 5th International Symposium on Radiative Corrections (RADCOR 2000), Carmel, California, September 2000.

[3] D. Abbaneo et al. [LEP Collaborations, LEP Electroweak Working Group and SLD Heavy Flavour and Electroweak Groups], preprint CERN–EP/2000–16; S. Wynhoff, hep-ex/0101016, to appear in the Proc. of 5th International Symposium on Radiative Corrections (RADCOR 2000), Carmel, California, September 2000.

[4] P. Igo-Kemenes, presentation given at the open session of the LEP Experiments Committee Meeting, 3 November 2000 (see: http://lepHiggs.web.cern.ch/LEPHIGGS/talks/index.html).

[5] R. Barate et al., ALEPH Coll., Phys. Lett. B 495 (2000) 1; M. Acciarri et al., L3 Coll., Phys. Lett. B 495 (2000) 18; P. Abreu et al., DELPHI Coll., preprint CERN–EP/2000–004; G. Abbiendi et al., OPAL Coll., preprint CERN–EP/2000–156 [hep-ex/0101014].

[6] M. Carena et al., Report of the Tevatron Higgs working group, hep-ph/0010338.

[7] CMS Coll., Technical Proposal, report CERN/LHCC/94–38 (1994); ATLAS Coll., ATLAS Detector and Physics Performance: Technical Design Report, Volume 2, report CERN/LHCC/99–15 (1999).

[8] H. M. Georgi, S. L. Glashow, M. E. Machacek and D. V. Nanopoulos, Phys. Rev. Lett. 40 (1978) 692.

[9] M. Spira, Fortsch. Phys. 46 (1998) 203.

[10] S. Dawson, Nucl. Phys. B 359 (1991) 283.

[11] A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B 264 (1991) 440.

[12] M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. B 453 (1995) 17.

[13] M. Kramer, E. Laenen and M. Spira, Nucl. Phys. B 511 (1998) 523.

[14] R. V. Harlander, Phys. Lett. B 492 (2000) 74.

[15] Z. Bern, V. Del Duca and C. R. Schmidt, Phys. Lett. B 445 (1998) 168; Z. Bern, V. Del Duca, W. B. Kilgore and C. R. Schmidt, Phys. Rev. D 60 (1999) 116001.

[16] S. Catani and M. Grazzini, Nucl. Phys. B 591 (2000) 435.

[17] J. M. Campbell and E. W. Glover, Nucl. Phys. B 527 (1998) 264.

[18] S. Catani and M. Grazzini, Nucl. Phys. B 570 (2000) 287.

[19] S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B 492 (1997) 338; A. Retey and J. A. M. Vermaseren, hep-ph/0007294.

[20] W. L. van Neerven and A. Vogt, Nucl. Phys. B 568 (2000) 263, Nucl. Phys. B 588 (2000) 345.

[21] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C 18 (2000) 117.

[22] J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B 106 (1976) 292; A. Vainshtein, M. Voloshin, V. Zakharov and M. Shifman, Sov. J. Nucl. Phys. 30 (1979) 711.

[23] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Phys. Rev. Lett. 79 (1997) 353, Nucl. Phys. B 510 (1998) 61.

[24] S. Catani, Phys. Lett. B 427 (1998) 161.

[25] R. Harlander and W. Kilgore, hep-ph/0012176.

[26] T. Matsuura, S. C. van der Marck and W. L. van Neerven, Nucl. Phys. B 319 (1989) 570.

[27] S. Catani, D. de Florian and M. Grazzini, in preparation.

[28] G. Sterman, Nucl. Phys. B 281 (1987) 310.

[29] S. Catani and L. Trentadue, Nucl. Phys. B 327 (1989) 323, Nucl. Phys. B 353 (1991) 183.

[30] S. Catani, E. D'Emilio and L. Trentadue, Phys. Lett. B 211 (1988) 335.

[31] S. Catani, B. R. Webber and G. Marchesini, Nucl. Phys. B 349 (1991) 635.

[32] S. Catani, M. L. Mangano and P. Nason, JHEP 9807 (1998) 024.

[33] J. Kodaira and L. Trentadue, Phys. Lett. B 112 (1982) 66.

[34] D. de Florian and M. Grazzini, Phys. Rev. Lett. 85 (2000) 4678.

[35] A. Vogt, hep-ph/0010146.

[36] H. L. Lai et al., Eur. Phys. J. C 12 (2000) 375.

[37] M. Gluck, E. Reya and A. Vogt, Eur. Phys. J. C 5 (1998) 461.

[38] S. Catani et al., hep-ph/0005025, in the Proceedings of the CERN Workshop on Standard Model Physics (and more) at the LHC, Eds. G. Altarelli and M. L. Mangano (CERN 2000-04, Geneva, 2000), p. 1.

[39] J. Huston et al., Phys. Rev. D 58 (1998) 114034.

[40] S. Dawson, A. Djouadi and M. Spira, Phys. Rev. Lett. 77 (1996) 16.

[41] R. Harlander and W. Kilgore, hep-ph/0102241.